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EXPERIMENTAL
ELECTRICAL ENGINEERING

AND

MANUAL FOR ELECTRICAL TESTING

FOR ENGINEERS AND FOR STUDENTS IN
ENGINEERING LABORATORIES

BY

V. KARAPETOFF

FIRST EDITION
FIRST THOUSAND

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PREFACE.

In preparing this book the author has aimed to produce a laboratory manual suitable for general electrical-engineering work such as is covered during the Junior and Senior years in most American colleges of engineering. The experiments described cover the principal types of electrical machinery and auxiliary devices, as well as the most important commercial applications of electricity. Some knowledge of physics is assumed on the part of the student, and at least some elementary practice in a physical laboratory; but, for completeness of treatment several experiments are described recalling to the student's mind the fundamental physical laws of electricity and magnetism in their simpler practical aspects.

The arrangement of the book is such as to make each chapter as far as possible independent; in this way the laboratory experiments may be performed in almost any desired order, to suit the equipment at hand and the schedule of the class-room exercises. For the same reason cross-references have been avoided as much as possible. Each chapter covers one particular class of machinery or electrical relations; the experiments of the chapter are described in an ascending scale of difficulty or importance. For instance, the chapters on direct-current machines and on alternators are subdivided into (1) operating features, (2) commercial tests, and (3) a more advanced study of the magnetic circuit and armature windings. For easy reference, the experiments in each chapter bear the name of the chapter. Thus, the experiments in the third chapter are numbered: 3-A, 3-B, 3-C, etc.

The laboratory schedule can be arranged, if desired, so as to cover all the experiments of a particular chapter in succession; but the author greatly prefers the so-called *concentric* disposition of the course. In accordance with this method the Junior-year course is made up of elementary experiments selected from nearly all the chapters of the book; the student being thus introduced to the whole domain of electrical engineering. Then during the first term of his Senior year he performs more advanced experiments relating to the *same* subjects. Finally, during the second term he is given still more special tests (again on the same subjects) requiring a more mature understanding of the theory.*

The concentric method has marked advantages over the usual method with which the student is first given a thorough training in one subject, say direct-current machinery, before he is allowed to take up the next subject, for instance, alternating-current machinery. The principal advantage is that the "concentric" method is more in accordance with human nature; we always desire a bird's-eye view of a subject before we care to go into the details of a particular branch. In this way the course is made more interesting and more correct from a psychological point of view. Another advantage is that the student is brought in contact with the same subject at least three times during the course, and not only is he not allowed to forget it, but he sees it each time from a more advanced standpoint.

There are also some minor advantages of the concentric arrangement. For instance, the student is better prepared for practical work during the summer between his Junior and Senior years if he has handled all classes of machinery in the Junior laboratory; he is prepared to read electrical periodicals; he can be given more delicate apparatus in the Senior laboratory, etc. The laboratory equipment may be utilized much better if various sections of students are allowed to work on entirely different subjects. However, as was mentioned above, the book may be used with any order in which the experiments might be performed in the laboratory.

The plan followed in each chapter is this: first the particular class of machinery is described and the practical needs for certain arrangements and procedures of operation are given; then the object and the method of each particular experiment are described in detail, and instructions given for the manner in which data should be taken. At the end of most experiments the requirements for the reports are stated so that the student will not omit to take all the necessary readings and dimensions while in the laboratory.

It is advisable to have printed data sheets for the more complicated experiments; this will lead the student to take readings neatly and systematically and to record the general information about the apparatus. In some cases diagrams of connections and the necessary precautions to be observed should be posted near the apparatus.

Some teachers may think this is too much guidance, and that no margin for original thinking is left the student. Experience shows, however, that it is advisable first to acquaint students with good and efficient methods of experimenting; more individual freedom may be given in more advanced stages of the work.

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* See also the author's paper on "The Concentric Method of Teaching Electrical Engineering," read before the annual convention of the American Institute of Electrical Engineers, at Niagara Falls, in June, 1907.

The author wishes to express his appreciation of the assistance rendered to him by the instructors in the Electrical Engineering Laboratory of Cornell University in giving him valuable suggestions from their experience. He also wishes to acknowledge the moral support which he received from Professors R. C. Carpenter, H. J. Ryan, and H. H. Norris in the preparation of this book. To several manufacturers of electrical apparatus he is under obligation for valuable information and electrotypes.

The author is also indebted to his friend Mr. B. C. Dennison, of the Department of Electrical Engineering of Cornell University, for reading the manuscript and proofs. His painstaking care and unselfish interest were of inestimable value in forwarding the publication of the book.

CORNELL UNIVERSITY, ITHACA, N.Y.,
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GENERAL REMARKS ON ELECTRICAL LABORATORY WORK.

The right attitude of mind is the most essential condition for success in any kind of work. In application to the laboratory the thought should be: "I am coming to the laboratory to get as much information and practical experience as possible; this is for my own interest, and I am going to use this opportunity to my best advantage." Without this feeling on the part of the student, the best equipment, instructions and instructors are of little value; it may even be said that the results of the work depend directly upon the right attitude of mind, and a man who wants to learn will find the necessary information in spite of very adverse conditions.

In order to get the maximum benefit from a laboratory experiment the student should (*a*) come to the laboratory sufficiently prepared, (*b*) perform the experiment in the proper way, and (*c*) work up the results intelligently. A few points of advice given below may be of benefit to the beginner in experimental work.

I. *Preparation for laboratory work.* Without sufficient preparation too much of the laboratory time is wasted in unnecessary questions, discussions, and in guess-work; the readings are taken hurriedly, and are likely to be unsatisfactory or incomplete. One of the results of an insufficient preparation is that many interesting phenomena escape attention, and those observed are not interpreted aright. Therefore, before you come to the laboratory prepare yourself as follows:

(1) Read carefully the instructions relating to the experiment, and if possible look up the general theory of the particular apparatus.

(2) Try to see the apparatus with which you expect to work, before your regular laboratory time; this will help you while reading the explanations, as the whole arrangement will be more real to you.

II. *Performing the experiment.* It is difficult to give general advice good for all experiments; special precautions to be observed are given in the corresponding instructions. A student, who is duly prepared for the experiment, and knows what he is doing and for what purpose, can be reasonably sure to get satisfactory results. To save time and trouble it is well to observe the following:

(1) Be careful with the machines and instruments intrusted to your care: Not only is it customary to charge the one at fault for the full

amount of damage, but you must remember that in many cases some of your colleagues and friends may be deprived of the benefit of a laboratory exercise because of apparatus having been damaged by you.

(2) Many direct-current ammeters have external shunts, and some voltmeters have separate multipliers. Connecting into a circuit an ammeter without its shunt, or a voltmeter without its multiplier, invariably results in burning out the instrument. It is hardly possible or even desirable to make these instruments proof against careless handling; at any rate they represent the actual commercial practice. The repair of a burned-out instrument is comparatively expensive, so that the student should be careful in handling electrical measuring instruments.

(3) Do not forget to protect the circuit from an accidental overload, by fuses or by a circuit-breaker of proper size; also be sure that you know exactly which switch to open in case of an emergency. When an electric motor is a part of your apparatus, make the connections in such a way that the motor cannot possibly run away and do damage.

(4) Some of the laboratory exercises naturally imply the use of high voltages dangerous to life; however, in educational laboratories the apparatus is usually arranged so that the student does not need to handle live wires, or even come near them. An accident is possible only as a result of gross negligence, and will be out of the question if the student will follow this simple rule: *not to touch any live parts of the circuit, and always to open the main switch before making any changes.*

(5) In coming to the laboratory, bring with you a slide-rule, an inch-rule or tape, a speed counter, a screwdriver and a pair of pliers. This will save you time and trouble of looking for them or borrowing from other students. Do not forget to have a pocket-knife for skinning off wire; a small wrench is also very useful. Provide enough clean paper for sketches, diagrams, and notes.

(6) In taking down readings, it is advisable to use carbon paper so as to get a copy for each partner; this saves copying. Have cross-section paper with you, and whenever possible plot approximate curves as soon as the complete set of readings has been taken. In this way doubtful and contradictory results are detected at once and their cause made clear. The more care and intelligent effort exercised in the laboratory, the less trouble one has in the preparation of the report.

(7) In performing an experiment, start whenever possible with the most difficult conditions: heaviest load, lowest power factor, highest voltage, etc. This for two reasons: First, if you can get satisfactory readings under such conditions you may be reasonably sure of being able to take the whole curve; at any rate there is enough time to change

the instruments, the scheme of connections, etc. Secondly, the points at a heavy load, or under other unfavorable conditions, are usually the most interesting to get: thus, should anything prevent your getting the complete curve, you will still have enough data for a satisfactory report.

(8) Should anything go wrong, an instrument be damaged a bearing or a winding heat, a rheostat burn out, etc., do not try to hide the fact, but report without delay to the instructor. We know that accidents will happen, and honesty is the best policy.

(9) Always put on your data sheets serial numbers of the instruments used during the experiment. This helps to locate a possible discrepancy or contradiction in the data, and will enable the instructor to supply you with the right constant after the laboratory period.

III. *Writing up reports.* The report must possess as much individuality as possible, in order to be of any use whatever. After having performed the experiment, discussed the problem during the recitation, and worked out the numerical results, you certainly ought to get some idea of the subject. All that is required of you, then, is to give the method used and the results arrived at; if you can add to it some personal observations and remarks, so much the better. The following points should be noted in particular:

(1) Make the report clear, concise, and *systematic:* The instructor has many reports to read, and his time is limited. Moreover, a confused or hurriedly written report is apt to prejudice the instructor's judgment of its value.

(2) Do not repeat in the report things already stated in the manual: a reference to the page number is sufficient. Where, however, the text contains the general principle only, it is well to explain its particular application in your experiment.

(3) In most cases, definite directions are given as to how to plot certain curves. If, however, you find that a particular point can be best illustrated in a different way, you are most welcome to do so. The directions are merely intended to facilitate your work, and not to limit your original thinking and initiative.

(4) Criticisms of all kinds are highly desirable, since they greatly increase the value of the report. Criticise anything you wish to — the subject of the experiment, the available laboratory equipment, the method of performing the experiment, laboratory rules, conduct of recitations, requirements for reports, etc. All these are capable of improvement, and the instructors are glad to have your suggestions and remarks. In doing so you profit by developing a critical point of

view in your specialty, while the laboratory management profits by having its attention called to the weak points of the work. Wherever in industrial and commercial establishments, the employees are encouraged to give their suggestions to the management, the results are beneficial to both parties; there is no reason to doubt the same good results in an educational institution.

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EXPERIMENTAL ELECTRICAL ENGINEERING

CHAPTER I.

MEASUREMENT OF RESISTANCES.

1. Experience shows that electrical conductors offer a certain opposition to the passage of an electric current. It is necessary, therefore, to apply a difference of potential, or an electromotive force, at the ends of the conductor to produce a flow of current in it. This is analogous to a difference of pressure necessary at the ends of a pipe in order that water may flow through it. It is also found that the electrical pressure E to be applied at the ends of a conductor is proportional to the current I desired to be produced in the conductor. In other words,

$$E = RI$$

where R is a coefficient of proportionality, called the *resistance* of the conductor. This experimentally determined relation between electromotive force and current is called Ohm's law. The greater the resistance R, the higher must be the electromotive force E to produce a certain current I. In this respect electrical resistance R is analogous to friction between the walls of a pipe and water flowing through it; the more this friction the higher the pressure required to produce the same flow of water in the pipe.

The resistance of a conductor is its most important electrical feature, and various methods have been devised for accurately measuring and comparing resistances. The practical unit of resistance is called *the ohm*. According to the definition of this unit established by an international agreement, it is represented by the resistance of a column of mercury of a definite length and cross-section and at a definite temperature. The reasons for selecting this unit, and the relation of the ohm to the absolute or c.g.s. (centimeter-gram-second) system, possess little interest for practical engineers. For them it is an arbitrary unit of which the prototype is carefully preserved by the respective governments, and to which prototype secondary standards in practical use are compared.

Two methods for measuring resistances are most often used in practice:

(a) Drop-of-potential method;

(b) Wheatstone bridge.

The first method is based directly on Ohm's law given above; the second is a zero, or balance, method, the unknown resistance being compared to a standard. These, and some other methods, are described below and illustrated in application to some important practical problems.

DROP-OF-POTENTIAL METHOD.

2. The resistance of a conductor is determined by this method as the ratio of a voltage at the terminals of the conductor to the current produced by this voltage. The necessary connections are shown in Fig. 1. *X* is the unknown resistance which is connected in series with a source of current, such as the battery Ba. The current strength is adjusted by the rheostat *K*, and is measured on the ammeter *A*. A voltmeter is connected across the terminals of the resistance *X*.

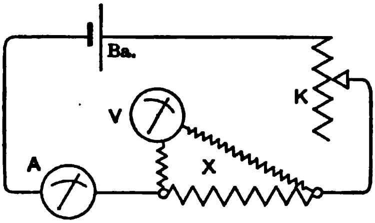

FIG. 1. Measurement of resistance by the drop-of-potential method.

If *E* is the voltmeter reading in volts, and a current of *I* amperes is read simultaneously on the ammeter, the resistance of the conductor $X = E \div I$ (in ohms). This is according to Ohm's law mentioned in § 1.

Usually several readings are taken with different values of the current, and an average calculated of the corresponding values of *R*. It is important not to use too large a current which might appreciably heat the conductor. Experience shows that the resistance of most conductors depends on their temperature; in making measurements it is therefore necessary to note to what temperature they refer.

As a further precaution, the current flowing through the voltmeter must be negligible as compared to that flowing through the resistance *X*; otherwise a correction may be necessary for the ammeter reading. Suppose, for instance, that the voltmeter shows 100 volts, and the ammeter reading be 0.5 ampere. Let the resistance of the voltmeter itself be 10,000 ohms. At a pressure of 100 volts, the current through the voltmeter is 100/10,000 = 0.01 ampere. Therefore, the true current through X is 0.50 − 0.01 = 0.49 ampere, and the unknown resistance $X = 100 \div 0.49 = 204$ ohms.

Employing this method for very accurate measurements, a potentiometer is used (§ 63), or a static voltmeter (§ 43) instead of an ordinary voltmeter; with either of these no current is shunted around X, and no correction is necessary.

A modification of the drop-of-potential method is shown in Fig. 2; here a standard resistance R is used in place of the ammeter, and the voltage drop is taken in succession, first across R, then across X. Let the readings be E_R and E_x. If the current I has not changed during the measurement, we have,

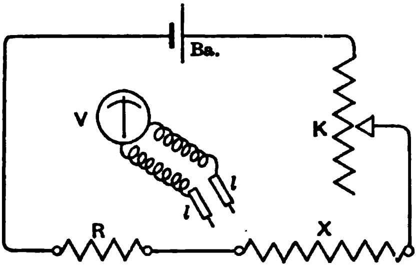

FIG. 2. Comparison of resistances, using a voltmeter.

$$I = \frac{E_R}{R} = \frac{E_x}{X},$$

or

$$X = R\frac{E_x}{E_R}.$$

The voltmeter is conveniently transferred from one resistance to the other by using flexible leads ll. The advantage of this method is that only one instrument, instead of two, needs to be calibrated, and even for this one instrument it is not necessary to know the actual values of divisions on its scale, since only the ratio of the readings enters into the result. Where extremely accurate results are required R and X are compared by means of a potentiometer (see § 43).

Experiments and practical tests to be described later illustrate the drop-of-potential method.

3. Influence of Length, Cross-Section and Material of a Conductor on its Resistance. — It is found that the resistance of a conductor (a) increases in proportion to its length, and (b) varies in an inverse ratio with its cross-section. Hence, the resistance may be expressed by the formula

$$R = k\frac{l}{q} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where l is the length of a conductor, q is its cross-section, and k a physical constant which characterizes the material of the conductor. This again is analogous to the flow of water through a pipe: the longer the pipe, the greater is its frictional resistance; the larger its cross-section, the easier it is to force through it a certain quantity of water per minute.

The constant k is called the *specific resistance* of the material of a conductor. Its physical meaning is: resistance of a conductor of unit length and of unit cross-section. The length is usually measured in feet, the cross-section in circular mils. A circular mil is the area of a circle having a diameter of 1 mil = 0.001 inch.

For copper, k = about 10 ohms at ordinary room temperature, which means that the resistance of a piece of copper wire 1 ft. long and of a cross-section of 1 circular mil is about 10 ohms. The electrical resistance of most metals increases appreciably with temperature, therefore, in stating a resistance, it is necessary to mention to what temperature it refers.

4. EXPERIMENT 1-A. — Determining Influence of Length, Cross-Section and Material of a Conductor on its Resistance. — The purpose of the experiment is to verify the relations stated in the preceding paragraph and to determine the specific resistance k for a few metals used in practice. The connections are shown in Fig. 1; or, if preferred, the method shown in Fig. 2 may be used. Insert the wire to be tested, in place of X, and adjust the current by the rheostat K. Read the current on the ammeter, and the voltage drop across a certain length of the wire. Use knife-edge contacts at the end of the voltmeter leads, in order to insure a good contact and to have a definite length of wire. Reduce the current in steps and read the corresponding voltages.

Repeat the same experiment on wires of (a) different length, (b) different cross-section, (c) different materials. The metals of chief importance in electrical engineering are: copper, aluminum, iron, German silver, and manganin. In each case vary but one factor entering into the formula (1); keep the other factors constant. With each sample of wire take several readings of volts and amperes in order to eliminate possible errors and to obtain more accurate results. Do not use large currents, which might appreciably heat the conductors under test; unless you have a means for ascertaining the temperature of the conductor. When accurate results are required, conductors are immersed in an oil-bath maintained at a definite temperature. Before leaving the laboratory measure the lengths and the cross-sections of the samples tested.

Report. (1) Figure out the resistances of the samples tested; this being done by dividing volts by the corresponding amperes. It is best to average the readings for each sample. A short way to do this is as follows: plot the observed volts to amperes as abscissæ on a sheet of cross-section paper, and draw a straight line passing through the origin and the points thus obtained. The average resistance is then calculated from any point on this line.

(2) Show from the results of the experiment that resistances are directly proportional to the length of a conductor and inversely proportional to its cross-section.

(3) Calculate the specific resistance k for the materials tested, using the foot and circular mil as the units.

5. Influence of Temperature on the Resistance of Conductors. — The electrical resistance of metals increases with their temperature; within practical limits the increase in resistance is usually assumed as proportional to the temperature rise. For copper the resistance increases 0.42 per cent for each degree centigrade, *the resistance at* 0°C. *being assumed as* 100 *per cent.* Thus, if the resistance of a copper conductor at 15 degrees C. (R_{15}) is 5 ohms, then, since

$$R_{15} = R_0 \,(1 + .0042 \times 15),$$

the resistance at 25° C. is

$$R_{25} = R_0 \,(1 + .0042 \times 25) = R_{15} \left(\frac{1 + .0042 \times 25}{1 + .0042 \times 15}\right)$$

Substituting the value of R_{15}, we find $R_{25} = 5.197$ ohms. A constant, such as 0.0042, is called the temperature coefficient of a conductor; it varies within wide limits with different substances.

Liquids have a negative temperature coefficient; their resistance decreasing with the increase in temperature.

In solving various practical problems it is of importance to know the temperature coefficient or per cent increase of resistance with temperature for various substances. Thus, for instance:

(1) Resistances used as standards must be made of materials having a negligible temperature coefficient.

(2) Some protective resistances (for instance, those used in Nernst lamps) are made of a material whose resistance increases rapidly with temperature. This protects the apparatus against excessive currents.

(3) Knowing the temperature coefficient of a material, temperature rise in electrical windings may be measured more accurately than with an ordinary thermometer; also in places where it would be impossible to reach with a thermometer.

Iron is interesting in that its resistance increases rather slowly at ordinary temperatures, and very rapidly when it is just beginning to glow dull red. This is the reason for using iron for protective resistances in Nernst lamps.

Resistance of carbon decreases with increasing temperature; therefore carbon is spoken of as having a negative temperature coefficient. For this reason, the current in an incandescent lamp increases more rapidly than the terminal voltage. This is an undesirable feature, since it makes the lamp particularly sensitive to voltage fluctuations.

6. EXPERIMENT 1-B. —Determination of Temperature Coefficient of Metal Conductors. —The experiment is performed in a way similar to Experiment 1-A; the materials tested must be placed in an oil-bath maintained at a definite temperature. The bath is heated gradually, and readings of volts and amperes are taken as in Fig. 1 every few minutes. The corresponding temperatures are read on a thermometer placed in the oil. Perform this experiment with materials such as iron or copper which have an appreciable temperature coefficient, and with manganin and German silver which have a negligible temperature coefficient.

Observe the rapid increase in resistance of an iron wire as it becomes dull red. The wire may be heated by the same current by which the resistance is measured. No oil-bath is used with this experiment, and only qualitative results are expected.

Demonstrate that the resistance of carbon decreases with an increase in temperature. This can be easily shown on the filament of an ordinary incandescent lamp. Gradually raise the voltage at the terminals of the lamp and read the corresponding values of the current. Refer the readings to the states of incandescence, thus: the lamp just begins to glow, dull red, red, bright red, yellow, white, brilliant white.

Report. (1) Calculate the values of the temperature coefficient for the materials tested.

(2) Give the results of the test on the iron wire at dull incandescence.

(3) Plot the resistances of the incandescent lamp to terminal volts as abscissæ; mark on the curve the stages of incandescence.

7. EXPERIMENT 1-C. —Determination of Temperature Rise in Windings by the Increase in Resistance. —This problem is the inverse of that in the preceding experiment. The temperature coefficient is now assumed to be known, and the temperature rise of a coil of wire is calculated from the observed increase in its resistance.

Suppose, for instance, the resistance of a coil of copper wire to be 4.573 ohms at 15 degrees C., while at some unknown temperature it was found to be 5.468 ohms. For copper, resistance increases 0.42 per cent for each degree centigrade; therefore, the resistance of the coil at 0°C. is

$$\frac{4.573}{1 + .0042 \times 15} = 4.302 \text{ ohms.}$$

The per cent increase of resistance to the unknown temperature is

$$\frac{5.468 - 4.302}{4.302} = 27.1 \text{ per cent.}$$

Therefore, the unknown temperature is

$$\frac{27.1}{0.42} = 64.5^\circ \text{ C.}$$

This is the method regularly used for the determination of temperature rise in windings of electrical machinery. A measurement by thermometers is not sufficient, because the temperature of the outside layers may be considerably below that in the center of the coil (Fig. 3). An excessive temperature rise inside the coil deteriorates the insulation and finally causes a short-circuit.

Temperature rise in a coil depends essentially upon the conditions of cooling. In some cases, as for instance in transformers, coils are cooled artificially by immersing them in oil or by subjecting them to a draft of air, (oil-cooled and air-cooled transformers). To illustrate this there is provided the following experiment.

Take three identical coils, connect them in series and investigate temperature rise under different conditions of cooling: Place one coil in a space protected from draft; put another coil in an oil-bath; subject the third coil to an artificial draft — that, for instance, produced by an electric fan. Adjust the current so that the coil without artificial cooling would be heated up to its safe limit in a reasonable amount of time — about an hour. Read the current through the coils and the voltage drop across each coil every few minutes. Note the temperature on the surface of the coils with ordinary thermometers (see Fig. 321).

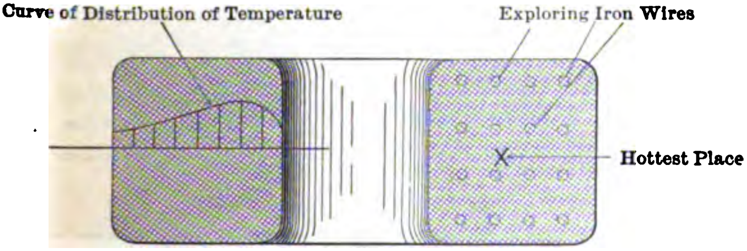

FIG. 3. Cross-section of a coil, showing exploring wires and distribution of temperature.

Such a test gives only an average temperature rise throughout the coil, but not the maximum temperature of the central layers (Fig. 3). The distribution of temperature within a coil may be investigated by placing exploring wires in various places, as shown by small circles. The temperature rise is determined by the increase in the resistance of the exploring wires. Iron is the most suitable material for such wires, because it has a considerable temperature coefficient. The terminals of each exploring coil are brought out separately; the resistance is measured from time to time while the coil under test is being heated by a current. It is not safe to take the value of temperature coefficient of iron given in standard tables; this coefficient must be determined on a sample taken from the wire used for the exploring coils.

Instead of measuring temperature rise by the increase in resistance, exploring wires may contain thermal junctions previously calibrated for temperature rise.

Report. Plot curves of temperature rise for the three coils tested under different conditions of cooling. Give the distribution of temperature inside of the coil, as shown in Fig. 3.

8. Resistances in Series and in Parallel. — Resistances are connected in electrical circuits in various combinations; it is sometimes required to determine the value of a resistance which would produce the same electrical effect as two or more given resistances. Such a resistance is called the *equivalent* or the *resultant* resistance. A few simple cases are here considered.

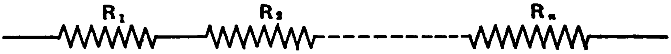

FIG. 4. Resistances in series.

(1) *Resistances in Series.* From the conception of electrical resistance, it follows that two resistances R_1 and R_2 (Fig. 4) connected in series are equivalent to a resistance $(R_1 + R_2)$. The same is true for any number of resistances in series, so that

$$R_{\text{equiv.}} = \sum_{1}^{n} R \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

(2) *Resistances in Parallel* (Fig. 5). The equivalent resistance in this case is less than either of the component resistances, because the addition of each resistance means a new path for the current. Let the common voltage across the resistances be e; the currents i_1, i_2, etc. We then have

$$i_1 = \frac{e}{R_1}\,;\; i_2 = \frac{e}{R_2}\,;\; \text{etc.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The equivalent resistance must by definition have such a value that the same total current Σi would flow through it with the same voltage e; hence

$$\sum_1^n i = \frac{e}{R_{\text{equiv.}}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

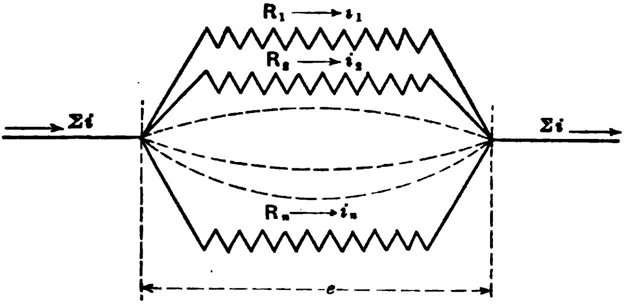
$R_1 \longrightarrow i_1$
$R_2 \longrightarrow i_2$
Σi
Σi
$R_n \longrightarrow i_n$
e

FIG. 5. Resistances in parallel.

Substituting the values of i_1, i_2, etc. from (3) we have

$$\sum_1^n \frac{e}{R} = \frac{e}{R_{\text{equiv.}}};$$

or, dividing both members of the equation by e:

$$\frac{1}{R_{\text{equiv.}}} = \sum_1^n \frac{1}{R} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

This formula gives the value of the equivalent resistance in terms of the component resistances. The reciprocal value of a resistance is sometimes called the *conductance;* the result (5) indicates that the equivalent conductance is equal to the sum of conductances of the branches connected in parallel. Denoting the conductances by G, we have

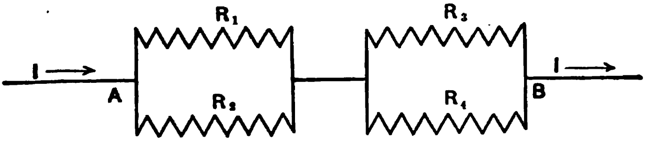

FIG. 6. Combination of resistances in series and in parallel.

$$G_{\text{equiv.}} = \sum_1^n G \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

which is analogous to the expression (2).

(3) *Series-parallel Resistances.* The problem (illustrated in Fig. 6) is to find a resistance, equivalent to the combination of resistances shown.

First find a resistance r' equivalent to the resistance offered by R_1 and R_2, by using formula (5); in the same way find the resistance r'' equivalent to R_3 and R_4. The equivalent resistance of the whole combination is $(r' + r'')$.

A more complicated combination of resistances in series and in parallel is shown in Fig. 7. It corresponds to the practical case of a transmission line $OeO'e'$ with current-consuming devices R_1, R_2, ... R_5 connected at different places. The problem is to find one single resistance which would give the same total current, with the same supply voltage between O and O'.

The problem is solved in steps: The resistances R_4 and $(2\,r_5 + R_5)$ are connected in parallel between the points d and d'. Their equivalent resistance R_4' according to the formula (5) may be calculated from the expression

$$\frac{1}{R_4'} = \frac{1}{R_4} + \frac{1}{R_5 + 2r_5};$$

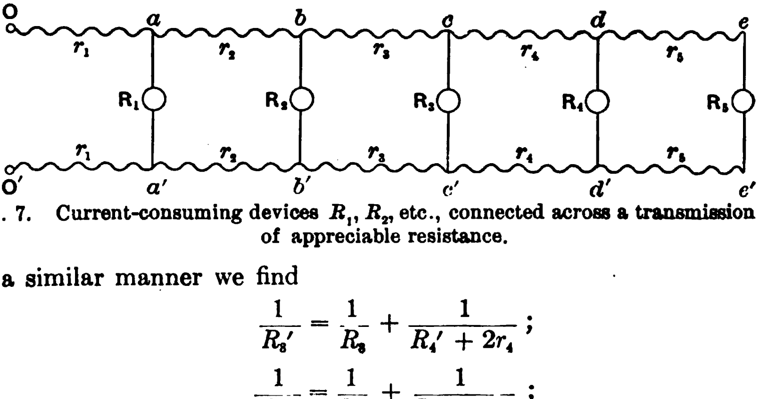

FIG. 7. Current-consuming devices R_1, R_2, etc., connected across a transmission line of appreciable resistance.

In a similar manner we find

$$\frac{1}{R_3'} = \frac{1}{R_3} + \frac{1}{R_4' + 2r_4};$$

$$\frac{1}{R_2'} = \frac{1}{R_2} + \frac{1}{R_3' + 2r_3};$$

$$\frac{1}{R_1'} = \frac{1}{R_1} + \frac{1}{R_2' + 2r_2};$$

and finally

$$R_{\text{equiv.}} = R_1' + 2r_1.$$

The problem is solved by calculating R_4' from the first equation and substituting its value into the second equation; then R_3' is calculated from this equation and its value substituted in the next equation, etc.

9. EXPERIMENT 1-D. —Exercises with Resistances in Series and in Parallel. — Take several resistances of suitable value and measure them separately by the drop-of-potential method. Connect the resistances as shown in Figs. 4 to 7, or in any combinations desired and measure the resultant (equivalent) resistance. See how closely the results check with the formulæ given in the preceding article.

10. EXPERIMENT 1-E. — Measuring Resistance of D. C. Armatures. — A current is passed through the armature in question from an external source, and the voltage at the armature terminals measured with a low-reading voltmeter. The ratio of the voltage to the current gives the resistance of the armature. In performing this measurement it is advisable to hold the voltmeter leads once on the terminals of the machine, and then on the commutator bars. This will make it possible to separate the armature resistance proper from that due to the contact resistance at the brushes, and to the resistance of the brushes themselves.

This contact resistance is different when the machine is at rest, and when it is running. To see the difference, the machine may be run at a low speed with the field circuit open, and the same measurement repeated. It must be remembered, however, that even when the field circuit is open, there is some residual magnetism in the field. This magnetism induces a voltage in the armature and may considerably affect the results. In order to eliminate its influence, the measurements must be repeated with the machine running in the opposite direction at exactly the same speed. The average of the two resistances will give the true resistance of the armature.

In stating the results it is necessary to note to which temperature they refer. The resistance of the armature, and therefore the voltage drop, will be somewhat higher when the machine is hot. In contracts and guarantees is usually stated, that voltage drop must not be above a certain figure, when the temperature of the machine is, say, 50 degrees C. above ordinary room temperature. If the resistance was measured when the machine was cold, and again after a continued run at some load (temperature test), the temperature rise in the armature winding may be calculated from the increase in resistance, as explained in § 7.

SUBSTITUTION METHOD.

11. Theory of the Substitution Method. — With this method a current of a definite value is produced in the circuit containing the *unknown* resistance, and then there is substituted for it a *known* resistance of such a value as to give the same current. It is evident then, that the known resistance is equal to the unknown resistance. The connections are shown in Fig. 8; a circuit is formed through the battery Ba, galvanometer Ga, regulating resistance K and the unknown resistance X. A double-throw switch S is provided, which allows a box R of calibrated resistances to be substituted for X. The switch S is first thrown down, and the current adjusted so as to obtain a certain deflection of the galvanometer. After this the switch is thrown up, and R adjusted until the galvanometer gives the same deflection; then the resistance of R is equal to X.

In using this method care should be taken, that the battery e.m.f. does not change (due to polarization) between the two readings, and also that the resistance K remains constant. Instead of connecting R and X in parallel, they may be connected in series, as shown in Fig. 9, and short-circuited in succession.

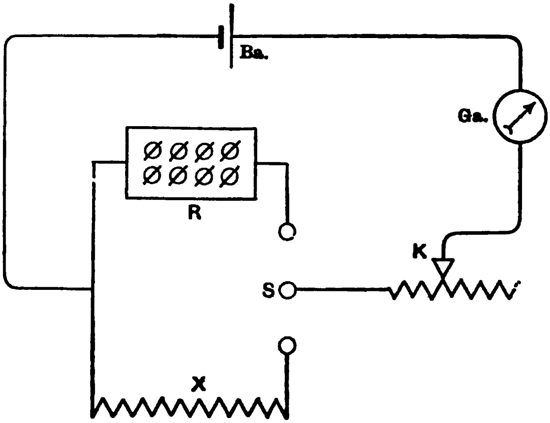

FIG. 8. Substitution method of measuring resistances.

The substitution method is sometimes used for measuring high resistances, such as insulation resistances; also for measuring resistance of liquid conductors, as explained in § 12.

12. Measuring Resistance of Electrolytes by the Substitution Method.—The difficulty in measuring electrical resistance of most liquids is that they are decomposed by current. Gases are deposited on the electrodes, the resistance being thereby increased, and a counter e.m.f. of polarization set up. One way out of this difficulty is to measure the resistance of liquids with alternating currents (see § 21). Another method based on the substitution of resistances is shown in Fig. 9.

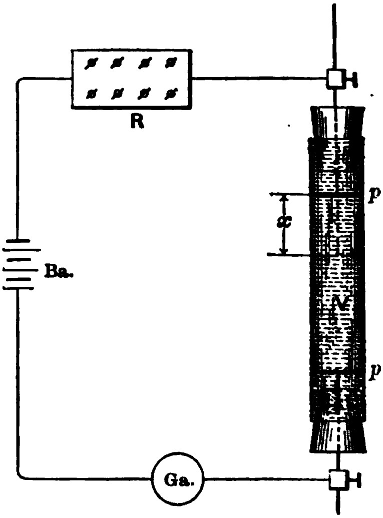

FIG. 9. Measuring resistance of a liquid by the substitution method.

The liquid under test is placed in a glass tube *V*, provided with plugs at both ends, and with the electrodes *pp*; the upper electrode being adjustable. *R* is a calibrated resistance box. A certain current is sent through the circuit, until the conditions in the liquid become constant. The deflection of the galvanometer is noted, then the upper electrode *p* is moved by the amount *x*, as shown by dotted lines. The current increases, but is brought back to its former value by introducing more resistance *R* into the circuit. The amount *R* is evidently equal to the resistance of the column *x* of the liquid. The effect of the polarization is thus eliminated, as it is present to the same extent with both positions of *p*.

The resistance of liquids is usually reduced to the resistance of a column 1 cm. high and having a cross-section of 1 sq. cm. This is called the *specific* resistance of the fluid. If the cross-section inside of the glass tube *V* is *Q* sq. cm., the specific resistance of the liquid under test is $RQ \div x$. This is evident since resistance is proportional to the length and inversely proportional to the cross-section of a conductor.

13. EXPERIMENT 1-F. — Measuring Resistances by the Substitution Method. — The theory of the method and its application for measuring resistances of solid conductors is given in § 11; the application to liquid conductors is explained in § 12. An experiment should be performed affording practice with the method as illustrated in Figs. 8 and 9.

WHEATSTONE BRIDGE.

14. The combination of conductors known as the Wheatstone bridge (Fig. 10), devised for measuring resistances, has the following features:

(1) The unknown resistance is compared directly to a standard resistance.

(2) No calibrated measuring instrument is required, the adjustment being made by bringing the galvanometer back to zero (null method).

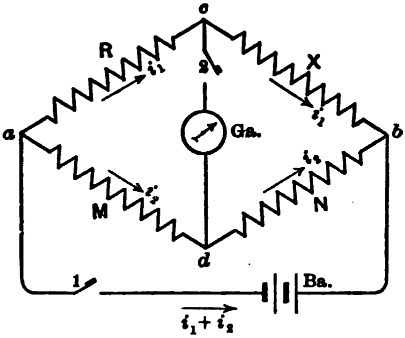

FIG. 10. Diagram of the Wheatstone bridge.

The unknown resistance *X* and a standard resistance *R* are connected in series with each other and with the battery *Ba*. The combination is shunted by two other resistances *M* and *N*, the ratio of which — $N \div M$ — is known. The galvanometer is bridged between the two sets of resistances, hence the name "bridge." Contact keys 1 and 2 are provided in the galvanometer and battery circuits.

The battery key 1 is closed first, and a moment later the galvanometer key 2 is closed. If the galvanometer gives a deflection, the "balancing" resistance R and the "ratio" resistances M and N are varied until the galvanometer comes back to zero. Then

$$X : R = N : M \quad . \ . \ . \ . \ . \ . \ . \ . \ . \quad (1)$$

Proof: Suppose that the four resistances satisfy the condition that no current flows through the galvanometer; in other words, the difference of potential between the points c and d is zero. From this condition it follows, that the voltage drop in the branch ac is equal to that in the branch ad; or, with the notations in the sketch,

$$Ri_1 = Mi_2.$$

Similarly

$$Xi_1 = Ni_2.$$

Dividing the second equation by the first we obtain the expression (1).

15. Slide-Wire Wheatstone Bridge. — A simple Wheatstone bridge is shown in Fig. 11; the ratio resistances M and N are formed by two parts of a straight wire; the ratio is varied by the sliding contact d. The slide-wire is made of a non-corrodible metal of high specific resistance and of a small temperature coefficient. The brass blocks a, b, c to which R and X are connected have a negligible resistance.

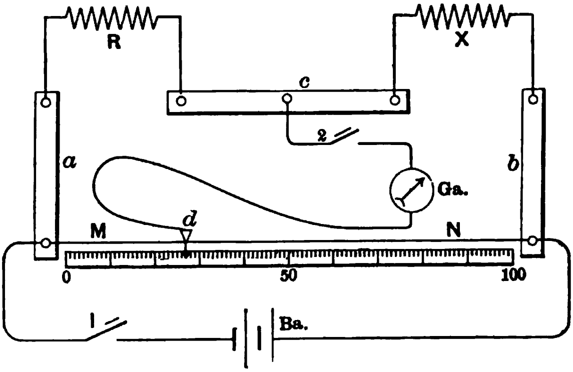

FIG. 11. Diagram of the slide-wire Wheatstone bridge.

To measure the resistance X, the key 1 is closed first, then the key 2, and the contact d moved along the wire until the galvanometer remains on zero when the key 2 is closed or opened. Suppose the standard resistance R be $= 10$ ohm; with the position of the contact shown in the sketch, $M = 27$ div., $N = 73$ div.; then, according to formula (1),

$$X = 10\frac{73}{27} = 27.04 \text{ ohm.}$$

The details of the construction of the bridge are shown in Fig. 12; the rider is provided with a vernier so as to read accurately the tenth parts of a division. Extra binding posts and the reversing switch in the center enable the bridge to be used for the Carey-Foster method, described in § 17.

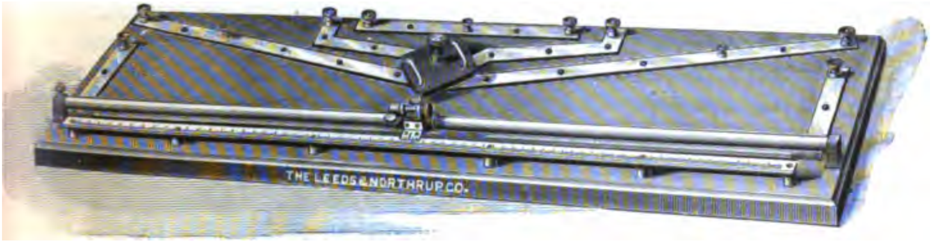

FIG. 12. Slide-wire Wheatstone bridge, also suitable to be used as a Carey-Foster bridge.

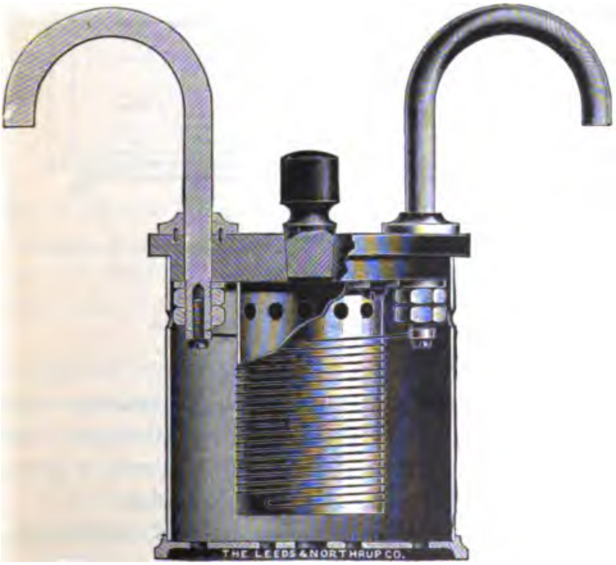

FIG. 13. A standard resistance.

The balancing resistance R may have various forms; usually it consists of a coil of wire (Fig. 13) wound on a metal spool and protected by a metal case. The heavy horn terminals are used for suspending the device from mercury cups, in order to minimize the error due to contact resistances. The greatest accuracy of measurement is obtained when X is about equal to R so that the contact d is near the center of the slide-wire.

16. EXPERIMENT 1-G. — Measuring Resistances with a Slide-Wire Bridge. — The theory and the use of the bridge are explained in the two preceding sections. Three kinds of problems may be solved with the bridge:

(a) Checking standard resistances against a primary standard.
(b) Determining values of unknown resistances.
(c) Adjusting resistances to desired values.

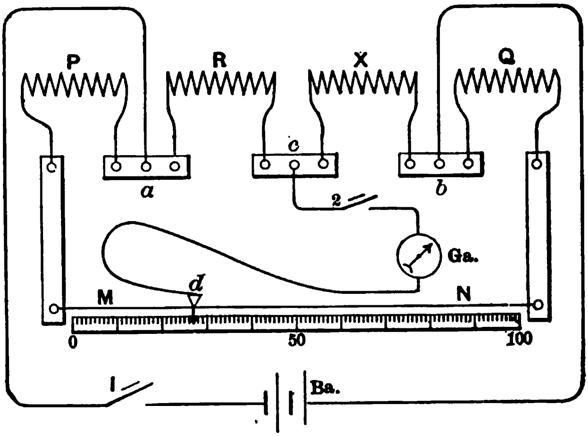

FIG. 14. Carey-Foster bridge.

The student is expected to make himself familiar with the use of the bridge, and to practice in these three problems. No particular accuracy is expected in this exercise, it being only a preparatory one for more advanced work with Wheatstone bridge.

17. Carey-Foster Method for Comparing Resistances. — This is a modification of the ordinary Wheatstone bridge, and is used for an accurate comparison of two resistances nearly equal to each other, as in determinations of per cent error of secondary standards. The connections are shown in Fig. 14; the bridge illustrated in Fig. 12 may be used with this method. P and O are two resistances to be compared to each other; R and X are two other resistances, of which it is not necessary to know the numerical values. It is advisable, however, to have R and X not very different from each other, in order to obtain the balance near the center of the slide-wire.

The bridge is balanced as usual; then the positions of P and Q are interchanged by means of the commutator shown in Fig. 12; a new balance is obtained by slightly moving the contact d. The resistance of the slide-wire between the two positions of the contact is equal to the difference of the resistances P and Q.

The reason for this is, that the branches R and X being the same in the two measurements, the total resistance of each of two other branches must also be the same. If P is smaller than Q, the difference must be compensated by the corresponding amount of resistance in the slide-wire.

The method presupposes, that the resistance of the slide-wire per one division of the scale is known. The calibration is made by measuring with the bridge the difference in the resistance of two standards P and Q, which difference has been determined before in some other way. This is best done by taking two equal resistances P and Q, each of perhaps 1 ohm, and to shunt P by a comparatively high resistance, for instance 100 ohms. This reduces the resistance of P to a certain value P'. According to the formula 5 of § 8 we have:

$$\frac{1}{P'} = \frac{1}{1} + \frac{1}{100}; \quad \text{or } P' = 0.9901 \text{ ohm.}$$

Knowing the difference between Q and P', the slide-wire may be calibrated, at least in the middle portion used for measurements. A balance may be obtained at any desired point of the slide-wire by changing the ratio $X : R$.

18. EXPERIMENT 1-H. —Comparing Resistances by Means of the Carey-Foster Bridge. — The method is explained in § 17, preceding. Special types of Carey-Foster bridge are on the market, with which an accuracy of comparison to within one one-hundredth of a per cent is possible. For ordinary practice with the method the bridge shown in Fig. 12 is sufficient.

(a) Connect four resistances as indicated in Fig. 14 and practice in comparing their values.

(b) Calibrate the slide-wire as explained above.

(c) Determine the limits of accuracy of the method by varying the difference between P and Q, the ratio $R : X$, and the absolute values of the four resistances.

19. Sage Ohmmeter. — A convenient portable Wheatstone bridge of the slide-wire type is shown in Fig. 15; it is known as the Sage ohmmeter, and is intended for all-around practical work, particularly for locating faults in the insulation of wiring installations. The slide-wire is divided into two parts connected electrically; this reduces the length of the case, and at the same time the wire is long enough to give sufficient accuracy. The contact *d* (Fig. 11) is formed by the stylus shown lying across the top of the instrument. Touching the wire with the stylus closes the galvanometer circuit. The instrument is entirely self-contained ; the galvanometer and a few dry cells are mounted in the case. The telephone receiver shown in front of the instrument may be used, if desired, in place of the galvanometer. Four standard resistances *R*, usually 1, 10, 100, 1000 ohm, are supplied with the bridge. They are mounted inside of the box, and the leads taken to the four sockets seen on the right side of the middle bar. Any one of them may be used according to the value of the unknown resistance; the desired value is selected by inserting the plug in the corresponding socket.

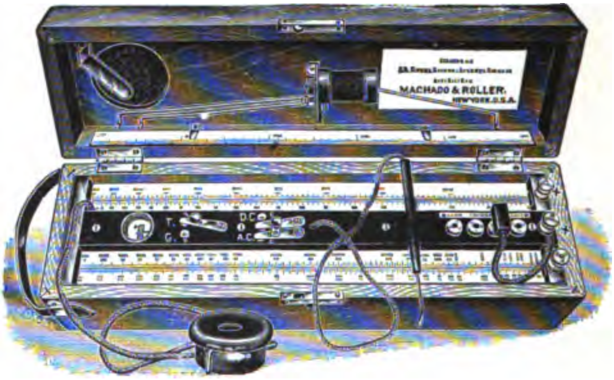

Fig. 15. Sage ohmmeter

The scale is calibrated directly in ohms, so that no calculations are necessary. Four sets of figures in different colors are printed on the scale, and the sockets are labeled accordingly. When the plug is in the " blue " socket, blue figures are read on the scale, etc. In this way even an unskilled person can use the bridge with little probability of making a mistake. Uniformly-divided scales are also provided with the instrument, and may be attached in place of the direct-reading scales, if so desired.

An induction coil is shown mounted on the lid of the box; by means of it the bridge may be used with alternating currents. The coil is supplied with current from the battery mounted in the box. The telephone receiver must be used instead of the galvanometer, when using the induction coil. Alternating current is applied where polarization would vitiate results obtained with direct current; as in locating grounds when an electrolytic action between the conductor and the earth is suspected; also for measuring resistance of liquids, as in § 21.

Switches are provided in the instrument for changing to direct or alternating current at will; also for connecting into the circuit either the galvanometer or the telephone receiver. The battery switch is built in the telephone receiver, so that the operator may close the switch with the same hand with which he is holding the receiver to his ear. With the other hand he touches the slide-wire with the stylus, and thus locates the point where the click in the receiver disappears, or is reduced to a minimum.

20. EXPERIMENT 1-I. — Practice with the Sage Ohmmeter. — (a) Inspect the instrument and make clear its connections and operations.

(b) Measure by means of it a box of calibrated resistances, within as wide a range as possible, paying particular attention to the sensitiveness of the instrument at different values of the unknown and the balancing resistances. (1) Perform the measurements first with direct current, using in succession the galvanometer and the telephone receiver; then (2) repeat some of the measurements with the induction coil and the telephone receiver.

(c) Determine the resistance of an electrolyte with direct and with alternating current, and note the difference.

(d) Measure the resistance between a line and the ground (resistance of the fault).

(e) Locate the place of a fault by one of the methods described in §§ 295 and 296. Form your own opinion about the instrument, its advantages and shortcomings, the limits of accuracy, the best range, the relative advantages of using the telephone receiver as vs. the galvanometer, and direct as vs. alternating current.

21. Kohlrausch Bridge. — A disadvantage of a stretched slide-wire, as in Fig. 12, is that it cannot be made long enough for accurate measurements without making the instrument itself too large. Kohlrausch solved this difficulty by winding the slide-wire on an insulating cylinder, as shown in Fig. 16, to the right. The cylinder may be rotated by a handle, contact being made by a roller. The balancing resistances are placed in the base of the instrument, and may be connected into the circuit by the plugs seen in the sketch.

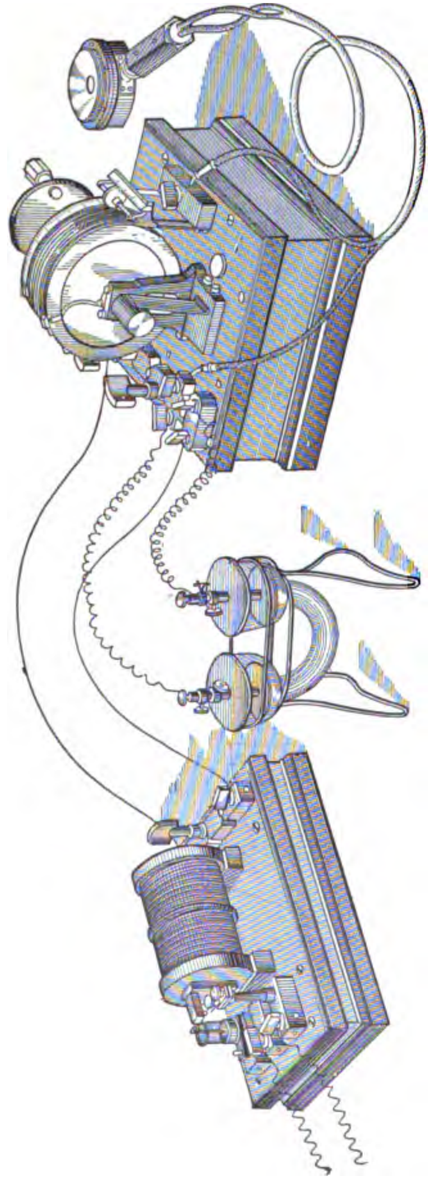

FIG. 16. Measuring resistance of an electrolyte with the Kohlrausch bridge, using alternating current.

The Kohlrausch bridge is particularly well adapted for measuring resistances of liquid conductors with alternating currents (see § 12). An induction coil for producing alternating currents is shown to the left; a telephone receiver is substituted for the galvanometer.

The liquid under test is put into the U-shaped vessel shown between the bridge and the induction coil, this vessel being provided with platinum electrodes on both ends. The form of the vessel is too complicated to allow the specific resistance of the liquid to be deduced as in § 12. This, however, is no serious objection; the constant of the vessel may be determined once for all by measuring in it the resistance of a standard fluid, whose specific resistance is known.

For instance, it is known that the specific resistance of a 25 per cent solution of zinc sulphate is 21.4 ohms per cu. cm. at 18 degrees C. If the resistance of such a solution, as measured in the Kohlrausch vessel to be calibrated, be 152 ohms, then the constant of the vessel is $21.4 \div 152 = 0.141$. The resistance of all other liquids measured in this vessel must be multiplied by this coefficient to get their specific resistances. The Kohlrausch bridge is more convenient for a quick and accurate determination of resistances of electrolytes than the substitution method described in § 12.

22. EXPERIMENT 1-J. — Measurement of Resistances with Kohlrausch Bridge. — See preceding paragraph.

23. Testing Rail Bonds. — On most electric roads the current for operating cars returns to the station through the track rails; this saves one conductor. In order to make possible the use of the rails for this purpose they must be " bonded " together, or connected by more or less flexible copper conductors as shown in Fig. 17. Rail bonds must be periodically inspected, as they get loose or broken, causing an excessive resistance between the rails. Poor bonding brings with it an excessive drop of voltage and loss of power, electrolytic corrosion of gas and water pipes in the vicinity, telephone troubles, etc.

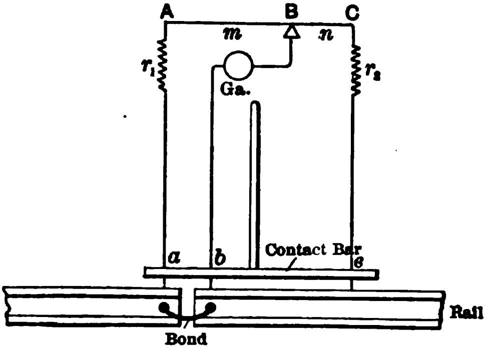

FIG. 17. Diagram of the Roller bond tester.

If rail bonds could be tested while the road is not in operation, the simplest method would be to send a current from the station and to measure the drop across each bond with a low-reading voltmeter. Usually, however, testing must be done with cars running on the same line, so that the current in the rails is widely fluctuating. In order to utilize this current special devices are used as described below. The resistance of the bond is expressed in feet of a solid rail; thus a resistance of 2 feet means that the voltage drop caused by the bond contacts is the same as would be caused by two additional feet of solid rail. Convenient methods of testing bonds are as follows:

(a) If no special bond tester is available, *two ordinary milli-voltmeters* may be used, one connected between *a* and *b* (Fig. 17), another between *b* and *c*. The milli-voltmeters are read simultaneously, so that the fluctuations in the rail current do not affect the ratio of the readings. This ratio is evidently equal to the ratio of the resistances of the bond *a-b*, and the rail *b-c*. It is advisable to have some protective resistance in series with the milli-voltmeter connected across the bond. The voltage drop across a loose or broken bond may be many times in excess of the range of the instrument. The protective resistance may be short-circuited with a push button. Good contacts with the rail may be obtained either by chisel-ended spikes struck into the rail faces or by using short sections of hack-saw blades.

(b) A convenient device for testing rail bonds — the so-called *Roller bond tester* — is shown in Fig. 18, while a diagrammatic view of its connections is represented in Fig. 17. It operates on the principle of a slide-wire Wheatstone bridge. The unknown resistance *a-b* of the bond is compared to a definite length *b-c* of the solid rail by means of the slide-wire *A-C* and the galvanometer connected between *b* and *B*. The slider *B* is moved back and forth until the galvanometer shows zero. Then the ratio of the resistance of the bond to that of the rail is equal to the ratio $(r_1 + m) \div (r_2 + n)$. It is not necessary to calculate this ratio every time; the scale of the instrument (Fig. 18) gives the resistance directly in feet of solid rail.

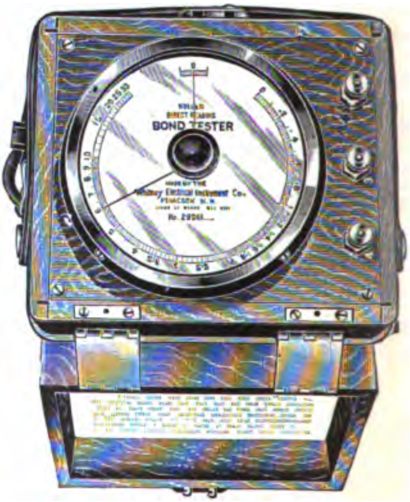

FIG. 18. The Roller bond tester.

The resistances r_1 and r_2 act as an extension of the slide-wire, making the useful part of it longer and therefore more accurate. Without these resistances, the short slide-wire would contain all the values from zero to infinity. Fluctuating currents in the rail do not affect the balance, because fluctuations occur in all the branches of the bridge simultaneously.

One observer is sufficient with this instrument; he holds the contact bar by the upright in one hand, and operates the knob of the slider with the other hand. The instrument itself is suspended from his shoulders. The large scale indicates the resistance, the small scale is for galvanometer deflections. In many cases, it is sufficient to know that the resistance of the bond is not beyond a certain limit; then the pointer of the slider is simply set at this limit. On all bonds having resistance below this limit the galvanometer deflects one way, on defective bonds it shows the other way. This makes the instrument very convenient for rapid testing, even with an unskilled observer.

(c) Another popular *bond tester* is that designed by *Conant.* In it the contact *c* (Fig. 17) is separate from the two other contacts, and the operator's assistant moves it along the rail, until a balance is obtained. A telephone receiver is used instead of a galvanometer, and the current is periodically interrupted by a wheel driven by a clock-work. The balance is reached when the click in the telephone disappears; the equivalent length of the rail is then measured directly with a foot rule.

24. EXPERIMENT 1-K. — Testing Rail Bonds. — The method and the apparatus are described in the preceding article. Become familiar with the devices in the laboratory before trying them on an actual track. Two rails should be provided in the laboratory, connected by a variable resistance; this resistance is to represent various states of a bond, from a perfect contact to a broken bond.

(a) Connect the rails to a D. C. circuit, placing a regulating resistance in series with them, to produce fluctuating currents as in practice.

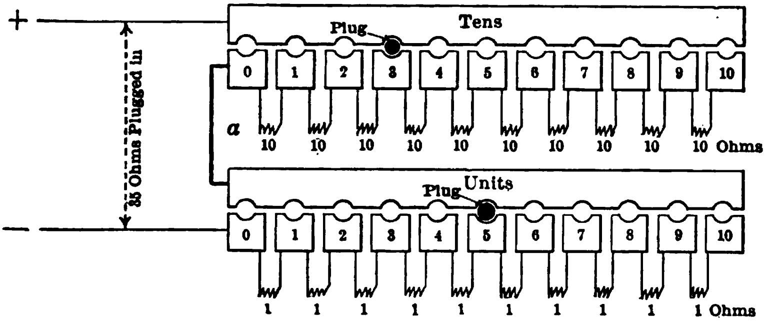

FIG. 19. Decade arrangement of resistances.

(b) Setting the resistance of the bond very low, measure it first with a steady current, then with fluctuating currents.

(c) Repeat the same tests with higher resistances of the bond.

(d) Having become thoroughly familiar with the instrument, measure with it a number of rail bonds on a street-car line in actual operation.

In performing this experiment, pay particular attention to the accuracy throughout its range of the instrument used, and to the influence of current fluctuations. See in how far a poor contact between the rail and the saw blade or the spike used for making contact may affect the results.

25. Plug-Type Wheatstone Bridge. — Series and Decade Arrangement of Coils. — For accurate work, bridges are used in which the three "known" resistance arms *R*, *M*, and *N* (Fig. 10) consist of carefully adjusted resistance coils (Fig. 20). These coils are mounted inside of a box and leads are taken to contacts on its top. Resistances are varied either by inserting plugs, as in Fig. 21, or by dial switches (Fig. 22).

The older method of connecting coils is that shown in Fig. 20; the coils are connected in series, and each plug short-circuits a coil. Thus, in the upper row all the coils are short-circuited, except 4000, 3000, and 200 ohm, the reading being 7200 ohms.

The new or the "decade" method is shown in Fig. 19. Here the resistances which are not in use are simply left out of the circuit. Therefore only one plug is necessary for each decade or row of ten coils.

The advantages of the decade arrangement are:

(1) A smaller number of plugs is required.

(2) Additional resistance between plugs and blocks at the contacts is less.

(3) The result is easier to read, as no summation is necessary.

(4) There is less liability to make mistakes, or to lose a contact plug.

The advantage becomes evident by comparing the two upper rows of contacts in the bridge shown in Fig. 20 with the four rows in Fig. 24. In the first case we have to add mentally

$$4000 + 3000 + 200 + 30 + 20 + 3 + 2 + 1 = 7256 \text{ ohms};$$

in the second case we read directly 6192 ohms. Moreover, in the first case 8 plugs are in the circuit, and 8 more are lying idle on the table. In the second case there are but 4 plugs, and they are always in use.

26. Examples of Plug-Type Bridges. — A simple bridge is shown in Fig. 20; the lettering is the same as in Fig. 10. The bridge has three arms: two ratio arms *M* and *N* in the lower row, and the balancing resistance *R* in the two upper rows of coils. The box is provided with three pairs of terminals — for the battery, for the galvanometer and for the unknown resistance *X*. The battery key 1 and the galvanometer key 2 are also mounted on the box.

Unlike the slide-wire bridge (Fig. 11), plug-type bridges have only a limited number of ratios $M \div N$, — usually in powers of ten. The balancing is done by the third arm *R*. In the instrument under consideration, the resistance of this arm may be varied from 0 to 10,000 ohms in steps of 1 ohm. Thus with the ratio $N \div M = 1000 \div 1$, resistances can be measured up to 10,000,000 ohms. On the other hand, by selecting $R = 1$ ohm and $N \div M = 1 \div 1000$, resistances may be measured down to 0.001 ohm. The practical range of the bridge is within considerably narrower limits, because it is neither sufficiently accurate nor sensitive enough near the apparent limits of its range.

To measure a resistance, connect it to the terminals marked X; select a ratio $N \div M$, say $10 \div 100$. Press the battery key, and then the galvanometer key; vary the resistance R, until the galvanometer remains at zero when its key is pressed. Best results are obtained when the ratio is selected so that balancing can be done with all four places of the resistance R — thousands, hundreds, tens and units. The right ratio $N \div M$ can always be selected, if the approximate value of X is known in advance; otherwise, the best ratio is found by trials. With the setting shown in Fig. 20 the unknown resistance

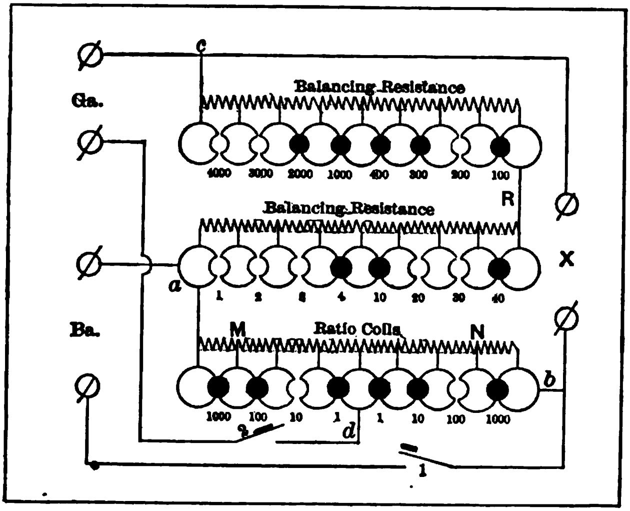

FIG. 20. Diagram of a plug-type Wheatstone bridge.

$$X = 7256.\frac{100}{10} = 72560 \text{ ohms.}$$

A Wheatstone bridge for precision work is shown in Fig. 21. The ratio coils M and N are in the last two rows to the right; the other five rows are connected to the balancing resistance R, the decade arrangement of coils being employed. The keys and the binding posts are clearly seen in the sketch. The coils may be joined in series or in multiple or in any combination of series and multiple. The coils may thus be checked against each other in many combinations. For example, all the ten ohm coils taken in parallel may be compared with any one ohm coil.

The precision of adjustment is $\frac{1}{20}$ per cent for the coils of the tenth ohm series, and $\frac{1}{50}$ per cent for the other coils of the rheostat.

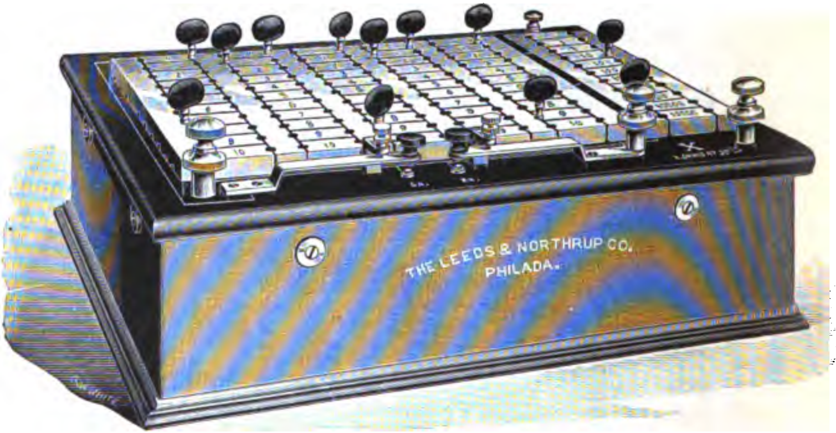

FIG. 21. A plug-type Wheatstone bridge for accurate measurements.

The ratio coils are certified to be like each other within $\frac{1}{100}$ per cent.

A similar box is shown in Fig. 22; dial switches being used on the balancing resistance instead of plugs. This makes possible a quicker adjustment; at the same time, with good workmanship, switch contacts are very little inferior to plugs.

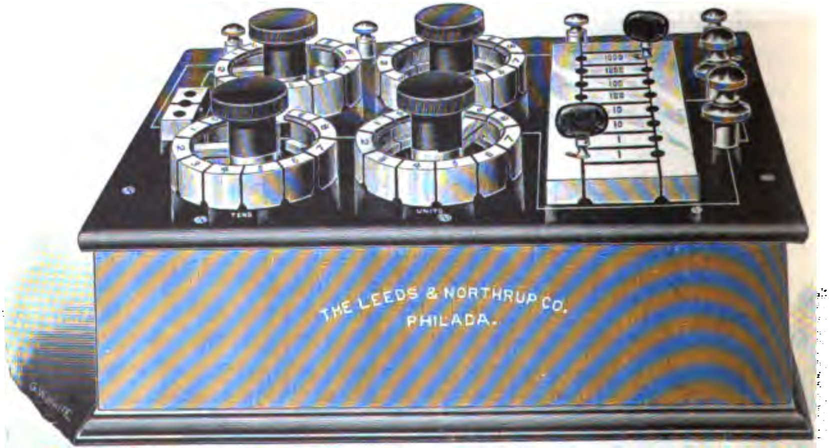

FIG. 22. A Wheatstone bridge with dial switches in place of plugs.

27. Portable Testing Sets. — The above-described bridges are primarily intended for *stationary* work. There is a demand for self-contained *portable* bridges, for locating faults and for other emergency purposes in the operation of electric plants. The Sage ohmmeter described in § 19 is one of this kind. A more accurate instrument is shown in Fig. 23, while its electrical connections may be followed in Fig. 24. The set consists of a Wheatstone bridge with a decade arrangement of resistances; a galvanometer and a battery of small dry cells are mounted in the same box, so as to make the instrument self-contained.

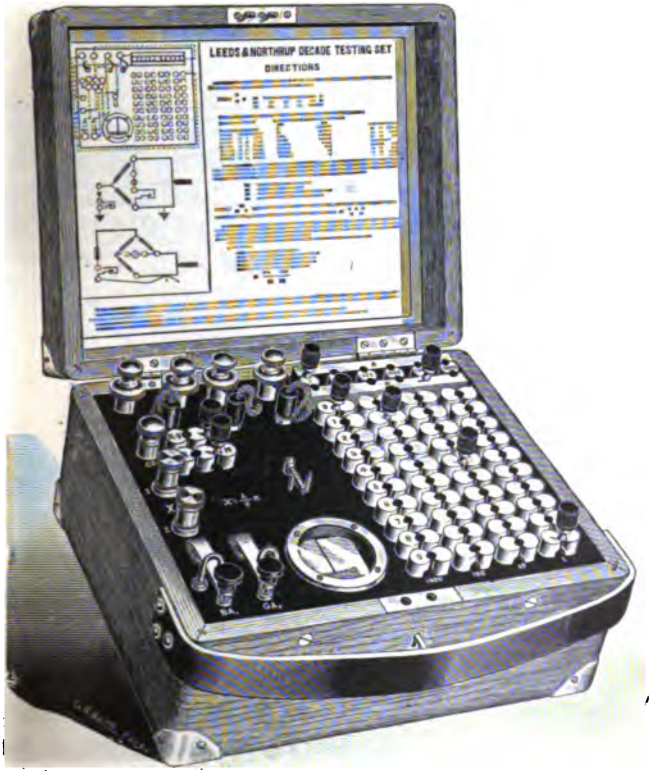

FIG. 23. A portable testing set.

Some features of this set are as follows:

(1) The terminal marked "Gr." (ground) and the extra blocks *MM* enable the bridge to be used for locating faults according to either the Murray or Varley method (§§ 295 and 296).

(2) The instrument may be used as an ordinary resistance box.

(3) The galvanometer may be used separately, or in series with a resistance, for any desired purpose.

(4) If the battery is exhausted, or not strong enough for a certain purpose, the flexible leads may be disconnected and an outside battery connected to the bridge.

(5) An outside galvanometer may be used if desired, by disconnecting the flexible galvanometer leads.

Explicit directions are supplied with the instrument, so that any one with a very elementary knowledge of electricity can use it. Similar testing sets are made by other leading manufacturers of measuring instruments.

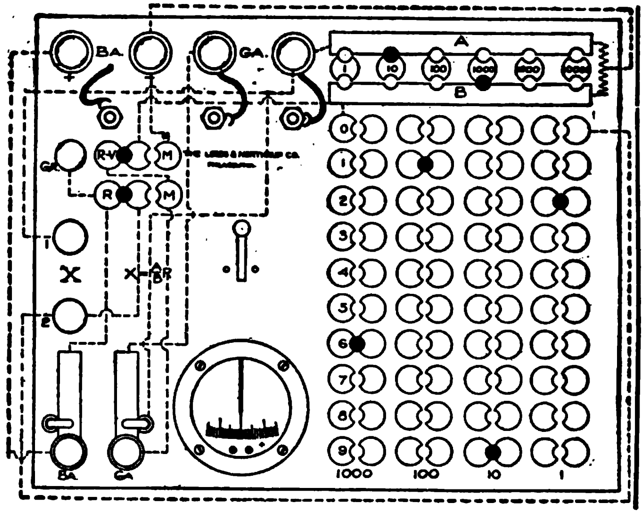

FIG. 24. **The diagram of connections of the testing set shown in Fig. 23.**

28. EXPERIMENT 1-L. — Measuring Resistances with a Plug-Type Wheatstone Bridge. — A few types of bridges are described in §§ 25 to 27. The problems which may be solved with these bridges are the same as enumerated in § 16. If possible the student should connect the coils of the bridge itself in various combinations, so as to check them against each other. The purpose of the exercise is not only to learn how to perform measurements, but also to study the bridge itself, ascertaining the limits of its accuracy, the best range, the necessary sensitiveness of the galvanometer, etc.

29. Measurement of very Small Resistances.—The Wheatstone bridge is not suitable for measuring small resistances, say below 0.1 ohm, because the resistance of the contacts at *a*, *b*, *c* and *d* (Fig. 10) may constitute an appreciable part of the resistances *X* and *R* themselves. The drop-of-potential method (§ 2) gives satisfactory results with resistances considerably below 0.1 ohm, if care is taken to have voltmeter leads properly applied, so as to eliminate the contact resistance (Fig. 35). For an accurate comparison of very low resistances, the potentiometer method may be used (see § 43).

Lord Kelvin combined the Wheatstone bridge and the drop-of-potential method into a scheme which is exceedingly accurate for measuring low resistances, say down to 0.0001 ohm. The diagram of connections of this so-called Kelvin double-bridge is shown in Fig. 25.

The resistances *R* and *X* to be compared are connected in series with a low-voltage battery *Ba.*, and some regulating resistance not shown in the Figure. The connections to the galvanometer are taken from the points *E*, *F*, *G* and *H* separate from the main terminals. This is a point of paramount importance, because the contact resistances at *A*, *B*, *C* and *D* should not enter into the result. Four ratio coils are used, *m*, *n*, *m′* and *n′*, instead of two ratio coils *M* and *N* of the ordinary Wheatstone bridge (Fig. 10).

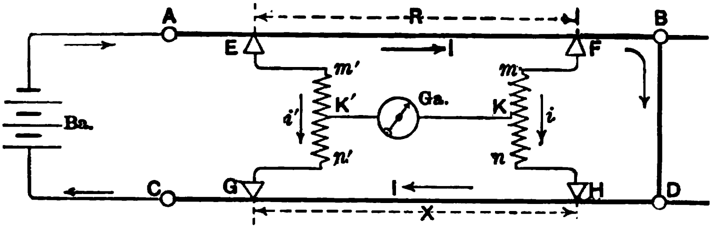

FIG. 25. Principle of the Kelvin double-bridge.

The unknown resistance *X* is measured by adjusting either *R* or the ratio coils, until the galvanometer gives no deflection. Both pairs of ratio coils are adjusted simultaneously, so that the relation

$$m \div n = m' \div n' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

is preserved all the time. When the balance is obtained the relation holds true

$$R \div X = m \div n = m' \div n' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

from which *X* is calculated.

Proof: When no current flows through the galvanometer, the points K' and K are at the same potential. Therefore the voltage drop from E to K' is the same as from F to K. With the notations in the sketch we have

$$i'm' = IR + im.$$

The same holds true with respect to the point G; hence,

$$i'n' = IX + in.$$

From these two equations we get

$$R : X = (i'm' - im) \div (i'n' - in),$$

or substituting for m' its value from (1) we obtain after a simple transformation the relation (2).

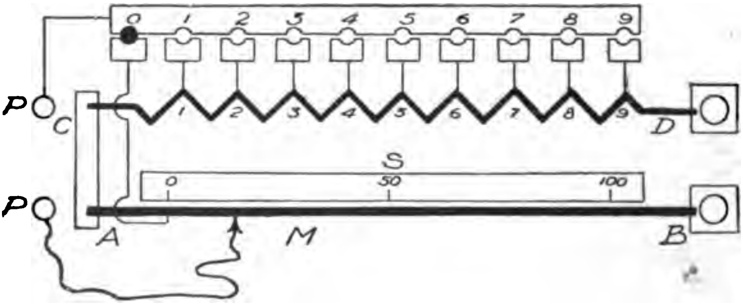

FIG. 26. Leeds and Northrup variable low resistance, for use as a standard in connection with the Kelvin double-bridge.

There are two types of Kelvin bridge: (1) Either a set of a few standard resistances R are supplied with the bridge, and the adjustment is made by the ratio coils; or (2) the ratio coils are made to give only a limited number of combinations, but the standard resistance R is made adjustable, so as to give practically any value of resistance within certain limits. The first is the older method, the second is coming into use.

The variable standard low resistance made by the Leeds and Northrup Co. for use with the Kelvin double-bridge is shown diagrammatically in Fig. 26.

AB is a heavy piece of resistance metal of uniform cross-section and uniform resistance per unit of length; CD is another piece of resistance metal of smaller cross-section, and the two are joined together by a heavy copper bar AC into which both are silver-soldered; LL are the current terminals and PP are the potential terminals. The resistance of *AB* between the marks 0 and 100 on the scale *S* is .001 ohm. From the point 1 on the resistance *CD* to 0 on *AB* is also .001 ohm, from 2 to 0 is .002 and so on, and from 10 to 0 is .01 ohm. The slider *M* moves along the resistance *AB* and its position is read on the scale *S* which is divided into 100 equal parts and can be read by a vernier to thousandths. Subdivided in this way the resistance between the tap-off points *PP* may have any value from .001 to .01 ohm by steps of .000001 ohm. Using a ratio of 1 to 10 it gives all values from .1 ohm to .01 ohm, by steps of .00001 ohm, and, using the inverse ratio, all values from .001 ohm to .0001 ohm by steps of .0000001 ohm.

It will be noted that there are no contacts in the main circuit between the terminals *LL*. The only contacts are at the tap-off points to the potential terminals *PP*. The resistance of these contacts is negligible, being in series with the ratio coils, which have several hundred ohms resistance. The ratio coils are mounted within ordinary resistance boxes, similar to the one shown in Fig. 21; the resistances may be adjusted to agree with each other within $\frac{1}{10}$ per cent.

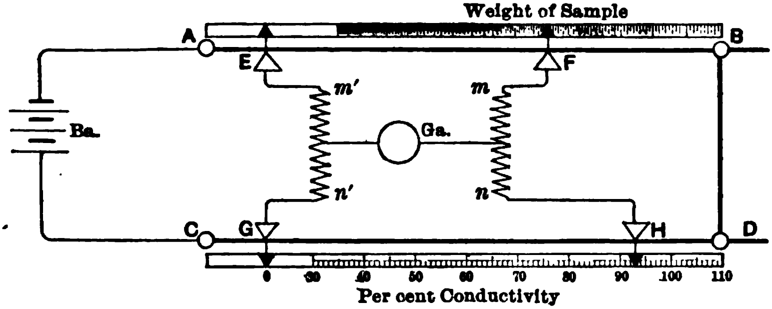

FIG. 27. Diagram of the Hoopes conductivity bridge.

30. Hoopes Conductivity Bridge.—The principal practical application of the Kelvin double-bridge is for determining conductivity of bars of copper and aluminum. This is done regularly in factories manufacturing wire for electrical purposes; wire is also tested by large consumers before accepting a consignment. It is desirable in such cases to modify the standard Kelvin bridge described in the preceding article, so as to fulfill the following requirements:

(1) The apparatus must be direct-reading in per cent conductivity of chemically pure metal, instead of giving results in ohms.

(2) The apparatus must be adapted for testing a large number of similar samples in a short time.

(3) It must be easily handled by a person who has but little electrical knowledge.

A bridge which satisfies these conditions was designed by Mr. Wm. Hoopes. It is shown diagrammatically in Fig. 27; a general view of the apparatus is given in Fig. 28. Comparing Figs. 27 and 25 it will be seen that the electrical connections in the Hoopes bridge are identical with those in the regular Kelvin bridge. *AB* is a standard wire made of the same material as the samples to be tested, in order to have the same temperature coefficient; *CD* is a sample under test. The operations for determining per cent conductivity are very simple. The sample wire is cut off to the standard length of 25 inches in a special cutting-off machine, and accurately weighed. Then it is clamped in the position *CD* and the contact *F* set at a place corresponding to the weight of the sample. The contact *H* is moved back and forth until the galvanometer shows zero; per cent conductivity is then read off directly on the lower scale.

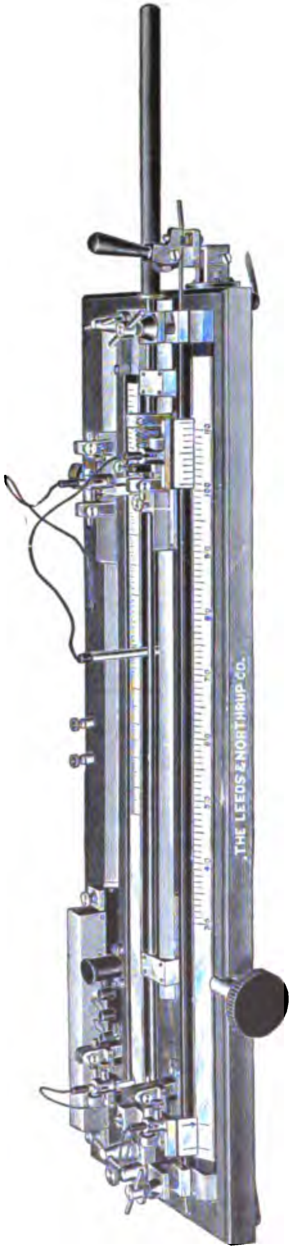

FIG. 28. The Hoopes conductivity bridge.

The theory of the method is easily understood from the fact that a sample of a certain metal, of a given length and weight, has a definite and known resistance if chemically pure (100 per cent conductivity). Therefore the scale of the standard wire may be made so as to balance a chemically pure sample with *H* on the division 100 of the lower scale. Then if the sample under test has a lower conductivity, a smaller length of it balances the standard wire, and the slide *H* has to be moved to the left. The decrease in length is evidently directly proportional to the decrease in conductivity. Therefore the lower scale may be calibrated directly in per cent.

The upper scale could be calibrated in circular mils, instead of weights; but it is easier to determine the weight of a piece of wire 25 inches long, than its average cross-section. A single standard wire covers with some overlap a range of three numbers of the Brown and Sharp scale; if a wider range is required several standards are used. Each standard with its scale is mounted on a separate piece of hard rubber, and may easily be clamped in place.

The details of the bridge may be seen in Fig. 28. The standard conductor is shown in the back and the two contacts on it are made by means of knife edges; one remains set at the zero position and the other slides up and down, so as to take any position on the scale. By a special locking device, the latter is arranged so that it cannot be moved along the standard except when the knife edge is raised. This is arranged to prevent wear on the standard. The sample wire is clamped in the front and also has two knife edges making contact on it, one at the zero position, which remains fixed, and the other at the other end, which is movable. Wires which are not perfectly straight can be stretched straight by means of the device shown at the right-hand end. The long rod also projecting from the right-hand end is used to give large movements to the contact knife edge, and the handle projecting from the front towards the other end works a rack and pinion which gives it small movements.

31. EXPERIMENT 1-M. —Measurement of Low Resistances with Kelvin Double-Bridge.—See §§ 29 and 30.

32. EXPERIMENT 1-N —Determination of Conductivity and of Temperature Coefficient of Wires with Kelvin or Hoopes Double-Bridge. —See §§ 29 and 30.

33. Other Methods of Measuring Resistances. —Three methods of measuring electrical resistances are described in this chapter: The drop-of-potential method, the substitution method and the Wheatstone bridge. Some other methods are used under special conditions; as such may be mentioned the direct-deflection method described in §§ 290 and 292, and the direct-reading ohmmeter described in § 291.

CHAPTER II.

AMMETERS AND VOLTMETERS—CONSTRUCTION AND OPERATION.

34. In dealing with electrical energy two of the most important quantities to be measured are: *current,* and *difference of potential.* Commerical instruments for measuring electric current are commonly called *ammeters,* because the practical unit of electric current is the "ampere." Instruments for measuring differences of potential, or electric pressures, are called *voltmeters,* since the practical unit of difference of potential is the "volt."

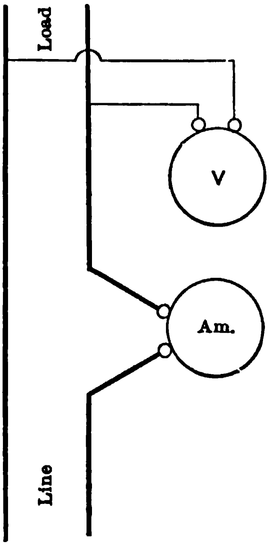

FIG. 29. Standard connections for a voltmeter and an ammeter.

Ammeters are connected in series with the circuit in which the current is to be measured (Fig. 29); voltmeters are connected at two points of the circuit, between which it is desired to determine the difference of potential. There is no fundamental difference in the construction of ammeters and voltmeters, for — with the exception of electrostatic voltmeters — all voltmeters are in reality ammeters calibrated in volts. In other words, they measure current passing through them, which current is made proportional to the voltage at the terminals of the instrument, and the scale is divided directly in volts.

To illustrate, suppose that an ammeter of a high resistance, say 1000 ohms, be connected *across* a line and that it show 0.1 ampere. According to Ohm's law it takes 100 volts to drive 0.1 ampere through a resistance of 1000 ohms; therefore the voltage of the line is 100 volts. To use the instrument as a voltmeter, the scale may be conveniently changed so as to have the division "100 v." correspond to what was formerly "0.1 amp."

DESCRIPTION OF COMMERCIAL TYPES.

35. Ammeters and voltmeters are built on various principles, as enumerated below; each system having its advantages and shortcomings, and a field of application of its own. Some instruments may be used with both direct and alternating currents; others are suitable for one kind of current only. The types in practical use at present are:

(1) Soft-iron core, or electro-magnetic instruments, based on the attraction between a stationary coil and a pivoted piece of soft iron (Figs. 30 and 32).

(2) Moving-coil instruments in which a light coil moves in the strong field of a permanent magnet (Figs. 33 and 34).

(3) Electro-dynamometer type instruments, based on the attraction between a moving and a stationary coil (Figs. 38 and 39).

(4) Hot-wire instruments, in which the current to be measured heats a wire, thus changing its length (Figs. 41 and 42).

(5) Induction-type instruments, based on the principle of revolving magnetic field (Fig. 43).

(6) Electrostatic voltmeters (Figs. 46 and 47) in which two metallic plates are mutually attracted because of opposite electric charges on them.

These types are described more in detail in the following articles.

36. Soft-Iron Instruments. — (*a*) One of the oldest instruments of this type is shown in Fig. 30. *C* is a stationary coil; *P* is a soft-iron plunger pivoted so that it can move freely up and down. The shaft which supports the plunger also carries the counter-weight *Q* and the pointer *N*. When a current flows through the coil, the plunger is drawn in, against the effect of the weight *Q*. The corresponding deflection is shown by the pointer on the scale. When the circuit is opened, the counter-weight brings the moving system back to zero.

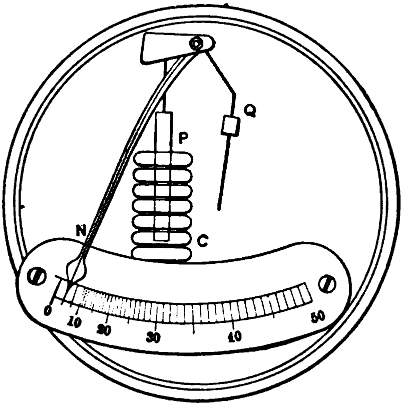

FIG. 30. Plunger-type soft-iron instrument.

In instruments of this kind used as ammeters, the coil consists of a few turns of heavy wire; in voltmeters it has a great many turns of fine wire. Even then the resistance of the coil is not sufficient to cut down the current, and it is necessary to have some resistance *R* (Fig. 31) connected in series with the instrument. This resistance is usually called the multiplier; by varying the resistance of the multiplier, the range of the instrument may be changed within wide limits.

Soft-iron instruments may be used on alternating as well as on direct current, because the attraction between soft iron and a coil does not depend on the direction of the current. In instruments intended for alternating current circuits the plunger is laminated, or made of iron wires; this is done to prevent the formation of eddy currents and subsequent heating (§ 189). The spool on which the coil is wound must be made of an insulating material, or, if made of metal, must be subdivided so that no secondary currents can be induced in it. The calibration of the same instrument is somewhat different on alternating and on direct current, because of the influence of hysteresis, eddy currents and saturation in the plunger; the calibration also depends to some extent on the frequency and on the wave-form of the alternating current.

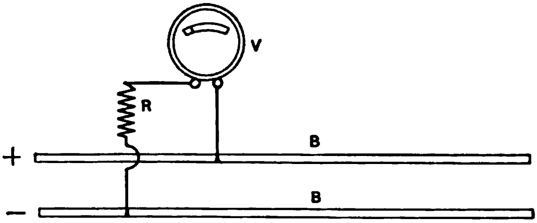

FIG. 31. The use of a voltmeter multiplier.

(*b*) A more perfect instrument of the same type, the so-called *Thomson inclined-coil ammeter*, is shown in Fig. 32. The coil is placed at about 45 degrees to the direction of the shaft; a piece of soft iron (iron vane) is mounted on the shaft in an inclined position. When the coil is energized, it produces a magnetic flux, as indicated by the arrows: the vane tends to move so as to embrace a maximum of lines of force. In doing so it turns the shaft, and the deflection is shown on the scale. The motion is opposed by a spiral spring, the counter-weight serving to balance the moving part.

This instrument is more compact than the above-described plunger-type instrument; the moving part is lighter, and the friction is much reduced, making the instrument more sensitive. Moreover, the spring control is more positive than the gravity control used in the plunger instrument. Thomson inclined-coil instruments are made practically

" dead-beat " by means of an aluminum vane fastened to the moving part. The movement is effectively damped by the resistance of the air to movements of this vane, and the pointer assumes its final deflection without swinging to and fro.

(c) By saying that an ammeter indicates an alternating current it is understood that it merely shows its effective value, and not instantaneous values. Let I be a certain value of direct current flowing through the coil of a soft-iron instrument, and M the corresponding magnetic flux in the plunger. The pull on the plunger is proportional to the product of the current times the flux, or

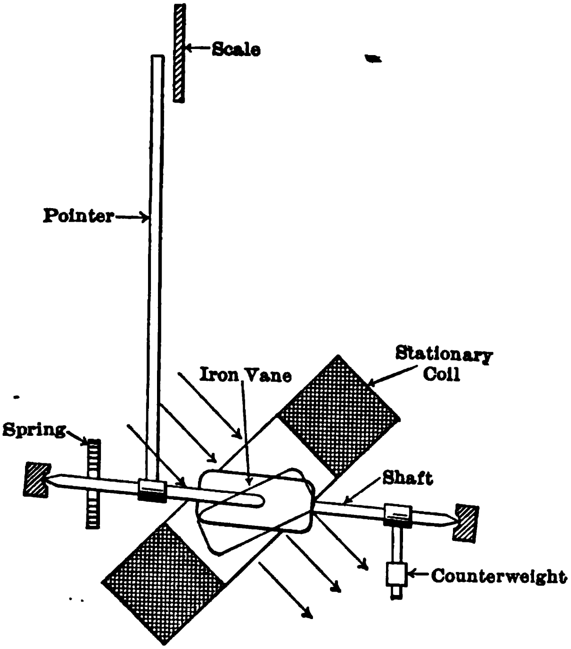

FIG. 32. Mechanism of the Thomson inclined coil ammeter.

$$\text{pull} = I \times M \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

The magnetism in the plunger is produced by the current I; if the saturation in iron is not carried too high, it may be assumed, that $M = kI$, where k is a constant; so that

$$\text{pull} = k \, . \, I^2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

In other words the pull is proportional to the square of the current. If an alternating current now flow through the coil and i and m be some instantaneous values of the current and of the flux, we have as before, that the instantaneous pull = ki^2. It is easy to see that the direction of the pull does not change with the change in direction of the current, because i and m change their sign simultaneously.

Since the inertia of the moving part of the instrument is sufficiently large to prevent it from following the fluctuations in the value of the pull, the plunger assumes the position corresponding to the average pull. Thus we have

$$\text{average pull} = k \,.\, \frac{1}{T}\int_0^T i^2 \,.\, dt = k \,.\, I^2_{\text{eff}} \quad . \quad . \quad . \quad . \quad (3)$$

where dt is an infinitesimal element of time, and T is the duration of one cycle of the alternating current.

The value of I_{eff}, defined by the expression (3) is called the *effective* value of the alternating current, or the square root of the mean square of the instantaneous values. It will be seen by comparing the expressions (3) and (2) that a soft-iron instrument calibrated with direct current should show effective values of alternating currents. In reality this is not quite true, because the magnetism M is not exactly proportional to the current (effect of saturation in iron); moreover the effect of hysteresis and of eddy currents is noticeable. At any rate a calibration made with direct current is true within a very few per cent with alternating currents of usual frequencies.

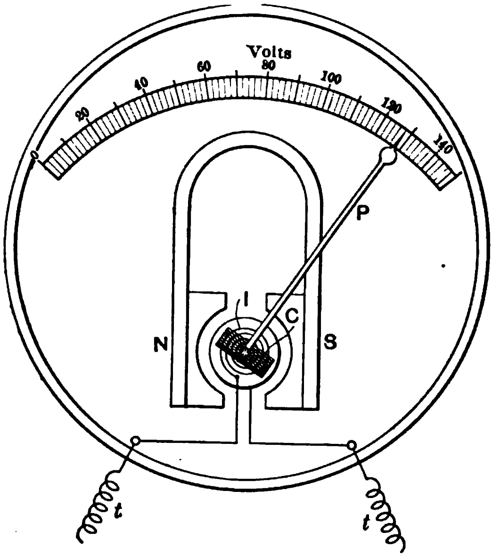

FIG. 33. Arrangement of parts in a moving-coil instrument.

37. Moving-Coil Instruments. — The principle upon which these instruments are constructed will be seen from Figs. 33 and 34. A light coil *C* of fine wire is pivoted or suspended in the strong field of a stationary steel magnet *NS*. When a direct current is passed through the coil, it tends to move so as to embrace the lines of force produced by the magnet, according to the fundamental law of electro-magnetism. The deflection is shown on the scale by the pointer *P*. A soft-iron cylinder *I* is placed inside the coil. It offers an easier path for the lines of force from pole to pole, and also makes the field in the air-gap uniform. This latter point is of particular importance, since it makes it possible to have a perfectly uniform scale, which is one of the good features of these instruments.

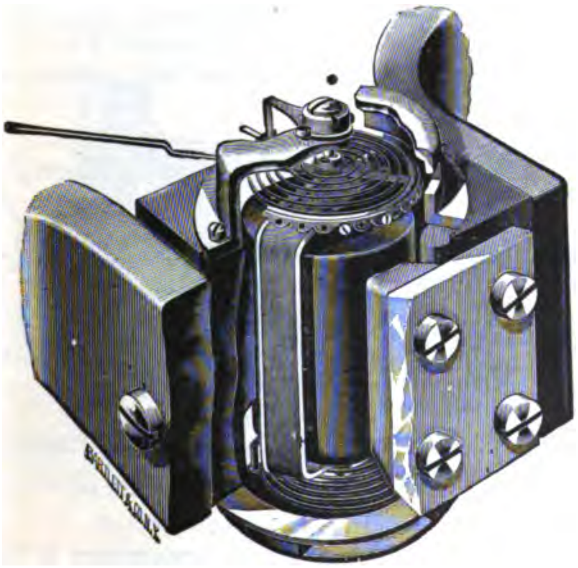

FIG. 34. Construction of Weston moving-coil instruments.

The movement of the coil is opposed by two spiral springs, one on top and the other on the bottom (Fig. 34); the springs are coiled in opposite directions so that one of them is twisted while the other is untwisted during the movement of the coil. This compensates for a possible non-uniformity in the material of the springs. In most instruments, the same springs are used for leading the current into and out of the coil. In some cases separate leads are used independent of the springs; such leads must offer no opposition to the movement of the coil. Without the controlling force of the springs, the needle would be deflected to the end of the scale with any current.

A little consideration will show that moving-coil instruments can be used for direct current only, because the direction of deflection depends upon the direction of the current. When an instrument of this type is connected to an alternating-current circuit, the needle merely trembles at its zero position without giving any deflection. This is because it receives opposite impulses in such rapid succession that it has no time to move in either direction.

Moving-coil instruments are easily made "dead-beat" by winding the moving coil on an aluminum frame. Eddy currents induced in the frame during the movement of the coil effectively check any tendency to swing about the point of equilibrium. The damping of the instruments made by the American Instrument Co. is adjusted so that the pointer passes a little beyond the final position and immediately returns to it. The purpose of this arrangement is to make sure that the movement is not impeded by friction.

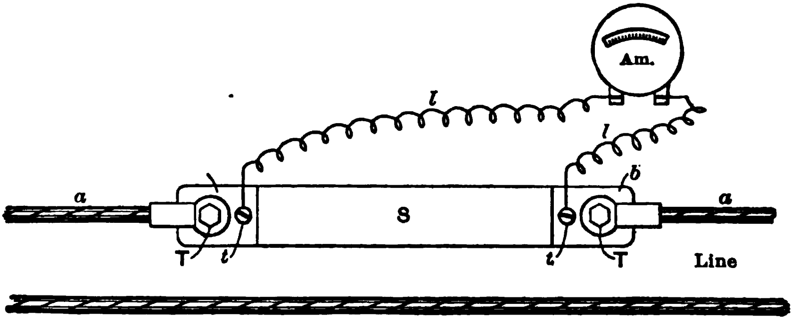

FIG. 35. Measurement of currents with an ammeter shunt and a milli-voltmeter.

The moving coil carries only very small currents — usually not more than 0.05 amp.; at the same time the resistance of the coil is comparatively low. Therefore instruments used as voltmeters are provided with high resistance multipliers connected in series with the coil (Fig. 31). These multipliers are either mounted within the case of the instrument itself, or else are placed outside the instrument in separate boxes.

38. Ammeter Shunts. — Moving-coil instruments used as ammeters are provided with so-called "shunts" which carry the main current to be measured. The shunt is connected into the line *aa* (Fig. 35) in which it is desired to measure current. The instrument (Am.) itself is but a sensitive voltmeter (milli-voltmeter) which measures the drop across the terminals *tt* of the shunt.

Assume, for instance, that the resistance of the shunt is 0.001 ohm, and that the instrument reads 50 milli-volts, or 0.050 v. According to Ohm's law the current in the line

$$i = \frac{0.050}{0.001} = 50 \text{ amperes.}$$

In commercial instruments, the shunt and the milli-voltmeter are usually calibrated together, and the scale reads directly in amperes instead of in milli-volts. In ammeters not over 25 amperes capacity, shunts may be placed inside of the ammeter case, so as to make the instrument self-contained. In larger instruments, the shunt is placed outside (Fig. 36) and connected to the milli-voltmeter by flexible leads.

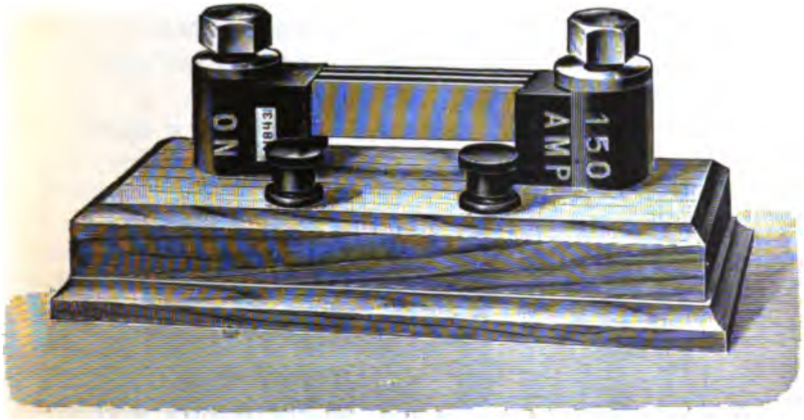

FIG. 36. An ammeter shunt (Weston).

Ammeter shunts are made of strips of manganin, German silver or other material of high specific resistance and low temperature coefficient. The strips *S* (Fig. 35) are sweated into heavy brass blocks *bb*. Two separate pairs of terminals are provided: Large terminals *TT* for the line connection, and small terminals *tt* for connecting the shunt to the milli-voltmeter. The latter terminals are placed so as to measure the drop across the body *S* of the shunt, independent of the distribution of current in the blocks *bb*. This distribution may vary with an uncertain contact at the main terminals *TT*.

One common mistake which the beginner is apt to make, when using moving-coil instruments, is to forget to connect a multiplier or a shunt. This invariably results in either the moving coil (Fig. 34) or the spiral springs being burned out, and the pointer being bent or broken.

39. Friction in Pivots.—The usual way of supporting the moving coil is by means of hardened steel pivots with sharp points; these rest in *V* or cup-shaped depressions formed in agate or sapphire bearings. While the moving system is made as light as possible, the area of contact between the end of the pivot and its journal is so minute, that the pressure per unit area is quite considerable, even with the instrument at rest. As a result some wear is produced and the friction increases with time, especially if the instrument is carried around or is subjected to continuous jar.

While such conical pivots are standard with most makers, there is a tendency on the part of a few to remedy the drawback of sharp points. One company uses with good success highly polished cylindrical pivots in which sharp points are eliminated, and the pressure per unit area considerably reduced.

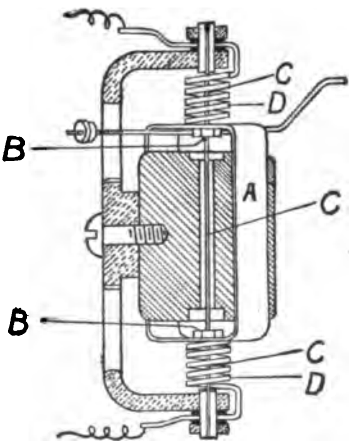

FIG. 37. Construction of Whitney moving-coil instruments; the coil is supported by the springs *DD*, instead of on pivots.

Another solution is shown in Fig. 37. Two ruby jewels *BB*, of the kind used in watches, are attached to the moving coil *A* of the instrument. Through the holes pierced through these two jewels is threaded a length, *CC*, of phosphor bronze or nickel steel wire, which thus guides the coil and holds it truly centered. To provide against endwise motion, spiral springs, *D*, *D*, are attached to the coil, the other ends of same being secured to brackets on a stationary portion of the instrument. These springs not only support the coil but furnish the force opposing its rotation when current flows. If an instrument so built is dropped, the coil, *A*, evidently will but slide up and down on its guide wire for a short distance without causing any damage. When in use and at rest, the coil moves as freely as that of a reflecting galvanometer, the only friction being the molecular one of the supporting springs.

40. Electro-Dynamometer Instruments.—These instruments have a stationary and a movable coil; the two coils being connected in series attract each other when a current flows through them (Figs. 38 and 39). Spiral springs used as the controlling force for the movable coil hold it at a certain angle with the stationary coil when no current is flowing. When a current is flowing through the coils, the moving coil tends to place itself in a plane parallel to that of the stationary coil, producing a deflection which is read on the scale.

The coils of a voltmeter based on this principle are shown in Fig. 38; the general make-up of the instrument is the same as in Fig. 34, except that stationary coils are substituted for permanent magnets. Because of this latter fact, the instrument becomes equally applicable for direct or alternating current, since — the coils being connected in series — the current changes simultaneously in both, and the attraction between them does not reverse. In fact, electro-dynamometer type instruments are calibrated with direct current and used on alternating currents.

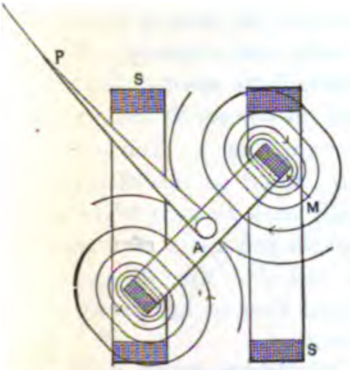

FIG. 38. Arrangement of coils in a dynamometer-type voltmeter.

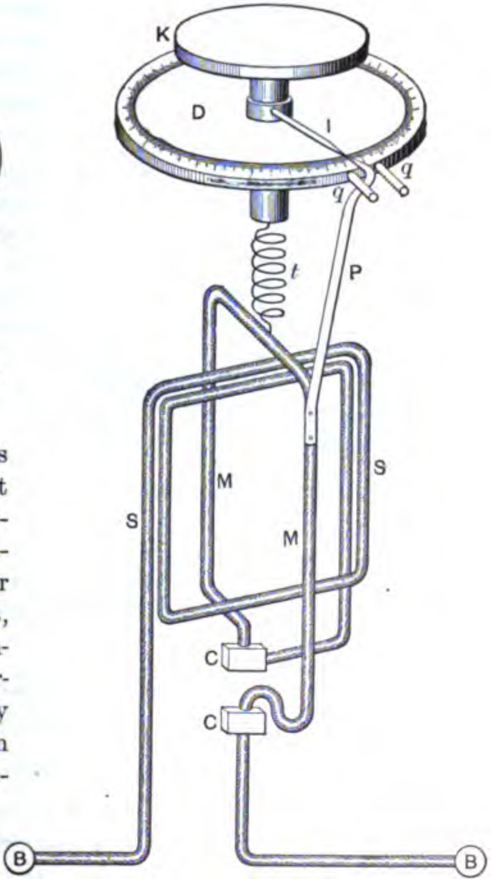

FIG. 39. Arrangement of parts in an electro-dynamometer (ammeter).

Instruments of this type are better adapted for voltmeters than for ammeters. In the latter, it is difficult to devise a satisfactory arrangement for leading heavy currents into the movable coil without interfering with its free motion. The usual construction where large currents are to be measured is shown in Fig. 39. *BB* are the terminals of the instrument; the current flows from the left terminal through the stationary coil *SS* to the upper mercury cup *C*; thence through the movable coil *MM* and the lower cup *C* to the right terminal *B*. The coil *MM* is suspended from the top of the instrument by a cocoon thread (not shown in Fig. 39), and is controlled by the spiral spring *t* operated by the torsion knob *K*; the zero of the scale is between the stops *qq*. When a current flows through the instrument the pointer *P* strikes against the right stop *q*. The knob *K* is then turned to the left, until *P* comes back to zero. The index *I* shows the angle of torsion on the dial *D*. Fig. 40 gives the general view of an electro-dynamometer.

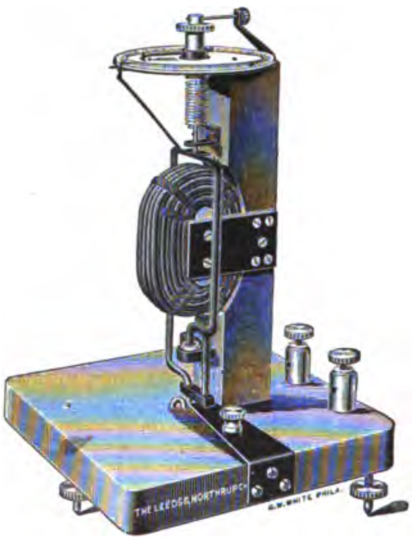

FIG. 40. An electro-dynamometer.

The instruments shown in Figs. 39 and 40 are not direct reading, the pointer indicating an angle only. If α is the angle of torsion in degrees, the current

$$i = k\sqrt{\alpha},$$

where k is a calibration constant of the instrument. This formula is deduced as follows: The torque between the two coils is proportional to the product of currents in them, or

$$\text{torque} = k_1 \,.\, i_{\text{sta}} \,.\, i_{\text{mov}}.$$

The coils being in series $i_{\text{sta}} = i_{\text{mov}} = i$; therefore

$$\text{torque} = k_1 i^2 \qquad \dots\dots\dots \qquad (4)$$

This torque is balanced by the torsion of the spring; the spring is wound so that the twisting force is proportional to the angle of torsion. Thus

$$\text{torque} = k_2\alpha,$$

hence

$$k_1 i^2 = k_2\alpha,$$

or

$$i = \sqrt{\frac{k_2}{k_1} \cdot \alpha} = k\sqrt{\alpha}.$$

With alternating currents the expression (4) becomes:

$$\text{torque} = k_1 \cdot \frac{1}{T}\int_0^T i^2 dt = k_1 \,.\, I^2_{\text{eff}}.$$

Hence the instrument shows true *effective* values of alternating currents (see § 36 *c*).

A good feature of electro-dynamometers used as ammeters is that they are accurate and sensitive; moreover, they may be calibrated with direct current and used with alternating current. Their serious drawback is that they are not direct reading and cannot be used on commercial switchboards. The presence of mercury, and the necessity for leveling make electro-dynamometers inconvenient to handle, even in ordinary testing work. Moreover, they have no provision for damping, and require some skill in taking readings, especially with fluctuating currents.

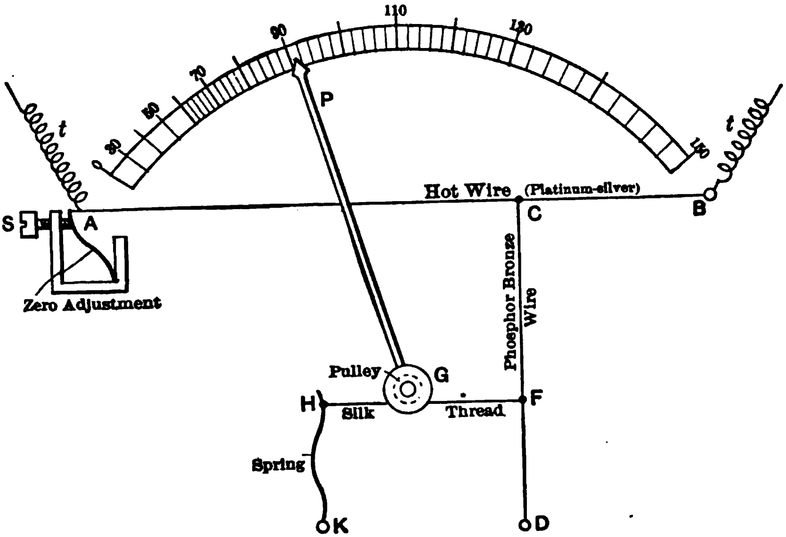

FIG. 41. Mechanism of Hartmann & Braun hot-wire instruments.

In spite of these drawbacks, they are largely used in testing, for lack of better A. C. ammeters. Attempts have been made to make the electro-dynamometer direct reading in order to do away with the awkward expression $\sqrt{\alpha}$. To make the scale more regular the moving coil is placed eccentrically, and both coils are bent in such a way as to increase deflections with small currents.

41. Hot-Wire Instruments. — A typical hot-wire instrument is shown in Fig. 41. The current to be measured, or a definite part of it, passes through the stretched platinum-silver wire AB and heats it so that it takes an appreciable sag. The resulting downward motion of the point C is transmitted to the pointer P. The deflection is magnified by the wire *CD* the point *F* of which is under the tension of the spring *KH*. When *C* moves downward, *F* and *H* move to the left, and the silk thread turns the pulley *G*; the pointer *P*, mounted on the same shaft with *G*, shows the deflection on the scale. Instruments of this construction can be used equally well with direct or alternating currents, as an ammeter, or as a voltmeter. Shunts are used with hot-wire ammeters (see Fig. 35); the current through the hot wire itself being usually about 5 amperes with a full scale deflection. The scale of hot-wire instruments is not uniform, because the heat energy put into the wire increases as the square of the current.

The instrument shown in Fig. 41 is made "dead-beat" as follows: An aluminum disk is mounted on the same shaft with the pulley *G* and placed between the poles of a strong horse-shoe magnet. Eddy currents are induced in the disk during its motion and effectually damp vibrations of the needle.

The hot-wire instrument shown in Fig. 42 has the following construction: a wire, *a-b*, of high resistance, low temperature coefficient and non-oxidizable metal, is secured at one end to a plate *c* and passed around a pulley *d* which is fastened to a shaft *e*. The other end of the wire is brought back again and mechanically — though not electrically — attached to the same plate *c*. Plate *c* is kept under stress by the spring *f*, which constantly tends to pull it in a direction at right angles with the axis of the shaft *e*; the plate is so guided that it can be moved in that one direction only. To the shaft *e* is likewise secured an arm *g*, bifurcated at one end and counterweighted at the other. Between the extremities of the bifurcated ends of the arm *g* is another shaft *h*, on which there is a small pulley and to which is attached the needle *i* that gives the desired indications. A fine silk fiber is attached at one end to one of the arms of *g*, then passes around the pulley on the staff *h* and has its other extremity secured to the other arm. The arms are springy and serve to keep the silk fiber taut. The current to be measured flows through the wire *a* only, entering and leaving as indicated by the arrows.

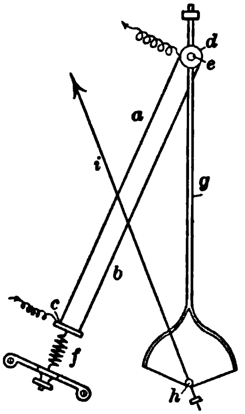

FIG. 42. Mechanism of Whitney hot-wire instruments.

When *a* is heated by the passage of a current, it expands; this makes *a*'s tension relatively less than that of *b*; the equilibrium can be restored only when the pulley *d* rotates sufficiently to again equalize the strain.

In its rotation, *d* carries *g* with it, and *g* in moving causes the silk fiber to rotate the shaft which carries the needle. If the temperature of the air surrounding the instrument changes, *a* and *b* are affected alike; their resulting equal expansion simply causes a movement of the plate *c* back or forth in its path without any tendency to rotate the pulley.

If *r* is the resistance of the "hot-wire," the *average* heat generated in it with alternating currents is

$$\frac{1}{T}\int_0^T (i^2 r)\, dt = r \,.\, \frac{1}{T}\int_0^T i^2 \,.\, dt = r\, I^2_{\text{eff}}.$$

Hence, a hot-wire instrument calibrated with direct current shows true effective values of alternating current (see § 36 *c*).

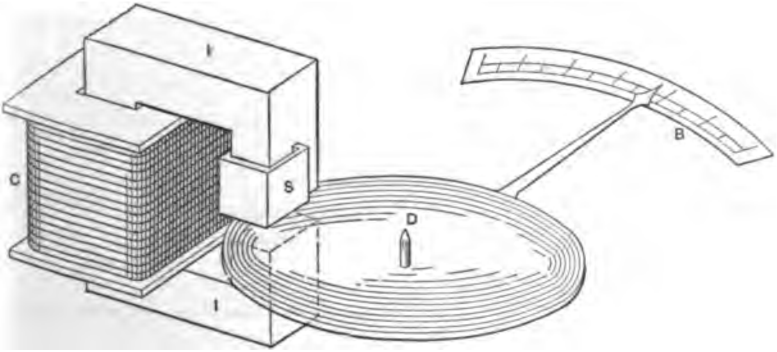

FIG. 43. Principle of action of Westinghouse induction-type instruments.

42. Induction-Type Instruments. — These instruments are based on the principle of a revolving magnetic field produced by two out-of-phase alternating currents (see § 520). In the construction shown in Fig. 43 the alternating current to be measured flows through the coil *C* of the laminated iron core *I* and produces in it a pulsating magnetic field. A pivoted aluminum disk *D* is subjected to the inductive action of this field in the air gap of the core; which disk can be made to move under the influence of eddy currents induced in it, if these currents are *unsymmetrical* in respect to the iron core. This is done by placing an electric screen, or a secondary coil *S*, on *one side* of the iron core. The currents induced in this coil oppose the primary currents in *C* and thus weaken the magnetic flux on one side of the core — the phase of the flux is also changed hereby. This combination of the two fluxes produces unsymmetrical currents in the aluminum disk and causes it to move, thus producing a deflection on the scale *B* against the action of spiral springs, used as the controlling force. In short, this instrument is a single-phase induction motor, the screening coil taking the place of a "split-phase" arrangement. In some instruments an aluminum drum instead of the disk is used as the moving part; the instrument has then the aspect shown in Fig. 83; however, the theory remains the same.

The instruments based on this principle are robust, convenient for use, and possess a fair degree of accuracy. Of course, they can be used on alternating currents only, and their indications depend to a considerable degree on the frequency of the supply. It is possible to have in these instruments a fairly uniform scale extending over 300 degrees, which adds considerably to their accuracy.

Ammeters based on this principle are usually wound for 5 amperes,

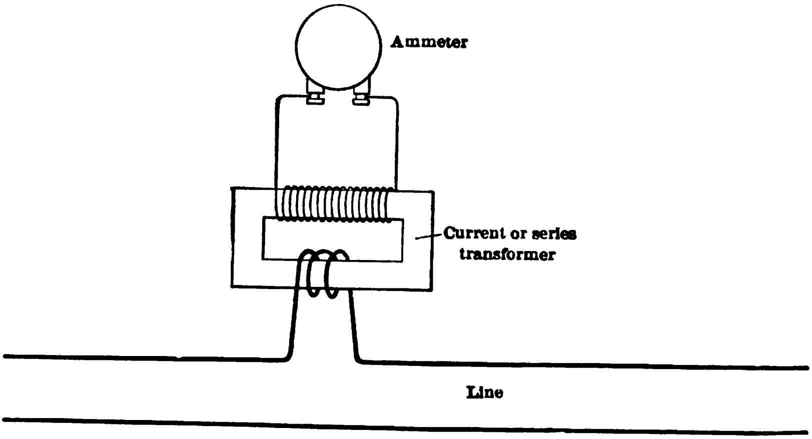

FIG. 44. Measurement of alternating currents through a series- or current-transformer.

and adapted for use in connection with series or current transformers (Fig. 44) by means of which currents of any value can be measured with the same instrument. Voltmeters are provided with multipliers, and for higher voltages also with shunt transformers (Fig. 45).

One detail in induction-type voltmeters deserves special mention. When the temperature of the aluminum disk increases, its resistance is increased, thereby decreasing the current flowing in the disk. This reduces the torque, and consequently the deflection of the pointer. Two methods are used for compensating the influence of temperature, by increasing the strength of the magnetic field:

(1) In meters intended for high frequencies, and having a large torque, a short-circuited coil is placed around one pole of the electromagnet *II* (Fig. 43). This coil acts as the secondary of a transformer,

and reduced the flux in the instrument. When the temperature increases, the resistance of this coil also increases; this reduces its choking effect on the magnetic field, and the flux increases.

(2) In meters for low frequencies a non-inductive resistance is shunted around the magnetizing coil C (Fig. 43). This shunt is selected of such a value that its resistance increases with temperature at the same rate as the resistance of the aluminum disk. Therefore, when the temperature rises, more and more current flows through the magnetizing coil of the voltmeter, increasing the field strength in the desired proportion.

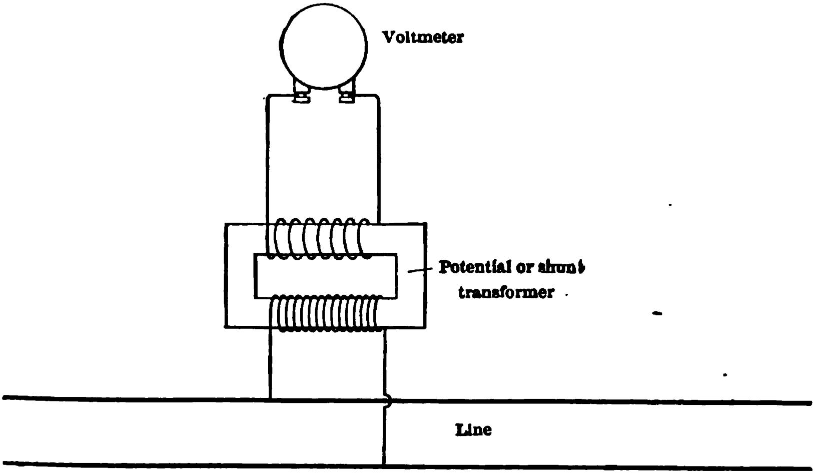

FIG. 45. Measurement of alternating voltages through a potential- or shunt-transformer.

43. Electrostatic Voltmeters. — These instruments are based on electrostatic attraction between two plates carrying opposite electric charges. The stationary and the moving plates are clearly seen in the voltmeter shown in Fig. 46. They are insulated from each other and are connected to opposite sides of the line, so that they are charged with equal and opposite quantities of electricity. The movable plate is attracted by the stationary, and the deflection is shown on the scale. In this particular instrument gravity is used as the controlling force. By changing the weights shown on bottom of the movable plate, the sensitiveness of the instrument is varied within wide limits. Electrostatic voltmeters give identical deflections with direct and with alternating voltages; in the latter case the sign of the electric charges is changed simultaneously on both plates, so that the force between them is always an attraction.

With low voltages the necessary area to be given the plates becomes rather large, in order to have an appreciable attraction. For this reason, electrostatic voltmeters are mostly used for high-tension work. A low-voltage static voltmeter designed by Lord Kelvin is shown in Fig. 47. The required surface is obtained by using several small plates, because of which the instrument is known as the *multi-cellular* voltmeter.

An ingenious electrostatic voltmeter, designed by Mr. S. M. Kintner is shown in Fig. 48. The working parts of the instrument are immersed in insulating oil, and the voltmeter may be built for voltages up to 200,000 volts. The stationary part consists of two curved plates *BB* connected to the lines, either directly or through the condensers *CC*. The moving element consists of two suspended hollow cylinders *MM*. The instrument may be used with both condensers in series, or with either or both short-circuited, thus giving a wide range of voltages. When an electric pressure is applied between the plates *BB*, the moving element becomes charged by induction, and its cylinders tend to approach the stationary plates. This they can do by moving counter-clock-wise, the shape of the stationary plates being such that this

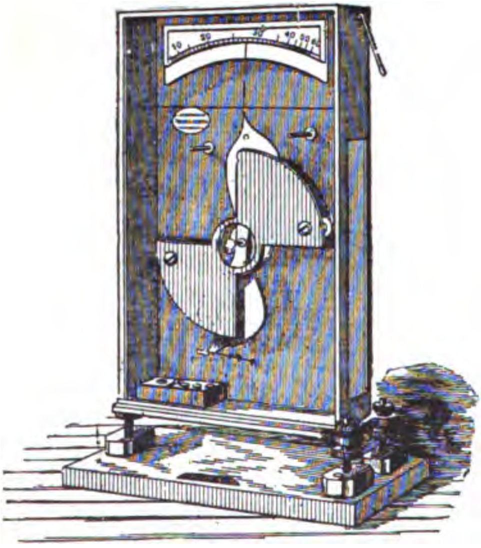
FIG. 46. Lord Kelvin static voltmeter for high voltages.

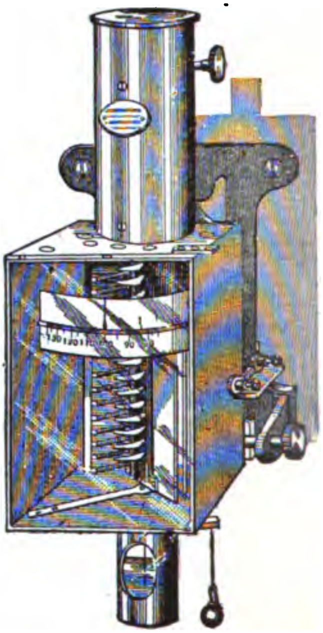
FIG. 47. Lord Kelvin multi-cellular (static) voltmeter for medium and low voltages.

movement reduces the distance between them and the moving element. The deflection is read on the scale, spiral springs being used as the controlling force.

The insulation is the most important feature in an instrument of this type intended for high voltages; the use of oil has many advantages which may be summarized as follows:

(1) The distance between the operating elements is lessened, and the actuating forces are greatly increased, due not only to the smaller distance between the parts, but also to the increased specific inductive capacity of oil over air.

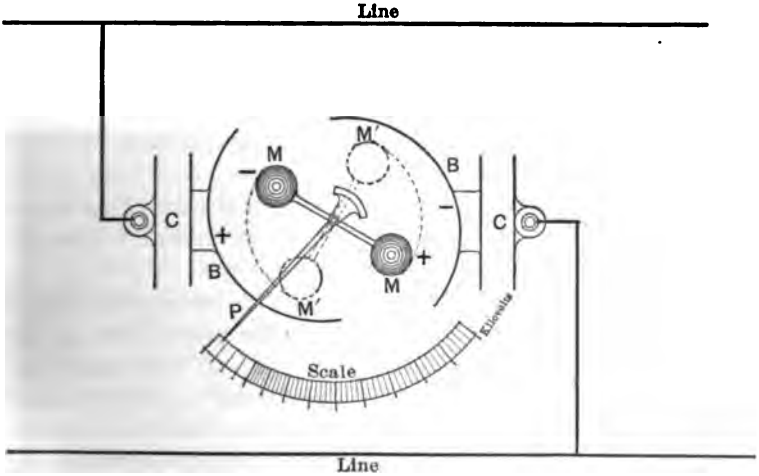

FIG. 48. Mechanism of the Westinghouse static voltmeter for extra high voltages.

(2) Oil acts as a dampener and makes the instrument nearly "deadbeat."

(3) The oil buoys up the moving element, practically removing all weight from the bearings. This does away with friction and makes the instrument much more accurate.

44. Summary of the Above Described Types of Instruments. — Thus, summing up the above types of voltmeters and ammeters, used in practical engineering work, we have: soft iron instruments are the cheapest and are used for both D. C. and A. C. work where no particular accuracy is required; for accurate D. C. work moving-coil instruments are employed exclusively; electro-dynamometer type and hot-wire instruments are used for accurate A. C. measurements. Recently, portable induction-type instruments are being introduced for this purpose, though so far they have been mostly used for A. C. switchboard work. Electrostatic voltmeters are used to some extent for high-tension measurements, and in special research work.

45. Requirements for Good Ammeters and Voltmeters. — The principal requirements for a good measuring instrument are:

(1) Permanency of calibration;

(2) Insensibility to stray magnetic fields;

(3) "Dead-beat" quality, i.e., the instrument should not swing too long before giving a definite indication;

(4) Suitable character of the scale.

All these requirements are met to a larger or smaller degree in modern ammeters and voltmeters. However, the more strict the requirements, the more expensive becomes the instrument; it is therefore advisable not to demand greater accuracy than a specific case may require. In discussion of the above requirements, it may be said that

(1) *The calibration* may be affected by a change of form or relative position of parts inside of the instrument; by an increased friction in pivots; by ageing of springs or of iron; by permanent magnets losing part of their magnetism.

(2) Some of the above-described types of instruments are inherently less affected by *external stray fields* than others, and all of them can be sufficiently protected by being inclosed in a cast-iron case. Hot-wire instruments are practically unaffected by stray fields, since their action is not based on a magnetic effect; induction instruments are affected very little, since their air gap is small and their own field quite strong. Electro-magnetic instruments are affected the most, and should never be placed near dynamos, or on a switchboard within a loop of bus bars and cables carrying strong currents. Electro-dynamometer type instruments are affected by terrestrial magnetism when used on direct current; therefore in accurate measurements it is essential to reverse the current in the instrument and to take the average of two readings.

(3) *The "dead-beat" quality* is always desirable in an instrument, though usually it can be attained only by the addition of an extra damping device. An aluminum vane is quite frequently used for this purpose, the damping being attained either by the resistance of the air to the movement of the vane, or by placing the vane between the poles of a permanent magnet, so as to induce in it eddy currents. Moving-coil instruments are easily made "dead-beat" by making the frame of the moving coil of aluminum. Eddy currents induced in it by the permanent magnet are sufficient to make the instrument dead-beat (the word "aperiodic" is sometimes used instead of the expression "dead-beat").

(4) Different types of instruments have a somewhat *different character of scale.* The scale of moving-coil instruments is usually perfectly regular, all divisions from zero to full scale being equal (Fig. 33). On the other hand, hot-wire instruments have rather an irregular scale, divisions being crowded on the lower part of the scale (Fig. 41), so that the instrument can be read with fair accuracy at not less than 40 per cent of its full range. Sometimes the scale is made suppressed on purpose in its lower part so as to have larger divisions in the useful range of the instrument. In instruments based on the hot-wire or dynamometer principle, the scale is, of necessity, irregular, since the deflecting force is proportional to the square of the current. *A regular scale is by no means always desirable in switchboard service,* though it is very convenient in testing and for experimental purposes, since it increases the useful range of the instrument.

46. EXPERIMENT 2-A. — Study of Ammeters. — The purpose of the experiment is to learn the construction, good qualities and limitations of the principal types of instruments described in §§ 36 to 43. This should help the student to handle instruments properly in his future work, and to select intelligently the right instruments for a given purpose. The instrument to be investigated must be connected into a circuit; if necessary another instrument, used as a standard, must be also connected into the same circuit, in order to observe the comparative behavior of the two. There are many points in regard to which two instruments of the same range and of different type may be compared to each other; the following are among the most important:

(1) Are there any defects in the construction, such that the instrument cannot possibly have a permanent calibration?

(2) Is the instrument "dead-beat," and if not, how can it be made dead-beat in the most convenient way? The degree of being "dead-beat" can be measured by suddenly closing the circuit on a current that gives a certain per cent of full scale deflection, say 60 or 70 per cent, and by observing the time which it takes for the pointer to become steady at this point. This method of defining the "dead-beat" degree is entirely arbitrary, but if the same procedure is used on *all* the instruments under test, the results are comparable among themselves.

(3) If the instrument is absolutely dead-beat (aperiodic) note how long it takes the pointer to assume its final deflection. Also see to what degree the moving part is capable of following rapid fluctuations of current. This point is particularly important in hot-wire instruments; they are somewhat sluggish on fluctuating loads, because it takes a certain length of time for a wire to change its temperature. Charac-

terize this sluggishness numerically, by varying the current up and down at a certain speed and by a certain percentage; observe the behavior of the instrument under test, as compared to that of the standard instrument.

(4) See if the pointer always returns to zero, or at least to the same point, when the circuit is opened; if not, investigate the cause. In hot-wire instruments a special screw, accessible from the outside, is provided for setting the pointer at zero; it is denoted by S in Fig. 41.

(5) Can the instrument be used indifferently in a vertical and horizontal position? Is the moving part sufficiently well balanced so that small differences in the position of the instrument do not affect the calibration?

(6) What kind of force is used to deflect the pointer from zero position, and what is the resisting force?

(7) How explain the character of the scale (regular, crowded on one side, etc.) by the character of the moving and resisting forces? What changes should be made in the instrument to get a scale more regular, or still more suppressed on one side?

(8) Is the instrument conspicuously bulky and heavy, as a result of a particular construction adopted? What are the principal materials used, and what, in your opinion, are the most important and expensive operations in manufacturing the instrument?

(9) Is the calibration affected by the temperature, and if so, how much? Is there in the instrument any compensation for the influence of temperature? Some alternating-current voltmeters have a small thermometer on the face of the instrument, and an adjustable rheostat in series with the multiplier. In some hot-wire instruments the plate on which the wire is stretched is made of a material having the same expansion coefficient as the wire itself; in this way no stresses are produced in the wire by changes in temperature of the surrounding air.

(10) If the instrument has a shunt, determine if the same has a negligible temperature coefficient, in other words, if the calibration of the instrument changes because of the heating of the shunt.

(11) Determine in how far the instrument is affected by stray magnetic fields and a proximity of large iron masses. Does the iron case of the instrument shield it entirely from these influences, and does it not introduce an error by itself?

(12) Is the instrument influenced by terrestrial magnetism? Is there any means for eliminating this influence, for instance, by reversing the current and taking an average of the two readings? Is there

any position of the instrument with respect to the meridian, such that the above influence becomes a minimum?

(13) Does the instrument show an appreciable hysteresis effect, so that indications on increasing and decreasing currents are different?

(14) Is the calibration the same on D. C. as on A. C., and if not, what is per cent difference and the cause of the difference?

(15) Is the calibration affected by the wave-form of alternating current?

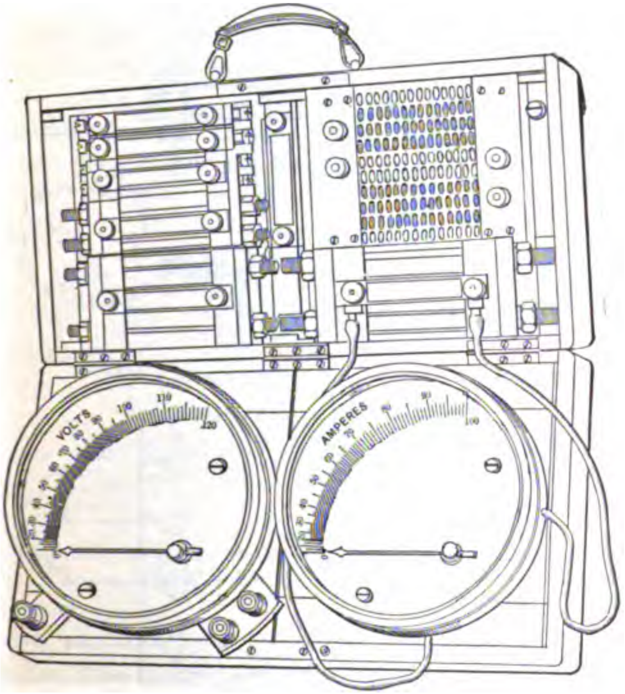

FIG. 49. A portable volt-ammeter with several ranges.

(16) What is the energy consumption in the instrument itself, and how does it compare with that of other instruments having the same range?

Report. Describe the general arrangement of the test, the construction of the instruments used, and give the calibration curves. Also answer for each instrument studied the above 16 questions. Do not repeat the questions; simply refer to them by number.

47. EXPERIMENT 2-B. —Study of Voltmeters. —The experiment is performed in the same way as experiment 2-A.

48. Combination Volt-Ammeters. —It is convenient in some cases to have a portable combination volt-ammeter (Fig. 49) for all-round testing purposes, or as a secondary standard. The ammeter in the set shown in the sketch is provided with six different shunts covering the range from 25 to 1000 amperes; the voltmeter is provided with two multipliers by means of which its original range can be increased two or four times. Of course, it is not necessary to have the same range in all cases; the shunts and the multipliers must be selected so that the set would cover as much as possible the total range of the work for which it is intended to be used.

49. Polyphase Boards.—In many practical cases, especially in polyphase work, it is necessary to measure currents and voltages in more than one part of the circuit. At the same time it is not always possible or convenient to have a sufficient number of ammeters and voltmeters. In such cases so-called polyphase boards may be used, by means of which one ammeter and one voltmeter may be connected in succession to several independent circuits or parts of a circuit. The polyphase boards described below may be used with any number of circuits up to four.

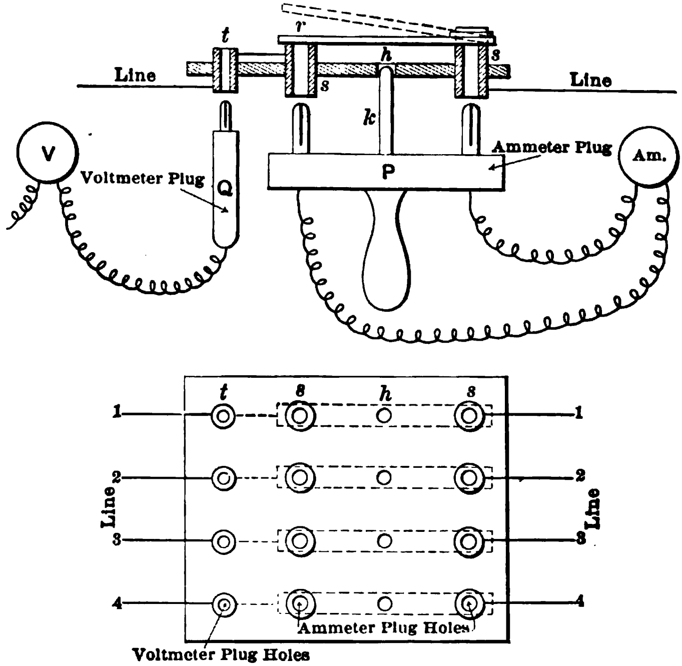

Fig. 50. A plugging arrangement for using one ammeter, one voltmeter, and one wattmeter in several circuits, without interrupting the main current.

(*a*) *Plug-type board.* The polyphase board shown in Fig. 50 is very convenient for laboratory purposes. The line wires, in which it is desired to measure currents, are connected to the terminals t and s of the board. When the plug P is out, the circuit of each wire is closed from t to the left socket s and through the spring r to the right socket s. If it is desired to measure current, say in line 1, the plug P is inserted in the corresponding receptacles ss as far as it will go. The pin p enters the hole h and opens the spring contact r, as shown by the dotted lines. But before the circuit is opened at r, it is shunted between the receptacles ss through the ammeter Am. When the plug is taken out, the spring closes the circuit at r before the ammeter circuit is opened. In this way the line circuit is never completely opened, and there is no sparking at r or fluctuation of the load.

The receptacles t are intended for two voltmeter plugs Q. By inserting these plugs in any two receptacles, the voltage may be measured between any two lines or phases. The same polyphase board accommodates, if necessary, a wattmeter. The current winding of the wattmeter is connected in series with the ammeter, the potential winding in parallel with the voltmeter. Inserting the plugs P and Q automatically connects the ammeter, the voltmeter and the wattmeter in the desired phases.

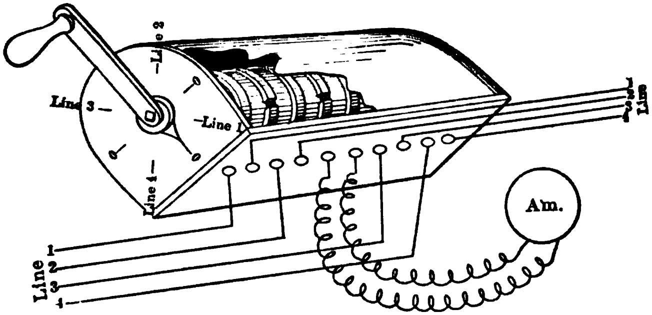

FIG. 51. A controller for connecting an ammeter and a wattmeter into several lines in succession, without opening the main circuit.

(*b*) *Controller-type board.* Another convenient device for the same purpose is shown in Fig. 51. It is intended for transferring the ammeter only from one circuit to another. A separate switch must be used for transferring the voltmeter, as with the polyphase boards shown in Figs. 52 and 53. The device is an ordinary drum-type controller; the lines and the ammeter leads are connected to stationary fingers. The revolving drum is provided with copper contacts which establish the required connections between any of the lines and the ammeter.

When the pointer on the handle shows "line 1," the ammeter measures the current in this line; when the handle is in one of the "O" positions, all the lines are connected direct, and the ammeter is out

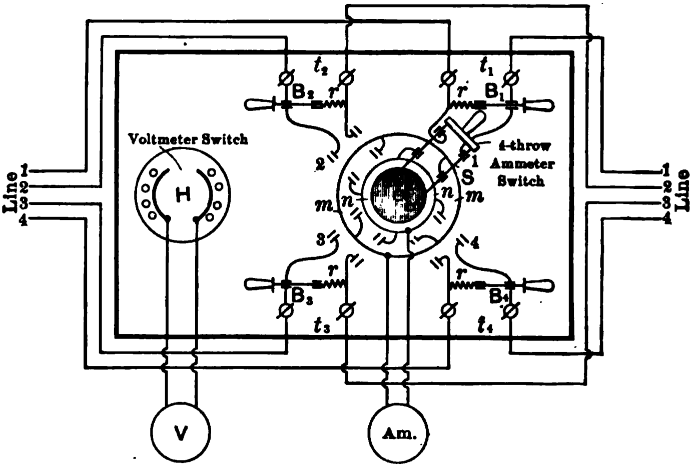

FIG. 52. A polyphase board with one multi-throw switch for currents.

of the circuit. This type of polyphase board is very convenient to handle, especially when readings must be taken in rapid succession.

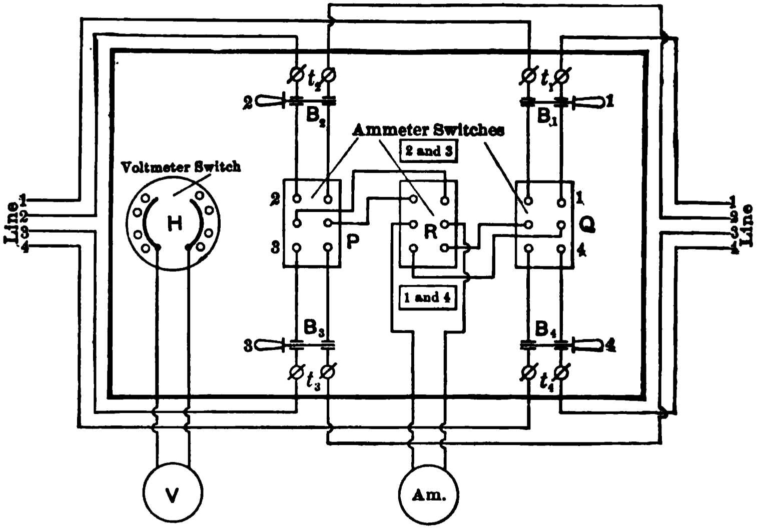

FIG. 53. A polyphase board with three double-pole double-throw switches for currents.

(c) *Boards with knife-switches.* The boards shown in Figs. 52 and 53 are not so convenient in operation as the two devices described above, but they can be built with less difficulty, by using ordinary knife-switches. The ammeter and the voltmeter switches are entirely separate. The lines are connected to four pairs of terminals t_1, t_2, t_3 and t_4. When the switches B_1, B_2, B_3 and B_4 are closed, the currents flow directly through the lines, outside the ammeter. To read amperes in phase 1, throw the central radial switch S (Fig. 52) in position 1, and open the switch B_1. The current from the left terminal t_1 flows through the left blade of S, connection n, ammeter Am, connection m and the right blade S to the right terminal t, and to the line. Resistances r in series with the switches B are intended to compensate for the resistance of the ammeter and the leads, so that the total resistance of the line remains the same, whether the switch B is open or closed.

The polyphase board shown in Fig. 53 differs from that of Fig. 52 in that the connections are accomplished in it by three ordinary double-pole double-throw switches P, Q and R, instead of by a special radial switch S.

50. Recording Instruments. — It is of importance in power stations to have a continuous record of the performance of electrical machinery, partly as data for economic and engineering calculations, partly as a check on station attendants. Recording ammeters, voltmeters, wattmeters, etc., are used for this purpose. The record is traced automatically on a strip of coördinate paper by a pen fastened to the end of the pointer of the instrument. The paper is moved at a constant speed by a clock mechanism. Any of the indicating instruments described in §§ 36 to 43 may be adapted to be used as a recording instrument. Fig. 54 shows a Bristol recording ammeter based on the electro-magnetic principle. A is the stationary coil through which the current to be recorded flows. The magnetic field produced by the coil attracts the soft iron disk B which rests on knife-edge supports C and D. The supports have a lateral spring action, which opposes the movement of the disk. The motion of the disk B is transmitted

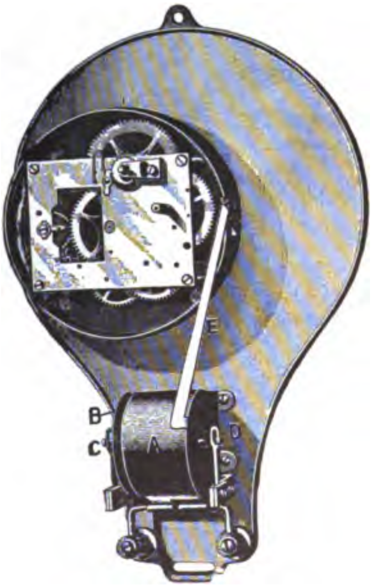

FIG. 54. The Bristol recording ammeter.

to the recording pen-arm *E*, which traces a record on a sheet of paper, as shown in Fig. 55. The paper is driven by the clock mechanism shown in the upper part of the instrument. One revolution of the paper may be had in 24, 6 or even 1 hour, as desired.

Fig. 56 represents a recording ammeter based on the moving-coil principle; the ammeter movement proper is in the upper part, the recording drum and the clock are underneath. A shunt connected into the main line is shown below the instrument. There are similar instruments on the market, based on the hot-wire and on the electrodynamometer principles.

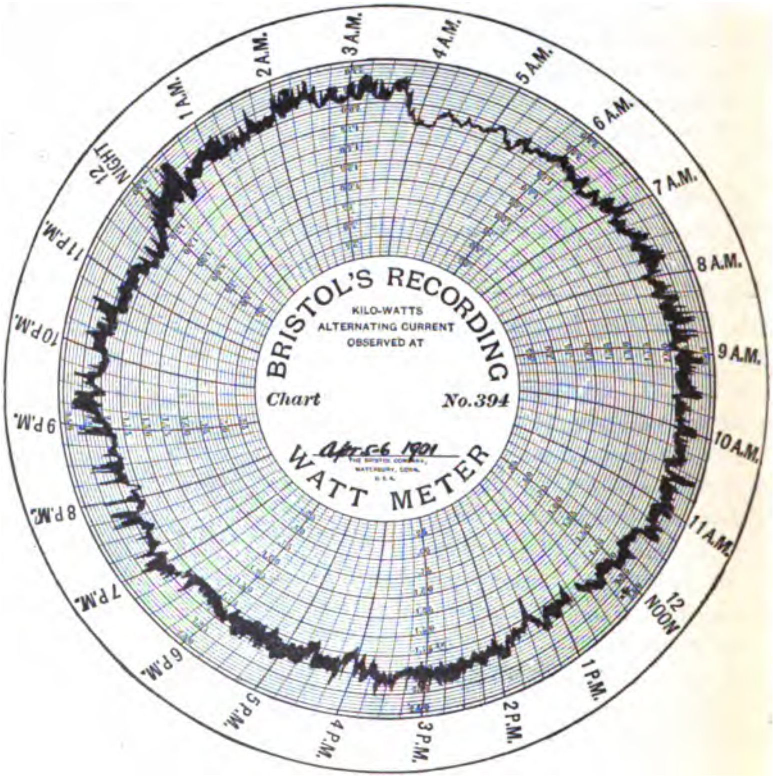

Fig. 55. A 24-hour record taken on a fluctuating load.

The imperfections common to the above recording meters are:

(1) The friction between the pen and the paper impairs the accuracy of the record; or else it necessitates making instruments with a high torque, consequently with a considerable power consumption.

(2) The clock must be periodically wound up.

(3) The records are made either on a circular sheet (Fig. 55) which has to be replaced each day, or on a roll of paper with curved coördinates (Fig. 56); in either case the records are in a form inconvenient for calculations or for the use of a planimeter.

These objections are eliminated in recording meters operating on the "relay" principle and described in the next paragraph. The improvement is, however, obtained at a considerably increased cost and complication of the instrument.

51. Recording Meters Operating on the Relay Principle. — The three above-enumerated objections are eliminated in the following way in meters operated on the relay principle:

(1) The movement proper of the meter operates merely a set of contacts (Fig. 57) which close an auxiliary circuit. This circuit energizes the solenoids which operate the pen; a comparatively large amount of energy is not objectionable in this case, nor does friction in any way impair the accuracy and sensitiveness of the instrument.

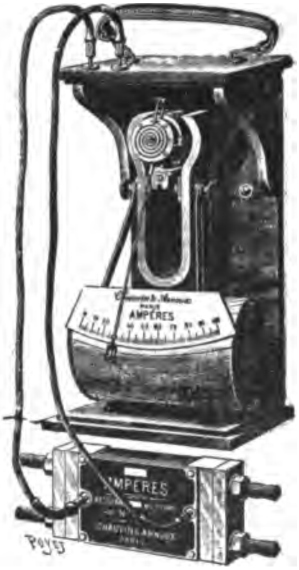

FIG. 56. A recording ammeter with a moving-coil mechanism.

(2) The clock mechanism which moves the paper is made electrically self-winding, say once an hour, and does not require any attention.

(3) The recording pen is made to move across the paper in a straight line, and the record is obtained on a continuous sheet of paper ruled with rectangular coördinates.

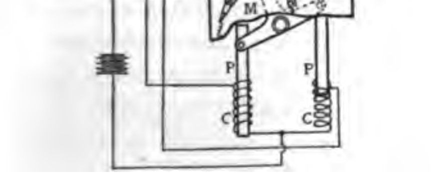

FIG. 57. A diagram of the Westinghouse recording voltmeter, showing the relay principle.

A complete set of recording instruments embodying these features are built by the Westinghouse Electric and Manufacturing Company. The ammeters, voltmeters and wattmeters operate on the dynamometer principle used in the Kelvin balance (Fig. 74); frequency meters, power factor meters, etc., are built with the same movement as in the corresponding indicating instruments (Figs. 375 and 85).

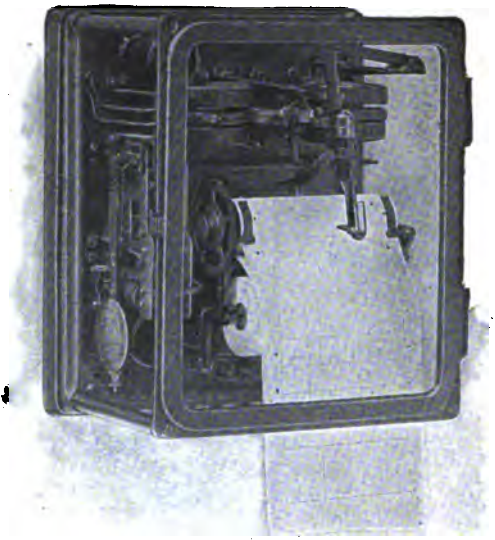

FIG. 58. The Westinghouse graphic recording voltmeter.

A recording voltmeter of this type is illustrated in Fig. 58; the details of connections are shown in Fig. 57. The instrument has four stationary coils and two movable coils between them, arranged exactly as in the Kelvin balance. The movable part is provided on the left side with a contact arm which closes one or the other of the two contacts between which it swings. The auxiliary D. C. source is represented schematically by a battery; the two solenoids *CC* which operate the pen are connected in parallel, and each is connected to one of the above mentioned contacts. The condensers *KK* and the resistance *R* are for the purpose of eliminating sparking at the contacts. The leverage *MM* of the pen is connected by the spiral spring *S* to the movable part of the voltmeter.

The six coils constituting the voltmeter itself are all connected in series across the circuit in which is desired to record the voltage; *VR* is the multiplier. Suppose that the position of the pen at a certain moment corresponds exactly to the voltage of the line, and both contacts are opened. If the line voltage drops, the spring *S* overcomes the attraction between the movable and the stationary coils and closes the upper contact. A current flows from the battery into the left solenoid *C*, a pull is exerted on the plunger *P*, and the pen begins to move to the left tracing a record. The movement of the pen reduces the tension of the spring *S*; when the pen arrives at the right point, the electro-magnetic pull between the stationary and the moving coils of the voltmeter becomes just sufficient to overcome the tension of the spring, and the contact is broken. The pen remains in this position until a new change in voltage occurs.

In reality the pen follows variations in voltage almost instantaneously. The pen (Fig. 58) is made of glass and is provided with a reservoir of a sufficient size to hold ink for a continuous record of two months. The quickness of motion of the pen is regulated by two dash-pots connected to the moving coils. The sensitiveness of the record may also be varied by adjusting the distance between the contacts. The rate of feed of paper may be varied within certain limits.

52. Recording Ammeters for Rapidly Fluctuating Loads. — The above-described recording instruments are not suitable for accurately recording rapidly fluctuating currents, such, for instance, as are taken by street cars during periods of starting and accelerating (Fig. 512). Instruments are required in this case which possess a powerful torque combined with a high rate of paper feed; in addition the instrument must be able to stand the vibrations of an electric car. An ammeter of this kind has been built and successfully applied to car tests by the General Electric Company. A description of the instrument may be found in an article by Mr. A. H. Armstrong, in the *Transactions of the A. I. E. E.*, Vol. XXII, p. 689. Another arrangement with which any ordinary ammeter may be used is shown in Fig. 511. The device is non-automatic; the observer has to follow the ammeter pointer with a handle which traces the record.*

53. EXPERIMENT 2-C. — Study of Recording Instruments. — Different types of recording instruments are described in §§ 50 to 52. The points to be investigated are:

(1) Construction of the indicating and of the recording parts.

(2) Promptness of response to sudden fluctuations.

(3) Pen friction and its influence on the sensitiveness of the instrument.

(4) Character of the scale of the record.

Report. State your findings and explain how you would figure out the average current or the average voltage, if the record has curved coördinates as in Fig. 55.

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* See also Ashe and Keiley, *Electric Railways*, pp. 48 and 50.

CHAPTER III.

AMMETERS AND VOLTMETERS—CALIBRATION.

54. Primary and Secondary Standards. — The absolute values of the ampere, volt and ohm are established by an international agreement, and primary standards of these quantities are maintained by the respective governments — in this country by the Bureau of Standards of the Department of Commerce and Labor. Whatever the theoretical reasons are for the selection of certain values as units, for practical purposes they are as arbitrary as the standard inch or the pound. The industry merely demands that the units be of a convenient size and easily reproducible with a sufficient accuracy.

It is not necessary to have primary standards of all the three quantities — the volt, the ampere and the ohm. According to Ohm's law, when two of these are known, the third is represented by their product or ratio. The unit of resistance, or the international ohm. is officially defined as being represented by the resistance of a column of mercury of a certain length and mass, and at a specified temperature. The international ampere is defined as the current which deposits in one second of time a specified quantity of silver out of a solution of nitrate of silver, under certain definite conditions. As a consequence, the international volt is the e.m.f. which, being applied at the terminals of a standard ohm, produces in it a current equal to one ampere (an act of July 12, 1894).

The mercury ohm and the electro-chemical ampere are standards hardly suitable for practical purposes. Secondary standards are therefore used in practice, calibrated to these. The most popular secondary standards are: Resistances made of wire or strip, and standard cells for producing known e.m.f's. Current is determined either as the ratio of volts to ohms, or standard ampere balances are used (§ 72).

The calibration of ammeters and voltmeters with a standard cell requires rather delicate and complicated arrangements and allows of an accuracy not needed for many practical purposes. In places where many instruments must be calibrated in a comparatively short time, and extreme accuracy is not required, it is customary to use good ammeters and voltmeters as intermediate standards; they are checked from time to time with the primary standards kept in the laboratory. We shall first describe simple calibrations by means of intermediate standards, and then give the methods for checking the standard instruments themselves.

55. Calibration of D. C. Ammeters with a Standard Milli-Voltmeter and a Shunt. — Standard D. C. instruments are invariably of the moving-coil type, because of the accuracy and sensitiveness of this construction. As was explained in § 38 ammeters based on this principle

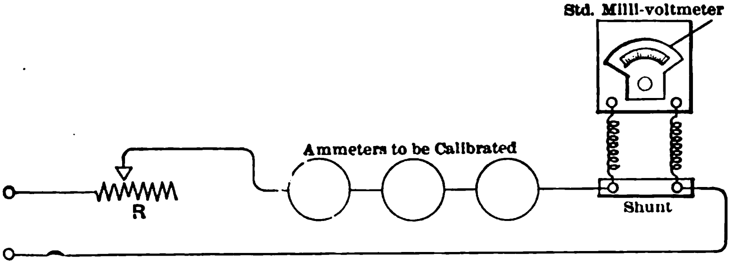


Fig. 59. Calibration of ammeters with a standard shunt and a standard milli-voltmeter.

are in reality milli-voltmeters which measure voltage drop across a shunt (Fig. 35). Therefore, a standard milli-voltmeter and a set of standardized shunts covering the required range of currents is all that is necessary for calibrating D. C. ammeters (Fig. 59). The ammeters to be calibrated are connected to a steady source of direct current in series with a standard shunt and the regulating resistance *R*. A standard milli-voltmeter is connected across the terminals of the shunt. The current is adjusted to a desired value, and the ammeters and the milli-voltmeter are read simultaneously; the current is changed, new readings taken, etc.

Fig. 60. The Weston standard semi-portable voltmeter (or milli-voltmeter).

A standard milli-voltmeter for accurate calibration is shown in Fig. 60; it is similar to ordinary portable instruments, but is several times larger and has a scale shown in Fig. 61. With this scale parts of divisions may be read much more accurately than with an ordinary scale, such, for instance, as in Fig. 33. The calibration of the instrument depends to a slight degree upon its temperature; a thermometer is provided on the base of the instrument so that the necessary correction may be taken into account. The scale is provided with a mirror (Fig. 61); in taking readings the observer's eye must be in such a position that the end of the pointer is seen to cover its image in the mirror.

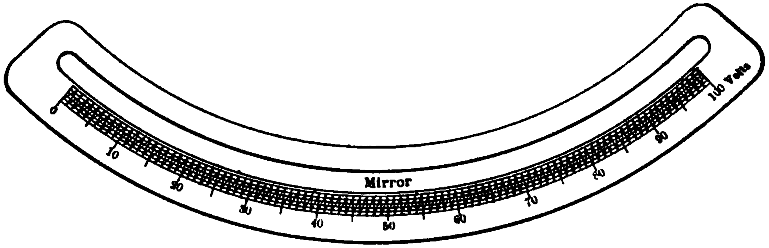


Fig. 61. The precision scale used in Weston standard instruments.

The results of the calibration are given either in the form of a table, or a curve is plotted giving true amperes to actual readings as abscissæ. Some prefer to plot calibration curves in the form shown in Fig. 62, instead of true readings, corrections only are given, in scale divisions or in per cent.

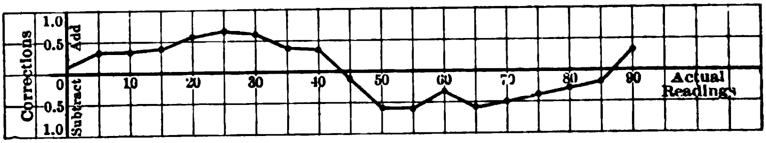


Fig. 62. Calibration curve of an instrument.

If a wide range of amperes is to be covered, one shunt is not sufficient. Suppose, for instance, that the milli-voltmeter gives a full scale deflection with a current of 1000 amperes flowing through the shunt. With 100 amperes the deflection will be only 10 per cent of the scale; it is hardly advisable to go below this, as the accuracy of the readings is impaired. Thus below 100 amperes another shunt of ten times the resistance is required; it will give a full scale deflection at 100 amperes and may be used down to 10 amperes. The next shunt will serve from 10 to 1 amperes, etc.

Not only ammeters, but milli-voltmeters and shunts may be calibrated in a similar way. To calibrate a shunt, connect it in series with a standard shunt (Fig. 59) and measure the voltage drop with a calibrated milli-voltmeter, first across the standard shunt, and then across the shunt to be calibrated. If the line current is constant, the ratio of readings will give the ratio of resistances of the two shunts. An ammeter should be kept in the circuit to be sure that the current remains unchanged between readings. It is well to use a double-throw switch with mercury contacts when changing the milli-voltmeter from one shunt to the other; ordinary knife-switches have an appreciable contact resistance, which may vitiate the results.

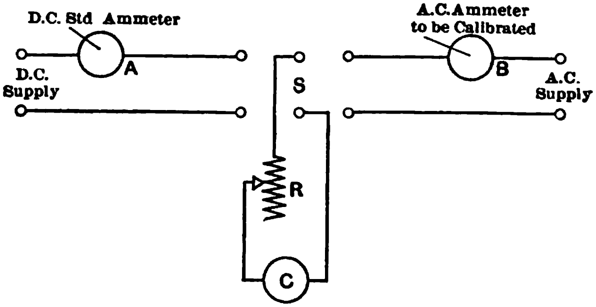
To calibrate a milli-voltmeter it is connected across a shunt in parallel with a standard milli-voltmeter, and simultaneous readings are taken on both instruments.

56. EXPERIMENT 3-A. — Calibrating D. C. Ammeters with a Milli-Voltmeter and a Shunt. — The method is explained in the preceding paragraph. Connect several ammeters as in Fig. 59 and calibrate them throughout their range. Then calibrate a shunt; give its temperature correction, if any. Calibrate a milli-voltmeter; see if the length of the leads and the presence of a switch in the circuit influence the indications.

Report some of the results in the form of curves showing true amperes to actual readings as abscissæ; other results in the form of correction curves, as in Fig. 62.

57. EXPERIMENT 3-B. — Calibrating A. C. Ammeters with Intermediate D. C. — A. C. Standards. — Hot-wire and dynamometer-type ammeters (§§ 40 and 41) may be calibrated with direct current, as in Fig. 59, and used with alternating currents. One precaution necessary when using an electro-dynamometer on direct current is to eliminate the influence of the terrestrial magnetism on the moving coil. This influence is reduced to a minimum by placing the moving coil in a plane perpendicular to the magnetic meridian. It is also well to take readings with the same current flowing through the instrument in both directions and to average the two deflections. Precision dynamometers are made astatic, that is to say, they have two stationary and two moving coils, one system above the other. The direction of the currents is such that the influence of the terrestrial magnetism is cancelled.

Soft-iron and induction-type ammeters (§§ 36 and 42) must be calibrated with currents of the same frequency for which the instrument is intended to be used. If an accurate A. C. standard, such as a Kelvin balance or a precision dynamometer, is not available, the instruments may be calibrated by using intermediate D. C.-A. C. standards (Fig. 63). Hot-wire instruments are particularly well adapted to be used as intermediate standards. The double-pole double-throw switch S is first thrown to the left, and the hot-wire instrument C is calibrated with the direct current standard instrument A. Then the switch is thrown to the right and the instrument B is calibrated with C, using alternating current. It is important that the hot-wire instrument should preserve its zero during the test; it may be set to zero by the regulating screw S (Fig. 41).



A. C.—D. C. Ammeter.

Fig. 63. Calibration of an alternating-current ammeter by means of an intermediate A.C. — D.C. instrument.

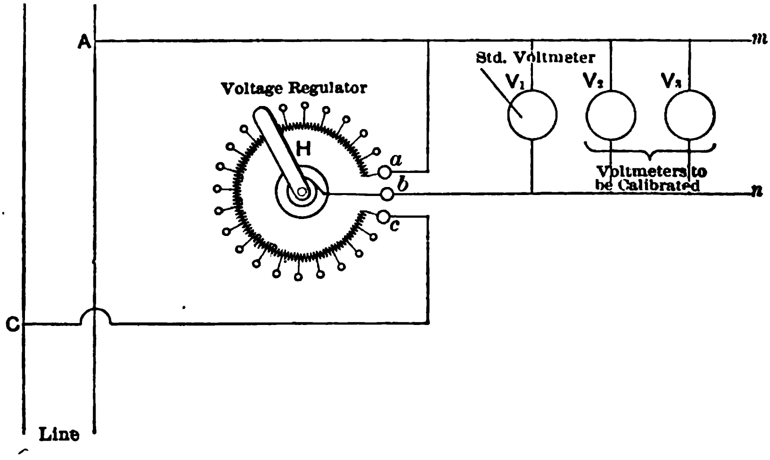


Fig. 64. Calibration of voltmeters.

58. EXPERIMENT 3-C. — Calibrating D. C. Voltmeters with a Standard Voltmeter. — The connections are shown in Fig. 64; the voltmeters V_1, V_2, V_3, etc. are connected in parallel between two busbars A-m and b-n. The terminals a and c of a high-resistance rheostat are connected across the source of supply; by means of the sliding arm

H connected to *b*, any part of the total line-voltage may be applied at the terminals of the voltmeters. When *H* is in position *a*, the pressure between *m* and *n* is = 0; when *H* is in the position *c*, the total line-voltage is applied between *m* and *n*. One of the voltmeters is a standard instrument with which the others are to be compared. An ordinary embedded-type field rheostat is convenient for use as a voltage regulator. It usually has only two terminals *a* and *b*; it is easy to solder to it a third terminal *c*.

Calibrate the voltmeters with and without multipliers; see if the instruments themselves or the multipliers have an appreciable temperature coefficient. Plot calibration curves as specified in § 56.

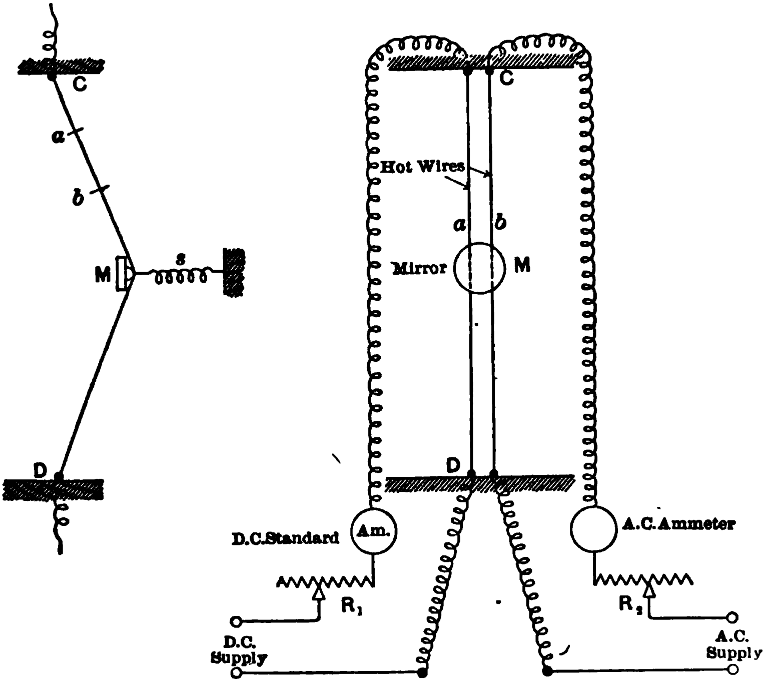


FIG. 65. The Leeds & Northrup comparator, for accurate measurement of alternating currents and voltages.

59. EXPERIMENT 3-D. —Calibrating A. C. Voltmeters with an Intermediate D. C. — A. C. Standard. —The arrangement of the test is similar to that of § 57, except that the instruments to be calibrated are connected in parallel as in Fig. 64.

60. Leeds and Northrup Comparator. — To avoid the necessity of using an intermediate standard, as in Fig. 63, Dr. E. F. Northrup devised an ingenious comparator shown diagrammatically in Fig. 65.

By means of it any A. C. ammeter or voltmeter may be calibrated directly with a D. C. standard. The comparator is essentially a differential hot-wire instrument, which shows zero when the alternating and the direct currents to be compared are equal.

Two identical wires *a* and *b* are stretched between the bases *C* and *D* and have a small mirror *M* fastened to them. A spring *s* pulls the wires backward, and keeps them taut. When a current is sent through one of the wires, that wire is heated and slackens; the spring *s* deflects the mirror, and the deflection is read by means of a telescope and a scale, as with an ordinary galvanometer. If the same current flows simultaneously through both wires, they recede by the same amount, and the mirror continues to show zero.

The actual zero of the instrument is determined by sending a direct current through both wires connected in series. Then an alternating current is sent through the wire to the right and the A. C. ammeter to be calibrated. A direct current is sent through the other wire and the standard ammeter. By regulating the rheostats R_1 and R_2, the values of the currents are adjusted until the mirror comes back to zero. Then the currents flowing through the two ammeters are equal to each other. For the sake of simplicity, total currents are shown flowing through the hot wires. In reality shunts are used on both sides, so that only a small part of total currents passes through the comparator. The same comparator provided with suitable multipliers may be used for calibrating A. C. voltmeters.

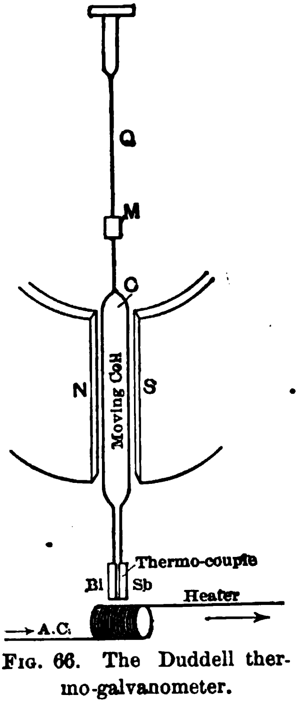


Fig. 66. The Duddell thermo-galvanometer.

61. Duddell Thermo-Galvanometer. — For a long time there has been felt the need of an instrument for measuring small alternating currents with the same accuracy with which small direct currents are measured with moving-coil galvanometers. In § 37 it is explained that moving-coil instruments cannot be used on alternating currents because of the unidirection effect of the permanent magnet. Mr. Duddell ingeniously combined the moving-coil and the hot-wire principles (Fig. 66) and obtained an extremely sensitive A. C.–D. C. instrument with a wide range. The alternating current to be measured is sent through a resistance or the "heater" shown on the bottom. The heat of the current affects the thermal junction *Bi-Sb* (bismuth-antimony) which produces a direct current in the loop *C*. The same is deflected by the permanent magnet *NS* against the torsion of the quartz fiber *Q*. The sensibility of the instrument depends upon the resistance of the heater and upon its distance from the thermal junction. The instrument can evidently be calibrated with direct current and used with alternating currents. There is no doubt that this thermo-galvanometer will become of importance in measuring high frequency A. C. currents, notably in wireless telegraphy, telephone transmission, X-ray tubes, etc. The instrument is well suited to be used as an intermediate D. C.-A. C. standard.

62. Standard Cells. — Standard voltmeters and milli-voltmeters used for calibration, as described in §§ 55 and 58, are sufficiently accurate for most practical purposes, provided they are periodically compared to a primary standard of e.m.f. The so-called "standard" cells, which are special primary batteries, are used at present for representing the normal value of the volt.

The standard cell used at present is the Weston cadmium cell; its construction and chemical composition are shown in Fig. 67. There are two forms of cadmium cell, in one of which the cadmium sulphate solution contains solid crystals of cadmium sulphate — this cell is known as the *saturated form*. The other form, in which the solution is saturated at 4 degrees C., is called the *unsaturated form*. It has practically no temperature coefficient. The normal value of the e.m.f. of this cell is 1.01985 volt, at any temperature between 10 degrees C. and 35 degrees C. The saturated form has a slight temperature coefficient; its electromotive force, at any temperature, is given by Jaeger as:

$1.0186 - 0.000038\ (t - 20 \text{ degrees}) - 0.00000065\ (t - 20 \text{ degrees})^2$ volts.

The unsaturated form is not so reliable for extremely accurate scientific research, as the standard form; but it is sufficiently accurate and more convenient for ordinary engineering work. With some skill and experience, a standard cell may be constructed with comparatively simple means at hand. For detailed specifications of materials and for other instructions see a paper by Profs. Carhart and Hulett in the *Transactions of the International Electrical Congress, St. Louis*, 1904, Vol. II, p. 109.

Previous to the introduction of the Weston cadmium cell, the Clark standard cell was universally used. It differs from the Weston cell in that zinc is used instead of cadmium. The e.m.f. of the Clark cell is 1.434 volt at 15 degrees C.; it decreases by 0.00115 volt for each 1 degree C. increase in temperature, between the limits of 10 degrees and

25 degrees. Prof. Carhart improved the Clark cell by saturating the solution of zinc sulphate at 0 degree C. The e.m.f. of the Carhart-Clark cell is 1.440 volt; its temperature coefficient is about half of that of the Clark cell.

A very important precaution should be observed in handling standard cells not to draw any appreciable current from them. The cell becomes polarized almost instantly, and it is doubtful if it ever sufficiently recovers, to be reliable as a standard. The Weston cell should never be closed on a resistance of less than 10,000 ohms, a greater resistance being preferable.

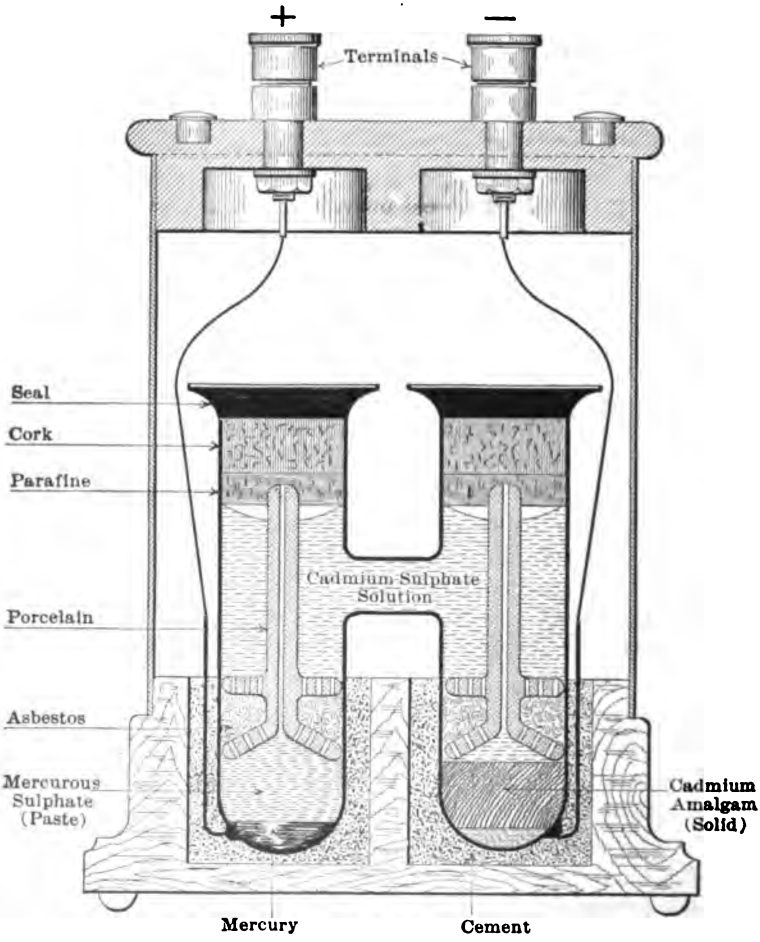


FIG. 67. Cross-section of a Weston standard cell.

63. Principle of the Potentiometer. — Instruments by means of which voltages are measured in comparison with a standard cell are called *potentiometers* (Figs. 68 and 69). They are used for very accurate measurements of currents, voltages and resistances and for calibration of the corresponding instruments. With a potentiometer, voltages from a fraction of a millivolt to several thousand volts can be accurately compared to the e.m.f. of a standard cell, and currents measured from a small fraction of an ampere to many thousand amperes.

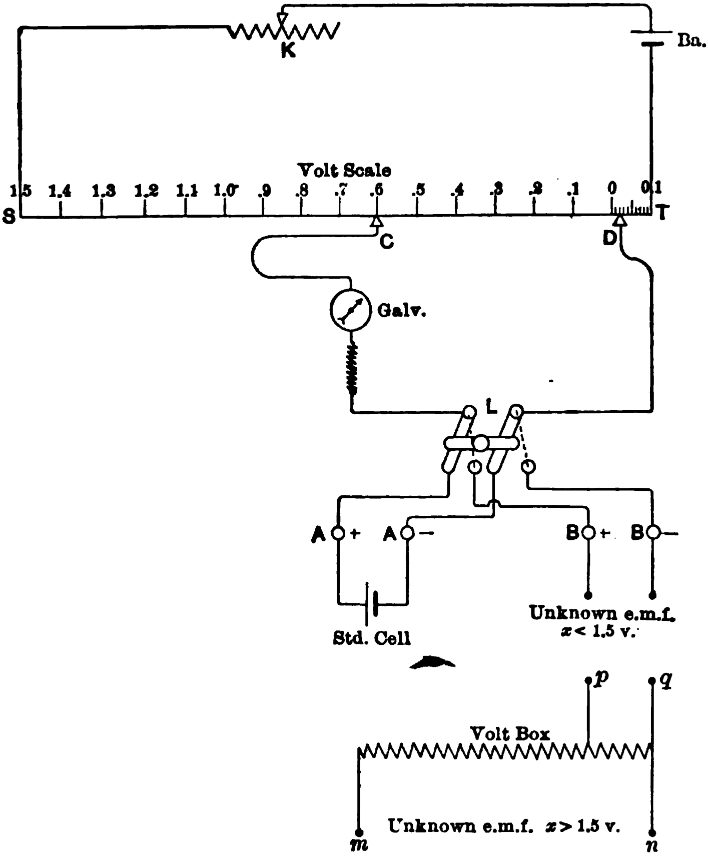


FIG. 68. The potentiometer principle of comparing voltages.

The ordinary potentiometer connections are shown in Fig. 68; for a clear understanding of the principle involved the student should keep in mind two points:

(a) The standard cell balances the unknown voltage, but is not used as a source of current.

(b) The "zero" method is used so that it is not necessary to know the galvanometer constant.

A slide-wire *ST* is connected in series with one or two storage cells *Ba.* and a regulating rheostat *K*; a constant current is thus established in *ST*. Any desired voltage drop may be had between the contacts *C* and *D* by moving them along the slide-wire, or by regulating the rheostat *K*. The contact *C* is for crude regulation, *D* for fine regulation. The drop between *C* and *D* is balanced against the standard cell and against the unknown e.m.f. in succession. A balance is recognized by the galvanometer needle returning to zero. The ratio of the lengths *C-D* in the two cases is equal to the ratio of the e.m.f.'s under comparison, provided the current in *ST* remains constant. By means of the double-throw switch *L* connections are conveniently changed from the standard cell to the unknown e.m.f., and back.

In order to make the arrangement direct-reading, the contacts *C* and *D* are set beforehand so as to balance the voltage of the standard cell at the proper divisions of the scale — in case of the Weston cadmium cell about 1.02 volt. (See § 62.) The switch *L* is thrown to the left, and the rheostat *K* adjusted until the galvanometer returns to zero. Then the switch is thrown to the right, and the galvanometer balance again obtained by shifting *C* and *D*. With the position shown in the sketch, the unknown voltage is 0.620 volt.

The potentiometer scale usually has a range of 1.5 volt so that only voltages below this limit can be directly compared to that of a standard cell. For voltages above 1.5 volt a multiplier, or the so-called "volt-box," is used, shown on the bottom of the sketch. It consists of a high resistance with one or more taps at a known part of it, say 1/10, 1/100, etc. The unknown e.m.f. is connected across the terminals *m* and *n*; the terminals *p* and *q* are connected to the terminals *BB* of the potentiometer. Let, for instance, the unknown voltage be about 110 volts and the resistance between *p* and *q* be 1/100 of the total resistance of the volt-box. Then the voltage drop across *pq* is only about 1.1 volt and can be directly compared to the e.m.f. of the standard cell. Multiplying the result by 100 we get the actual unknown voltage.

64. EXPERIMENT 3-E. — Study of a Simple Potentiometer. — Before attempting to use the accurate and delicate potentiometers it is desired that the student make clear to himself the working principle of the potentiometer, according to the simple diagram shown in Fig. 68. A German-silver wire stretched along a uniform scale may be used as the slide-wire; ordinary voltmeter leads with knife-edge terminals for contacts *C* and *D*.

(a) Connect the circuits as shown in Fig. 68 and practice in comparing two e.m.f.'s small enough to be within the range of the slide-wire. It is not necessary to use a standard cell; any dry cell of which the e.m.f. has been previously determined will serve the purpose.

(b) Arrange the connections for measuring amperes by means of a standard shunt (Fig. 59); use the potentiometer in place of a millivoltmeter.

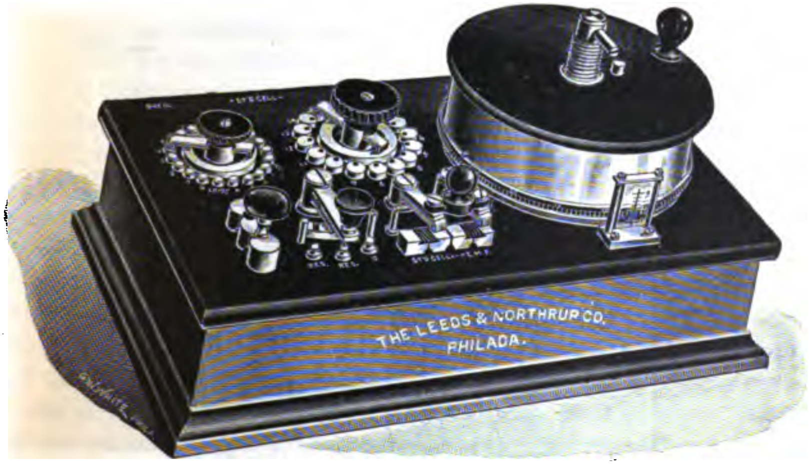


FIG. 69. The Leeds & Northrup potentiometer.

(c) Compare two resistances by measuring the drop at their terminals (Fig. 2).

(d) Finally improvise a volt-box and make clear to yourself the method of measuring voltages beyond the range of the slide-wire.

65. Leeds and Northrup Potentiometer. — A convenient form of potentiometer for ordinary engineering work is the Leeds and Northrup instrument (Fig. 69). The connections are shown in Fig. 70; the same notations are used as in Fig. 68. The larger portion *SO* of the slide-wire is replaced by 15 five-ohm resistance coils in series, with their terminals connected to contact buttons. This arrangement permits of having a considerable resistance without increasing the size of the instrument. The dial and the handle corresponding to the contact *C* are visible in Fig. 69 behind the switches. The part of the slide-wire between O and T is wound on a marble cylinder and is about 16 feet long, its resistance being exactly 5 ohms. The contact D is mounted inside of an aluminum hood and moves up and down with the hood as it turns on a heavy screw projecting from the center of the marble cylinder. The slide-wire makes 10 turns on the cylinder, so that each turn corresponds to 0.01 volt. It is easy to read on the circular scale at least 1/100 of a turn, so that voltages can be accurately read down to 0.0001 volt or 1/10 millivolt.

For measuring very low voltages a 10 times greater accuracy may be obtained by introducing the shunt 10 : 1 (Fig. 70); this is done by changing the plug shown to the left, from the hole 1, to that marked 0.1; the plug and the receptacles are clearly seen in Fig. 69. The resistance of the shunt and that of R bear such a ratio that when the shunt is applied, the voltage drop across the circuit SOT is reduced 10 times, provided the battery current remains the same. This gives the potentiometer a range from 0.15 volt to 0.00001 volt, or 1/100 millivolt. The connections to the standard cell and to the unknown e.m.f. are similar to those shown in Fig. 68. The switch-key V in the galvanometer circuit has three positions; in the two left positions some protective resistance is connected in series with the galvanometer. This is to prevent violent deflections of the galvanometer before the

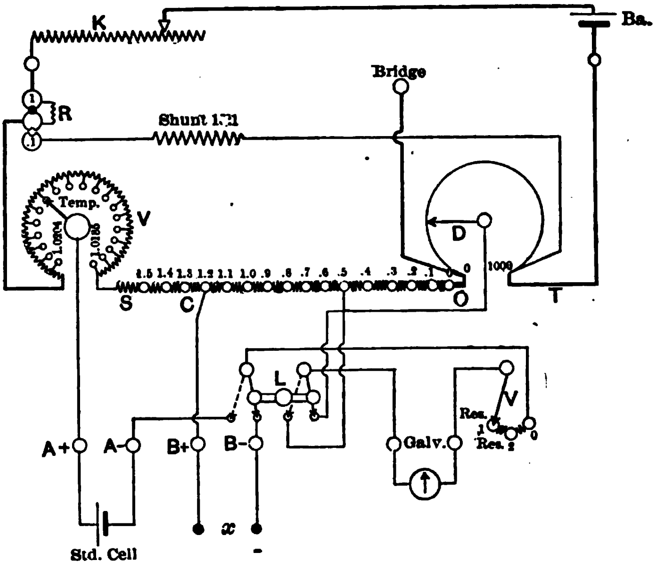


FIG. 70. Diagram of connections of the Leeds & Northrup potentiometer.

balance is obtained, and also to protect the standard cell against excessive currents which might polarize it.

The connections are such that the contacts C and D are used for obtaining balance with the unknown e.m.f. only, but not when adjusting the current — this being done by means of the resistance K. It will be seen from the sketch, that the standard cell is connected between the point .5 of the potentiometer resistance and the sliding contact on the dial to the left, marked " Temp." (temperature). The e.m.f. of the Weston cell being about 1.02 volt, the resistances connected to this dial permit the operator to set the potentiometer exactly at the certified e.m.f. of the cell, taking into account small variations depending on temperature or any other causes (§ 62).

By throwing the switch L to the left the balance is obtained by regulating K, *with any position of C and D.* This is the distinctive feature of the Leeds and Northrup potentiometer, and has the following advantage: Suppose the resistance K to be adjusted so as to give a balance with the standard cell, the connections being as in Fig. 68. Now the switch L is thrown to the right, and a new balance obtained by moving C and D. In the meanwhile the battery current might have changed; in order to check this, it is necessary to set C and D back into the position corresponding to the e.m.f. of the standard cell, and to throw the switch L to the left. This means setting C and D every time anew, and losing the previous setting; also a loss of time. With the circuits shown in Fig. 70 the setting of C and D may be left intact, and the battery current checked by throwing L to the left and pressing the galvanometer button.

66. Checking Potentiometer Resistances. — Provision is made in the potentiometer shown in Fig. 70 for checking the resistances of the dial and of the slide-wire. The coils in the series SO are each five ohms, and between each there is a brass block with a reamed hole (Fig. 69). A pair of flexible cords, with taper plug terminals to fit the reamed holes, is furnished. These coils can be measured with an ordinary Wheatstone bridge and thus compared with each other to a high degree of accuracy even if the bridge is not accurate. For potentiometer work the essential point is that they should be like each other, not that they should be accurately any particular value. In the same way the resistance of the extended wire can be compared with the resistances of the coils in SO and should be found exactly equal to them. The heavy connector leading out to the binding post marked " Bridge " is provided for this purpose; also that the extended wire may be used by itself as a Kohlrausch bridge (Fig. 16).

The calibration of the extended wire OT on the cylinder may be checked against the dial resistances *SO* by converting the potentiometer itself into a Wheatstone bridge. To do this, short-circuit the two outside upper binding posts with a heavy copper connector, not smaller than No. 10 B. & S.; also short-circuit the terminals *BB* and throw *L* to the right. Connect the battery between the terminals marked " Ba " and "Bridge." The contacts *C* and *D* form one junction of the two arms on each side of the bridge with the galvanometer between, while the posts " Bridge " and " Ba " form the second junction with the battery between. On one side there are 16 equal resistances of 5 ohms each, the resistance of the temperature dial being supplemented with an extra resistance, so as to make it exactly 5 ohms. If the extended wire be read in its 1000 parts, each of the 16 coils of the potentiometer circuit will be equal to $\frac{1}{16}$ of 1000 or 62.5 divisions when balanced against the wire.

67. Directions for Using Leeds and Northrup Potentiometer. — In view of the popularity of the above-described potentiometer, it was deemed advisable to reprint here the explicit directions for its use, as given by the makers.

Setting up. — All connections must be made as indicated by the stamping on the potentiometer. Particular attention must be given to the polarity of the standard cell, battery and e.m.f., the corresponding + and − signs being marked.

If used with a wall galvanometer having a telescope and scale, it will be found convenient to place the potentiometer so that the telescope is directly over the glass index of the extended wire, thus permitting the observer to read the galvanometer deflections and potentiometer settings without change of position.

Potentiometer current. — A medium-sized storage cell will be found advantageous, producing a steady current. Errors of measurement are frequently made by using an unsteady source of e.m.f.

Setting for standard cell. — Set the standard cell switch to correspond with the certified e.m.f. of the standard cell, as given in its certificate. *Place plug in hole* 1, *and see that it is always in this position when checking against the standard cell.* Place the double-throw switch at Std. Cell. The switch-key should be at the left of the three contacts marked RES., RES. and O, so as to include the greatest extra resistance in series with the galvanometer. This precaution is always to be taken before the final balance is obtained to prevent any violent deflections of the galvanometer.

Adjust the regulating rheostat until the galvanometer shows no deflection. The final galvanometer reading should be taken with the switch-key at zero, so as to remove the series resistance and thus increase the sensibility. The middle "RES." contact can be used as an intermediate point of contact as the galvanometer balance is approached.

Measurement of unknown e.m.f. — The potentiometer gives direct readings for voltages up to and including 1.6 volt, therefore no voltage exceeding this amount must be connected to the e.m.f. posts of the instrument. For voltages higher than the direct range of the instrument, a volt-box or multiplier must be used.

The standard cell balance having been obtained as described under "Setting for Standard Cell," proceed as follows:

Place the double-throw switch in position E.M.F. and the galvanometer key to the left of the contacts so as to again include the highest series resistance. The balance for the unknown e.m.f. is now obtained by manipulating the tenths switch and rotating the contact on the extended potentiometer wire. The final position of the two contacts in conjunction with the position of the plug at left of instrument indicates the voltage under test as described later.

Plug at 1 *or* .1. — Plug at 1 gives readings for voltage direct from settings of tenths switch and extended wire contact.

Plug at .1 shunts the potentiometer circuit so that the voltage measured is one tenth. Therefore, the readings taken from the settings of the tenths switch and slide-wire contact must be divided by 10.

To balance galvanometer for unknown e.m.f.

Place plug in hole 1 for voltages up to 1.6.
Place plug in hole .1 for voltages up to .16.

Rotate the tenths switch (having the galvanometer key at first RES.) until a condition of balance is obtained exactly or approximately. To secure an exact balance, rotate the contact on the extended wire. The unknown e.m.f. can now be read directly from the position of the tenths switch and the extended wire contact if plug is at 1, or by dividing by 10 if plug is at .1.

Example. — A balance was obtained with the tenths switch at 1.3 and the extended wire contact at 176 and the plug at 1. The voltage under test, therefore, is 1.3176. If the plug at .1 had been used the same reading would have indicated .13176.

To ascertain if current in the potentiometer circuit has altered during a measurement, it is only necessary to plug in at 1, place the double-throw switch on Std. Cell and close the galvanometer key. No deflection indicates that current is constant. If previous balance does not exist, the regulating rheostat must again be adjusted until the galvanometer shows no deflection.

68. Nalder Potentiometer. — Another popular potentiometer is that manufactured in England by Nalder Bros. & Co. The instrument is illustrated in Fig. 71; the circuits are shown in Fig. 72. No slide-wire is used, all the resistance consisting of coils of wire connected to contact buttons. The dial to the left corresponds to the part *SO* of the scale (Fig. 68); the dial to the right is for fine adjustments and corresponds to the part *OT*. The regulating resistance *K* is mounted in the case with the rest of the apparatus; it consists of a wire rheostat *K* for crude adjustment, and of a carbon disk rheostat *H* for fine regulation.

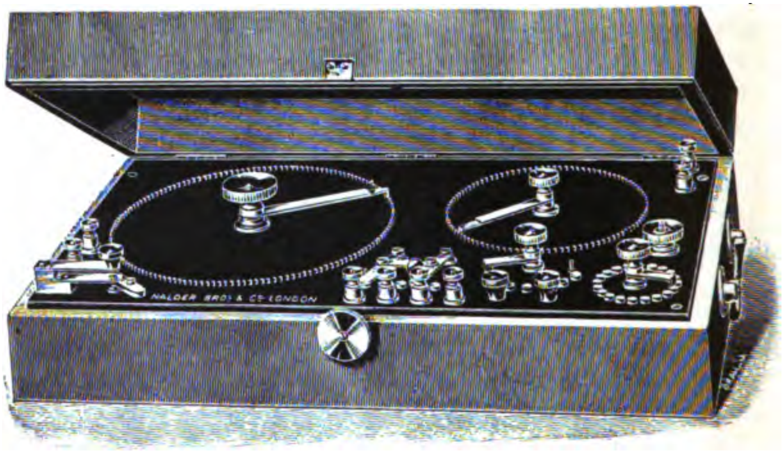


Fig. 71. The Nalder Bros. potentiometer.

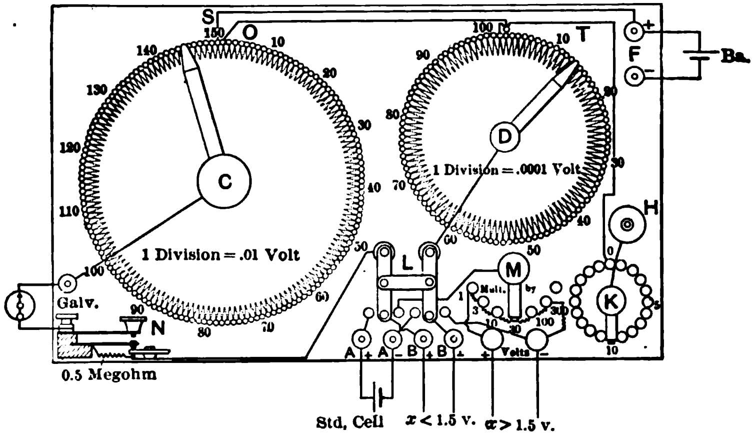


Fig. 72. Diagram of connections in the Nalder potentiometer.

The multiplier or the volt-box for e.m.f.'s above 1.5 volt is also a part of the instrument itself; the regulating handle of the multiplier is denoted by *M*. The switch *L* has three positions, instead of two, as in Figs. 68 and 70; these positions correspond to standard cell, $x < 1.5$ volt, and $x > 1.5$ volt. The galvanometer and the standard cell are protected by a 0.5 megohm resistance; this resistance is in the circuit when the key *N* is slightly pressed on; by pressing the key as far as it will go, the protective resistance is short-circuited. The circuits are exactly the same as in Fig. 68. The current from the + terminal of the battery flows to *S*, and through all the coils of the two dials to the regulating rheostat *K*; thence to *H* and back to the battery. The potential arm *D* is connected to the switch *L* directly — the arm *C* through the galvanometer and its protective resistance.

To use this potentiometer with a Weston cell set the large dial on 101, the small dial on 98 or 99 according to the certified value of the e.m.f. of the cell (see § 62). Throw *L* to the left and adjust the current (usually from a large storage cell) by *K* and *H*, so that the galvanometer remains on zero when the key *N* is pressed. Now the potentiometer is ready for measuring e.m.f.'s between the terminals *BB* or " volts." To measure a voltage above 1.5 volt *L* is thrown in its extreme right position, and the arm *M* set at the proper value. The contacts *C* and *D* are adjusted until the galvanometer shows zero. With the position of the contacts shown in the sketch, the reading is 1.4514 volts; the multiplier is in position "30," consequently the unknown potential is $1.4514 \times 30 = 43.542$ volts.

If it is desired to determine whether the battery current has changed, the contacts *C* and *D* are set back to 101 and 98 (or 99) and the key *N* pressed. If the galvanometer shows a deflection, an adjustment is made with *H*, and the unknown voltage measured again. This necessity of destroying the setting of *C* and *D* is a drawback of this instrument; it is eliminated in the Leeds and Northrup potentiometer by a somewhat different disposition of the circuits, and by adding the " Temp." dial (Fig. 70).

69. EXPERIMENT 3-F. —Calibrating Voltmeters with a Potentiometer and a Standard Cell. — The theory of the potentiometer is given in § 63; two commercial types are described in §§ 65–68. Study carefully the connections in the potentiometer available; then calibrate with it several voltmeters of different range. Pay particular attention to the sensitiveness and accuracy obtainable and to the factors which influence these. If possible, check the resistances of the potentiometer itself, as explained in § 66; or devise another method more suitable for the apparatus at hand.

70. EXPERIMENT 3-G. —Calibrating Ammeters with a Potentiometer and a Standard Resistance. — The experiment is similar to that described in § 56, except that the voltages across the shunt (Fig. 59) are measured with the potentiometer instead of a milli-voltmeter.

71. EXPERIMENT 3-H.—Comparing Resistances by a Potentiometer. — The method is the same as in Fig. 2, except that a potentiometer is used instead of an ordinary voltmeter or milli-voltmeter. A potentiometer with three pairs of terminals, as shown in Fig. 72, is particularly convenient for the purpose. One of the resistances is connected to the terminals *BB*, the other to "volts." If the resistances are very different from each other, it may be necessary to use the volt-box with one of them. It is essential that the current through the resistances under comparison should not change during the test. A standard low resistance is shown in Fig. 73. It has two pairs of terminals, as does an ammeter shunt (Fig. 35). Current is led in through the bent terminals, potential drop is measured between the straight terminals.

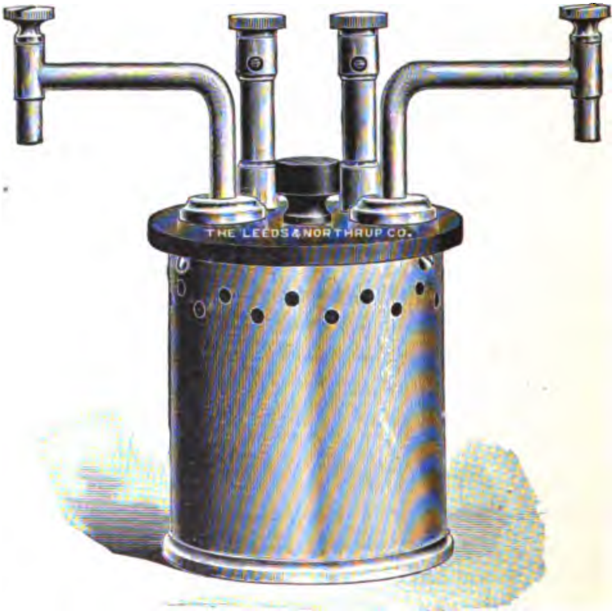


FIG. 73. A standard resistance for large currents.

72. Ampere Balance. — When using a potentiometer and a standard cell, amperes are measured indirectly as the values of the e.m.f.

and the resistance are found. A direct standard of current is offered by the Kelvin balance (Figs. 74 and 75). It is based on the electromagnetic attraction between stationary and moving coils, and is calibrated by means of a silver voltameter, according to the legal definition of the ampere.

The instrument consists of four stationary coils A, and two movable coils B suspended by metal strips C, which strips serve also as current leads for the coils B. The six coils are connected in series; a current flowing through the coils produces attractions and repulsions, as marked in Fig. 74. The influence of terrestrial magnetism is neutralized, since the current in the two movable coils is flowing in opposite directions.

A weight is placed in the trough D; with this weight the movable coils are in balance without current when the sliding weight w is in its extreme left position; this position corresponds to the zero of the scale. When a current is flowing through the instrument, it is necessary to move w to the right in order to keep the movable coils in balance, and current is indicated on the scale by the position of the weight w. It will be seen in Fig. 75, that the instrument is provided with two scales: one is the accurate uniform scale; the other, the so-called inspectional scale, is calibrated directly in amperes. The ampere divisions are proportional to the square roots of the distances from the left end of the scale, because the attraction between the coils is proportional to the square root of current (§ 40). The inspectional scale is accurate enough for ordinary purposes; if a greater accuracy is required the uniform scale is used; the current is then calculated by extracting the square root of the reading and multiplying it by the constant of the instrument.

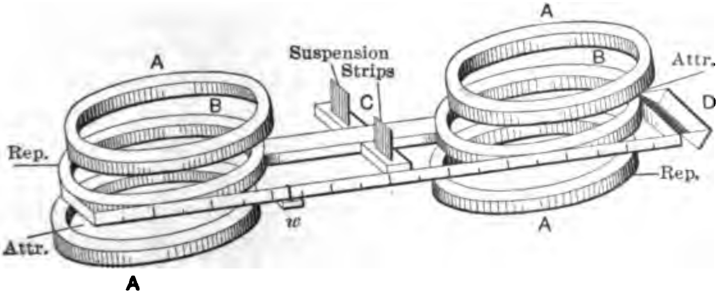


FIG. 74. The principle of the Kelvin balances.

The slipping of the weight w into its proper position is performed by means of a self-releasing pendant, hanging from a hook carried by a sliding platform, which is pulled in the two directions by two silk threads passing through holes to the outside of the glass case (Fig. 75). For the fine adjustment a small flag is provided, as in an ordinary chemical balance. The flag is actuated by a fork having a handle visible below the base. The round knob in the base, fastened to the chain, lifts the moving coils off the suspension, when the instrument is not in use. Four pairs of weights (sliding and counterpoise) are supplied with each instrument; of these the sledge of the moving weights and its counterpoise constitute the first pair. These weights are adjusted in the ratios of 1 : 4 : 16 : 64, so that each pair gives a round number of amperes, or half-amperes, or quarter-amperes, or of decimal subdivisions or multiples of these magnitudes of current, on the inspectional scale.

These instruments can be used equally well with direct or alternating currents, and are made in seven sizes covering the range from 0.01 ampere to 2500 amperes. The smallest size may also be used as a voltmeter, when provided with a suitable high resistance in series (multiplier). Balances are built on the same principle for measuring watts; also composite balances for measuring amperes, volts and watts with one and the same instrument. Detailed instructions for using the balance always accompany the instrument.

73. Absolute Electro-Dynamometer. — The above balances in spite of all their accuracy are but secondary standards, which need to be calibrated with a silver voltameter. There is a continuous effort on the part of physicists to build a satisfactory *primary standard of current* based on the electro-dynamometer principle, the constant of which standard may be deduced theoretically in C. G. S. units *from its geometrical dimensions*. One such "absolute" electro-dynamometer, designed by Pellat, is shown in Fig. 76. It consists of a large stationary coil shown to the left, and a small coil fastened to the beam of a delicate balance. When the instrument is in use the large coil is moved along to inclose the small one. The two coils are connected in series and the electro-magnetic couple of the current is balanced by a weight on the scale pan. The current corresponding to a certain *net* weight on the pan is calculated theoretically from the dimensions of the coils, so that the instrument is a primary standard in the true sense of the word. Standard electro-dynamometers are very expensive; the coils must be finished very accurately, and the whole instrument made of materials which will not change their dimensions with time or temperature.

Three primary standards: (a) standard cell, (b) standard resistance, (c) standard electro-dynamometer, are shown in Fig. 77, con-

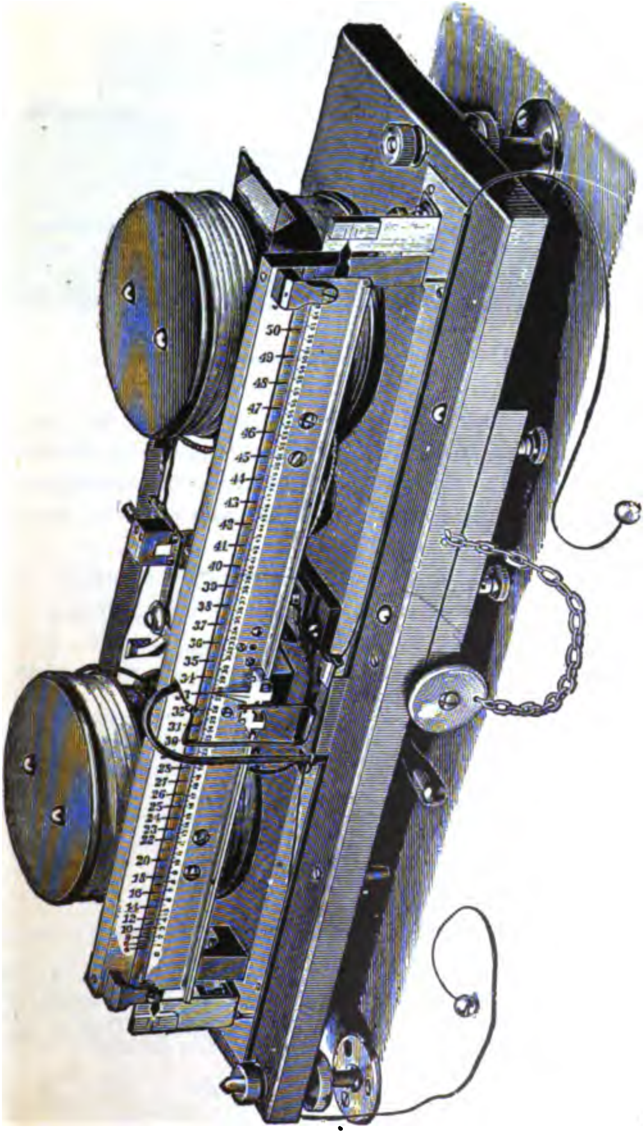


FIG. 75. The Kelvin balance for measuring currents.

nected so that any two of them may be checked by the third. The current is measured by the absolute electro-dynamometer, and also by

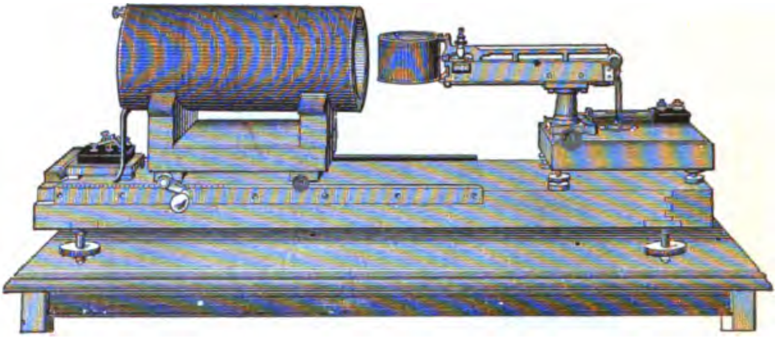


Fig. 76. The Pellat absolute electro-dynamometer.

the standard resistance and the cell. The value of the current is adjusted by the rheostat *R*, so that the galvanometer shows no deflection when the switch *K* is closed. Assuming, for instance, that the electro-dynamometer is correct, and the exact value of the resistance

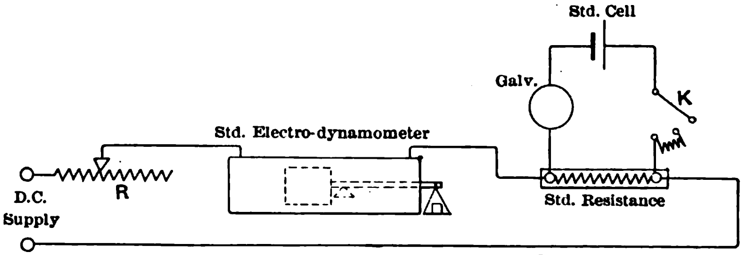


Fig. 77. The mutual check of the three standards; viz., those of current, electromotive force, and resistance.

of the shunt is known, the e.m.f. of the standard cell may be calculated as the product of the current and the resistance. This is the method by which the latest determinations of the e.m.f. of Weston and Clark standard cells were made — see an article by Dr. Guthe in *Bulletin No.* 1 (*Vol.* 2) *of the Bureau of Standards.*

CHAPTER IV.

WATTMETERS.

74. WATTMETERS are instruments for measuring the electric power developed in a circuit. The name "wattmeter" comes from the name "watt," it being the practical unit of power in the volt-ampere-ohm system. Three types of wattmeters are in use: (a) indicating, (b) recording, (c) integrating. Indicating wattmeters show instantaneous values of power, or the rate at which energy is consumed in a circuit; recording wattmeters trace a curve of these instantaneous values on a record sheet. Integrating wattmeters show *total* energy delivered to a circuit during a certain period of time.

To make this distinction clearer, take the case of a 100-volt direct-current circuit, in which the current regularly fluctuates between 50 and 70 amperes every 10 seconds. Imagine a wattmeter of each of the three above types connected to such a circuit, and observe their behavior — say for five minutes. The needle of the indicating wattmeter will fluctuate regularly between the divisions of the scale, corresponding to 5000 and 7000 watts. The recording wattmeter will trace a wavy line between the same values (Fig. 55). The dial of the integrating wattmeter will show at the end of five minutes

$$\frac{(7000 + 5000)}{2} \times 5 = 30000 \text{ watt-minutes},$$

or

$$\frac{30000}{60} = 500 \text{ watt-hours}.$$

INDICATING WATTMETERS.

75. Power expended in a direct-current circuit (Fig. 29) is equal to the product of the current delivered times the pressure at which it is supplied; in practical units

$$\text{watts} = \text{amperes} \times \text{volts}$$

or, with the customary notation,

$$w = i \times e \quad . \; . \; . \; . \; . \; . \; . \; . \qquad (1)$$

Thus in direct-current circuits power may be measured with an ammeter and a voltmeter. In alternating-current circuits, the presence of self-induction and capacity so modifies the relations, that the product "effective volts" times "effective amperes" is not equal to the true average watts expended in the circuit. The relation (1) is, however, always true for instantaneous values of current and voltage; in other words,

$$i.e.\,dt$$

represents the electrical energy delivered to an alternating-current circuit during an infinitesimal element of time dt; i and e are instantaneous values of the current and the voltage. The average energy delivered in one second, or the power in watts,

$$w = \frac{1}{T}\int_0^T i.e.\,dt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

where T is the time of one cycle of the alternating current. Assuming that both current and voltage vary according to the sine law, we have

$$\begin{aligned} i &= I \operatorname{Sin} mt \\ e &= E \operatorname{Sin} (mt \pm \phi) \end{aligned}$$

where ϕ is the phase angle by which the e.m.f. leads or lags behind the current, and $m = 2\pi/T$ (see Fig. 102). Substituting in (2) we obtain

$$w = \frac{IE}{T}\int_0^T \operatorname{Sin} mt \,.\, \operatorname{Sin} (mt \pm \phi) \,.\, dt$$

or

$$w = \frac{EI}{2} \operatorname{Cos} \phi.$$

Substituting the *effective* values of the voltage and the current, as shown by the ammeter and the voltmeter (§ 36 *c*), we get

$$w = i_{eff} \,.\, e_{eff} \,.\, \operatorname{Cos} \phi. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

Equation (3) shows that true power delivered to an alternating circuit depends not only upon the effective values of the current and the voltage, but also upon the phase angle between the two. The expression

$$i_{eff} \,.\, e_{eff}$$

is sometimes called *the apparent power;* the ratio of the true power to the apparent power, or Cos ϕ, is called *the power factor* (of the load).

It follows from the above considerations that in order to measure the true power in an alternating-current circuit, it is necessary to have an instrument which automatically accomplishes the integration required by the expression (2) and gives the average value of *w*. Such instruments are called indicating wattmeters.

76. Weston Wattmeter. — The most popular indicating wattmeter is the Weston instrument shown diagrammatically in Fig. 78; the electrical connections and the general view of the instrument are shown in Fig. 79. The Weston wattmeter is based on the electro-dynamometer principle (§ 40). It has two stationary coils and a moving coil between them. The details of construction are similar to those shown in Fig. 34, except that stationary coils are substituted for permanent magnets.

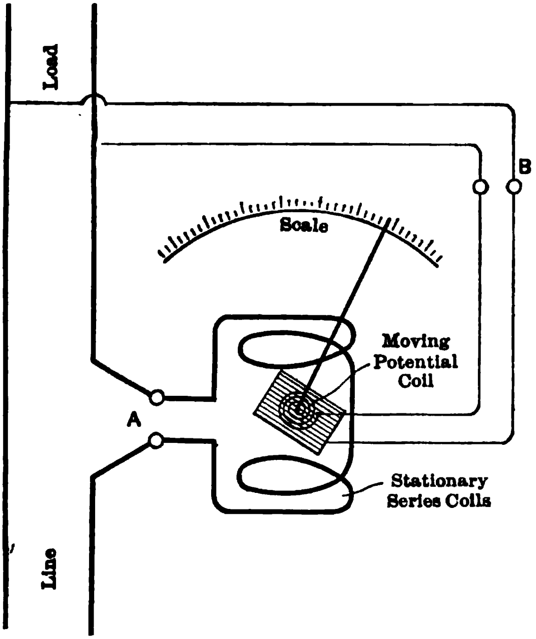


FIG. 78. The arrangement of parts in an indicating wattmeter.

The stationary coils consist of a few turns of heavy wire or strip connected in series with the circuit, as in an ammeter. The moving coil consists of many turns of fine wire, and is similar to the moving coil of D.C. voltmeters; it is connected across the circuit with a high non-inductive resistance (multiplier) in series with it. The instantaneous force of attraction between the coils is thus proportional to the product of current by voltage, or to the instantaneous power in the circuit. If the moving part of the instrument had no inertia it would vibrate with the fluctuations of the instantaneous values of energy.

But with usual frequencies of alternating currents these fluctuations follow in such quick succession that the moving element assumes a position corresponding to the *average* impulse; in other words, it automatically integrates the power according to expression (2).

The scale is divided directly in watts or in kilowatts (1 kilowatt = 1000 watts). The instrument is calibrated with standard direct-current ammeters and voltmeters, and may be used on either direct or alternating current. The large terminals *dd*, Fig. 79, are connected in series with the circuit in which it is desired to measure the power; the small terminals *ee* are connected across the line. The button shown below the scale closes the potential circuit when pressed down. This is a necessary precaution in order not to overheat the moving coil and the multiplier.

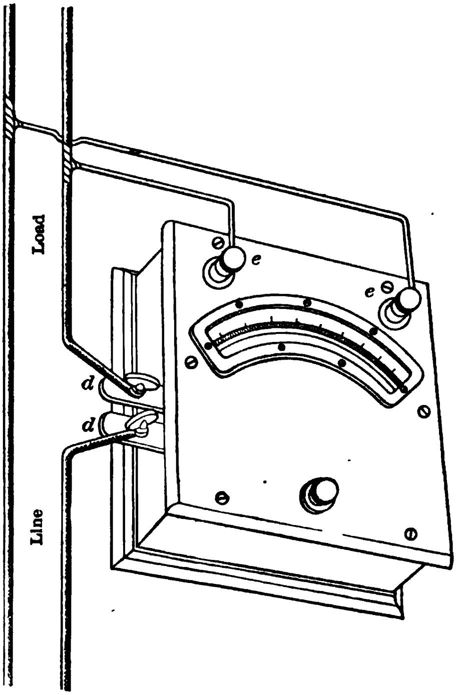


FIG. 79. External connections and general view of an indicating wattmeter.

77. Compensation for Power Consumption in the Wattmeter Itself.—An indicating wattmeter, such as is shown in Fig. 79, consumes some power in its series and potential windings. The inaccuracy introduced thereby is in some cases sufficient to make necessary a correction. It will be easily seen in Fig. 79, that the current flowing through the series winding of the wattmeter is a sum of the load current and of the current consumed in the potential circuit of the wattmeter itself. Therefore, the instrument indicates more power than is actually consumed in the load. The corresponding correction is

$$i^2R = \frac{E^2}{R}$$

where i is the current in the potential circuit, R the ohmic resistance of this circuit, and E is the line voltage.

Connecting the potential leads of the wattmeter on the " line " side of the series winding, brings in another inaccuracy; namely, the voltage across the potential terminals of the instrument is then higher than the load voltage, by the amount of voltage drop in the series winding of the instrument. If the resistance of the series winding is r and the load current is I, the corresponding correction is

$$I^2r.$$

It is usually preferred to have the wattmeter connected as in Fig. 79, and to correct for the loss of power in the potential winding. The reason is that it is more difficult to measure the resistance r of the series winding, and, moreover, it depends on the contact resistance at the terminals A.

Weston wattmeters are provided with a compensating winding, which makes a correction unnecessary, and thus simplifies the use of the instrument. Such a compensated wattmeter is shown in Fig. 80, the compensating coil being connected in series with the potential coil. When the load is zero, a current flows through the series and the potential windings of the wattmeter (Fig. 79), and causes the pointer to indicate some power, though in reality the power consumption is zero. The compensating winding gives a number of ampere-turns equal and opposite to that created by the series coils, and thus makes the pointer indicate zero at no load. The compensation is good at all voltages, since the current in the compensating coil is always the same as in the series winding (at no load). The potential circuit is completed between the terminals C and B, through the compensating coil, the movable potential coil, and a high resistance (multiplier) R. The terminals C and B are usually marked in Weston instruments " ± " and " 150 volts."

In cases in which the series winding and the potential winding are supplied from two independent sources, the compensating coil should be left out of the circuit, and the terminal D used, instead of C. The terminal *D* is marked " Ind.," or independent, and is connected to the potential coil through a low resistance *r* equal to the resistance of the compensating coil. This is done in order to have the same total resistance between *B* and *D*, as between *B* and *C*. In this way the instrument calibrated between one pair of the terminals reads correctly between the other pair of the terminals.

The practical cases in which the "independent" terminal *D* should be used, instead of *C*, are:

(1) When the wattmeter is connected through series and shunt transformers (when connected to a high-potential circuit).

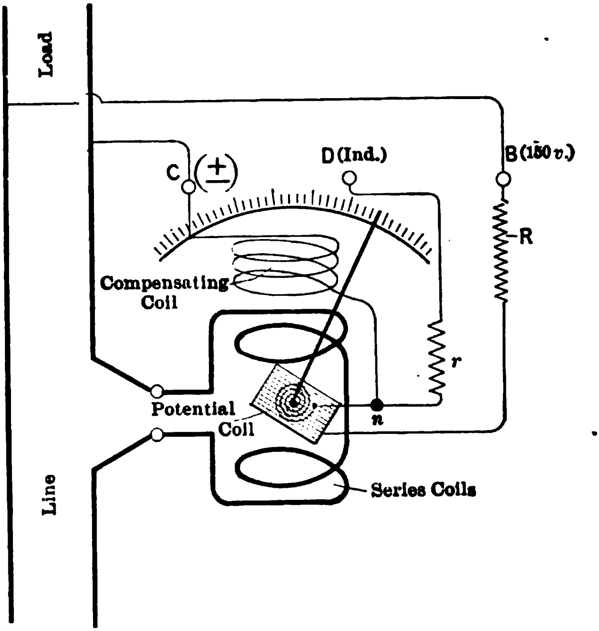


FIG. 80. Compensation for the power consumption in the wattmeter itself (Weston).

(2) When the instrument is calibrated with a separate source of current for the series winding. The latter is the case when a large storage cell is supplying the current, while a battery of small cells gives the required voltage. With this arrangement, large wattmeters are calibrated with a comparatively small expenditure of power.

78. Inaccuracy at Low Power Factors.—The above given theory of the wattmeter presupposes that there is no inductance in the potential circuit, so that the current in the moving coil is in phase with the line voltage. In reality the coil and the multiplier have some small inductance, so that, strictly speaking, the current in the potential circuit lags slightly behind the applied e.m.f. The inaccuracy thus introduced is usually negligible especially at high values of power factor, but with very low power factors may become quite noticeable.

The influence of the power factor is shown in Fig. 81. I is the vector of the current; E_1 the vector of the applied e.m.f. with a low power factor of the load (large phase displacement ϕ_1). If the potential circuit had no inductance, the current in the moving coil of the wattmeter would be in phase with E_1, and the indications of the instrument would be proportional to the working component $On_1 = E_1 \cos \phi_1$ of the voltage. In reality, the current in the potential coil is lagging behind E_1 by an angle α. The result is the same as if the applied voltage were reduced to Oa_1, with a working component Om_1. Applying the same reasoning at a high power factor corresponding to the applied voltage OE_2, the points n_2 and m_2 are obtained. It is easy to see that with the same current I, same voltage E and the same angle of lag α in the potential circuit, per cent error $n_1\ m_1$: On_1 is much larger at the low power factor than per cent error $n_2\ m_2$: On_2 at the high power factor.

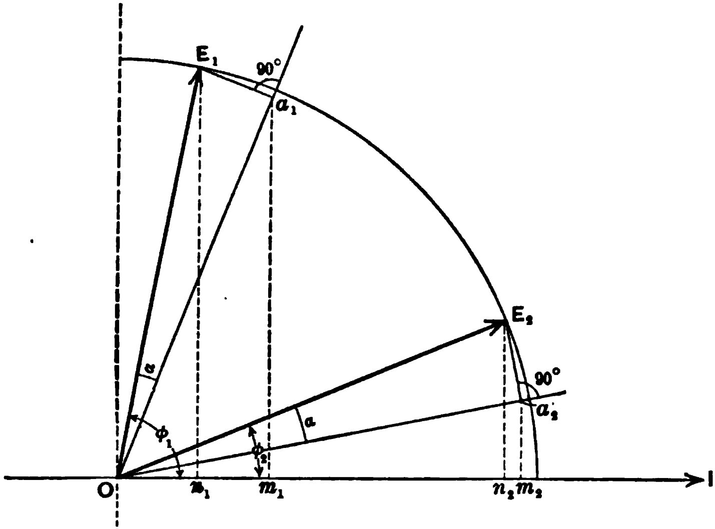


FIG. 81. Vector diagram, showing the influence of inductance in the potential circuit of a wattmeter.

Under ordinary circumstances, and with good instruments, the inaccuracy due to this cause is negligible, but under extreme circumstances may entirely destroy the value of the results. One way out of the difficulty is to measure "wattless" power as in Fig. 136; another way is to measure the power by some means other than a wattmeter, for instance by a calorimeter, or by the three-voltmeter method (§ 10). A hot-wire wattmeter may also be used; see Roller, "Electric and Magnetic Measurements," p. 223.

79. Whitney Wattmeter. — The instrument (Fig. 82) is based on the same principle as the above-described Weston wattmeter, an electromagnetic torque being produced between the stationary series-coil *SS* and the moving potential-coil *MM*. The power is indicated by the angle of torsion necessary to bring the moving coil back to zero, and not by the direct deflection of the moving coil, as in the Weston instrument. In this respect the Whitney wattmeter is similar to the electro-dynamometer shown in Fig. 39.

The instrument is connected to the line as in Fig. 78, the polarity being selected so that the pointer *P* deflects from *O* to the side marked +. Turning the knob *K* clockwise brings *P* back to zero; the angle is shown by the index *I*. The small scale on both sides of *O* permits one to read watts differing slightly from those shown by the index *I*; this is convenient on fluctuating loads. Suppose, for instance, the reading on the large scale be 125 watts, and the pointer *P* indicate 4 watts clockwise; the actual reading is then 121 watts. If *P* had shown 4 watts on the + side, the reading would be 129 watts.

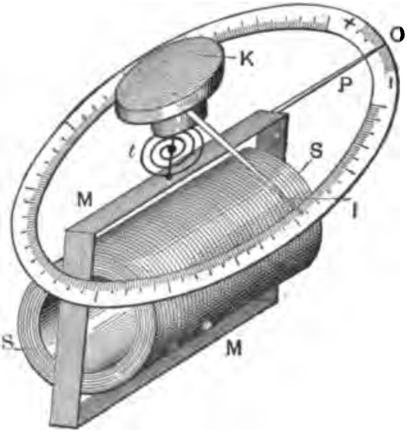


FIG. 82. The Hoyt (Whitney) wattmeter.

80. EXPERIMENT 4-A. — Calibration and Study of Indicating Wattmeters of Electro-Dynamometer Type. — The instruments are described in §§ 74 to 79; they are calibrated with direct current, and may be used on either direct or alternating current. Connect the wattmeter under test to a direct-current line and provide a suitable load; connect into the same circuit an ammeter and a voltmeter which have been previously standardized. Begin the calibration with the largest load —

read volts, amperes and watts. Gradually reduce the load, taking similar readings. In connecting the wattmeter keep in mind the explanations of § 77 and avoid errors due to energy consumption in the instrument itself. To see the influence of this factor, use the "Ind." (independent, Fig. 80) terminal instead of the regular one and note the difference in the indications with the same load. See if the correction I^2R accounts for the discrepancy; try this with a large and a small load. Observe the influence of terrestrial magnetism by reversing both currents in the instrument and take an average of the two readings to eliminate this effect. This is necessary with direct currents only.

Having calibrated the instrument try it on an A. C. circuit; change if necessary the ammeter and the voltmeter. First measure a non-inductive load, preferably incandescent lamps; then connect some inductance in parallel with it (as in Fig. 120), keeping total current constant. You will find that the watts decrease gradually, while the apparent power (volts × amperes) remains the same.

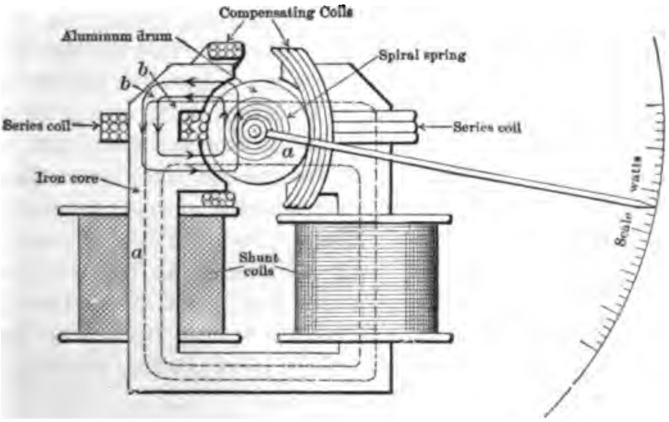


FIG. 83. The Westinghouse indicating wattmeter.

Report. Give a calibration curve in the form shown in Fig. 62. State your findings in regard to the energy consumption in the instrument itself and in regard to the influence of the terrestrial field. Give data showing that the true power is less than the apparent power when the load is inductive. Figure out the power factor of the load, according to equation (3) in § 75.

81. Induction Wattmeter. — An indicating wattmeter based on an entirely different principle is shown in Fig. 83. In its principle it is a single-phase induction motor (§ 534) the armature of which is not allowed to revolve, but is merely deflected, overcoming the tension of a spring. The action is the same as in the induction-type instrument shown in Fig. 43, except that the split phase is produced by series coils carrying the line current, instead of a coil short-circuited upon itself. Moreover, the moving element is an aluminum drum and not a disk. This last difference is, however, not essential: there are induction ammeters and voltmeters on the market, with the moving part in the form of a drum, and vice versa. The shunt coil is connected across the line, in series with a high inductance (choke coil). Therefore the current in the shunt coil lags practically 90° behind the applied e.m.f., and so must also the flux (shown by dotted lines) produced by the coil. The series coil carries the line current and produces a flux in phase with it; this flux is shown by lines drawn in full.

When the current is in phase with the applied voltage the aluminum drum is threaded by two fluxes at 90° to each other geometrically and also displaced in phase by 90° electrically. These are exactly the conditions necessary for producing a revolving flux (see § 520). This flux induces secondary currents in the drum, and the drum tends to follow the flux. The action is proportional to the magnitude of both component fluxes; consequently it is proportional to the product "volts times amperes," or to the power in the circuit. If the current is out of phase, only its working component contributes to the revolving flux, being 90° out of phase with the shunt flux; the flux produced by the wattless component does not increase the torque exerted on the aluminum drum. Thus with any value of the power factor, the action on the moving part is proportional to true watts.

In reality the current in the shunt coil lags less than 90° behind the applied voltage, because of some ohmic resistance in the potential circuit. Nevertheless it is possible to keep the shunt flux in exact quadrature with the applied e.m.f. by placing on the poles of the iron core a "compensating" winding. The compensating coils are short-circuited upon themselves and act as the secondary of a transformer of which the shunt coil is the primary. The diagram (Fig. 84) explains the corrective action of the compensating winding. Let E be the vector of the applied voltage, and i_1n_1 the ampere-turns on the shunt field; they are not quite in quadrature with E. The ampere-turns on the secondary or compensating winding are represented by i_2n_2; they are not exactly in opposition to the primary ampere-turns, because of the resistance of the compensating coils and of the magnetic leakage between the two windings. The total ampere-turns which produce the magnetic flux F in the pole-pieces are represented by the vector in, which is a geometrical sum of i_1n_1 and i_2n_2. By properly selecting the number of turns and the resistance of the compensating winding, the flux F is thus made to lag exactly 90° behind the applied voltage E.

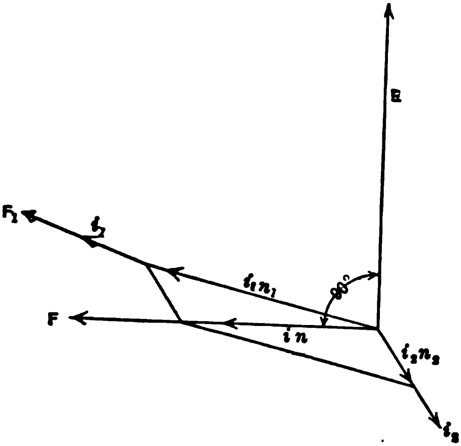


FIG. 84. Corrective action of the compensating coil of a wattmeter.

Induction wattmeters are robust, have quite a high torque, and are sufficiently accurate for ordinary practical purposes. Their accuracy is increased by their being usually provided with a long scale extending over 300°; a feature impossible in dynamometer-type wattmeters, unless a torsion knob is used (Fig. 82). Connecting the current and the potential coils in series or in parallel considerably increases the useful range of the instrument. Wattmeters of this type are built either as portable instruments or for switchboard service. For polyphase service two independent wattmeter movements are mounted on a common shaft, and the deflection on the scale measures total power of the system, with any distribution of load between the phases. With large outputs and high voltages, series and shunt transformers are used with wattmeters (Figs. 44 and 45).

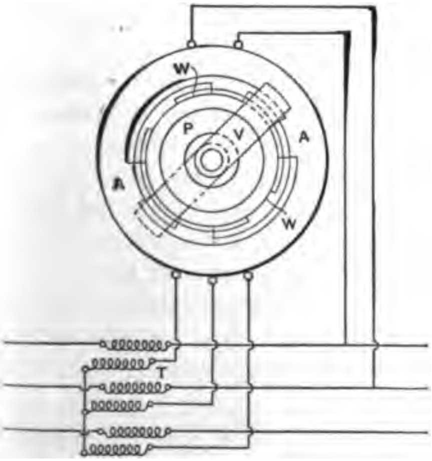


FIG. 85. The electrical circuits of the Westinghouse power-factor meter.

Induction wattmeters can evidently be used on alternating-current circuits only; moreover, their indications depend to some extent on the frequency of the supply. They are calibrated through the medium of a dynamometer-type wattmeter which in its turn is previously standardized with direct current.

82. Power-Factor Meter. — Some operating features of A. C. power plants and transmission lines depend essentially on the power factor of the load. By having on the switchboard the necessary ammeters, voltmeters and wattmeters the power factor of the output can be easily calculated at any moment. But it is much more convenient to have an instrument which shows power factor *directly*, without any computation. The principle of one type of such power-factor meters is shown in Fig. 85. The device can be used on polyphase lines only; this is, however, no objection, because practically all transmission plants are either two-phase or three-phase.

The stationary part consists of a laminated iron core A, provided with a regular two- or three-phase winding, and connected to the line through the series transformers T. The moving part consists of a soft-iron vane V, of a shape shown in Fig. 86. The stationary winding produces a rotating magnetic field (§ 520) under the influence of which alone the vane would revolve as the rotor of an induction motor. But this vane is also actuated upon by a stationary coil P, connected across one of the phases of the circuit. The pulsating field produced in this coil weakens the revolving field in a certain direction, and leaves it intact in the perpendicular direction. The moving vane then assumes this direction of maximum magnetic field; its position is indicated by a pointer on the scale.

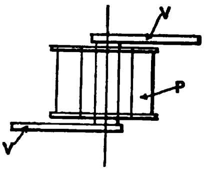


FIG. 86. The potential coil and the movable vane of the power-factor meter, shown in Fig. 85.

The currents in the polyphase winding of the instrument, being produced through the series transformers, are in phase with the line currents. The current in the coil P is in phase with the line voltage. Therefore the position of the vane will depend *on the phase relation* between the line current and the line voltage, or, which is the same, *on the power factor* of the load. The instrument is calibrated directly in per cent power factor, the calibration extending over all the four quadrants of the scale. This is necessary because currents may be leading or lagging, and the phase angle between them and the voltage will be less or more than 90 degrees, according to whether the machine is working as a generator or as a motor.

The power-factor meter is by no means an accurate measuring instrument; its indications are affected by the magnitude of the line current, and by the line voltage. But for a comparative judgment of the variations of power factor during the day it is a very useful instrument, and deserves its place on every well-equipped switchboard.

83. EXPERIMENT 4-B. — Study and Calibration of a Power-Factor Meter. — The instrument is described in the preceding paragraph. Provide a three-phase load as in Fig. 337 or Fig. 345; have an ammeter, a voltmeter and a wattmeter connected into the circuit by means of a polyphase board (§ 49). Connect the power-factor meter as in Fig. 85 and apply the largest inductive load possible with safety. Read the four instruments and gradually increase the power factor of the load, keeping the same total current, until the load becomes non-inductive. Repeat the same test with two or three smaller values of current. If an over-excited synchronous motor is available the calibration may be extended on the quadrants corresponding to leading current.

Before or after the calibration investigate a few features of the construction of the instrument:

(1) Open the potential circuit and observe the vane rotate under the influence of the field produced by the series winding alone. Reverse two of the series leads, whereupon the vane will revolve in the opposite direction.

(2) Reverse the potential leads; the pointer will turn by 180 degrees. This corresponds to the change from output to input, in other words, from alternator to synchronous motor.

(3) The instrument is intended to be read with balanced load only; unbalance the load and see what effect this has on indications of the power-factor meter.

INTEGRATING WATTMETERS.

84. Watt-Hour as a Unit of Consumption of Electrical Energy. — A certain amount of coal is required to generate electrical energy at the rate of one watt during one hour. The energy made use of by the consumer of electrical energy depends on the number of watts consumed, and on the time during which the power has been used. For these reasons it is proper to charge for electric power on the basis of watt-hour or kilowatt-hour consumption. A 16 candle-power incandescent lamp consumes about 56 watts; hence it uses during one hour 56 watt-hours of electrical energy. Two lamps, if burned half an hour each, or 4 lamps used a quarter of an hour each, also consume together the same amount of energy. Within certain limits it does not make any difference to the company supplying the power during what period of time a certain amount of energy is consumed. If the price is ten cents per kilowatt-hour, the cost to the consumer is in either case

$$10 \times \frac{56}{1000} = 0.56 \text{ cent.}$$

Expressed more accurately, the cause for measuring electrical energy in watt-hours, instead of simply in watts, is that one watt represents a certain amount of energy liberated or absorbed *in one second;* in other words, it is only a *rate* of expenditure of energy. This follows from the definition of watt, it being a product of "volt × ampere," where the ampere is a quantity of electricity *per second.* On the other hand, the energy contained in one lb. of coal is measured in B.T.U.'s, calories, joules, etc., units independent of time. Therefore watts must be multiplied by time in order to reduce them to the same physical conception, and to the same dimensions as the heat units.

Instruments which measure watt-hours energy consumed in a circuit during a certain period of time, are called integrating wattmeters — sometimes they are referred to simply as "electric meters." The monthly bill of a customer is usually made up on the basis of indications of the integrating wattmeter on his premises.

85. Types of Integrating Wattmeters. — Meters used at present are of the motor type almost exclusively, a small motor being arranged to revolve at a speed proportional to the rate at which energy is passing through it. The number of revolutions is recorded on a dial; by knowing the constant of the meter this number of revolutions can be reduced to kilowatt-hours. In most meters the shaft of the motor is geared to the recording mechanism in such a way that the meter reads directly in kilowatt-hours.

Two types of *indicating* wattmeters are used, — dynamometer type (§ 76) and induction type (§ 81); to these there correspond two types of *integrating* wattmeters, — commutator meters (Fig. 87) and induction meters (Fig. 92). A direct attraction between stationary and revolving coils is utilized in the first type, and such integrating meters may be used on either direct- or alternating-current circuits. Induction meters are based on the principle of the revolving magnetic field and can be used on alternating-current circuits only. With the advent of the induction meter, the commutator meter is relegated to direct-current circuits only, the induction meter being simpler and more accurate. The so-called Sangamo meter described in § 89 below, was produced as an effort to do away with the commutator in direct-current integrating wattmeters.

On D. C. circuits on which the voltage is kept practically constant, watts are proportional to amperes, and it is sufficient to measure ampere-hours, instead of watt-hours. Multiplying the former by the actual or agreed voltage of the supply gives watt-hour consumption. Electrolytic meters are to some extent used in such cases, especially abroad. The current flowing through the meter decomposes a certain chemical contained in it. The amount of material decomposed is proportional to coulombs or ampere-hours which pass through the meter; the scale can be made direct-reading.

86. Commutator-Type Integrating Wattmeters. — The commutator meter (Fig. 87) consists of a small direct-current, vertical shaft motor, without iron in its magnetic circuit. The armature *A* is connected across the line with some additional resistance *R* in series, according to the voltage of the supply; the fields *F* are in series with the line. Thus the current flowing through the armature is proportional to the line voltage, and the current flowing through the field is equal to the line current; hence the attraction between the field and the armature is proportional to the product "line voltage times line current," or proportional to the power delivered. In this respect it is similar to the indicating wattmeter shown in Fig. 78. Fig. 88 gives the general view of this meter.

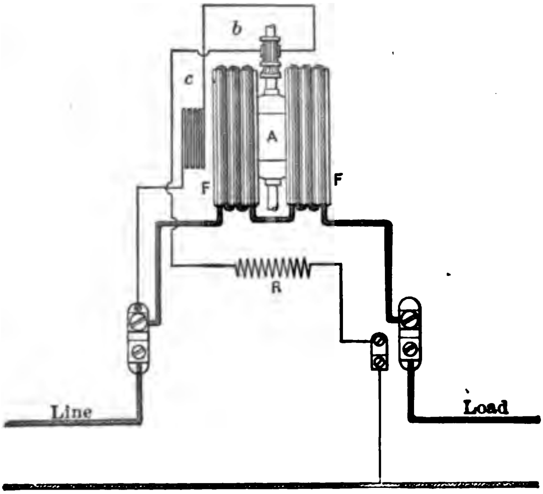


FIG. 87. The electrical connections in a Thomson integrating wattmeter.

A motor connected in this way would run away without a load. To make its speed proportional to the power transmitted, the meter is provided with a Foucault-current brake, consisting of a copper or aluminum disk *D* mounted on the armature shaft and placed between the poles of permanent magnets *MM*. To understand the action of this brake, assume first that the meter has no friction; at the full-rated load the armature revolves at a certain speed, determined by the counter-torque of eddy currents in the brake. Now if the load, or the product volt-amperes, falls, say, to $\frac{3}{4}$ of its former value, the attraction between the field and the armature of the motor drops in the same ratio and the motor slows down. This decreases the eddy currents in the brake

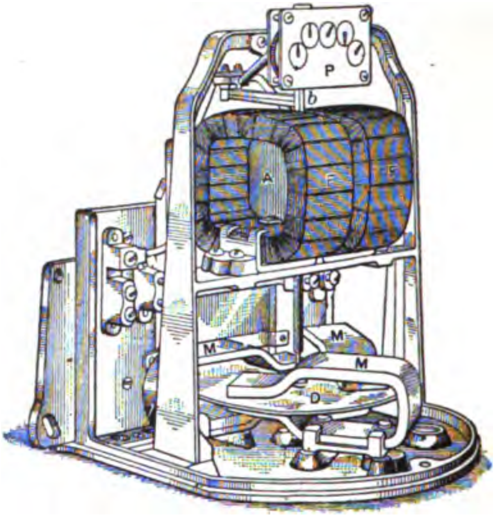


FIG. 88. A general view of the Thomson integrating wattmeter.

(which currents are proportional to the speed); at $\frac{3}{4}$ of the initial speed the counter-torque is again equal to the motor torque. Thus, neglecting friction, the speed of the meter is proportional to the load, and it should register correctly at all loads.

The speed of the meter is usually adjusted so as to make it direct-reading, or at least have simple multiples as 2, 10, etc. This is done by adjusting the position of the permanent magnets of the brake so as to get the required amount of eddy currents induced in the disk *D*. A meter dial is shown in Fig. 89; with the position of the hands shown

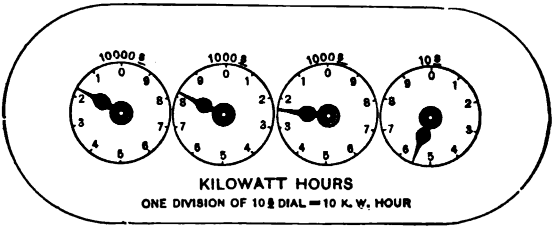


FIG. 89. The scale of an integrating wattmeter.

there the reading is 18255 kw.-hours. If the reading next month is, for instance, 18490 kw.-hr. the consumption during the month was

235 kw.-hr., and at a price of, say, 10 cents per kw.-hr. the monthly bill will amount to $23.5.

It can be seen from Fig. 87 that one of the series or current terminals is used at the same time as one of the potential terminals. The connection is made inside of the meter so that only three leads come out of the cover.

87. Compensating for Friction. — Friction causes the meter to run slow on small loads; as in a great majority of cases meters run on light load most of the time, this would represent an appreciable loss to the company supplying the current. For instance, a 20-lamp (10 amp.) meter would not start at all (without the compensating device described below) if only one lamp of usual size were burning; this might induce some customer to leave a lamp permanently in the circuit.

To remedy this, a compensating coil *c* (Fig. 87) is provided in the armature circuit; the field produced by this coil gives an additional torque just sufficient to balance the friction of the meter. As it is impossible to predetermine the exact amount of friction, the more that it varies with the pressure of the brushes on the commutator, with the wear of the jewel supporting the shaft, etc., this compensating coil is made adjustable. The complete arrangement is shown in detail in Fig. 90. *FF* are the main field coils, *C* is the compensating coil. Its distance from the armature can be varied by means of the screws *A* and *B* and the guide *D*.

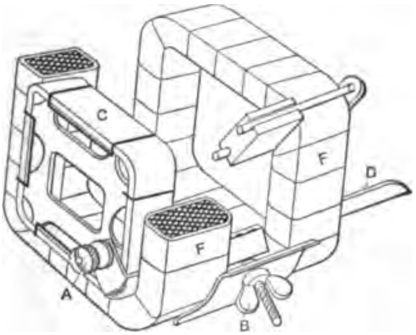


Fig. 90. The friction compensation in Thomson integrating wattmeters.

In some meters the compensating coil is stationary, but is provided with several taps connected to the buttons of a dial. By means of a small lever more or less turns of the compensating coil may be introduced into the circuit, and in this way the amount of compensation varied at will.

If the meter is not sufficiently compensated, the company is losing money on light loads; if the meter is over-compensated, it *creeps* at no load and registers energy even when no current is used. This naturally leads to a complaint on the part of the customer; he refuses to pay the bill, becomes prejudiced against using electric power, and the company is again losing money. It is commonly appreciated now that honesty in regard to meter calibration and adjustment is the most profitable policy on the part of illuminating companies, and the best advertisement for obtaining new customers.

88. EXPERIMENT 4-C. — Calibration of Commutator-Type Integrating Wattmeters. — Integrating wattmeters are calibrated by comparison with a good indicating wattmeter. To perform the calibration by maintaining a constant load until the reading on the dial of the integrating wattmeter had changed a reasonable amount would, however, consume too much time, and involve a useless loss of electrical energy; therefore the following method is employed. A desired load is put on both meters and read on the indicating wattmeter, or on an ammeter and a voltmeter; the number of revolutions of the armature shaft of the integrating wattmeter during a minute or so is counted (use a stop watch). The reduction ratio of the recording train is usually an even number, for example, 100, 200, etc., and knowing this ratio the wattmeter error can be easily calculated.

Suppose, for instance, that a constant load of 180 kw. was put on an integrating wattmeter and kept constant by means of an indicating wattmeter. Suppose that 15 revolutions of the armature disk be counted during 28 seconds, and that the reduction ratio of the recording gear be 1000 ÷ 1. If the value of one complete revolution of the pointer on the lowest dial is 100 kw.-hr. (see Fig. 89), the armature disk must complete one revolution while 100 ÷ 1000 kw.-hours, or 0.1 kw.-hr. is delivered to the load circuit. During the test, an energy equal to 180 × 28 kw.-seconds, or

$$180 \times \frac{28}{3600} = 1.4 \text{ kw.-hrs.}$$

has been delivered to the circuit. Therefore, the disk should have completed

$$1.4 \div 0.1 = 14 \text{ revolutions.}$$

In reality it made 15 revolutions; thus, at this particular load the meter runs about 7.1 per cent fast, and consequently registers 7.1 per cent more energy than is actually consumed.

Before beginning the calibration, investigate the action of the friction compensating device, and the behavior at light loads of the meter without the compensating coil connected in. The test consists in determining at what per cent of full load the meter can be made to positively start without danger of "creeping" at no load. Also try suddenly throwing off a heavy load and see if the meter stops in a short time. Sometimes a meter correctly adjusted on a testing rack begins to creep when put on a wall where it is subjected to jarring from the street or from an engine working near by. Try the effect of slightly jarring the meter under test and see what margin should be allowed for this effect. *Before beginning the calibration proper, adjust the meter so that it reads as accurately as possible on light loads.*

(a) Calibrate the meter with direct current; begin with an overload of about 25 per cent and gradually reduce the load to zero.

(b) Then calibrate with alternating current at a high and at a low power factor, to see if the calibration constant remains the same. When running at a low power factor, the error caused by the self-induction of the potential circuit is more noticeable (see § 78). The following form of data sheet will be found convenient, especially if more than one meter are calibrated together.

Meter No.__________ **Name**__________ **Type**__________
" " __________ " __________ " __________
" " __________ " __________ " __________
" " __________ " __________ " __________
" " __________ " __________ " __________

Rated Volts____ **Rated Amps.**____ | **Gear ratio: one rev. of disk** = ____ **watt hrs.**
" " ____ " " ____ | " " " " " = ____ " "
" " ____ " " ____ | " " " " " = ____ " "
" " ____ " " ____ | " " " " " = ____ " "
" " ____ " " ____ | " " " " " = ____ " "

				Wattmeter No.____		Wattmeter No.____		Wattmeter No.____	
	Volts.	Amps.	Watts	Rev.	Sec.	Rev.	Sec.	Rev.	Sec.
Inst. No.									
Const.									

(c) Determine the torque per watt per unit weight of the moving element, and the friction-torque ratio as explained in § 92.

Report. (a) Plot calibration curves of the meter, for D. C. and A. C. Use current in per cent of full-load current as abscissæ, and per cent "slow" or "fast" as ordinates.

(b) Give your results as to light-load adjustment and creeping at no load.

(c) Give the specific torque and the friction-torque ratio of the meter.

(d) Mention features of construction different from those described above.

89. Sangamo Meter. —The above-described commutator meters have two drawbacks:

(1) The commutator with its brushes is liable to give trouble, or get out of order. This is particularly annoying in the case of integrating meters which are sealed and placed on customer's premises, and thus cannot be conveniently watched.

(2) Absence of iron necessitates a comparatively large number of turns on both windings; this makes the moving part heavy, increases bearing friction and causes jewel troubles.

These two objections are eliminated in an original integrating meter recently put on the market under the name of "Sangamo" meter. In its principle (Fig. 91) it is a homopolar electric motor, the armature D of which floats in mercury M and is subjected to a comparatively strong field of a shunt coil S provided with an iron core CC. The main current passes from the terminal T_1, through mercury M to the armature D, which consists of a copper disk, floating in the mercury; thence the current passes to the terminal T_2 again through the mercury. The energizing coil S is connected across the line. Thus the field of this meter is proportional to the voltage of the supply, and the armature current to the load current. The attraction between the armature and the field is thus proportional to volts times amperes, or to the power to be measured. This meter is provided with an eddy-current brake (not shown in Fig. 91) similar to that in Fig. 88, and so its speed is proportional to the watts consumed in the load circuit. Meters of this type intended for large currents are provided with outside shunts similar to ammeter shunts (Fig. 35). Friction is com-

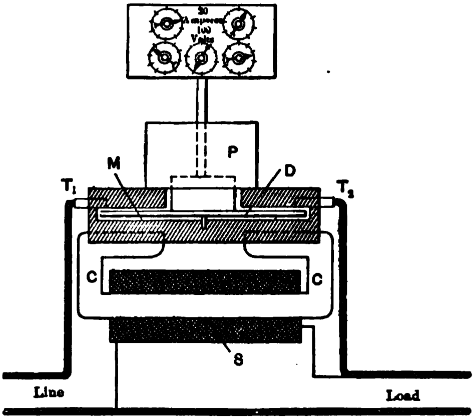


FIG. 91. The general arrangement of Sangamo integrating wattmeters.

pensated for by an adjustable shunt, or by-pass, which allows a small current to flow constantly through the revolving disk, even at no load, thus creating an additional torque.

It should also be mentioned that the meter is provided with a pocket *P* of such a form that the mercury cannot possibly be spilled out whatever the position of the meter during transportation.

The Sangamo meter has no commutator or brushes, so that this source of trouble is removed; the armature is much simpler and lighter than that of commutator meters. Moreover, its weight does not rest on the lower bearing, since the disk is floating in mercury and is balanced so that there is just a little buoyancy upward. A strong field makes it possible to obtain a higher torque; this makes the action of the meter more positive on light loads. Whether these advantages are counterbalanced by the presence of mercury, and how satisfactory the meter will prove under severe conditions of actual service, remain to be seen.

Sangamo meters can also be used on alternating-current circuits. As, however, they are based on the principle of direct attraction between the armature and the field, the field flux must be in phase with the armature current (at non-inductive loads), instead of being in quadrature with it, as in induction meters (§ 81). The shunt winding *S* possesses some self-induction; it is neutralized by connecting some capacity (a condenser) in series with it, to make the circuit practically non-inductive; the iron core must of course be laminated. In all other respects D. C. and A. C. Sangamo meters have the same construction.

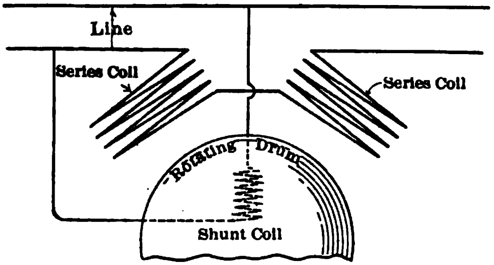


FIG. 92. Arrangement of coils in an induction type wattmeter.

90. Induction Meters. — Alternating-current integrating wattmeters operate on the principle of the revolving magnetic field, as single-phase induction motors. The general arrangement of the circuits in one of the popular makes of such meters is shown in Fig. 92. The wattmeter has two windings, one in series with the line, another shunted across the line. Both windings are stationary, and produce together a rotating magnetic field in which a light aluminum armature revolves, as a squirrel-cage rotor of an induction motor. The two windings are clearly seen in Fig. 93; Fig. 94 represents the revolving part, or aluminum armature of the meter. The meter is similar to the indicating wattmeter described in § 81, and the explanation there given of its action is fully applicable here. In order to make the speed of the meter proportional to the load, an eddy-current brake is provided, as in commutator meters (§ 86). Only here it is not necessary to have a separate disk on which permanent magnets act. The same aluminum rotor can be used as the brake armature (Fig. 95);

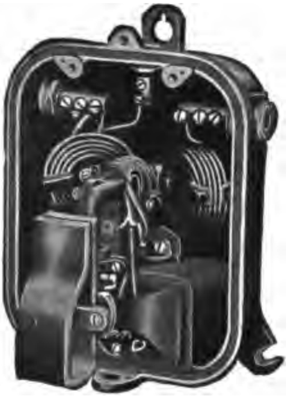


Fig. 93. A view of the Fort Wayne integrating wattmeter, showing the series and the potential windings.

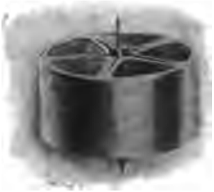


Fig. 94. Revolving aluminum armature.

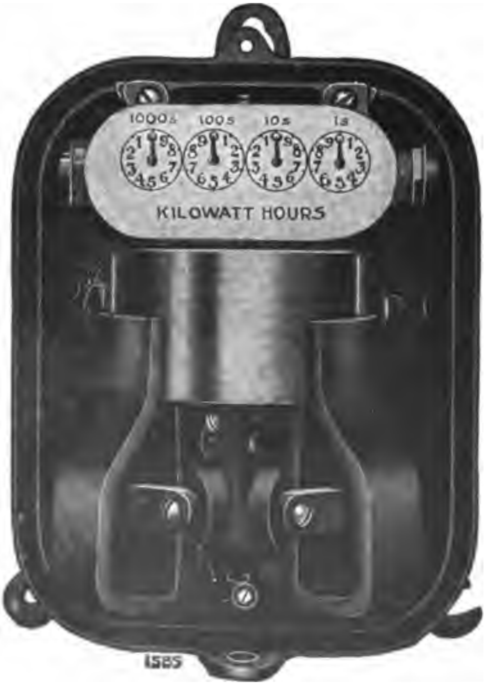


Fig. 95. General view of a Fort Wayne induction type integrating wattmeter.

this makes the meter less expensive and more compact than direct-current meters.

An induction meter of different construction is shown in Fig. 96. The lower coil is the series coil; *BB* are two potential coils. The armature has the form of a disk (not shown in figure) and revolves in the space between the series and the potential coils. Friction is reduced to a negligible amount by having the moving element exceedingly light; moreover, the lower end of the shaft rests on a steel ball instead of a jewel, thus substituting rolling friction for the ordinary jewel friction.

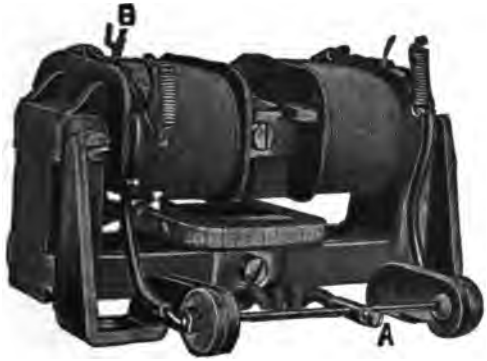


Fig. 96. Magnetic circuit of the Westinghouse integrating wattmeter.

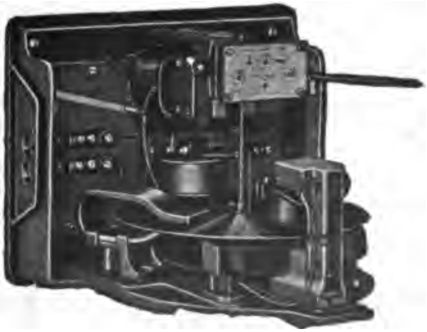


Fig. 97. General Electric polyphase integrating wattmeter.

Polyphase integrating meters are merely a combination of two single-phase meters, whose armatures are mounted on the same shaft and act on the same recording train, thus automatically adding together the indications of the two component meters. Some polyphase meters have separate armatures for each component meter; in some constructions, however (Fig. 97), both revolving fields act on the same aluminum disk. The brake magnet also acts on the same disk, thus the whole construction is made light and compact.

The method for connecting polyphase meters into a three-phase circuit is shown in Fig. 98; the scheme of connections is based on the fact that each of the three line wires, for instance *B*, can be considered as a common return wire for two other line wires *A* and *C*. The three upper connections belong to the upper component meter, the three lower connections to the lower meter. There are no electrical connections between the two component meters inside of the case. The upper meter registers the energy between the wires *A* and *B*; the lower meter, that between *C* and *B*. Three terminals are used instead of four, because one of the current terminals serves at the same time as a potential terminal (see Fig. 87). For the theory of power measurement in three-phase system, see § 430.

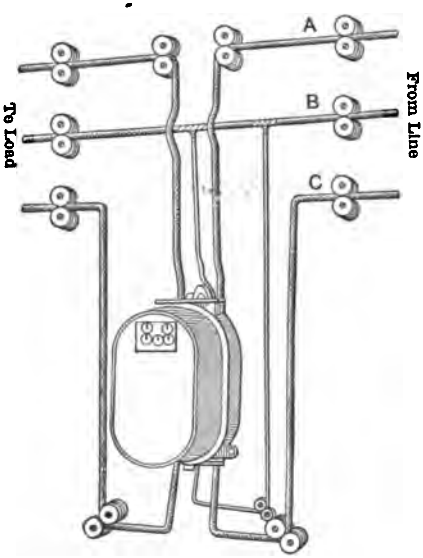


FIG. 98. Connections between a three-phase line and an integrating wattmeter.

91. Correction for Phase-Angle. — A compensating winding is necessary in induction-type integrating wattmeters in order to bring the shunt flux in exact quadrature with the terminal voltage; the explanation for this is the same as given in § 81. In the meter shown in Figs. 92 and 93 this is accomplished by an additional shunt winding *G* (Fig. 99). *CC* is as before the series winding, *D* is the regular shunt winding connected across the line through a reactance coil *I*. The compensating coil *G* is connected across a few turns of the reactance coil *I* and is placed inside the shunt coil *D*, on a separate iron core. Resistance *H* in series with the compensating coil (lagging resistance) is adjusted so that the meter stands still at a full inductive load; this is a check for the compensation.

A third winding *E* is shown, short-circuited upon itself through an adjustable resistance *L*. This winding makes possible the use of the same meters with two standard frequencies: 60 cycles and 140 (or 133) cycles. When the meter is used on a high-frequency circuit, the coil *E* is inoperative; but when the meter is connected into a 60-cycle circuit the compensation offered by the coil *G* is not sufficient. Then the circuit of the coil E must be closed, and the resistance L adjusted until the meter again stands still at a purely inductive load.

It would be hardly possible in practice to obtain a load at the power factor zero; therefore this adjustment is made on a two-phase circuit. The series winding of the meter is connected to one phase through a non-inductive load; the potential winding is connected to the other phase, which is exactly in quadrature with the first phase. In this way conditions are created that would take place in a purely inductive circuit.

In the meter shown in Fig. 96 compensating coils are wound on the same spools with the shunt coils B, and are short-circuited upon themselves through a resistance. This resistance can be adjusted by means of the slide-contact A so as to bring the shunt flux into the exact quadrature position.

In describing commutator meters a friction compensating winding or the light load adjustment (§ 87) was mentioned. Such a compensation is of much less importance in induction meters, since their moving element is lighter, and the torque per unit weight of the moving element much higher, so that the influence of the pivot friction is not so marked. The necessary compensation in the meter shown in Fig. 99 is obtained by making the field initially somewhat unsymmetrical, so as to give the armature a one-sided pull, just sufficient to balance the friction. Namely, the phase-correcting coil G is wound on a movable arm A, and this arm can be set in such a position that the meter will start at a certain small percentage of full load. The meter shown in Fig. 96 has no friction compensation whatever; the moving part is supported by a steel ball, and the friction is reduced to a negligible amount.

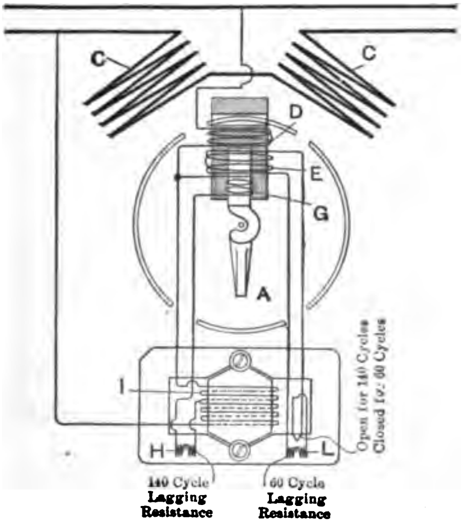


FIG. 99. Phase adjustment in an induction-type integrating wattmeter.

92. Torque and Friction. — It is now generally agreed that an integrating meter is better, the higher its torque per unit of weight of the moving element. This is because the disturbing influence of the pivot friction is easily overcome, and the meter registers more accurately at light loads. There is a special instrument on the market, the so-called *torque balance*, by means of which the torque on the shaft of a meter can be directly measured in gram-centimeters, and two meters of different make compared.

The instrument is shown diagrammatically in Fig. 100. A light arm T is clamped to the shaft S of the meter under test, and is connected by a link L to a sensitive balance GH. The balance has a knife-edge support at C; the position of equilibrium is indicated by the pointer CO. A definite weight G is attached to the balance, causing it to tip toward the left. Then the meter is loaded electrically until the pull on the link L brings the pointer O back to zero. The critical load is measured on an indicating wattmeter; the mechanical torque is calculated from the weight G and the leverage $TLVC$. From these data the torque is calculated per watt of load and per ounce weight of the revolving part of the meter, supported by the lower bearing. This gives a basis for the comparison of competitive meters of similar construction. The rods V and T are provided with several

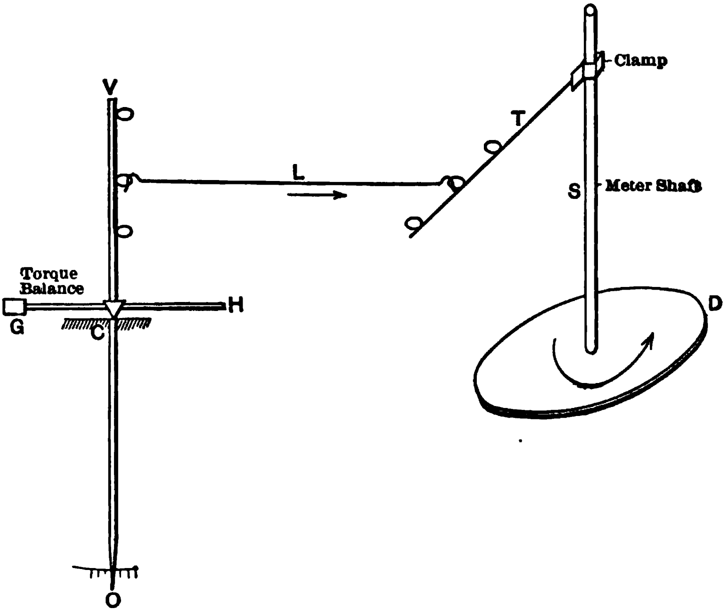


FIG. 100. A torque balance, for testing integrating wattmeters (General Electric Co.).

loops, so as to vary the torque in steps; two weights are supplied with the instrument, to cover a wide range of meters to be tested.

Meters can also be compared in this respect by determining their so-called *friction-torque ratio.* The friction compensator is adjusted at no-load so that the meter is just balanced, i.e., so that it will creep under the slightest vibration. When this is done, a load is applied equivalent to the full capacity of the meter and the speed of the revolving part measured. Then the friction compensator is removed and the speed measured again at the same load. The percentage decrease in speed is proportional to the friction-torque ratio; thus two per cent would mean a friction-torque ratio of 1: 50; one per cent 1: 100, etc. The higher this ratio, the better is the meter, at least as regards the disturbing influence of its friction.

93. EXPERIMENT 4-D. — Calibrating Induction-Type Integrating Wattmeters. — For instructions see Exp. 4-C. In addition to the tests specified there, it is interesting to calibrate one of the induction meters at various frequencies and with different wave-form. Theoretically the calibration must differ under these circumstances, but it is claimed for good modern meters that they are "practically" accurate within certain limits of wave-form and frequencies. It is not necessary to repeat the complete calibration curves; one or two points accurately observed are sufficient to enable the observer to judge if the calibration remain the same. When calibrating an induction meter, it is advisable to take at least three separate curves: at non-inductive load, at a moderate load of constant power, but varying power factor, and at a load of constant-current value, with varying power factor. There are meters on the market, which register correctly at non-inductive loads only, and run too slow at inductive loads.

94. Wright Maximum-Demand Indicator. — The actual cost of supplying a certain number of kilowatt-hours of power to a customer depends on the rate at which he consumes his energy. A customer who uses one kilowatt regularly for eight hours a day is more desirable to the operating company than another who uses 4 kilowatts for two hours. The amount of energy per month is the same in both cases; but the first customer requires less capacity of the generating apparatus and a smaller transmission line than the second one; he also causes smaller fluctuations of the load. Therefore the company may give him a better rate per kilowatt-hour. This principle of "discrimination" in charging for electrical energy is used in some cities where the customer is charged so much per kilowatt-hour of the actually consumed energy, and then an extra charge is made for each kilowatt of his *maximum demand.* Instruments for measuring the maximum load or maximum demand during a month, or other agreed period of time, are called maximum-demand indicators.

The Wright maximum-demand indicator, used to some extent in this country and in England, is shown schematically in Fig. 101. It records the maximum current which has passed through it at any time since it was last set. A liquid is hermetically sealed in a glass tube having a bulb at each end. Around the left, or heating bulb, is placed a band of resistance metal, through which passes the current to be measured. The passage of the current heats the air in the bulb, and the expansion of the air forces the liquid up into the right-hand side of the tube, causing it to overflow into the middle or indicating tube. The liquid deposited in the indicating tube remains there until the indicator is reset. The glass tube is carried on a backing, so hinged that the meter can be reset by tipping the tube and allowing the liquid to run out of the indicating tube into the side tubes. It is the difference in temperature of the air in the two bulbs, that causes the indicator to register. Any change in the temperature of the external air causes equal air expansion in both bulbs, and hence does not affect the reading.

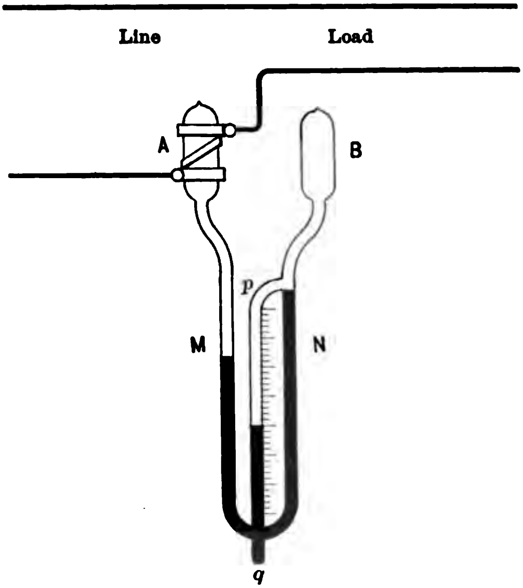


FIG. 101. Wright maximum-demand indicator.

The instrument is purposely made slow-acting; standard indicators are so designed, that if the maximum load lasts only five minutes, the meter will register about 80 per cent; if ten minutes, 95 per cent, and the full 100 per cent is registered when the load has continued about forty minutes. In this way the customer is not penalized for short overloads which do not inconvenience the supply station in any way.

95. EXPERIMENT 4-E. —Testing Maximum-Demand Indicators. —The test consists in calibrating the scale of the instrument for loads carried indefinitely (more than 40 minutes). While this is being done, per cent indication may also be determined for loads carried for shorter periods of time. Begin with a slight load and read the level of the fluid in the middle tube say every five or ten minutes until it becomes stationary. Tip the instrument over to reset the liquid, and repeat the same experiment with a somewhat larger load, etc. The left bulb must be cooled off before putting on a load.

Report. Plot a calibration curve for the final levels of the fluid, and a few curves showing per cent indication on loads carried for a shorter time.

CHAPTER V.

REACTANCE AND RESISTANCE IN A. C. CIRCUITS.

96. Physical Conception of Inductance.* — When a current flows through a conductor, a magnetic field or flux is created around it. As long as the current remains constant, this field does not react in any way upon the electric circuit. But when the current varies, the flux also necessarily changes; in doing so it induces in the conductor an electromotive force, in other words, reacts on the circuit and affects the rate of change of the current. This induced e.m.f., in accordance with the fundamental law of electromagnetic induction, has such a direction as to oppose the change in current, consequently to retard the change in flux. The action is very much as if the magnetic field had some kind of inherent *inertia.*

Sir Oliver Lodge gave the following striking picture, or analogy of this phenomenon. Imagine the magnetic field surrounding a conductor as consisting of whirls in ether driven by the current; assume these whirls to be endowed with some inertia. As long as the current is steady, the whirls are spinning at the same speed, and the effect of their inertia does not come into play. If now the applied e.m.f. be reduced, tending to reduce the current, the whirls by virtue of their inertia tend to spin at the same speed, and thus oppose the decrease of current. The current gradually decreases, and the field returns some of its stored energy to the source of the e.m.f. On the contrary, should the current be increasing, the whirls oppose its rise; the applied e.m.f. has to perform some additional work in accelerating the whirls. The opposing action of the whirls, or of the magnetic field, is stronger, the greater the flux; also the quicker occur the changes in the applied e.m.f. (the greater its frequency). This is analogous to the reaction of a heavy fly-wheel driven at a non-uniform speed — the reaction is proportional to the inertia of the wheel, and to the rate at which it is accelerated.

Whatever the explanation of the phenomenon, the observed fact is that electrical conductors oppose changes in current by the generation of a counter-e.m.f. *This reaction of conductors, expressed numerically, is called their inductance.* The reason for the name is that the reaction consists in "inducing" a counter-e.m.f. The practical unit of inductance in the volt-ampere-ohm system is called the henry. An electrical device is said to have an inductance of 1 henry, if 1 volt of counter-e.m.f. is induced in it, when the current changes at a rate of 1 ampere per second. The older name for inductance is the coefficient of self-induction.

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* The property of conductors called *reactance* is a composite quality which includes the frequency of the circuit, and a physical property of the conductor itself, called its inductance. Therefore, in order to understand reactance, it is necessary first to form a clear physical conception of inductance.

97. Difference between Inductance and Ohmic Resistance. — The presence of inductance in an alternating-current circuit necessitates an increase in the applied e.m.f. in order to produce the same current; similarly, an increase in resistance has the same effect. Thus, it may at first seem that, from a practical standpoint, resistance and inductance produce the same effect on the relations in the circuit, and do not need to be distinguished. There is, however, a vast difference between the two, as a consideration of the following points will show:

(1) Ohmic resistance is noticeable to the same extent whether current is steady or variable; the drop caused by it is at any moment proportional to the instantaneous value of the current. Inductance becomes apparent only when the current is varying; the induced e.m.f. or the resulting drop in potential, being proportional to the rate of change of the current and not to its absolute value.

(2) Resistance is due to some molecular friction in the conductor itself; inductance is caused by the inertia of the magnetic flux surrounding the conductor.

(3) Energy spent in overcoming ohmic resistance is converted into heat, and is lost electrically. Energy applied for overcoming inductance is stored in the form of electromagnetic energy of the field, and is periodically given back to the circuit through the medium of induced e.m.f.

(4) Ohmic resistance does not change with the shape of a conductor, inductance depends essentially on the form of the conductor. A wire, whether straight or wound into a coil, has the same ohmic resistance, while its inductance in the second case is increased many times because of the concentration of magnetic flux.

98. Inductance and Reactance. — Inductance is usually considered in electrical engineering as to its effect on alternating-current circuits of a constant frequency. It is convenient, therefore, to include the element of frequency into the value of inductance; this simply means a change in the unit and the dimension of the quantity, which expresses the inertia of the magnetic field. The new physical quantity is called the *reactance* of a circuit or of a device, and is expressed in ohms like ordinary resistance.

The following considerations lead to the conception of reactance. According to the above definition of inductance, the counter-e.m.f. set up in a coil having an inductance of L henrys is

$$e = L\frac{di}{dt} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

where $di \div dt$ is the rate of change of current with the time. We assume first that the coil has no ohmic resistance to be overcome. In this case the induced counter-e.m.f. is at any moment equal and opposite to the applied e.m.f. Otherwise a finite difference of e.m.f.'s in a conductor of zero resistance would produce an infinitely large current.

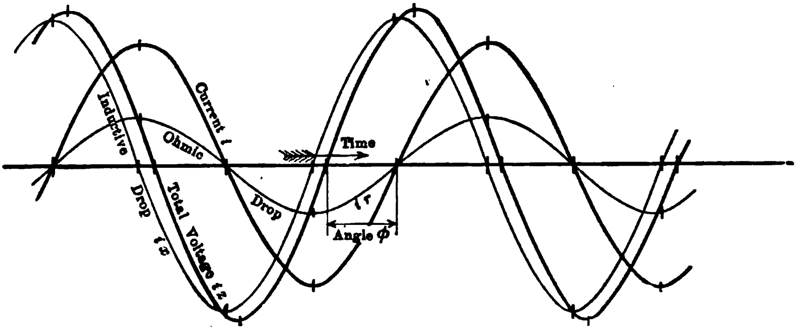

FIG. 102. Instantaneous values of current and voltages in an alternating-current circuit containing resistance and inductance.

Let the current vary according to the sine law, Fig. 102, at a frequency of n periods per second. The instantaneous values of the current may be expressed by the formula

$$i = I \text{ Sin } 2\pi nt \quad . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

where I is the amplitude of the wave, and t is time in seconds. Substituting (2) in (1) we get

$$e = 2\pi nLI \text{ Cos } 2\pi nt \quad . \; . \; . \; . \; . \; . \; . \quad (3)$$

This result interpreted means, that a sinusoidal current sets up a sinusoidal counter-e.m.f. of the same frequency, and of the amplitude

$$E = 2\pi nLI.$$

Denoting $$2\pi nL = x \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (4)$$

we get $E = xI$; the same relation holds true for the effective values of current and voltage (see § 36 *c*); thus we obtain

$$E_{eff} = xi_{eff} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (5)$$

The quantity x is called the *reactance* of the coil, or of any other electrical device under consideration. Equation (5) shows that reactance, being a ratio of voltage to current, is expressed in ohms like ordinary resistance. It will be seen from (4) that reactance is a composite conception; its value depends on the inductance of the device, and on the frequency of the supply. As frequency usually remains constant in practice, reactance also remains constant (save when the conditions surrounding the conductor are changed).

Pulsations of current i and of voltage e do not reach their maxima simultaneously; the current passes through its zero value when the e.m.f. reaches its maximum, and vice versa. This is shown by the equations (2) and (3), for — while the current is expressed by a sine function — the voltage varies according to a cosine function. The two waves are shown in their relative phase positions in Fig. 102; see the curves marked "current i" and "inductive drop ix." The waves are said to be displaced in phase by 90 electrical degrees, since

$$\text{Sin } 2\pi nt = \text{Cos } (90° - 2\pi nt).$$

The above discussion shows the convenience of using reactance x instead of inductance L, and the relation between the two.

99. Experimental Determination of Reactance. — Reactance of a coil AB of low resistance (Fig. 103) may be determined experimentally, by measuring the current flowing through the coil and the A. C. voltage at its terminals. The reactance is equal to the ratio of the voltage to the current, according to equation (5). The current is measured on the ammeter $Am.$, the voltage by the voltmeter V connected across the terminals of the coil. The rheostat R is for the purpose of regulating the current.

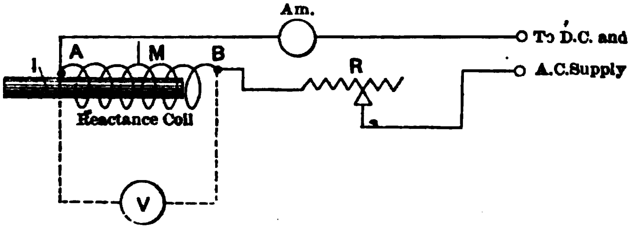

FIG. 103. Resistance and reactance in series.

An iron core I is shown within the coil; the presence of iron intensifies the flux produced by the coil, and thus greatly increases its reactance. By moving the core in and out the reactance may be varied within wide limits. This method is often used for regulating current in A. C. circuits.

Three factors influence the accuracy of determination of reactance by this method: The ohmic resistance of the coil, the iron loss in the core (hysteresis and eddy currents), and higher harmonics in the supply voltage. These factors may either be kept down by a suitable choice of conditions, or their influence may be determined and corrected for, as is explained below.

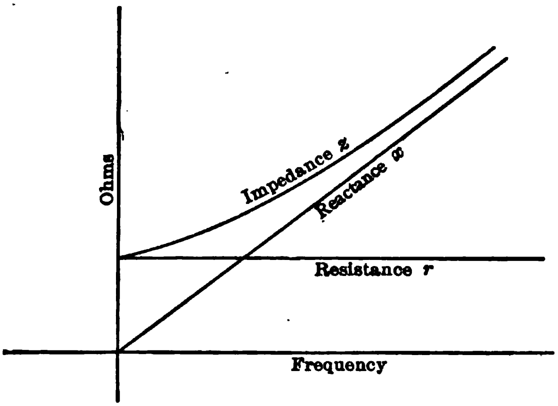

FIG. 104. **Influence of frequency on reactance and impedance (resistance and inductance are kept constant).**

100. Factors which Affect the Value of Reactance.—The reactance of a coil depends on the following factors:

(a) Frequency of the supply (Fig. 104).
(b) Presence of iron (Fig. 105).
(c) Intensity of current (Fig. 106).
(d) Number of turns in the coil.

In these diagrams the student is asked to pay attention, for the time being, only to the curves marked "Reactance x." It will be seen from these curves that:

(a) Reactance is directly proportional to the frequency of the supply, in accordance with the definition of reactance; see equation (4).

(b) Reactance depends essentially upon the presence and the position of the iron core, or plunger. The reactance decreases as the plunger is drawn out of the coil; the limiting value is that which the coil has without iron.

(c) With iron, the value of reactance depends on the intensity of the current flowing through the coil. This is because the magnetic flux is *not* proportional to the current, the iron approaching its saturation limit (§ 141). The reactance reaches its maximum at the point of maximum permeability of iron, in other words, where the flux per 1 ampere of current is a maximum. Without iron, there is no reason why reactance should depend on the value of the current, as both the

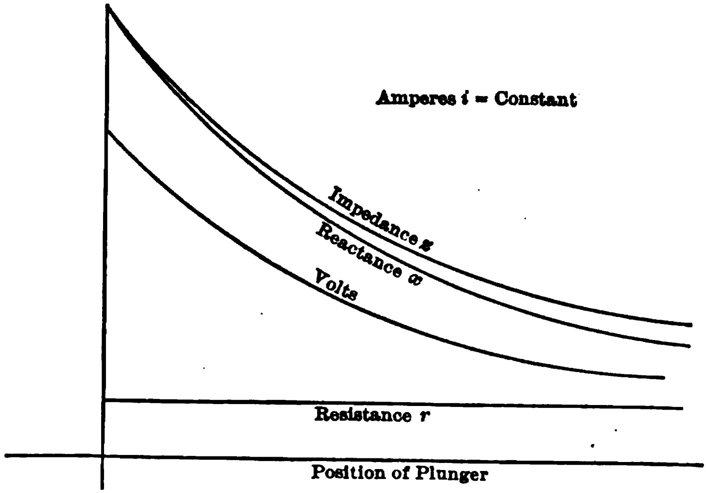

FIG. 105. Influence of the position of the plunger (Fig. 103) on reactance and impedance of the circuit.

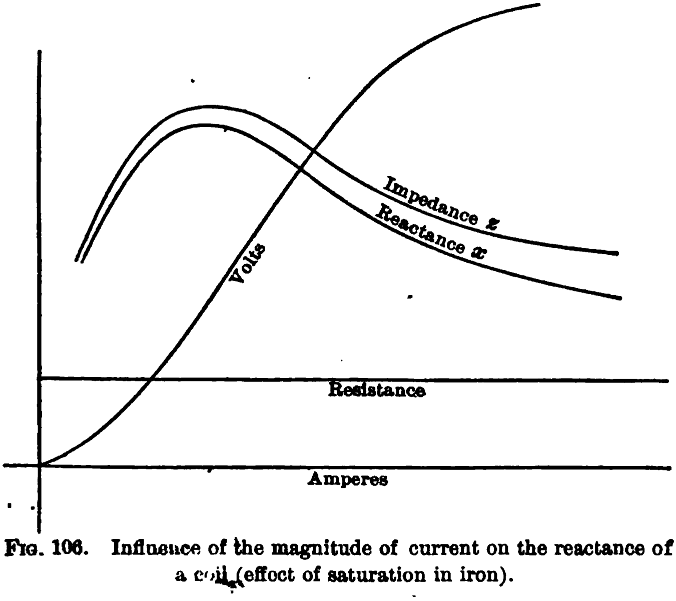

FIG. 106. Influence of the magnitude of current on the reactance of a coil (effect of saturation in iron).

flux in the coil and the applied voltage are proportional to the current.

(d) The reactance of a coil is proportional to the square of the number of turns in series, other factors being identical, as is evident from a consideration of two coils. The two coils under comparison should have the same outside dimensions and the same space available for winding. Let one of the coils be wound with a finer wire, so as to accommodate *m* times more turns in the same space; assume, that the currents in the two coils are adjusted so as to produce equal fluxes in the respective cores. Let the induced voltage and the current in the first coil be E and I respectively — those in the second coil E' and I'. If the flux in the second coil is the same as in the first, I' must be $= I \div m$, in order to have the same number of exciting ampere-turns. The flux in the second coil is interlinked with m times more turns than an equal flux in the first coil; consequently $E' = mE$. Thus, the reactance of the second coil

$$x' = \frac{E'}{I'} = \frac{mE}{I \div m} = \frac{m^2E}{I} = m^2x,$$

where x is the reactance of the first coil. This proves that reactance increases as the square of number of turns. With the same number of turns, reactance increases as the cross-section of the coil, or at least as the cross-section of its iron core, for the reason that the flux produced by the coil increases in this proportion.

101. EXPERIMENT 5-A. — Study of a Reactance Coil without Iron. — The influence on reactance of frequency and of the number of turns, as discussed in the preceding article, may be conveniently studied on a coil without an iron core. The connections are as shown in Fig. 103; for the experiment a coil with a considerable number of turns of heavy wire should be used in order to have an appreciable inductance with a negligible resistance. (a) First investigate the influence of frequency. Connect the circuit to the terminals of an A. C. generator the speed of which may be varied at will. Keep the current through the coil constant, and measure the voltage across its terminals at various frequencies of the current. Take readings with two or three different values of the current. (b) Investigate the influence of the number of turns. It is convenient for this purpose to have a coil wound with two or more wires in parallel. By using separate sections of the winding, or combining them in series and in parallel, different numbers of active turns are obtained. The experiment may be performed at a constant frequency; the reactance should be determined with several different values of current. (c) In cases where inductance is harmful, coils are wound *non-inductively*, or so as not to produce any magnetic flux. This is accomplished by connecting two halves of a winding so that each tends to produce a magnetic flux in a direction opposite to that of the other. Try this experimentally by sending a current through the coil with two sections of the winding connected in opposition. If the coil has only one winding, take a tap *M* (Fig. 103) as one terminal, and use the points *A* and *B* connected together as the other terminal. Before putting on current, insert enough resistance *R* to prevent an inrush of current. You will find the voltage drop across the coil very small, and consequently its reactance low.

Report. Plot values of reactance in ohms to cycles (of frequency) as abscissæ; figure out the inductance of the coil (in henrys) according to formula (4). Give the results showing that inductance increases as the square of the number of turns. Describe the experiment with the coil wound non-inductively.

102. EXPERIMENT 5-B. — Study of a Reactance Coil with an Iron Core. — The factors to be investigated are enumerated in § 100. The influence of the frequency and of the number of turns may be omitted, if experiment 5-A has previously been performed. To investigate the influence of the position of the plunger, first remove it (Fig. 103), and raise the current in the coil to its maximum safe value. Then gradually move the plunger in, at the same time cutting the resistance *R* out of the circuit, so as to keep the current constant (Fig. 105). Another set of curves may be taken, keeping the voltage across the coil constant, and allowing the current to drop as the plunger is inserted into the coil.

To investigate the influence of the intensity of current (Fig. 106), shove the plunger into the coil and gradually increase the current by regulating the rheostat *R*; read volts and amperes. Repeat the test with two or three different positions of the plunger.

Report. Plot curves showing variation of the reactance of the coil with the position of the plunger and with the intensity of the current in the coil, as in Figs. 105 and 106. Explain the character of the curves.

103. Impedance, or Combination of Reactance and Resistance. — Reactance coils have been considered heretofore as devoid of ohmic resistance, or at least having a negligible resistance. In many practical cases, however, the ohmic resistance of a circuit is of equal, if not of greater importance, than the reactance. It is necessary, therefore, to investigate the combined action of a reactance having a resistance in series with it.

Experience and theory show, that a reactance x and a resistance r connected in series, as in Fig. 107, cannot be added arithmetically, though both are expressed in ohms. They must be added geometrically, at right angles, as in Fig. 108. For instance, let an inductance of 4 ohms be connected in series with a resistance of 3 ohms. To force a current of 10 amperes through the reactance alone an e.m.f. of 40 volts is necessary; to force the same current through the resistance alone, 30 volts are required. But when the two are connected in series, only 50 volts are necessary, instead of 40 + 30 = 70 volts. In other words, the combined action of 4 ohms and 3 ohms is seemingly equivalent to 5 ohms, and not to 7 ohms.

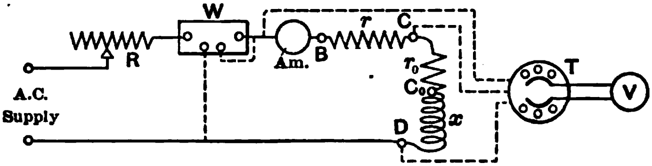

FIG. 107. Diagram of connections for investigating electrical relations in a circuit containing resistance and reactance in series.

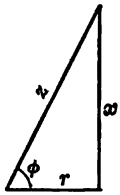

FIG. 108. Geometrical addition of a resistance and a reactance, the result being an impedance.

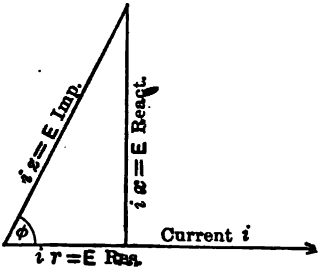

FIG. 109. The triangle of voltage drop, corresponding to Fig. 108.

This peculiar relation can be explained by means of the equations established in § 98. When a coil — in addition to an inductance L — has some ohmic resistance r, the e.m.f. necessary to be applied at its terminals, in order to produce a current i, is

$$e = L\frac{di}{dt} + ir \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

compare with equation (1). Substituting i from (2), and remembering the relation (4), we obtain

$$e = xI \operatorname{Cos} 2\pi nt + rI \operatorname{Sin} 2\pi nt \quad . \quad . \quad . \quad . \quad . \quad (7)$$

This shows that the applied voltage at any moment is a sum of two voltages, each varying as a sine wave (Fig. 102). The wave of the inductive drop ix is the same as before; the wave of the ohmic drop ir is in phase with the current, both being represented by the same sine function. The total voltage $e = iz$ is also represented by a sine wave, for the sum of two sine waves of the same frequency is also a sine wave.

It will be seen from Fig. 102 that the amplitude of the wave marked "total voltage" is less than the sum of the amplitudes of the two component waves. Its phase position is also intermediate between the two. We may express this voltage by

$$e = zI \operatorname{Sin} (2\pi nt + \phi) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (8)$$

where z and ϕ are two new quantities which determine the magnitude and the position of the resultant e. Equating (7) and (8) we obtain

$$z \operatorname{Sin} (2\pi nt + \phi) = x \operatorname{Cos} 2\pi nt + r \operatorname{Sin} 2\pi nt \quad . \quad . \quad . \qquad (9)$$

This relation is true at any moment t; applying it first for $nt = 0$, and then for $nt = \frac{1}{4}$, we obtain

$$\left.\begin{aligned} z \operatorname{Sin} \phi &= x \\ z \operatorname{Cos} \phi &= r \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (10)$$

whence

$$\left.\begin{aligned} z &= \sqrt{x^2 + r^2} \\ \tan \phi &= x \div r \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (11)$$

For given values of x and r, the values of z and ϕ may either be calculated from (11) or constructed as in Fig. 108. The "combined resistance" z is called the *impedance* of an apparatus or of a circuit; it is thus equal to the geometrical sum, and not to the arithmetical sum, of the component resistance and reactance in series.

Returning now to the above numerical example, we see that a 4-ohm reactance and a 3-ohm resistance act together as an impedance $z = \sqrt{4^2 + 3^2} = 5$ ohms. The equation (8) may be represented symbolically thus:

$$E_{eff} = z i_{eff} (\phi) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (12)$$

where (ϕ) means that the wave of the current lags behind that of the applied voltage by the angle ϕ (Figs. 102 and 109). This equation compared to the equation (5) clearly indicates the influence of an ohmic resistance in series with a reactance.

104. EXPERIMENT 5-C. —Study of a Reactance Coil with an Appreciable Ohmic Resistance. —We assume now that the coil A-B (Fig. 103) has an appreciable ohmic resistance r. For experimental purposes, it is better to have the coil itself of a negligible resistance, but have some resistance connected outside the coil, so as to be able to vary it (Fig. 107). The purpose of the experiment is: (1) To prove that a resistance and a reactance in series must be added geometrically; (2) To investigate the influence of the factors enumerated in § 100, as modified by the presence of resistance. To prove the triangle of resistances, Fig. 108, send an alternating current through the coil and the resistance connected in series. Measure the current and the voltage and determine the impedance z as the ratio of the voltage to the current, according to equation (12). Then measure the reactance x of the coil alone, also as a ratio of volts at its terminals to amperes flowing through the coil. Finally put on a direct current, and measure the ohmic resistance r. The three quantities thus determined must form a rectangular triangle, as in Fig. 108, so that any two of them check the third. Repeat this experiment with different values of current.

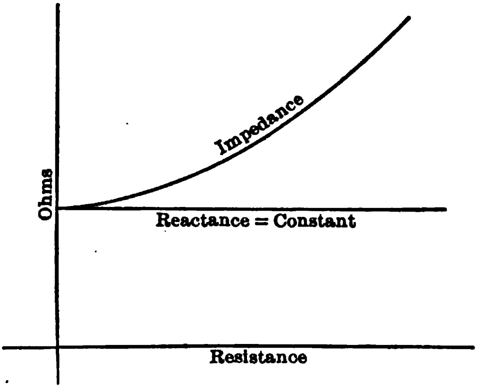

FIG. 110. Influence of variable resistance on the value of an impedance.

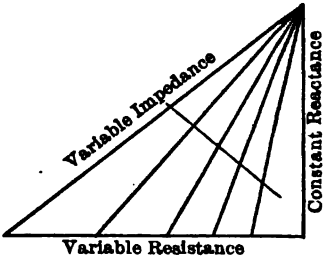

FIG. 111. Explanatory diagram to the curves shown in Fig. 110.

The influence of various factors on the impedance should now be investigated, as shown in Figs. 104 to 106. The two values to be measured are: The total impedance z and the ohmic resistance r. The reactance x is figured out either graphically, as in Fig. 108, or from the relation $x = \sqrt{z^2 - r^2}$. During this investigation the resistance r is kept constant. It will be noted that its influence becomes more pronounced, the smaller the reactance. It is also advisable to investigate the converse case (Fig. 110), the reactance being kept constant and the resistance gradually increased. It will be seen that the total impedance increases more slowly than the resistance. The geometrical reason for this may be seen from Fig. 111.

105. Triangle of Voltages.—It was shown in § 103 that a resistance and a reactance cannot be added arithmetically, but must be added geometrically at right angles; also that a drop of 40 volts across a reactance, and of 30 volts across a resistance in series with it, gives a total drop of only 50 volts, instead of 70 volts. This latter relation is deduced directly from Fig. 108, by multiplying the three sides of the triangle by the effective current i (Fig. 109). This gives a triangle of the effective values of voltage, across the resistance, across the reactance and across the total impedance. The triangle, Fig. 109, gives the same information in regard to the electrical relations in the circuit, as the set of sine waves in Fig. 102. The angle of lag ϕ is also represented in the triangle in its true magnitude. The diagram is completed by drawing a line which represents the current i. This line is in phase with the ohmic drop ir, since the two corresponding sine waves reach their maxima simultaneously.

Lines showing effective values and phase relation of alternating electrical quantities are called *vectors;* thus Fig. 109 is a vector diagram of currents and voltages in an inductive circuit. Such diagrams are now used almost exclusively in place of sine waves. They give the same information, and are much easier to read and to understand.

106. Phase Angle and Power Factor.—The phase angle ϕ, Fig. 109, is of great importance in determining the average power spent in a circuit. The actual power varies from moment to moment with the instantaneous values of current and voltage. When inductance is present, the power supplied becomes at times even negative; this is when the inductance gives back the stored electromagnetic energy (see §§ 96 and 97). For practical purposes only the average value of power, during one complete period of alternating current, is of importance.

If e is an instantaneous value of the voltage and i the corresponding value of the current, as in § 103, the average power during the time T of one cycle is

$$w = \frac{1}{T}\int_0 ei\,dt.$$

Substituting the value e from the equation (6), we get

$$w = \frac{L}{T}\int_0^T i\,di + \frac{r}{T}\int_0 i^2 dt,$$

or

$$w = \frac{L}{T}\left(\frac{i^2}{2}\right)_0^T + ri^2_{eff}.$$

The first term on the right side is = 0 because i is the same for any two moments, differing by T. This shows that with periodically fluctuating currents, no power is permanently lost or gained in a reactance; it is merely stored part of each cycle and returned to the circuit during another part of the cycle. The power lost in the resistance is represented by the familiar expression ri^2_{eff}.

For circuits in which voltage and current vary according to sine law another expression for power is obtained by substituting E Cos ϕ in place of ir (Fig. 109). We find

$$w = E_{eff} \cdot i_{eff}. \ \text{Cos}\,\phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

If the circuit is non-inductive Cos ϕ = 1 and the power is represented by the product $E_{eff} \cdot i_{eff}$. This latter product is also called the apparent power. Cos ϕ is called the power factor, for it is the factor by which the apparent power must be multiplied in order to get the true power w. Another proof for the expression (13) will be found in § 75.

By having in the circuit an indicating wattmeter W (Fig. 107) true watts may be read directly in addition to the current and the voltages, and the power factor calculated from (13). Another way of determining Cos ϕ is from the triangle of voltages (Fig. 109). If the instruments are in calibration, the two methods yield the same result.

107. EXPERIMENT 5-D. —Power Relations with Resistance and Reactance in Series. —The purpose of the experiment is to make clear the relations deduced in §§ 105 and 106. The connections are shown in Fig. 107. A voltmeter switch T is used, by means of which the voltmeter may be made to indicate at will either the total voltage, the voltage across the inductance, or across the resistance. The connections to this switch are shown more in detail in Fig. 113. The resistance r_0 in Fig. 107 is meant to represent the small ohmic resistance unavoidable in the reactance coil x.

Select by trials such values of r and x as will give about the same drop across BC as across CD; read amperes, volts and watts. The voltmeter connection is shown to include the drop in the ammeter; this is advisable in order to have the same total voltage, as is impressed across the potential coil of the wattmeter. The resistance of. the ammeter must thus be included in the value of r. Ohmic resistance is measured at the close of the experiment by the drop-of-potential method, using direct current. A double-throw switch D, shown in Fig. 113, is convenient for changing from alternating to direct current. Take several sets of readings, varying the applied voltage by the rheostat R. Repeat the same test with different values of r and x.

Report. (1) Plot amperes, watts and component volts to total volts as abscissæ (Fig. 112).

(2) Plot corrected values of pure resistance drop and pure reactance drop, as shown by dotted lines. This is done by adding ir_0 drop to ir and subtracting the total ohmic drop geometrically from the applied voltage.

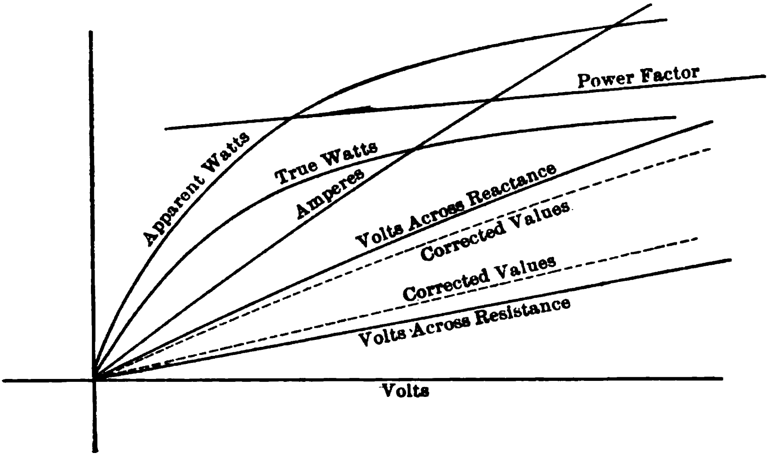

FIG. 112. Variations in amperes, watts, etc., consumed in the circuit shown in Fig. 107, with the change in impressed voltage.

(3) Show for a few points selected from the curves, that the power read on the wattmeter checks with the calculated expression $(r + r_0) \cdot i^2$.

(4) Check the values of Cos ϕ calculated as the ratio of true watts to apparent watts [expression (13)] with those determined from the triangle of voltages (Fig. 109).

108. Impedances in Series. —The relations deduced in §§ 103 to 107 may now be extended to the case of two or more resistances and reactances connected in series (Fig. 113). Such conditions are met in practice when, for instance, both the line and the current-consuming devices possess resistance and reactance, and it is desired to calculate the station e.m.f. and the power factor of the generator load.

The relations are shown in Fig. 114; the diagram is constructed for a given current i (horizontal vector). The total voltage drop across AC (Fig. 113) does not depend on the order in which the resistances and the reactances are connected. Therefore, we may substitute an equivalent resistance $R = r_1 + r_2$ for the two separate resistances r_1 and r_2. In a similar way an equivalent reactance $X = x_1 + x_2$ may be introduced. This gives th triangle APC analogous to that shown in Fig. 108; *AC* is the combined, or equivalent impedance Z of the circuit. Multiplying Z by i, according to equation (12), the total voltage *AC′* is obtained. The triangle *AMB* shows the resistance, the inductance, and the impedance between the points *A* and *B* of the circuit; the triangle *BNC* gives the same values for the part of the circuit between *B* and *C*. The triangle *APC* is a combination of these triangles. It may also be said, that the impedance Z is the geometric sum of the impedances z_1 and z_2.

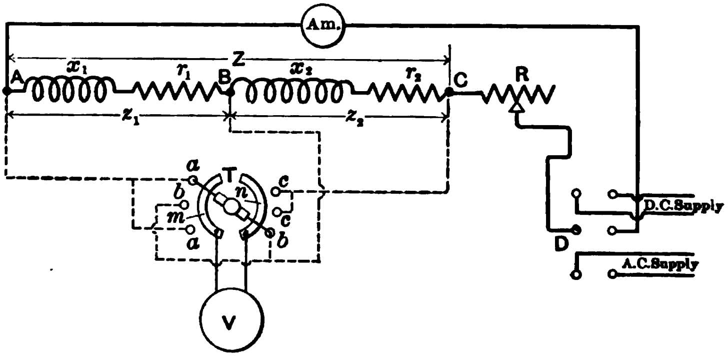

FIG. 113. Impedances in series.

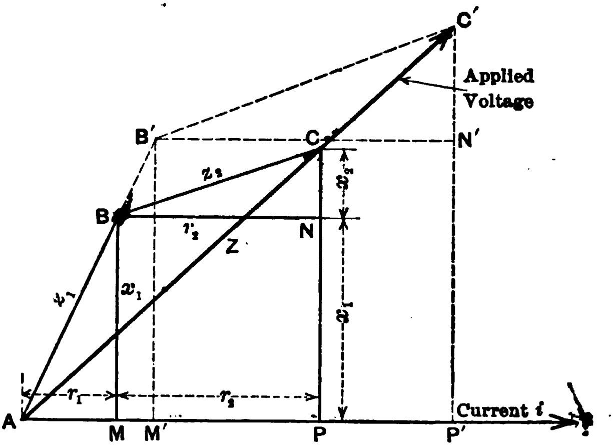

FIG. 114. Electrical relations with two impedances in series.

By changing the scale of the diagram in the ratio of *AC′* ÷ *AC* a diagram *AB′C′* of component voltages is obtained, as shown by dotted lines. AM' is the voltage across r_1; $M'P'$ is that across r_2. $M'B' = P'N'$ is the voltage across x_1 and $N'C'$ is that across x_2. The voltage across AB (Fig. 113) is represented by the vector AB'; the voltage across BC by $B'C'$. It may be said, that AC' is the geometric sum of AB' and $B'C'$. The angles of phase displacement between the current and the various voltages may also be measured on the diagram.

If the terminal voltage AC' is given instead of the current, the diagram is constructed on the basis of a current arbitrarily assumed; then the values obtained are reduced or increased in the ratio of the given voltage to that found by construction.

109. The Three-Voltmeter Method for Measuring Power.—A special case of the diagram shown in Fig. 114 is represented in Fig. 115. The reactance $x_2 = 0$, so that one of the impedances is reduced to a non-inductive resistance r_2; in other respects both diagrams are identical. This case is of considerable practical importance, as the method can be used for determining power in an inductive circuit, without the use of a wattmeter. Let BC (Fig. 113) be an apparatus, such as a single-phase motor, and let it be necessary to determine watts power consumed in it, without using a wattmeter. According to equation (13), the power depends on the current, the voltage, and the phase displacement between the two. The current and the voltage are measured directly; the phase displacement may be determined from the diagram, Fig. 115.

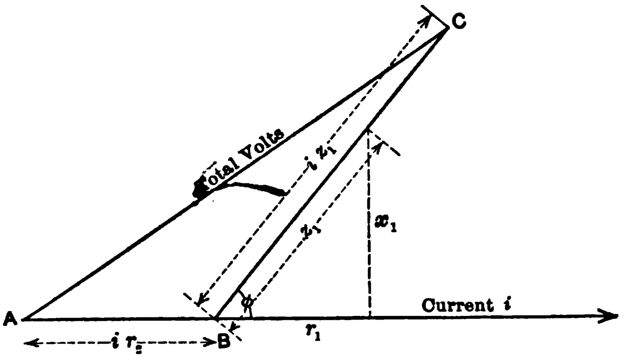

FIG. 115. Specific case of the diagrams shown in Figs. 113 and 114, when the reactance x_2 is zero.

For this purpose, a non-inductive resistance r_2 is connected in series with the apparatus BC; the three voltages AB, BC and AC (Fig. 115) are measured, as is the current i. The resistance r_2 being non-inductive, its drop AB is in phase with the current, so that the triangle ABC may be constructed in its true phase relation to the current i. This gives the desired phase-angle ϕ at B. Projecting BC on i gives $E \cos \phi$; this value being multiplied by i gives the required power.

If the value of r_2 is known, it is not necessary to have an ammeter, as the current can be calculated as the ratio, — voltage ÷ resistance.

This method is called *the three-voltmeter method*, because power, or at least the phase-angle, is determined from three voltmeter readings. It has been used to some extent in former years, but is now applied in exceptional cases only, since indicating wattmeters have come into universal use.

110. EXPERIMENT 5-E. — Voltage and Power Relations with Impedances in Series. — The connections are shown in Fig. 113, save that a wattmeter should be added as in Fig. 107. (a) Adjust the desired values of resistances and reactances, and take readings of volts, amperes and watts. Read total watts, and watts across each impedance, separately. Gradually increase the applied voltage by regulating the resistance R; take similar readings with each setting of the rheostat. Put on direct current and measure the ohmic resistances by the drop-of-potential method. (b) Repeat the same experiment with different values of resistances and reactances. (c) For the same impedance determine the power consumed in it, first using a wattmeter, and then by the three-voltmeter method.

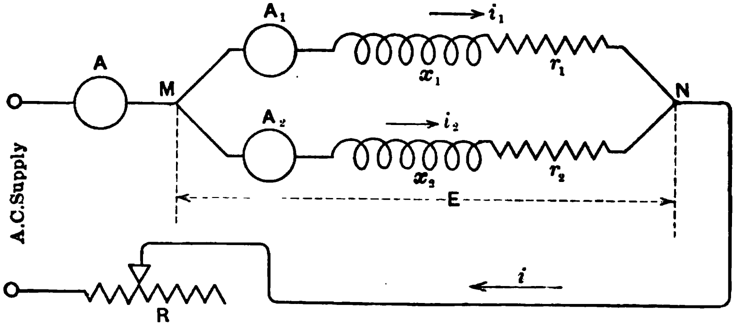

FIG. 116. Two impedances in parallel.

Report. Plot the results to total volts as abscissæ. Draw the diagram of voltages $AM'B'N'C'$ (Fig. 114) and check the cosines of the angles with those calculated from the wattmeter readings. Check the power determined by the three-voltmeter method with that read on the wattmeter, and with calculated i^2r.

111. Impedances in Parallel. —The connections are shown in Fig. 116. It is required to find the total line current i and its phase relation to the voltage E applied between M and N. Let us first consider the simplest case, when one branch has a pure inductance x only; the other merely an ohmic resistance r (Fig. 117). The current in the first branch is $i_1 = E \div x$ and lags 90 degrees behind E; the current in the second branch is $i_2 = E \div r$, and is in phase with E. The total current i is the sum of the two and is represented in the diagram by the diagonal of the rectangle. It will be easily seen that with z the equivalent impedance of the circuit

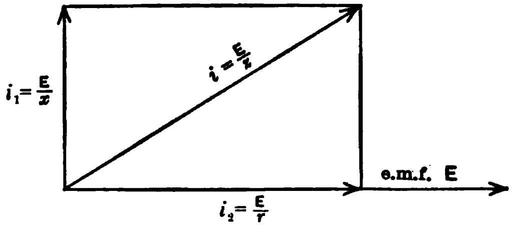

FIG. 117. Vector diagram of currents, with resistance and reactance connected in parallel.

$$\frac{1}{z} = \sqrt{\left(\frac{1}{x}\right)^2 + \left(\frac{1}{r}\right)^2} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (14)$$

as distinguished from the expression (11), when a resistance and a reactance are connected in series.

When resistance and reactance are present in both parallel branches, the two component currents i_1 and i_2 are lagging behind the impressed e.m.f. OA (Fig. 118) by certain angles ϕ_1 and ϕ_2. The total current $i = OC$ is the geometric sum of the two and lags behind OA by the angle ϕ.

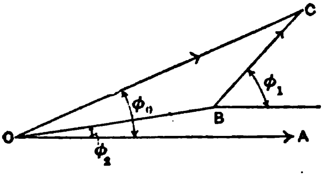

FIG. 118. Component currents and total current in the case of two impedances connected in parallel.

The same relations are shown in greater detail in Fig. 119. The applied voltage between the points M and N (Fig. 116) is represented by the vector OE. The voltages across the resistance and across the reactance in branch 1 are represented by the triangle OP_1E, as in Fig. 109. OP_1 is the drop in the resistance r_1; P_1E is that in the reactance x_1. The current i_1 being in phase with the ohmic drop is represented by a vector OC_1. The triangle OP_2E gives similar relations for the branch 2. If there are more branches in parallel, a triangle may be constructed for each branch. The apexes P_1, P_2, ... of all such triangles lie on a semicircle drawn on OE as a diameter. This is because all the triangles are rectangular, and all have OE for a hypothenuse. The total current i in the line is a geometrical sum of i_1 and i_2 and is represented by the vector OC. If there are more than two branches in parallel, the vectors of the component currents are added together as if they were mechanical forces radiating from the point O.

112. EXPERIMENT 5-F. — Study of Impedances in Parallel. —

The purpose of the experiment is to illustrate the relations described in

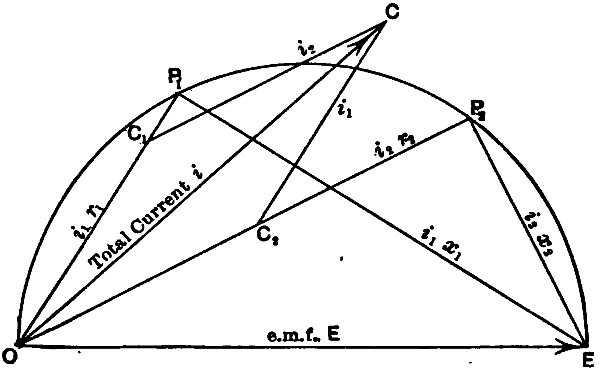

FIG. 119. Vector diagram of currents and voltages, corresponding to the connections in Fig. 116.

§ 111. The connections are shown in Fig. 120; they are identical with those in Fig. 116, except that one ammeter is used for all branches, being connected to a polyphase board (see § 49). A wattmeter is provided for determining phase relations. (a) Begin with the simplest case, illustrated in Fig. 117, when one branch contains only inductance, the other only resistance. Measure the three currents, volts and watts. Gradually increase the voltage and take similar readings. Finally measure the resistance r with direct current. (b) Repeat a similar experiment with other values of resistance and reactance. (c) Take the more general case of r and x in both branches, as in Fig. 116. Read amperes, watts and volts; also component voltages in each branch.

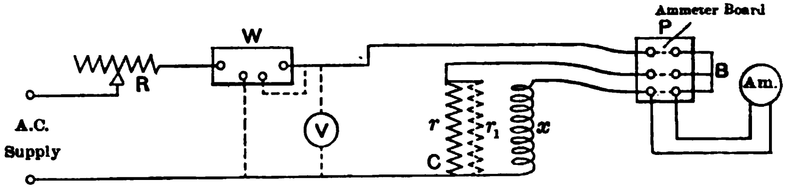

FIG. 120. Diagram of connections for investigating two impedances in parallel.

Report. (1) Show that the three currents form a rectangular triangle, when pure resistance is connected in parallel with pure inductance; check the phase-angle ϕ with that determined by the wattmeter. Also show that the total power corresponds to i^2r in the resistance, and that no power is lost in the reactance.

(2) Draw several diagrams, as in Fig. 118, and show that the relations obtained from the volt and ampere readings check with those calculated from the wattmeter readings.

(3) Construct, for one set of readings, a complete diagram, as in Fig. 119.

NOTE. Resistance r_1, shown in Fig. 120, by dotted lines, is meant to represent the influence of the core loss in the reactance coil. A coil with iron loss in its core is electrically equivalent to an ideal coil with a high resistance shunted around it.

113. EXPERIMENT 5-G. — Motors and Lamps Connected in Parallel to the Same A. C. Supply. — This experiment is a practical illustration of the case discussed in §§ 111 and 112. The load in most power plants consists partly of incandescent lamps, which constitute a practically non-inductive load, and of induction motors having a lower power factor. The resultant load in the power house is the sum of the power taken by the lamps and of that taken by the motors; the resultant power factor lies somewhere between the average power factor of the motors, and 100 per cent.

In studying the electrical relations between inductive and non-inductive load it is *not necessary* to have the experiment arranged on a large scale, since the relations are the same at 5 amperes as they are at 5000 amperes. A laboratory experiment may be considered as representing the actual conditions when performed, for instance, on a ½ hp. motor and some 10 to 15 incandescent lamps connected in parallel with it. By multiplying the results, say by 1000, we get the relations taking place in a city having about 500 hp. in motors and from 10,000 to 15,000 incandescent lamps connected to the electric distributing system.

The load and the power factor vary in actual service in innumerable combinations. To make the problem more definite select two extreme cases: (1) All the lamps are turned on; motor load increases from zero to maximum. (2) All the motors are running at full load; the lamps are gradually turned on. The relations taking place in intermediate cases can be readily understood from the results of these two extreme conditions.

Connect the lamps and the motor in parallel to the power supply, as in Fig. 120, and arrange switches and instruments to read watts and amperes supplied to the lamps, to the motor, and to both. The voltage must be kept as nearly constant as possible during the whole experiment. Take two sets of readings, under the conditions described in (1) and (2) above.

Report. The results should be plotted to total watts as abscissæ; plot kw. lamp load, kw. motor load, lamp current, motor current, total current, resultant power factor and motor power factor; also calculate and plot wattless amperes. Two separate sets of curves should be plotted: one for constant motor load, the other for constant lamp load. The diagram, Fig. 118, is simplified in this case to that in Fig. 121 because the load in one of the branches is non-inductive. Check a few points of the above curves by means of this diagram. For instance, take the observed component currents, find from the diagram the total current and the resultant power factor, and compare them with the values directly observed. Or take the observed total current and one of its components, and determine from the diagram the other component current.

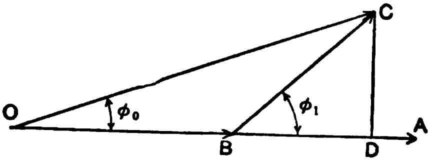

FIG. 121. Current relations when an inductive load and a non-inductive load are connected in parallel (specific case of Fig. 118).

114. Mutual Induction. — The reactance of a coil, such as C_1 (Fig. 122), is reduced by the proximity of another coil C_2, if the latter is short-circuited upon itself. The reason is, that the flux produced by the first coil induces currents in the secondary coil. These currents oppose the action of the primary currents, reduce the flux, and consequently the counter-e.m.f. produced by the first coil.

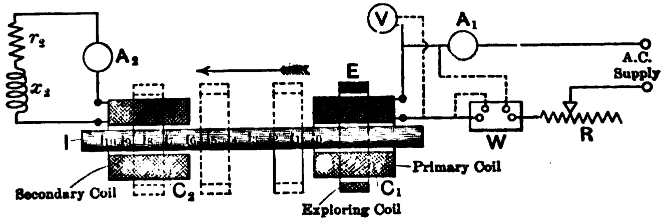

FIG. 122. Effect of mutual induction on the electrical relations in a circuit.

The effect of the secondary coil depends, among other factors, upon its distance from the primary coil and on the character of the circuit on which it is closed (resistance r_2 and reactance x_2). The action is greatly intensified if both coils are mounted on an iron core, *I*. This interaction of two coils, without direct metallic connection, is called *mutual induction.*

In some practical cases, mutual induction is highly desirable; thus the action of transformers (Chapter XI) is based entirely upon it. In other cases it has a harmful effect; for instance, when a power-transmission line parallels a telephone line, currents are induced, which interfere with the transmission of speech.

The electrical relations between two circuits having a mutual induction are usually too complicated to be expressed by practical formulæ or diagrams; it is merely desired that the student understand the physical character of the phenomenon, and the influence of the factors entering into it. This can be done as is explained in the following experiment.

115. EXPERIMENT 5-H. — Effect of Mutual Induction on the Reactance of a Circuit. — The phenomenon described in the preceding paragraph may be studied experimentally with apparatus as shown in Fig. 122. The coil C_1 under test is denoted the "Primary Coil." It is connected to an A. C. supply with an ammeter A_1, a voltmeter V, and a wattmeter W in the circuit, as usual. Another coil C_2 marked "Secondary Coil" is closed on a circuit consisting of a resistance r_2, reactance x_2 and an ammeter A_2. The secondary coil may be placed at any desired distance from the primary coil. In order to increase the inductive action between the two coils, they may both be put on a common iron core I. To investigate the influence of the position of the secondary coil, short-circuit it upon itself through the ammeter A_2, and bring it as close as possible to the primary coil (position " O "). Adjust the primary current by the resistance R to the maximum safe limit; read volts, amperes and watts. Gradually move C_2 away, keeping the primary volts constant. Take readings until the secondary coil is removed so far that its influence on the primary circuit is hardly noticeable. Finally open its circuit and again read the primary values.

As the coils are separated, the flux between them varies. The values of the flux can be ascertained by an exploring coil E connected to a separate voltmeter. The coil E is moved as shown by dotted lines, and the variations in the flux measured by the variations in the induced e.m.f. The voltmeter current is so small, that the presence of the exploring coil does not affect the electrical relations between the primary and the secondary coils.

Now investigate the influence of the character of the secondary circuit on the mutual inductance. Place the secondary coil at a short distance from the primary coil, and take several readings of volts, amperes and watts, varying the values of x_2 and r_2. Maintain either primary volts or primary amperes constant, in order to have a basis for comparison of results.

Report. Plot results to positions of the secondary coil as abscissæ

(Fig. 123). Plot the effect of varying secondary inductance and resistance; use as abscissæ the quantity which was varied. Give a physical explanation of the observed relations.

116. Measurement of Inductance with Wheatstone Bridge.—Small inductances, such as are used in telephone and telegraph work, and in scientific research, are usually measured by means of a Wheatstone bridge, with direct or interrupted currents. The various methods employed may be found in physical-laboratory manuals and in textbooks on electricity and magnetism.

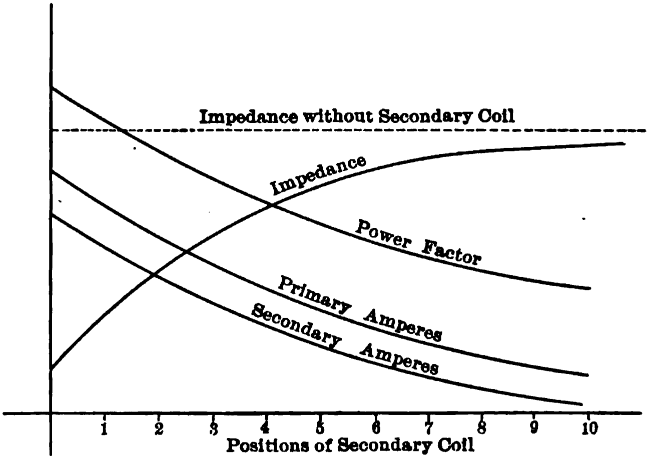

FIG. 123. Curves showing the effect of mutual induction, with the connections as per Fig. 122.

CHAPTER VI.

ELECTROSTATIC CAPACITY.

117. Physical Conception of Electrostatic Capacity.—Imagine two metallic plates C (Fig. 124) brought into proximity to each other, but separated by an air space.

If the plates are connected to a source of electric potential, they will be charged with equal and opposite quantities of electricity. This combination of two plates with a dielectric intervening is called *an electric condenser.* The quantity of electricity which it is able to store or "condense" under definite conditions is called its *electrostatic capacity.* The unit of capacity in the ampere-volt-ohm system is called the "farad." A condenser is said to have a capacity of one farad, if one coulomb of electricity is stored on each plate, at a pressure of one volt between the plates. The coulomb is the quantity of electricity supplied by a current of one ampere during one second. Originally the ampere was defined as a coulomb-per-second, but of late years the ampere has become so much more generally used than the coulomb, that it seems legitimate to define the coulomb by means of the ampere.

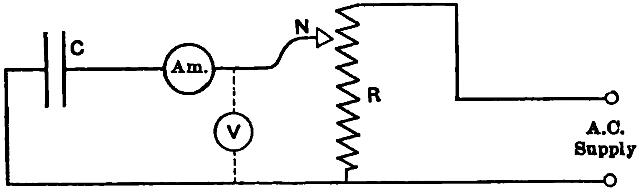

FIG. 124. A convenient method for varying the voltage at the terminals of a condenser.

The farad may also be defined by the volt and the ampere without introducing the conception of the coulomb. *A condenser has the capacity of one farad if it takes a charging current of* 1 *ampere when the voltage at its terminals varies at a rate of* 1 *volt per second.* The farad is too large a unit for practical purposes; therefore capacity of condensers is usually measured in millionth parts of a farad, or in *microfarads.*

The presence of capacity in an electric circuit is manifested by some of the current being used for charging it, instead of flowing through the circuit; if the applied voltage is alternating, the charge periodically absorbed by the condenser is given out during the periods of low e.m.f.

The following analogy may make clearer the action of a condenser. Imagine a water supply pipe with a branch opening to which an elastic bag is connected — bag as well as the pipe being filled with water. As long as the pressure remains constant, the presence of the bag does not interfere with the regular flow of water through the pipe. But should the pressure in the pipe increase, part of the water, instead of flowing through the pipe, enters the bag and stretches it, until its elasticity balances the increased pressure. Should the pressure in the pipe decrease, the elasticity of the bag forces part of the water back into the pipe. Thus, if the pressure in the pipe varies periodically, there is a periodic flow of water in and out of the bag, and the conditions of flow become more complicated than if the pipe were solid.

In a similar way, the flow of current in alternating circuits is modified by electrostatic capacity. When the voltage increases, extra current flows through the line to charge the condenser; when the voltage is on the decrease, the condenser is partly discharged through the line. In long cables and overhead transmission lines, capacity is not concentrated in one place, but is distributed along the line. Each element of the line acts as one plate of a small condenser, the earth, or another wire of the same line acting as the other plate. To extend the above analogy to this case, the pipe itself must be imagined as made of an elastic material, so that its cross-section varies with the applied pressure.

118. Electrostatic Capacity in Practice. —The most important practical cases in which capacity is brought into play are:

(1) In submarine telegraph cables, where capacity is so large that an appreciable time elapses before any current reaches the further end of the cable. The first few moments after the circuit is closed, all the current is used for charging the cable itself, so as to bring it up to the required potential. This circumstance limits the speed of transmission of signals.

(2) In high-tension cables and very long overhead transmission lines, capacity has quite a noticeable disturbing effect on the transmission of power, causing at times an abnormal rise of potential.

(3) Electrostatic capacity in telephone lines considerably affects the quality of speech. To minimize this effect, telephone cables are constructed so as to have as little capacity as possible. Inductance coils are sometimes inserted in long telephone lines to neutralize the effect of capacity.

(4) Condensers are useful in cases where direct current should not flow through a certain part of the circuit, while alternating currents should pass through it with as little opposition as possible. Condensers are used for this purpose in central energy telephones (see Figs. 534 and 536).

(5) Condensers are an indispensable part of wireless telegraph outfits; also in other cases where it is necessary to produce electric oscillations.

(6) Condensers are useful in many electrical investigations as auxiliary means for measuring various quantities, eliminating higher harmonics, etc.

119. Factors Affecting the Capacity of a Condenser. —The capacity of a condenser, or its ability to store electric charges, depends on its form and dimensions, and on the nature of the dielectric between the plates. Increasing the surface of the plates *C* (Fig. 124) increases the capacity in the same proportion, for the reason that each element of area of one plate binds a certain quantity of electricity on the corresponding element of the other plate; therefore increasing the surface of the plates increases the total charge which the condenser can hold for a certain pressure at its terminals. Bringing the plates closer together also increases the capacity, because the attraction between opposite charges is increased; this permits larger amounts of electricity to be held mutually bound.

Capacity is also increased by substituting various insulating compounds for air. It seems that electric charges attract and hold each other more easily through these compounds than through the air. Thus, substituting paraffine for air increases the capacity of a condenser about twice; the use of hard rubber increases it 2.5 times, etc. The number of times by which capacity is increased by the substitution of some other insulating material for air is called the specific inductive capacity of the insulating material. It is about 2.5 for rubber, about 2 for paraffine, and varies between 2 and 3 for many other substances.

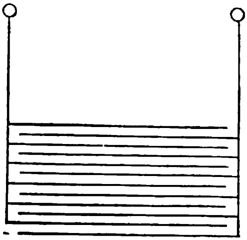

FIG. 125. Interposition of metal sheets in a condenser, to increase its capacity.

120. Construction of Condensers. — The capacity of condensers used in practical work is too large to be obtained by two plates, unless the plates be given enormous dimensions. It therefore becomes necessary to use several plates (Fig. 125) connected alternately to opposite terminals. An air-insulated condenser of this kind is shown in Fig. 126; it is used as a standard for measuring small capacities. For larger capacities, plates must be brought much closer together, and their number greatly increased. Besides, it becomes necessary to use a dielectric other than air, since the thin metal sheets are not strong enough to support themselves. This also increases the capacity of the condenser from two to three times, because of the higher specific inductive capacity of dielectrics other than air.

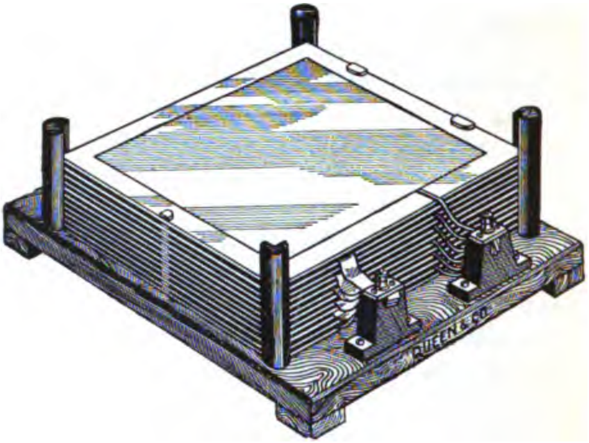

Fig. 126. A standard air condenser.

Mica is used for insulation in standard condensers; also in those intended for high voltages. Such condensers consist of sheets of tinfoil with sheets of mica between them. The whole is firmly pressed together in order that the distance between the plates and consequently the capacity shall remain unaltered. A more satisfactory method than using tinfoil is to have each sheet of mica coated chemically with a thin film of silver. A standard mica-condenser is shown in Fig. 127; it

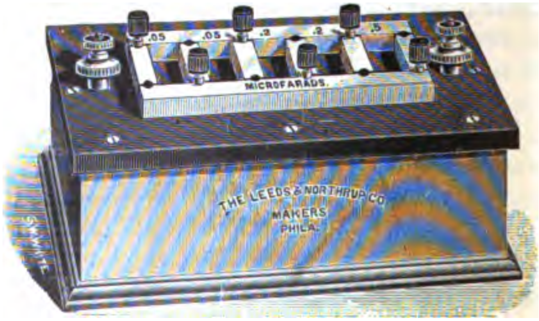

Fig. 127. An adjustable mica condenser.

is subdivided into sections of 0.05, 0.05, 0.2, 0.2 and 0.5 microfarad. By simple combinations of plugs, several values of capacity may be obtained between 0.05 and 1 microfarad (Fig. 128).

Mica condensers are quite expensive; therefore, where high accuracy and particularly high insulation are not required, sheets of tinfoil are insulated with paper instead of mica. Each sheet of paper is dipped in hot paraffine during the construction, and the whole is immersed in paraffine after completion, in order to exclude air. This makes the condenser a solid mass, and insures constant distances between the sheets of tinfoil. A paper condenser for voltages up to 500 volts, adjusted within 3 per cent, is shown in Fig. 129. Paper condensers may also be subdivided into sections, as in Fig. 127.

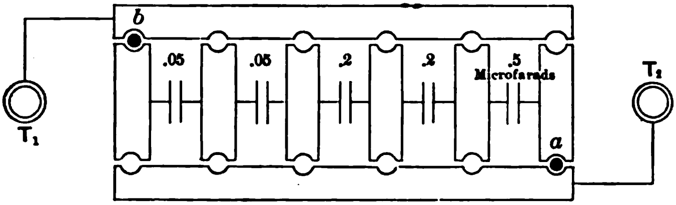

FIG. 128. Arrangement of sections in the condenser shown in Fig. 127.

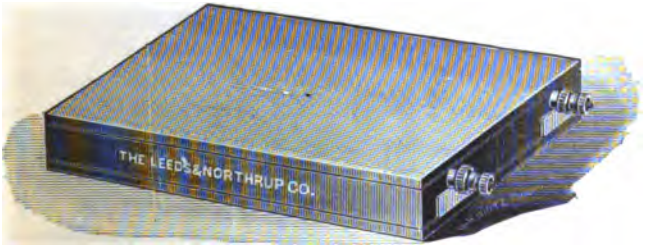

FIG. 129. A simple paper condenser.

121. Condensers in Series and in Parallel. — Condensers may be connected in series and in parallel, like resistances, and it is necessary to know how to calculate their combined capacity.

(*a*) *Condensers in parallel.* Suppose several condensers (Fig. 130) of capacity c_1, c_2, c_3, ... to be connected in parallel across a line having a pressure E. The problem is to find the capacity C of an equivalent condenser, such as can store at the same pressure E the same quantity of electricity as the given condensers together. Let the quantities stored be q_1, q_2, q_3. . . . According to the definition of capacity (§ 117), we have

$$q_1 = c_1 E;\ q_2 = c_2 E;\ \text{etc.}$$

For the equivalent condenser

$$q_1 + q_2 + \dots\dots = CE.$$

Substituting we find

$$C = \sum c\,;$$

in other words, the combined capacity of condensers in parallel is equal to the sum of their capacities. This result could be foreseen from the physical conception of capacity.

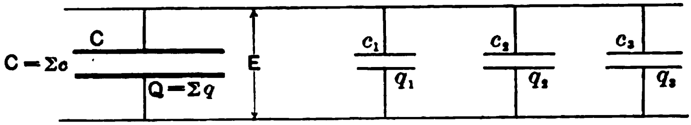

Fig. 130. Addition of capacities in parallel.

(*b*) *Condensers in series.* Condensers connected in series are shown in Fig. 131. The electric charge q in each of them is the same, though they may have different capacity. This must be, since each two direct-connected plates of adjacent condensers can be charged only with equal and opposite quantities q of electricity, since the plates are not connected to the line. These charges must be the same in all

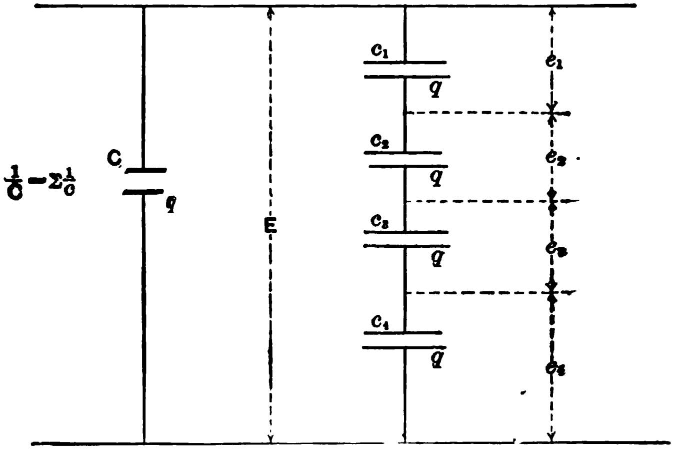

Fig. 131. Addition of capacities in series.

condensers, because only two equal charges can keep each other bound. The problem is to find the capacity C of *one* condenser that would hold the same charge q when connected across the line. Let the voltages across the given condensers be $e_1, e_2, e_3, \ldots$; we have then

$$q = c_1 e_1 = c_2 e_2 = c_3 e_3 = \ldots .$$

For the equivalent condenser

$$q = CE = C\,(e_1 + e_2 + e_3 + \ldots);$$

Substituting the values of e_1, e_2, e_3, etc., we find

$$q = C\left(\frac{q}{c_1} + \frac{q}{c_2} + \frac{q}{c_3} + \ldots\right),$$

or

$$\frac{1}{C} = \sum \frac{1}{c}.$$

This can be expressed in words by saying, that the sum of the reciprocal values of capacities connected in series is equal to the reciprocal value of the equivalent capacity. This is analogous to the expression for ohmic resistances in parallel (see § 8).

Let us apply these results to the standard condenser shown in Figs. 127 and 128. The largest capacity is obtained by connecting all the sections of the condenser in parallel, with plugs inserted alternately in the two rows of holes, as in Fig. 127. The smallest value of capacity corresponds to all the sections in series, with plugs at a and b (Fig. 128). Intermediate values are obtained by other suitable combinations of plugs. For instance, to get 0.15 microfarad, two sections of 0.2 mf. each, are connected in series, which gives 0.1 mf.; then 0.05 mf. is connected in parallel with them. With all the sections in series, we have

$$\frac{1}{C} = \frac{2}{0.05} + \frac{2}{0.2} + \frac{1}{0.5} = 52,$$

or $C = 1/52$ mf.; this is the smallest value obtainable with this condenser. The largest capacity with all the sections in parallel is

$$2 \times 0.05 + 2 \times 0.2 + 0.5 = 1 \text{ mf.}$$

122. Methods for Comparing Capacities. — Most practical methods for measuring capacity are based on comparing the capacity under test to that of a standard condenser. The calibration of standard condensers themselves is done by special methods, and is outside the scope of practical engineering. The following are the methods of comparison commonly used:

(1) Direct-discharge method;
(2) Method of mixture;
(3) Thomson zero method.

In addition to these, capacity may be measured in terms of volts and amperes with sinusoidal alternating currents. All these methods are described below.

123. Direct-Discharge Method. — A standard condenser of capacity C_1 is connected across a battery and charged at a certain voltage. Then it is discharged through a ballistic galvanometer (§ 139); let the deflection be d_1. The same is repeated with the condenser under test, the unknown capacity of which, C_2, is to be determined; let the deflection be d_2. Then, if the charging voltage is the same in both cases:

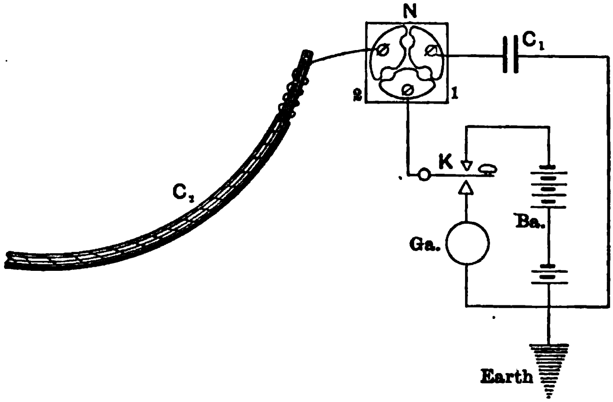

FIG. 132. Direct-discharge method for comparing capacities.

$$C_1 \div C_2 = d_1 \div d_2.$$

This is true since electric charges on two condensers, at the same voltage, are proportional to their capacities (§ 121 *a*). On the other hand, deflections of a ballistic galvanometer are proportional to discharges through it; consequently, in the case under consideration, galvanometer deflections are proportional to the capacities of the condensers.

When the capacity of a long cable laid underground is measured by this method, the connections are as shown in Fig. 132. The lead sheathing of the cable serves as one plate of the condenser; as it is connected to the ground, one pole of the battery must also be grounded to complete the circuit. N is a commutator; inserting a plug into the hole 1 connects the standard condenser C_1 to the battery. The key K is normally pressed against the upper contact, so that C_1 is charged. Pressing the key discharges C_1 through the galvanometer. Then the plug is inserted into the hole 2, when the cable is connected in place of C_1, and the same test repeated. The ratio of the deflections gives the ratio of the capacities.

124. Method of Mixtures.—This method is illustrated in Fig. 133 and used for measuring small capacities, such as the capacity of telephone cables, or of short pieces of power cables. It consists in adding the unknown capacity C_2 to a standard condenser C_1 which has been previously charged; C_2 is then calculated from the resulting drop in voltage of C_1. A mechanical analogy, shown in Fig. 134, may help to understand the method. C_1 is a vessel of a known cubical (volume) capacity; C_2 is another vessel the capacity of which is to be determined.

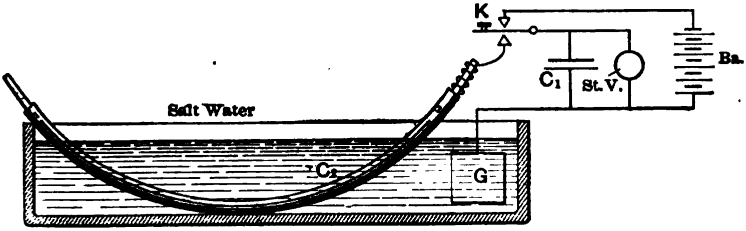

FIG. 133. The method of mixtures, for comparing capacities.

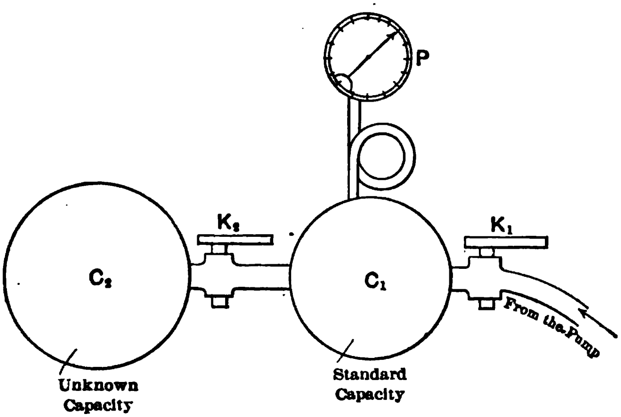

FIG. 134. Mechanical analogy illustrating the method of mixtures.

Close the valve K_2 and connect C_1 by means of the valve K_1 to a source of compressed air. A certain pressure will be indicated on the gauge P. Now close K_1 and connect C_1 with C_2. The air expands into C_2 and the pressure is reduced. The resultant pressure depends on the capacity of C_2, which thus may be calculated from the pressure-volume law of gases.

In a similar way, the standard condenser C_1 (Fig. 133) is charged from the battery, and the voltage measured on the multicellular static voltmeter "St. V." (see Fig. 47); let the indication be E volts. The key K is then pressed, so that the charge is distributed between C_1 and C_2 connected in parallel; let the voltmeter indication drop to e. As the total charge is the same in both cases, we have

$$\text{charge} = EC_1 = e(C_1 + C_2),$$

from which C_2 can be calculated.

The capacity of the voltmeter itself can usually be neglected. If such is not the case, the above formula becomes

$$E(C_1 + c) = e(C_1 + c + C_2)$$

where c is the capacity of the static voltmeter. The capacity c may be determined by performing the above measurement with a condenser C_2 of a known capacity.

When testing telephone cables by this method, the specification usually requires that the test be made under the most unfavorable conditions (maximum capacity). These are obtained when the lead sheathing and all the conductors of the cable, but the one under test, are grounded.

125. Thomson Zero Method for Comparing Capacities. — This method, illustrated in Fig. 135, depends upon the two following operations: (1) The ratio of capacities is made equal to a ratio of resistances, which ratio can be determined with great accuracy; (2) Balance is obtained when the galvanometer returns to zero; this adds to the accuracy of the method, and makes it unnecessary to have a calibrated galvanometer.

Let the condensers C_1 and C_2 be connected as in Fig. 135; the key K_o is closed first, and a steady current established through the resistances R_1 and R_2. The keys K_1 and K_2 normally touch the upper contacts, so that the condensers C_1 and C_2 become charged. The voltage at the terminals of each condenser — and consequently the magnitude of the charge — depends upon the position of the slider N.

Now K_1 and K_2 are pressed down simultaneously. This allows the positive charge of C_1 to neutralize all or a part of the negative charge of C_2; the other charges, no longer bound, neutralize each other to the same extent. The remaining charges are equalized through the galvanometer *Ga.*, by pressing the key K_3. The adjustment consists in finding such a position of the slider N that both condensers are charged with the same quantities of electricity. When such is the case, pressing the keys K_1 and K_2 completely discharges the condensers, so that the galvanometer does not deflect, when the key K_3 is closed.

When the correct position of the slider N is found, — with the voltages across R_1 and R_2 equal to e_1 and e_2, — the following relations will exist:

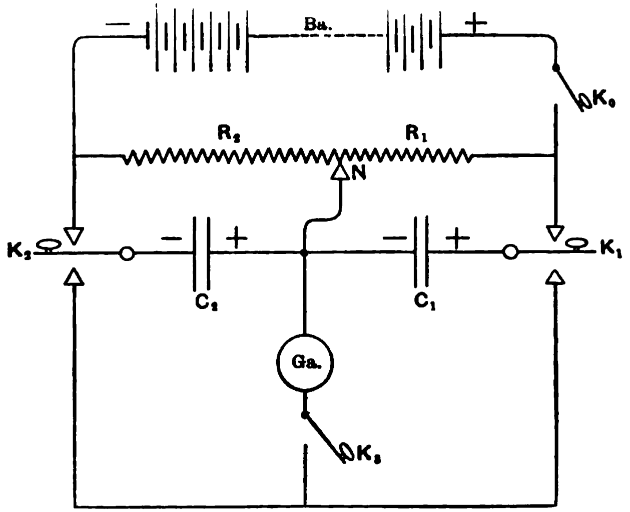

FIG. 135. The Thomson zero method for comparing capacities.

$$\text{charge} = e_1\,C_1 = e_2\,C_2.$$

But $e_1 : e_2 = R_1 : R_2$; eliminating e_1 and e_2 we obtain

$$C_1\,R_1 = C_2\,R_2.$$

From this formula the unknown capacity C_2 is calculated.

It must be remembered that some cables take quite an appreciable time, sometimes several minutes, to become fully charged. In the same way sufficient time should be allowed for the charges to equalize before closing the key K_3. When this method is used for determining the capacity of a cable whose sheathing is grounded, the necessary connections must be established through the ground. This is done by grounding the connection between the galvanometer and the slider N;

the sheathing of the cable acts as the positive terminal of the condenser C_2. The rest of the connections are the same.

In addition to the above three methods for comparing capacities, there are some others, mostly based on the use of a Wheatstone bridge. These methods, while seldom used in engineering, are well adapted for accurate scientific measurements. For a description, the reader is referred to standard works on physics.

126. EXPERIMENT 6-A. —Comparing Capacities of Two Condensers. —The methods are described in the three preceding articles. Try such of them as the apparatus at hand and the nature of the capacity to be measured allow. In each case take several readings, modifying the conditions (voltages, resistances and capacities) in such a way as to have a thorough check on the result. Ascertain the limits of accuracy of each method, and determine the conditions necessary for obtaining maximum accuracy. Connect condensers, or sections of a condenser, in series and in parallel, and check the relations derived in § 121.

√

127. Capacity in A. C. Circuits. —Consider a condenser C connected, as in Fig. 124, to a source of A. C. supply. As the main current flows through the rheostat R, by moving the slider N, any desired voltage may be had at the terminals of the condenser (potentiometer principle). This voltage is measured by the voltmeter V, — the current through the condenser is indicated on the ammeter Am. It may at first seem that no current can flow through the condenser, the circuit at the gap being open between the plates. A little consideration will show, however, that a condenser will become periodically charged with opposite quantities of electricity when an alternating voltage is applied at its terminals. This alternate flow of charging current back and forth. at the frequency of the supply, produces the same effect on the ammeter, as if the circuit were actually closed through a resistance.

We now desire to determine the value of the charging current which will flow through a condenser of a given capacity C, with a given alternating voltage E at its terminals, the frequency of the supply being n cycles per second. According to the definition of capacity given in § 117, a condenser takes in a charging current of 1 ampere, when the pressure rises at a rate of one volt per second, provided the capacity of the condenser is 1 farad. Therefore, with a capacity of C farads and the voltage rising at the rate of $de \div dt$, the charging current is

$$i = C\frac{de}{dt}.$$

Let the applied voltage vary according to the sine law, so that

$$e = E \text{ Sin } 2\pi nt.$$

Substituting this value of e in the above, we find

$$i = 2\pi nC \,.\, E \text{ Cos } 2\pi nt \quad . \;.\;.\;.\;.\;.\;.\;. \quad (1)$$

This equation shows that the charging current also varies according to the sine law. The wave of the current leads that of the voltage by 90 degrees; when the voltage is passing through its zero value, the current has already reached its maximum. These relations may be expressed by the equation

$$i_{eff} = 2\pi nC \,.\, E_{eff} \;(90°),$$

where (90°) denotes symbolically the phase relation between the current and the voltage. In this formula the capacity is expressed in farads; in practice, capacity is always expressed in microfarads. One microfarad is equal to 10^{-6} farads, so that

$$i_{eff} = 2\pi n\, C \,.\, 10^{-6} .\, E_{eff} \;(90°) \quad . \;.\;.\;.\;. \quad (2)$$

From this formula the charging current through a condenser of a given capacity may be calculated for a given voltage and frequency. Conversely, by measuring the current and the voltage, capacity C may be calculated. This method of determining capacity is strictly accurate with sinusoidal voltages only; the presence of higher harmonics may appreciably affect the value of the current. This follows from the fact that each harmonic gives a charging current of its own, and the values of charging current increase in proportion to the frequency. Thus, in the case that the seventh harmonic of the e.m.f. wave amounts to only 5 per cent of the fundamental wave, the charging current due to the seventh harmonic will be $0.05 \times 7 = 0.35$, or 35 per cent, of the current due to the fundamental wave. This circumstance should be kept in mind when using alternating currents for measuring capacity.

128. EXPERIMENT 6-B. — Condensers in A. C. Circuits. — The purpose of the experiment is to illustrate the relations derived in the preceding article. Connect a condenser C, as in Fig. 124, using a resistance R and the slider N for varying the voltage at the condenser terminals. Take curves showing variations of the charging current with:

(a) Capacity of the condenser;
(b) Applied voltage;
(c) Frequency of the supply.

Connect a wattmeter so as to measure the power taken by the condenser; show that the charging current is practically wattless. A small amount of power may be consumed in the condenser, due to imperfect insulation, and to what is called " dielectric hysteresis " in the insulation. If possible, vary the wave-form of the current (see § 587) and observe its effect on the charging current.

Report. Plot curves showing the effect of the factors (a), (b) and (c) on the charging current. Check a few points on these curves by the formula (2). Describe the test showing that the charging current is practically wattless. Give the effect of the wave-form on charging current.

129. Capacity Reactance. — Equation (2) shows that with alternating currents the expression $2\pi nC$, and not the capacity C alone, determines the charging current of a condenser. This is similar to the effect of reactance $2\pi nL$ (§ 98) which was there denoted by x. We shall denote accordingly

$$10^6 \div 2\pi nC = y \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

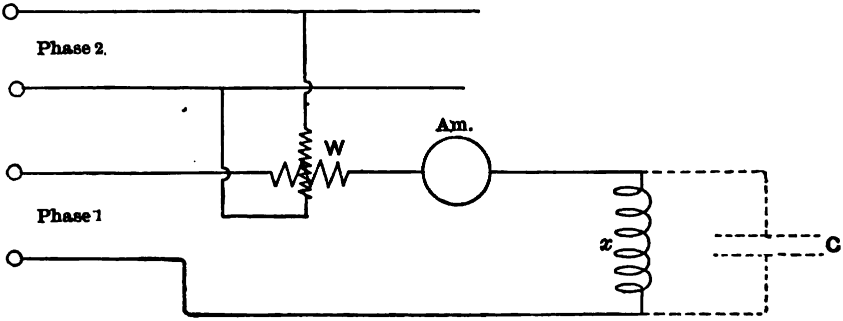

Fig. 136. **An experimental proof that capacity acts as a negative inductance (measuring wattless power).**

and call y the " capacity reactance " of a condenser, as distinguished from the magnetic reactance x. The capacity reactance is also expressed in ohms.

The considerations of §§ 103 to 112 may be applied to the case of capacity reactance by substituting $-y$ for x. The sign minus is necessary, because current is lagging with magnetic reactance and is leading in the case of capacity. In this respect, capacity acts in A. C. circuits as if it were a *negative* inductance.

A good way to demonstrate this experimentally is by means of a double oscillograph (Fig. 450). A reactance coil is connected in an A. C. circuit, and the waves of the current and the voltage projected on a screen. The current wave appears in a certain position, lagging behind that of the e.m.f. by nearly 90 degrees. Substituting a condenser for the reactance coil shifts the current wave by almost 180 degrees and makes it leading instead of lagging.

Another experimental proof is shown in Fig. 136. A reactance coil x is connected in phase 1 of a two-phase circuit, and the "wattless" power consumed in it measured by a wattmeter. The current and the voltage being displaced by almost 90 degrees, the wattmeter connected in the usual way would hardly give any indication at all. But connecting the potential coil of the wattmeter to phase 2, displaced by nearly 90 degrees, brings the two currents in the wattmeter almost into phase, and the instrument gives a considerable deflection. Now if a condenser C is substituted for the reactance coil — without changing the wattmeter connections — it will be found that the wattmeter gives a *negative* deflection, so that it becomes necessary to reverse either its current or potential terminals. This shows that at the same point on the e.m.f. wave the currents through the inductance and through the capacity flow in the opposite directions. If current in the inductance is lagging, the capacity current must of necessity be leading.

These relations are represented vectorially in Fig. 137. The current i_x lags behind the voltage e_1 by a considerable angle; the capacity current i_c leads e_1 by almost 90 degrees. With regard to the voltage e_2 in the second phase, the two currents flow practically in opposite directions; this is the reason why the wattmeter terminals must be reversed in the above experiment when changing from an inductance to a capacity.

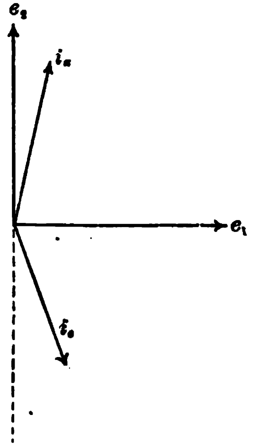

FIG. 137. Current and voltage relations, according to Fig. 136, shown vectorially.

When capacity and ohmic resistance are connected in series or in parallel, current and voltage relations and equivalent impedances may be determined from the triangles shown in Figs. 108 and 117, keeping in mind that capacity reactance y may be considered as a negative magnetic reactance x. The following experiment gives an example of these relations.

130. EXPERIMENT 6-C. — Capacity and Resistance in Parallel. — The connections are the same as in Fig. 120, save that a condenser of variable capacity is substituted for the reactance x. This corresponds to the practical case of a transmission line having an appreciable capacity, and supplying power to a non-inductive load. Keep the resistance constant and gradually increase the capacity; read component amperes, total amperes, volts and watts. Repeat the same experiment with different values of resistance and of voltage, and at a different frequency. Measure — with direct current — the values of resistances used.

Report. Plot to microfarads capacity as abscissæ the curves shown in Fig. 138. Check the following relations: (a) Total current must be a geometrical sum of the component currents, as in Fig. 117. (b) Power factor calculated from wattmeter readings must be the same as determined from the triangle of amperes. (c) Wattmeter readings must check with the calculated i^2r. (d) The charging current must satisfy the expression (2), in § 127.

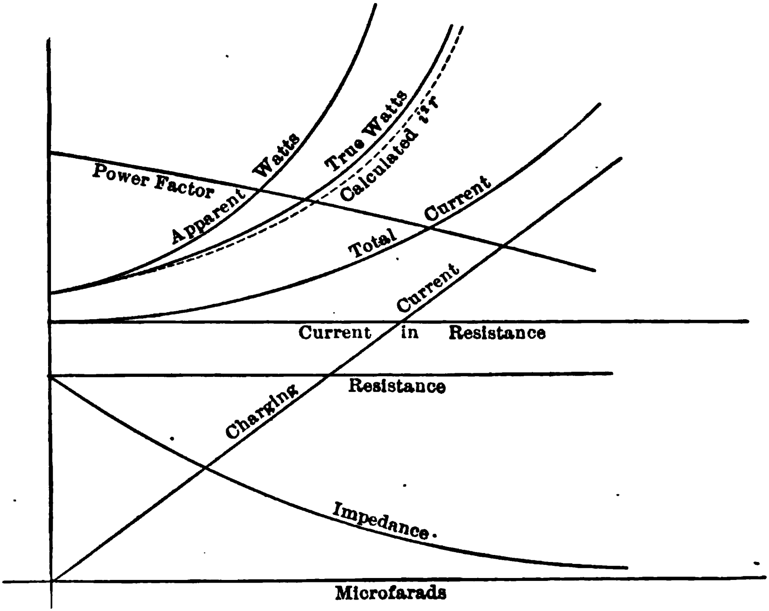

FIG. 138. Electrical relations with a resistance and a capacity connected in parallel to an alternating-current supply.

ELECTRIC RESONANCE.

131. Peculiar electrical relations arise in A. C. circuits, when capacity and inductance are present simultaneously. The phenomenon manifested is an abnormal rise of current, or of voltage, in part of the circuit, sometimes far above the values supplied by the source of power. This is due to the neutralizing effect of capacity on inductance, and vice versa (§ 129); the phenomenon is called *electric resonance*. The name is derived from a similar phenomenon of mechanical vibrations produced by the inertia and the elasticity of a system. It has been pointed out in § 96 that inductance is analogous to mechanical inertia, while electric capacity may be likened to elasticity (see mechanical analogy in § 117). Thus a certain combination of inductance and capacity gives rise to electrical oscillations of a definite frequency. If the frequency of the supply happens to be a submultiple of or is equal to that of these natural oscillations, the latter are greatly intensified, producing what is called electric resonance.

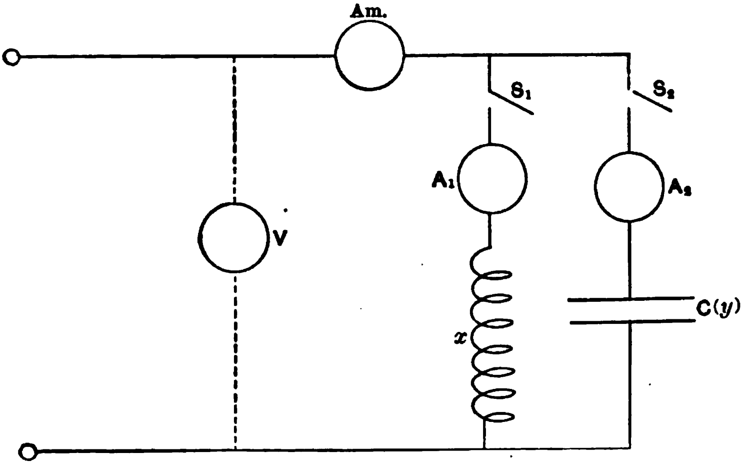

FIG. 139. Conditions necessary for producing a current resonance.

The phenomenon is different according to whether inductance and capacity are connected in parallel, or in series. When they are in parallel (Fig. 139), current resonance is produced; when they are in series (Fig. 142), voltage resonance is the result.

132. Current Resonance. — In the experiment illustrated in Fig. 136 capacity and reactance were supposed to be connected to the supply in succession, the currents being represented in Fig. 137. Should both be connected to the line, as in Fig. 139, the current in the main line becomes smaller than that in either branch. This is because the line current is the geometrical sum, or the short diagonal of the parallelogram built on i_x and i_c. Thus we obtain a seemingly paradoxical result, that either of the component currents is larger than their sum.

This is actually observed when both switches S_1 and S_2 are closed at one time. In the ideal case, when no ohmic resistance is present, the current in the reactance is lagging 90 degrees behind the voltage, and is equal to $E \div x$. The current in the condenser is equal $E \div y$, and leads the voltage by 90 degrees. The line current i is the difference of the two, or

$$i = \frac{E}{x} - \frac{E}{y} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (4)$$

When the magnetic impedance and the capacity impedance are equal to each other, in other words, when $x = y$, the line current i is reduced to zero, while a large current may circulate between x and C. The line voltage merely determines the value and the frequency of the resonance current, without actually supplying it. The condition $x = y$ means

$$2\pi n L = \frac{10^6}{2\pi n C} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (5)$$

or

$$\sqrt{LC} = \frac{10^3}{2\pi n} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (6)$$

In this formula L is in henrys, C in microfarads, and n in cycles per second. For instance, with $C = 50$ mf. and $n = 60$ cycles per second, perfect resonance takes place at a value of L deduced from the equation

$$L = \frac{1}{50}\left(\frac{1000}{2\pi.60}\right)^2,$$

or

$$L = 0.14 \text{ henry}.$$

With this value of inductance

$$\text{magnetic reactance } x = 2\pi \times 60 \times 0.14 = 52.8 \text{ ohms};$$

$$\text{capacity reactance } y = \frac{10^6}{2\pi.60.50} = 52.8 \text{ ohms};$$

the two being equal. Let the applied voltage be 220 volts; the current through the magnetic reactance

$$i_x = 220 \div 52.8 = 4.17 \text{ amp. (lagging)};$$

the current through the capacity reactance

$$i_y = 220 \div 52.8 = 4.17 \text{ amp. (leading)}.$$

The total line current

$$i = i_x - i_y = 0.$$

Let the inductance now be 0.09 henry; the magnetic reactance

$$x = 2\pi . 60 \times 0.09 = 33.9 \text{ ohms},$$

and $$i_x = 220 \div 33.9 = 6.5 \text{ amp.}$$

The total line current

$$i = 6.5 - 4.17 = 2.33 \text{ amp. (lagging).}$$

The line current is less than each of its components; this is a partial current resonance.

The relations become more complicated when ohmic resistance is present in either branch. The currents are then displaced less than 90 degrees from the line voltage and must be added geometrically. The diagram shown in Fig. 140 corresponds to the case when magnetic reactance remains constant, while the capacity is gradually increased. When the charging current is I_c', the resultant current I' in the line is lagging behind the impressed e.m.f. With a large capacity current I_c''' the resultant current I''' is leading. At a certain value I_c'' of the capacity current the total line current I'' is in phase with the e.m.f., as if the circuit possessed no inductance or capacity. The same relations are shown in Fig. 141 in the form of curves. As the capacity is increased, the line current first drops, then increases again as it becomes leading.

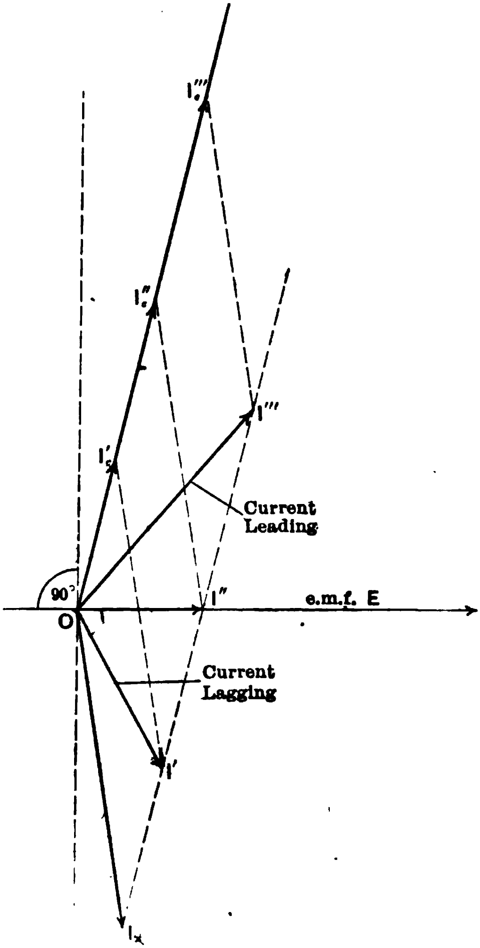

FIG. 140. A vector diagram showing current resonance.

133. EXPERIMENT 6-D. — Capacity and Reactance in Parallel. — Current Resonance. — The connections are shown in Fig. 139; it is well to have a wattmeter in the circuit, so as to be able to calculate the power factor of the line current. Close the switch S_1 and adjust the reactance current to a desired value. Then close S_2 and gradually increase the capacity. Read component amperes (in A_1 and A_2), total amperes, volts and watts. Note that the total current decreases and then increases again. Instead of using three ammeters, it may be more convenient to use one ammeter, and a polyphase board (§ 49).

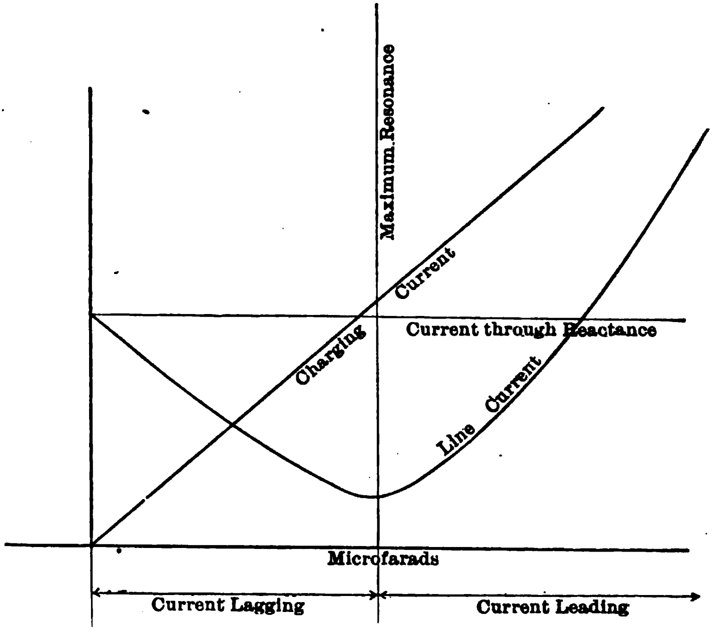

FIG. 141. Curves illustrating the gradual rise of current resonance.

Introduce some resistance in series or in parallel with the inductance, and repeat the same test. Also shunt the condenser with some resistance. Finally insert resistance in both branches, always taking readings with a gradually increasing capacity.

Report. Plot the results as shown in Fig. 141; construct several diagrams as in Fig. 140 and check the angles with those calculated from the wattmeter readings. See how closely the values of charging current check with those calculated from the expression (2) in § 127; also if equation (7) is satisfied when the capacity current is equal to the current through the reactance.

134. Voltage Resonance. — We shall now consider the resonance produced by a reactance x and a condenser C connected in series (Fig. 142) in an A. C. circuit; the resistance r may at first be left out of consideration $(r = 0)$. Suppose a certain current i flow through the system; the voltage across the reactance leads the current by 90°; the voltage across the condenser lags behind it by the same amount. This means that the waves of voltage across MN and across PQ are opposite and partly neutralize each other. Therefore the total voltage MQ is smaller than one of the component voltages, either across the reactance, or across the condenser. It may also be smaller than either of them. This peculiar condition is called *voltage resonance.*

An example may make this clearer. Let the magnetic reactance have a value of 10 ohms and the capacity reactance be 2.5 ohms, both values at a frequency of 60 cycles per second. The equivalent reactance is $10 - 2.5 = 7.5$ ohms.

At a terminal voltage of 150 volts between M and Q, the current is = $50 \div 7.5 = 20$ amperes. The drop across the magnetic reactance is

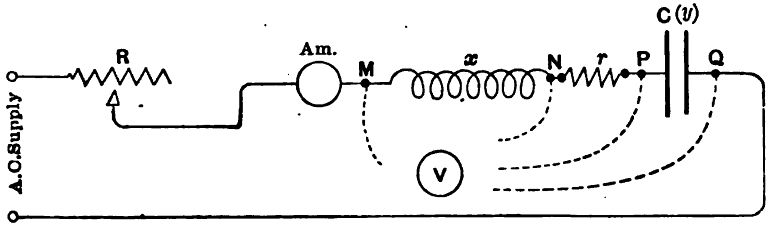

FIG. 142. Conditions necessary for producing voltage resonance.

$20 \times 10 = 200$ volts; that across the capacity $20 \times 2.5 = 50$ volts. Thus the drop across the magnetic reactance is larger than the applied voltage. If the capacity reactance were 5 ohms the current would rise to 30 amperes and the drop across the magnetic reactance would be 300 volts. With a capacity reactance of 10 ohms, the combined reactance becomes zero; the current and the voltage rise indefinitely. When such a combination actually happens either the condenser breaks down, or the inrush of current opens the circuit-protecting device.

Reducing the capacity still further, or, which is the same, increasing the capacity reactance, makes the voltage drop across the condenser larger than that across the reactive coil. For instance, with a capacity reactance of 15 ohms the equivalent reactance becomes $15 - 10 = 5$ ohms, and the current 30 amperes; in this case the current is leading. The voltage drop across the condenser becomes $30 \times 15 = 450$ volts.

Instead of varying the capacity, a perfect resonance could be obtained by changing the frequency. Let us assume again the same values of reactances, 10 ohms and 2.5 ohms at 60 cycles per second. At a frequency of 30 cycles, the equivalent reactance becomes

$$10 \div 2 - 2.5 \times 2 = 0.$$

The current and the voltage rise indefinitely.

135. Vector Diagrams of Voltage Resonance. — Resonance is made less harmful in practice, due to the presence of some resistance r (Fig. 142), — resistance limits the inrush of current and the rise of potential. Assume that the resistance r and the reactance x remain constant, while the capacity C is being gradually increased. This makes both the current in the line and the voltage across MQ vary. To make the conditions more definite we shall consider two limiting cases: (1) line current kept constant; (2) line voltage kept constant. The regulation of current and voltage is made by the rheostat R.

(*a*) *Line current constant* (Figs. 143 and 144). The drop $OA = ix$ in the inductive reactance is leading the vector of the current by 90°. The ohmic drop $AB = ir$ is in phase with the current. The drop $BE' = iy$ in the capacity is lagging behind the current. The total voltage across the combination of the three is represented by the vector $OE' = iz$, where z is the equivalent impedance of the circuit. From the figure $OABE'$

$$Z = \sqrt{(x - y)^2 + r^2},$$

the capacity reactance y opposing the magnetic reactance x. The current is lagging behind the line e.m.f. the action of the inductance being preponderant. With a larger capacity reactance (smaller capacity) the drop across the condenser may become as large as BE'''.

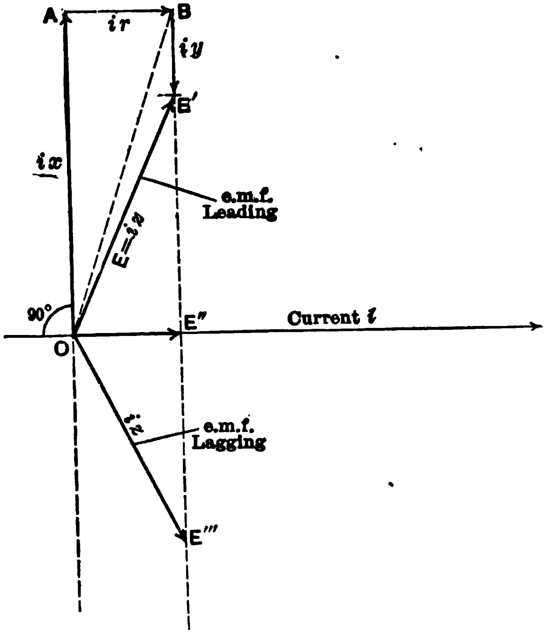

FIG. 143. Vector diagram of voltage resonance, at a constant current.

The applied voltage becomes OE''', the current being in this case leading. The equivalent reactance z of the circuit may have the same value as before, except that $(y - x)$ must be used instead of $(x - y)$.

At a certain value of capacity, the drop across it becomes equal to the drop across the reactance x, and the resultant e.m.f. OE'' is in phase with the current. The line voltage is, in this case, just sufficient for overcoming the ohmic resistance $(OE'' = ir)$; the capacity and the inductance neutralize each other. If the circuit contained no resistance, the current and the voltage would rise indefinitely.

Thus with variations of capacity, the extremity of the vector of the applied voltage travels on BE''', the current being lagging for large values of capacity (small values of y), and vice versa. The corresponding relations are plotted in Fig. 141.

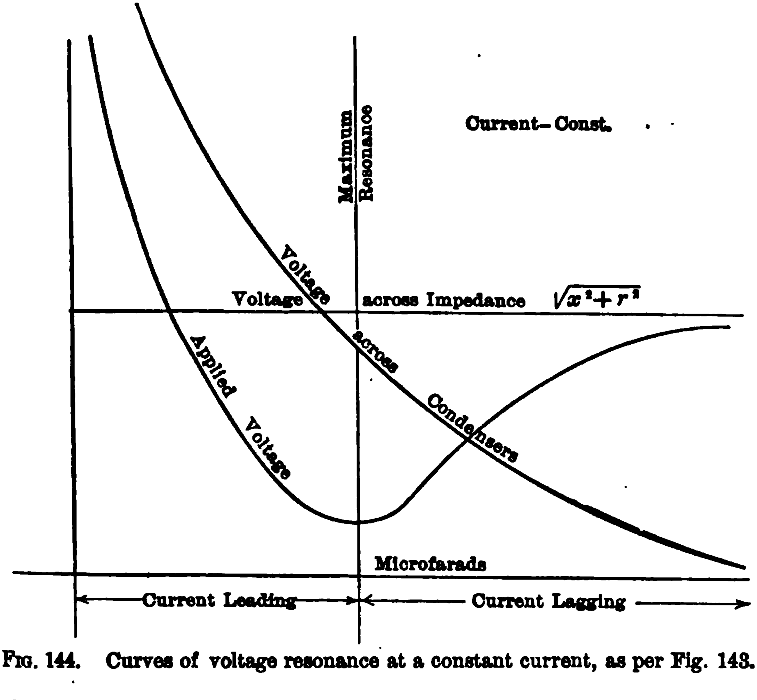

FIG. 144. Curves of voltage resonance at a constant current, as per Fig. 143.

(b) *Line voltage constant* (Figs. 145 and 146). The polygon $OA'B'E'$ is the same as $OABE'$ in Fig. 143. The current being variable, the triangle of drop $OA'B'$ increases and decreases with the value of the current, remaining similar to itself. At the same time the point E' moves on the circle drawn with the line voltage E as a radius; the assumption being that the line voltage is maintained constant by means of the rheostat R (Fig. 142).

For small values of capacity (large values of y) the figure assumes the position $OA'''B'''E'''$, the current becoming leading. At a certain value of capacity, such that $x = y$, the e.m.f. $= E''$ is in phase with the current, as if the circuit consisted of ohmic resistance r only. The relations are represented by the curves in Fig. 146.

136. EXPERIMENT 6-E. — Capacity and Reactance in Series. — Voltage Resonance. — The connections are shown in Fig. 142;

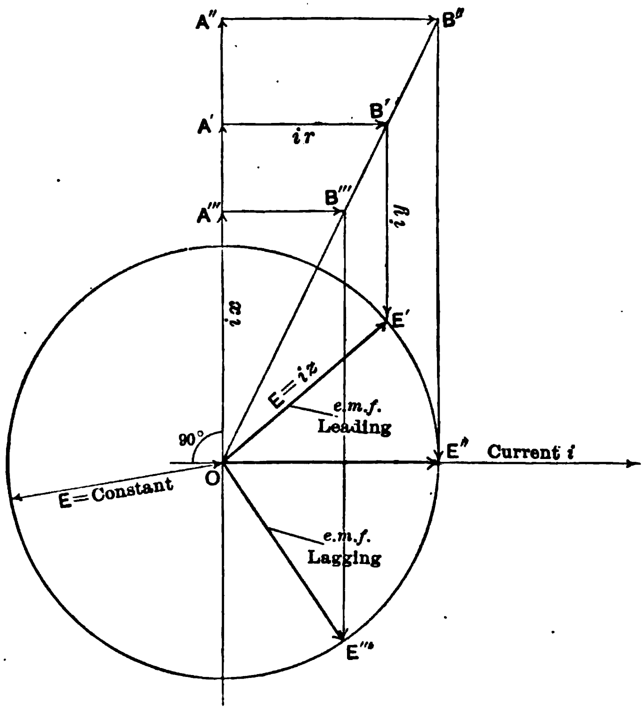

Fig. 145. Voltage resonance at a constant applied voltage.

it is well to have in addition a wattmeter for determining the phase displacement between the line voltage and the current. First try a resonance in its pure form, without the resistance r. Introduce enough resistance R in the circuit so that the current would not exceed a predetermined value, even when $x = y$; or else protect the circuit by a reliable circuit-breaker. Connect in as much capacity as possible, and a large inductive reactance, so as to have a rather small lagging current. This corresponds to the right end of the curves in Fig. 146. Gradually reduce the inductive reactances measuring voltages across x and across

C; also the line voltage and the current. The voltage curve (Fig. 146) rises very abruptly; therefore the student should be careful not to break down the insulation of the condenser, — the current may not be very large before the safe limit of voltage for the condenser is reached. When the limit is reached, open the circuit and reduce the reactance x considerably beyond the resonance point, so as to have a small leading current. This corresponds to the left end of the curves in Fig. 146. Close the circuit, and approach the point of maximum resonance from the other side. During this preparatory experiment neither line current nor the voltage needs to be kept constant.

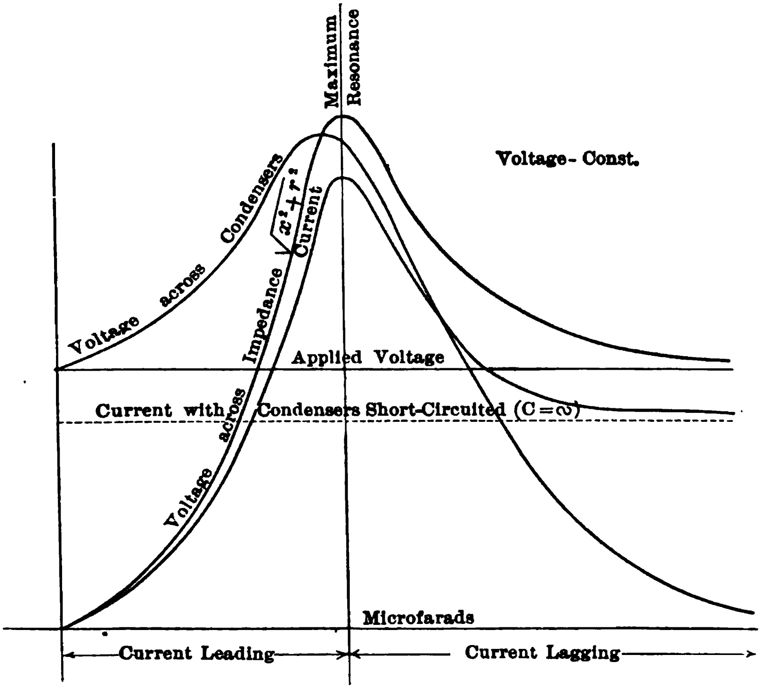

FIG. 146. Curves of voltage resonance at a constant applied voltage, as per Fig. 145.

Now connect some resistance r, such as to give, say, 10 to 20 per cent ohmic drop at the desired current; take curves shown in Fig. 144, that is to say, keeping the line current constant. Keep the resistance r and the reactance x constant and vary the capacity within as wide limits as possible. Finally short-circuit the condenser to realize the limiting condition with an infinitely large capacity (horizontal line in

Fig. 144). Perform a similar test, keeping the terminal voltage constant (Fig. 146).

Take readings with different values of reactance, capacity, resistance, and frequency, in order to see the influence of each factor.

Report. Plot curves, as in Figs. 144 and 146; check some of the results by means of diagrams shown in Figs. 143 and 145.

CHAPTER VII.

THE MAGNETIC CIRCUIT.

137. THE fundamental electromagnetic relations may be conveniently studied on the apparatus shown in Fig. 147. It consists of several sets of U-shaped pieces *m*, *m′*, of steel or iron and of magnetizing, or exciting coils *pp*, connected to a source of direct-current supply.

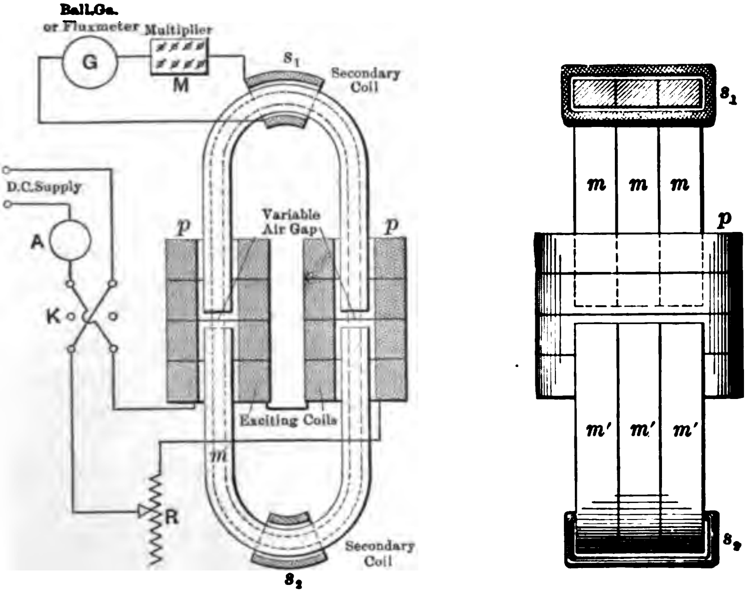

FIG. 147. **An apparatus for studying the fundamental relations in a magnetic circuit.**

K is a reversing switch by means of which the iron cores may be magnetized in either direction. When *K* is closed, a current flows through the coils *pp* and produces a magnetic flux, as shown by dotted lines. The intensity of magnetization — or the flux — may be varied:

(a) By regulating the exciting current with the rheostat *R*.

(b) By changing the number of coils in the circuit.

(c) By increasing or decreasing the air-gap between the upper and the lower cores.

(d) By changing the number of sections *m*, *m'*.

(e) By replacing the cores by others made of a better or of a poorer magnetic material.

The exciting current is read on the ammeter *A*. The amount of magnetism produced in the core is measured by the exploring coils s_1 and s_2, connected to a ballistic galvanometer (§ 139) or a fluxmeter (§ 140).

138. Ampere-Turns and Magnetic Flux. — The amount of magnetism produced in the cores *m*, *m'*, depends on the magnitude of the exciting current, and on the number of turns in the exciting coils; but the magnetism remains constant so long as the product "current times turns" remains constant. For instance, a current of 5 amperes, flowing through a coil with 20 turns, produces the same magnetizing effect, as 20 amperes flowing through a coil having 5 turns, or 1 ampere with 100 turns. The magnetizing, or the *magnetomotive* force is the same in each case and is equal to 100 *ampere-turns*. The reason is easy to see. Imagine a current of 1 ampere flowing through 1 turn of wire, and take 100 such elements. Combine them in series and you get 1 ampere flowing through 100 turns; combining them in parallel gives 1 large turn with 100 amperes. The magnetic effect *outside* the coil cannot depend on whether the turns are connected in series or in parallel, but merely on the number of elements producing the action; therefore it depends on the number of ampere-turns only.

The magnetic state in a medium may be imagined as a state of tension or attraction along certain lines; these imaginary directions are called "magnetic lines" or "lines of force." They are shown in Fig. 147 by dotted lines. In this connection the reader may be reminded of the familiar physical experiment, in which iron filings arrange themselves around a steel magnet in certain directions; these are the directions of lines of force. Lines of force, apart from their direction, are purely imaginary; it is convenient, however, to measure magnetism by assigning a certain *numerical* value to each line of force. With this supposition, lines of force are considered closer to each other in places where magnetization is stronger, and vice versa. This leads to the conception of *density of lines of force*, so important in electrical engineering.

The sum total of lines of force within a certain space is called the magnetic flux. The usual way of measuring and defining the magnetic flux within a coil, such as the coil s_1 in Fig. 147, is by the electric effects produced by the flux. When the magnetic flux, passing through a coil, increases or decreases, an e.m.f. is induced in the coil, proportional to the rate of the variation of the flux. The rational unit of magnetic flux in the volt-ampere-ohm system should therefore be based on the following definition: When a magnetic flux varies uniformly at the rate of one line of force per second, the e.m.f. induced in a coil which surrounds it is one volt per turn. Such a unit of lines of force is too large for practical purposes; the unit actually used is based on a similar definition, except that a C. G. S. unit of electromotive force is taken instead of the volt. This unit is 10^8 times less than the volt; this coefficient enters therefore into all formulæ which express an induced e.m.f. in volts in terms of flux.

To make the above definition clearer, suppose that a certain magnetism is produced in the cores m, m' by exciting the coils p, p with a certain current. Let it be required to express this magnetism numerically by the number of lines of force. Let the secondary or exploring coil s_1 consist of 100 turns of wire and be connected to a very sensitive voltmeter. So long as the flux remains constant, the voltmeter shows no deflection. Let now the exciting current be gradually reduced to zero by the rheostat R, at such a rate, that the voltmeter shows all the time 0.05 volt. Suppose 30 seconds of time elapse before the current is reduced to zero and the flux disappears. From these data the original flux is figured out as follows: The e.m.f. induced in one turn of the secondary coil is, in C. G. S. units,

$$\frac{0.05 \times 10^8}{100} = 50{,}000 \text{ (C. G. S.)}$$

This, according to the above definition of the flux, shows that the flux was decreasing at the rate of 50,000 lines of force per second. As it took 30 seconds for the flux to be reduced to zero, the original flux was $50{,}000 \times 30 = 1{,}500{,}000$ lines of force.

In accordance with the modern fashion of naming magnetic and electric units after famous scientists, the unit of flux, or one line of force, as above defined, was named *the maxwell*, so that fluxes are expressed in maxwells. While the unit of flux defined on the basis of one volt per second is too large, the maxwell is too small a unit for many practical purposes. A compromise is reached by using the following derivative units:

1 kilo-maxwell (or kilo-line) = 1000 maxwells.
1 mega-maxwell (or mega-line) = 1,000,000 maxwells.

It would be rather troublesome to measure magnetic fluxes by gradually reducing them to zero at a rate to give a constant voltmeter deflection, as described above. The following two methods, though based on the same principle, are more convenient.

139. Measurement of Magnetic Flux with a Ballistic Galvanometer. — A ballistic galvanometer is a measuring instrument whose deflections are proportional to very brief electric discharges through it. An electric discharge through such a galvanometer may be made proportional to a sudden change in a magnetic flux. For this reason the ballistic galvanometer is widely used for measuring magnetic fluxes. Like most galvanometers, so widely used in physics, a ballistic galvanometer (Fig. 149) consists of a coil of fine wire suspended in the field of a permanent magnet. When a current passes through the coil, it is deflected according to the strength of the current. The torsion of suspension wires, or spiral springs (Fig. 34) return the coil to zero, when no current flows.

The only difference between the usual type and the ballistic galvanometer is, that the moving part of the latter has a much greater inertia. This is necessary in order that the coil should not begin to move, until the total discharge has passed through it. Only under this condition does the instrument give deflections, which depend on total discharge, rather than on its duration or the instantaneous values of the current. If, for instance, the moving part is made so heavy, that the period of its swing is 10 seconds, the coil will not move appreciably with any discharge that lasts, say, less than 0.5 second.

For measuring a magnetic flux, the ballistic galvanometer is connected to the secondary coil s_1, — if necessary through a resistance M to reduce deflections. As long as the flux remains constant, the galvanometer shows zero, but if the flux is suddenly changed, an electric discharge is induced in the galvanometer circuit, and the coil begins to move. The final deflection multiplied by a constant of the instrument is a measure for the change in flux. Let, for instance, the galvanometer constant be 50,000 maxwells per 1 cm. scale deflection. Then should the deflection be 15 cm., the change in flux is $50{,}000 \times 15 = 750{,}000$ maxwells, or 750 kilo-lines.

The theory and the calibration of ballistic galvanometers are explained in §§ 177 and 178.

140. Grassot Fluxmeter. — The disadvantages of the ballistic galvanometer, described in the preceding article, are:

(1) It is necessary to change the flux in order to measure it.

(2) The change must be sudden, in order that the electric discharge be completed before the galvanometer coil begins to move.

(3) The indications are only instantaneous, as the moving coil immediately begins to return.

These disadvantages are eliminated in the Grassot fluxmeter, shown in Fig. 148. The details of the moving part are shown in Fig. 149. It is essentially a ballistic galvanometer, except that the moving coil is suspended from a single cocoon fiber of negligible torsional stiffness. In an ordinary ballistic galvanometer the coil is returned to zero under the influence of the torsion of the suspension wire; while in the fluxmeter the coil remains at rest in any position, and creeps only excessively slowly towards zero. The upper end of the fiber is attached to the flat spiral spring *R* to minimize the effect of shocks. The stiff wire frame *E* fixed upon the coil *B* allows of a central attachment of the very thin silver strips *s* and *s′*, serving to lead the current to and from the coil *B*. The soft iron core *A* is supported within the coil, between the pole-pieces *NS* of a permanent magnet, the field in the air-gap being of a great and uniform intensity. An arrestment, controlled by a milled button in front of the instrument, serves to lock the coil when not in use; at the same time it brings the pointer back to zero.

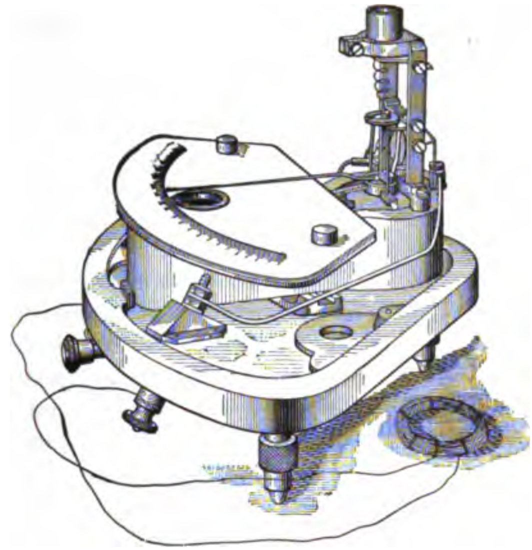

Fig. 148. The Grassot fluxmeter.

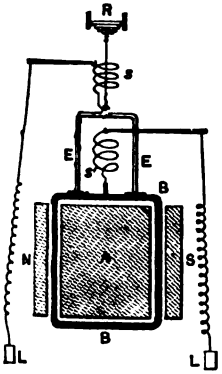

Fig. 149. Arrangement of parts in the Grassot fluxmeter.

The instrument is connected as shown in Fig. 147. When the switch *K* is closed, the coil of the fluxmeter starts into motion and comes to rest in a new position, the deflection being determined solely by the magnitude of the flux (§ 177 *b*). Whether the flux is increased suddenly or gradually, the deflection is precisely the same. When the flux is varied, the needle of the instrument follows the variations. When the exciting circuit is opened, the coil returns once more to its zero position. The fluxmeter may be calibrated directly in maxwells; it permits the reading of magnetic fluxes with the same facility with which currents and voltages are read on ordinary ammeters and voltmeters.

141. Saturation in Iron. — Experience shows, that magnetic flux in air and in other non-magnetic substances is strictly proportional to the number of ampere-turns which produce it. On the contrary, in iron and steel the flux, while intensified many times, increases more slowly than the exciting ampere-turns (Fig. 150). This peculiar property of iron is expressed, for the lack of a better term, by saying that the iron becomes gradually *saturated* with lines of force, and only reluctantly allows their number to be increased. The following example may make this clearer.

Let the iron cores in the apparatus, shown in Fig. 147, be removed, and a current of 10 amperes sent through the exciting coils. Let the fluxmeter register a small flux of 2 kilo-lines (in the air). With an exciting current of 20 amperes it will register 4 kilo-lines. Now put the cores in place and excite the coils again with 10 amperes; a much larger flux will be found, say 800 kilo-lines. But at 20 amperes the flux will probably be only 1200 instead of 1600 kilo-lines, showing that the iron is approaching the saturation limit. The influence of saturation is seen by the curve (Fig. 150) gradually bending toward the axis of abscissæ; for a given per cent increase in excitation, the flux increase becomes less and less.

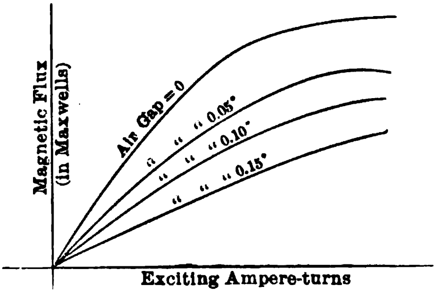

FIG. 150. Influence of small air-gaps on the magnetic flux.

The curves in Fig. 150 illustrate clearly the difference in the magnetic properties of air and iron. The addition of a very small air-gap — a few thousandths of an inch — decreases appreciably the flux produced by a given number of ampere-turns. This is expressed by saying that the *permeability* of air is much lower than that of iron.

For some practical purposes it is desirable to express the degree of saturation numerically. The revised Standardization Rules (1907) of the American Institute of Electrical Engineers specify the so-called *saturation factor* as a criterion of the degree of saturation, attained in a magnetic circuit. The saturation factor, by definition, is the ratio of a certain (small) per cent increase in exciting ampere-turns to the corresponding percentage increase in magnetic flux thereby produced. In the lower part of the curve (Fig. 150) one per cent increase in exciting ampere-turns produces one per cent increase in the flux; therefore, the saturation factor here is unity. As the curve bends towards the axis of abscissæ the saturation factor gradually increases to infinity. The reciprocal of the saturation factor, deducted from unity, is called *percentage of saturation*. For a graphical method of determination of the saturation factor and of the percentage of saturation see Articles 57 and 58 of the *Standardization Rules*.

142. EXPERIMENT 7-A.— Magnetization Curves and Influence of Air-Gap. — The purpose of the experiment is to illustrate the fundamental relations between a magnetic flux and its exciting ampere-turns. The connections are shown in Fig. 147. The constant of the ballistic galvanometer or of the fluxmeter is supposed to be known; otherwise the instrument is calibrated as described in § 178. Before beginning the experiment, thoroughly demagnetize the cores from the residual flux which they may contain after a previous experiment. To do this, excite the cores to a considerable degree and then gradually reduce the current to zero by the rheostat R, at the same time continually reversing the current by the switch K.

Eliminate the air-gap as thoroughly as possible, by bringing the cores together, and excite the circuit with a small current, keeping the galvanometer circuit open.

(a) With the circuit of the galvanometer closed, but with the coil at rest, read the exciting current and suddenly reverse K; observe the galvanometer deflection. It is preferable in practice to reverse the exciting current, instead of opening the circuit, on account of some indefinite residual magnetism which remains when the circuit is opened. By reversing the current the flux is changed by a definite amount $N - (-N) = 2N$. Again open the galvanometer circuit, increase the exciting current, and produce a second discharge through the galvanometer by reversing the switch K. Proceed in this way as far as the exciting coils will stand the increasing current. To keep galvanometer deflections within reasonable limits either the resistance M is varied, or the number of turns in the coil s_1.

(b) Again demagnetize the cores and repeat the same experiment with a definite air-gap, say 0.010 inch. Such an air-gap may be easily obtained by interposing a thin piece of fiber or paper between the cores; these materials being non-magnetic have the same effect as so much air. Take a magnetization curve as before. Make similar tests with larger air-gaps.

(c) Substitute cores made of other kinds of magnetic materials, say wrought iron, and cast iron.

(d) Check the statement made in § 138, that the exciting force depends on the number of ampere-turns, and not on the number of turns and the current separately.

If a fluxmeter is used instead of a ballistic galvanometer it is not necessary to reverse the exciting current. After the cores have been demagnetized, connect the fluxmeter to the secondary coil s_1, and gradually increase the magnetizing current. Read exciting amperes and fluxmeter deflections.

Report. (1) Plot curves as in Fig. 150.

(2) Plot values of saturation factor (§ 141) for an upper and a lower curve.

(3) See if the increase in ampere-turns, with a given value of flux, is proportional to the length of the air-gap.

(4) Determine for two or three values of flux, whether the increase in ampere-turns due to the air-gap is proportional to the magnitude of the flux.

143. Influence of the Length and Cross-Section of a Magnetic Circuit on its Reluctance. — Imagine the magnetic circuit, shown in Fig. 147, to be entirely closed, that is to say, to consist of a solid piece of iron without any air-gap. Experience shows, that in this case the number of ampere-turns necessary to produce a given flux is proportional to the length of the magnetic circuit; this length is the average length of the lines of force, as indicated by the dotted lines. This fact is analogous to a similar relation in the electrical circuit, namely, that the e.m.f. necessary to produce a certain current in a conductor, is proportional to the length of the conductor, other conditions being identical. Here the exciting ampere-turns, or the magnetomotive force is analogous to electromotive force, and the magnetic flux to electric current. The magnetic circuit may thus be said to possess a certain resistance to the passage of lines of force. To distinguish this magnetic resistance from electrical resistance, the former is called "reluctance." Thus we may say that reluctance is proportional to the length of a uniform magnetic circuit.

Experience shows also, that with non-magnetic materials, reluctance is inversely proportional to the cross-section of the path of the flux. This is again analogous to electrical resistance, which is inversely proportional to the cross-section of a conductor. The rule is not altogether true for steel and iron, on account of the peculiar phenomenon of saturation described in § 141 (unless referred to a constant flux density). When the number of lines of force per square inch, or the magnetic density, increases, the required number of ampere-turns increases more rapidly than the flux. This is equivalent to saying that the permeability (magnetic conductivity) of iron decreases, as the flux density increases.

144. Flux Density and Ampere-Turns per Inch. — The above considerations show that the number of ampere-turns per unit length, necessary for producing a given flux, in a given uniform magnetic circuit, is proportional to the length of the circuit, and depends on the magnetic density, but not on the value of the flux itself.* Thus, the necessary number of ampere-turns may be expressed with the aid of a single experimental coefficient: *Ampere-turns per unit length at a given flux density.* In the above example (§ 141) let the cross-section of the iron be 10 square inches, so that the flux density with 10 amperes exciting current is 80 kilo-lines per square inch. Let the number of turns in the exciting coils be 320, and the length of the magnetic circuit 100 inches. Then, with the iron used,

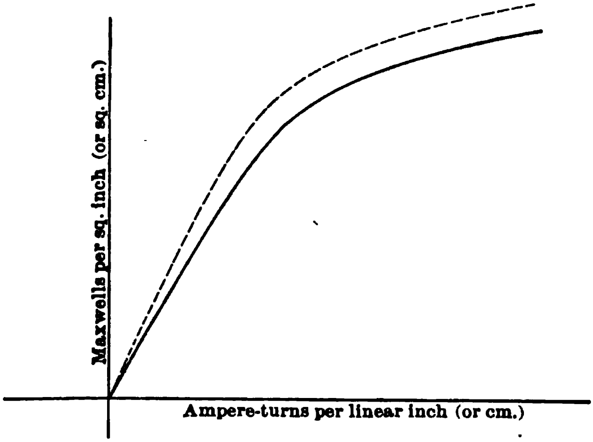

FIG. 151. Magnetization curve of a sample of iron, and the influence of an imperfect magnetic contact (the dotted line represents the ideal curve).

$$\frac{320 \times 10}{100} = 32 \text{ ampere-turns per linear inch}$$

necessary to produce the above flux density. This number, 32, characterizes the material at the density of 80 kilo-lines, without regard to the length or the cross-section of the circuit. The curve shown in Fig. 151 is plotted in this way from the upper curve in Fig. 150. The

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* The magnetic circuit is no different in this respect from the electric circuit. The familiar Ohm's law may be written in the form

$$E = I . k \frac{l}{q}$$

or

$$\frac{E}{l} = k \frac{I}{q}$$

which means, that the e.m.f. per unit length of conductor is proportional to current density, but does not depend on the value of the current itself.

dotted curve shows the ideal case when the cores consist of one solid piece of iron; the curve below it is the one obtained with the arrangement shown in Fig. 147, where a small air-gap is unavoidable.

145. EXPERIMENT 7-B. —Influence of the Length and Cross-section of a Magnetic Circuit on its Reluctance. —The arrangement of apparatus and the method for measuring the flux is the same as in § 142. (a) Take one section of the cores *m, m′* (Fig. 147) and bring the ends together as accurately as possible, so as to reduce the air-gap to a minimum. Take a magnetization curve, as in Fig. 150. (b) Now take the cores apart and fasten between them two straight pieces of iron, of the same cross-section and same quality of material as the rest, so as to increase the length of the magnetic circuit. Taking a similar curve, you will find that more ampere-turns are necessary to produce the same flux, because of the greater length of the magnetic circuit. (c) Take out the straight pieces and add more sections *m, m′* in parallel, so as to increase the cross-section of the magnetic circuit. Again take a magnetization curve. It will be found that the same flux is obtained with a considerably smaller number of ampere-turns. (d) Repeat the same experiment with cores made of different material.

Report. Show from the results of the test that the number of ampere-turns necessary to produce a given flux is proportional to the length of the circuit, but increases faster than the decrease in the cross-section of the magnetic circuit, due to saturation in the iron. For each material tested plot a curve, as in Fig. 151.

146. Magnetic Leakage. — When two or more paths in parallel are offered to an electric current, it is divided into parts inversely proportional to the resistances of the paths. In a similar way, a magnetic flux is divided when two paths are offered to it, as in Fig. 152. The apparatus shown there is the same as in Fig. 147, only the exciting coils *pp* are placed unsymmetrically. The flux produced in the lower core in part goes through the upper core, in part finds its way directly through the air, as shown by dotted lines. This latter portion is called the *leakage flux,* because in most practical cases it is not utilized for the purpose for which the magnetic circuit is produced. Thus, in electric generators and motors, part of the flux — instead of going through the armature of the machine — is shunted around it, and passes uselessly from pole to pole, directly through the air. The lower core in Fig. 152 corresponds to the field frame of a machine, the upper one to its armature.

The harm occasioned by magnetic leakage consists in the necessity for a larger flux to be produced than is actually utilized. This means more exciting ampere-turns, consequently more copper, and more expenditure of energy for excitation. The case is made worse by saturation in iron, since any increase in flux means a disproportional increase in the number of exciting ampere-turns. Suppose, for instance, that the leakage in a certain case amounts to 20 per cent of the useful flux (this is not an uncommon figure in practice). The increase in the exciting ampere-turns necessary for producing 20 per cent additional lines of force may be 50 per cent or more, due to a higher saturation in iron.

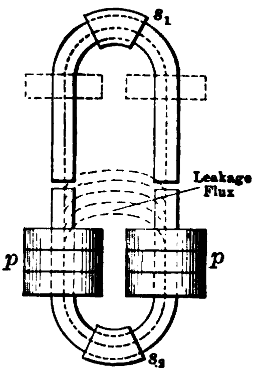

FIG. 152. Arrangement of coils for studying magnetic leakage with the apparatus illustrated in Fig. 147.

For this reason it is of importance to know how to measure magnetic leakage, and to learn the principal factors upon which it depends. In the apparatus shown in Fig. 152, the leakage flux, or the difference between the total and the useful fluxes, may be measured by a ballistic galvanometer or a fluxmeter as before, by taking discharges first through the coil s_2 (total flux), then through s_1 (useful flux). It is more difficult to study accurately the actual distribution of the leakage flux in space, because this means measuring its magnitude and direction from point to point. At the same time, a study of the map of magnetic leakage is of a considerable practical importance, because it permits the designer to modify the form and dimensions of the frame of a machine so as to minimize the leakage.

A preliminary study of distribution of the leakage flux may be made with a small magnetic needle freely suspended from a silk thread. The direction of the needle in different places of the stray field gives an idea as to the directions of the leakage lines of force. After this, the flux may be investigated either with a ballistic galvanometer, a fluxmeter, or a bismuth spiral (§ 185). When a ballistic galvanometer is used, a small exploring coil is connected to it, and is brought into the place in which it is desired to measure the leakage. The coil is held on a pivot by a retaining spring. Relieving the spring, the coil is snapped by 180 degrees. The flux cut during this movement produces a discharge in the galvanometer proportional to the flux. If this is impracticable, for instance, because of a narrow space, the coil is made flat and after having been brought into the desired place is withdrawn quickly from the field. In some other cases the exciting current must be broken or reversed in order to produce the desired galvanometer deflection.

When using a fluxmeter or a bismuth spiral, the mere fact of bringing the exploring coil into the field gives the desired deflection.

147. Factors on which Magnetic Leakage Depends. — Generally speaking, the magnitude of a leakage flux depends on the relative values of the reluctances of the useful path and of the stray paths. Any factor which decreases the reluctance of the stray path or increases the reluctance of the useful path creates conditions favorable for leakage. Thus, for instance, making the U-shaped cores (Fig. 152) narrower, in other words, bringing the two legs of each core closer together, is sure to increase the leakage, as it shortens the path for the leakage flux through the air. Increasing the air-gap between the two U's also increases per cent leakage, because it increases the reluctance of the useful path. For the same reason per cent leakage increases with the increase in saturation in iron. Moving the exciting coils lower, increases the cross-section of the leakage flux (in the vertical plane perpendicular to the paper), consequently increases the leakage flux itself.

An important case of increase in leakage is that due to some counteracting ampere-turns (here put on the upper core, as shown by dotted lines). This corresponds in practice to armature currents which counteract the excitation produced by the field coils. The phenomenon is known in generators and motors as *the armature reaction.* That armature reaction increases magnetic leakage, may be understood from the following example. Let it be required to maintain a certain flux of N lines of force in the upper core (Fig. 152), whether or not a current flows through the upper coils shown by dotted lines. Suppose that each section of the coils has 40 turns, and that at no load the necessary flux N is produced with 10 amperes exciting current, or $10 \times 6 \times 40 = 2400$ ampere-turns. Let the leakage flux at no load be 20 per cent of N. Now suppose that an opposing current of 15 amperes is sent through the upper coils, creating an armature reaction of $15 \times 2 \times 40 = 1200$ ampere-turns. Without leakage, the lower exciting coils would have to produce $2400 + 1200 = 3600$ ampere-turns in order to give the same flux as before. With leakage this increase is not sufficient, as is shown by the following simple calculation. The leakage flux in the lower core at no load was $0.2\,N$; the increase in ampere-turns to 3600 increases the leakage to

$$0.2\,N \times \frac{3600}{2400} = 0.3\,N.$$

Consequently, with the same total flux $1.2\ N$ as before, the flux in the upper core is now only $1.2\ N - 0.3\ N = 0.9\ N$, even though 50 per cent increase in ampere-turns were added on the lower core. This shows the importance of keeping the magnetic leakage as low as possible.

148. EXPERIMENT 7-C. —A Study of Magnetic Leakage. — The purpose of the experiment is to illustrate the relations explained in the two preceding articles. The apparatus used is shown in Figs. 147 and 152. Magnetic leakage depends on several factors, and it is best to investigate the influence of each factor separately. The total flux is measured by discharge through the coil s_2 (Fig. 152), the useful flux by that through the coil s_1.

(*a*) *Influence of air-gap and of saturation.* Set the exciting coils *pp* as shown in Fig. 152, and place the upper core as close as possible to the lower core (no air-gap). Take an accurate curve of fluxes in s_1 and s_2 with various values of the exciting current (Fig. 150). Read the fluxes as in experiment 7-A (§ 142). Repeat similar tests with two or three different values of air-gap.

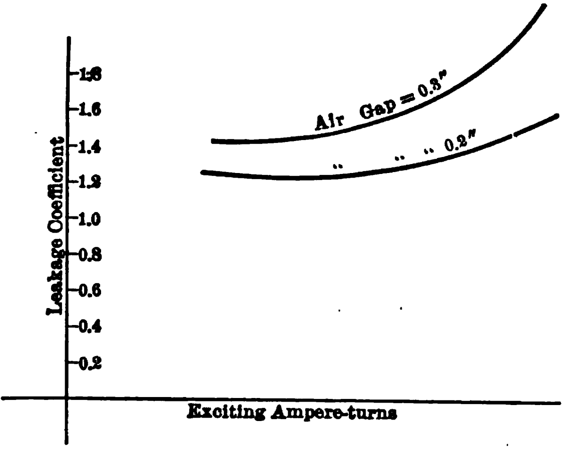

FIG. 153. Increase in per cent leakage with increased saturation in iron.

(*b*) *Influence of the position of the magnetizing coils.* Select a value of air-gap, and place the exciting coils symmetrically with respect to the two cores, as in Fig. 147, and measure the fluxes in both cores. Then gradually move the coils lower, as in Fig. 152, and take similar readings. Repeat the same experiment with different values of air-gap and of the exciting current.

(*c*) *Influence of the shape of pole-pieces.* Put pieces of iron on the inner side of the ends of the lower core, so as to assist the leakage through the air. Investigate the influence of the shape of these pole-pieces with different degrees of saturation in the iron and with different lengths of the air-gap.

(*d*) *Effect of armature reaction.* Excite the coils on the upper core to oppose the lower coils, as shown in Fig. 152. Determine the value of the flux in s_2 necessary for producing a given flux in s_1, with and without the opposing coils. Reverse the current in the upper coils, so as to make them assist the lower exciting coils; observe the change in leakage. Repeat similar tests with different positions of the coils, different degrees of saturation in the iron and various lengths of air-gap.

Report the results in such a form as to make clear the influence of each factor. The ratio of total flux to useful flux is called *the leakage coefficicnt.* Make use of this coefficient in order to bring out the results of the experiment in a simple form. Sample curves showing the effect of air-gap and of saturation on the value of this coefficient are given in Fig. 153.

149. EXPERIMENT 7-D. —Maps of Stray Flux. —This experiment supplements the preceding one, the purpose being to determine the actual paths of leakage lines of force. The methods of observation are described in § 146. Establish certain magnetic conditions, as in Fig. 152, and investigate as closely as possible the direction and the magnitude of the stray flux at various places. Change the conditions, as indicated in the preceding experiment, and see the effect on the stray flux. Preferably select the same conditions for which values of the leakage coefficient were determined in the preceding experiment.

Report. Plot the principal directions of the stray flux in several vertical and horizontal planes; mark the magnetic densities per square inch or square cm. State whether the densities increase proportionately in all places, with the increase in current. Discuss the influence of the factors enumerated in § 148, and show how they should theoretically affect the distribution and the intensity of the stray flux.

MAGNETIC FIELD OF DIRECT-CURRENT MACHINES.

150. The fundamental properties of the magnetic circuit investigated above will now be applied to a study of magnetic fields in electric generators and motors. In laying out a machine, the designer makes certain assumptions by which he determines the value and the distribution of the magnetic flux in the machine. A knowledge of the experimental methods by which the magnetic quantities are measured on actual machines is of primary importance, since the test data serve as a foundation for design. The principal values of importance are:

(1) Total flux in the armature and field (Fig. 264).

(2) Leakage coefficient, or the ratio between the total and the useful flux (§ 148).

(3) Distribution of the flux in the air-gap (Fig. 154). These three points will now be taken up more in detail.

151. Total Flux in Armature and Field.—Magnetic flux in the armature of a machine is smaller than that in the field, since part of the flux finds its way from pole to pole through the air surrounding the armature, constituting what is called the leakage or the stray flux (Fig. 152). Both fluxes may be measured by a calibrated ballistic galvanometer, as in Fig. 147. To measure the flux in a pole-piece a known number of turns of an exploring winding are wound on the pole and connected to the ballistic galvanometer. A current is sent through the regular field winding, and then the circuit is opened, or, still better, reversed (to eliminate residual magnetism). Knowing the ballistic constant, the flux can be figured out directly in maxwells from the instrument reading. A fluxmeter (§ 140) may be conveniently used instead of the ballistic galvanometer.

It is hardly practicable with large machines to suddenly open the field circuit, or to reverse the current, on account of the very high self-induction of the windings. The induced e.m.f.'s may reach several thousand volts, are dangerous to the operator, and harmful to the insulation of the winding. In such cases the flux is varied in steps. A resistance is connected in series with the field winding, and is short-circuited by a switch. By suddenly opening the switch the flux is changed, due to the decrease of field current, and the ballistic impulse (often called kick) may be observed. If a fluxmeter is available, the field may be varied slowly, and a total magnetization curve taken in a very few minutes.

The flux in the armature is measured by putting on it an exploring winding, which embraces the useful flux passing from one pole to the next through the armature iron. The exploring coil is connected to the ballistic galvanometer as before and impulses produced by varying the field current. Another simple method for determining the total flux in the armature is to drive the machine at no load, and to calculate the flux from the induced e.m.f. by the familiar formula

$$E = \frac{\Phi \times n \times \text{(r.p.m.)}}{60} \times 10^{-8}.$$

n is the total number of conductors on the armature, and Φ the useful flux (per pole) of the machine, in maxwells. The formula is applicable to bipolar machines and such multipolar armatures as are multiple-wound (see § 580). For series-wound multipolar armatures

$$E = p\frac{\Phi \times n \times \text{(r.p.m.)}}{60} \times 10^{-8},$$

where p is the number of pairs of poles.

When the value of the leakage coefficient of the machine is known, the corresponding flux in the pole-pieces can be calculated, or at least estimated.

One of the most important problems in connection with flux calculations is that of estimating the influence of armature reaction, or the armature demagnetizing ampere-turns. This influence may be determined experimentally by sending a current through the armature from an outside source; the armature must be clamped so that it cannot revolve. The flux existing in the field with various currents in the armature, and different settings of brushes, is measured as before. With the brushes set on one side of the neutral, the field is weakened by the armature reaction; on the other side it is strengthened.

152. EXPERIMENT 7-E. — Measurement of Magnetic Flux in D. C. Machines. — A calibrated ballistic galvanometer (§ 139) or a fluxmeter (§ 140) must be provided for the purpose. Put a certain number of turns of fine wire on one of the field coils, and the same number of turns on the armature. Begin the experiment with the field at zero value, having previously demagnetized the iron (§ 142). Increase the flux in steps, observing each time the ballistic throw. It is best to go over the complete cycle several times to be sure that the deflections are correct. Make similar runs with the ballistic galvanometer connected to the exploring coil on the armature.

After this, investigate the influence of the armature reaction. Clamp the armature, so that it cannot revolve; set the brushes on the neutral. Send a comparatively large current through the armature, and take the same magnetization cycles as before. This part of the experiment can be much more conveniently performed with a fluxmeter than with a ballistic galvanometer. Shift the brushes by one commutator segment, and repeat the same run, shift the brushes again, etc. Carry this operation as far as the brush-holder can be moved. Make similar runs with the brushes shifted in the opposite direction.

If possible, perform this test on a machine provided with compensating poles (§ 340), in order to see the influence of the compensating winding.

If the number of turns in the armature winding and the connections are known, take a no-load saturation curve (Fig. 266) in order to check the values of the flux by the formula given in § 151. Or, this method may be applied for determining the constant of the ballistic galvanometer.

Report. Plot to exciting current as abscissæ, values of flux in the pole-pieces and in the armature; plot to the same abscissae the ratio of the fluxes, or the leakage coefficient of the machine (>1). Check the values of the armature flux obtained ballistically with those calculated from the induced e.m.f. Plot to brush positions as abscissæ values of flux as influenced by the armature reaction; draw on the same sheet a horizontal line showing the flux with the armature circuit open. Explain the cause of the demagnetizing action of the armature, and the influence of the position of the brushes.

153. Leakage Coefficient. — The leakage coefficient, by definition, is the ratio of the total flux produced in a pole to the flux actually used in the armature for inducing e.m.f.'s. This ratio varies in modern machines from 1.20 to 1.50, according to the type, proportions, and saturation in the iron. A leakage coefficient of 1.30 means that out of every 130 lines of force produced in pole-pieces, only 100 actually pass into the armature; the rest find their path directly through the air from pole to pole.

It is important to keep the leakage flux as low as possible, since any increase in flux means a disproportionate increase in field copper, on account of saturation of the iron (§ 147). Moreover, machines with large leakage give a poor voltage regulation, because the armature reaction more easily deflects the flux into leakage paths. The leakage coefficient increases with saturation, because of the increase in the reluctance of the useful path in the armature, while that of the leakage paths in the air remains constant.

With the same useful flux, the leakage is larger when the machine is loaded than at no load, because the magnetomotive force between the poles is larger (§ 147).

The leakage coefficient may be determined as explained in § 152 by measuring ballistically the fluxes in the field and the armature and taking their ratio. The galvanometer does not need to be calibrated; the ratio of deflections gives directly the value of the leakage coefficient.

Mr. R. Goldschmidt suggested a compensation (or zero) method which does not require a ballistic galvanometer. According to this method a smaller number of exploring turns is put on the pole-piece than on the armature; the two exploring coils are connected in opposition, and in series with an ordinary low-reading voltmeter (in place of a ballistic galvanometer). When the flux is varied, the impulses induced in the two coils are in opposite directions, and the voltmeter receives the difference of the two. The ratio of the number of turns in the two coils is varied until the voltmeter needle remains at zero — when the flux is increased or decreased. Then the inverse ratio of the number of turns gives the ratio of the fluxes, or the leakage coefficient.

Suppose, for instance, that 20 turns were put on the armature. With 15 turns on the field, the voltmeter gave a small deflection in one direction; with 16 turns, in the opposite direction. The ratio of the fluxes, or the leakage coefficient, is between

$$\frac{20}{15} = 1.33 \text{ and } \frac{20}{16} = 1.25.$$

The true value may be estimated by noting the relative values of the deflections. This method is not as accurate as the ballistic method, but is accurate enough for technical purposes, and more convenient for testing floor work.

When using Goldschmidt's method the moving system of the voltmeter must be slightly weighted, so as to increase its moment of inertia. Otherwise the pointer moves both ways while the flux is being changed. This is because the flux often does not vary at the same rate in the field and in the armature.

In some cases, especially with new designs, it is desired to know not only the value of the leakage coefficient, but also the actual distribution of the leakage flux. This enables the designer to find where most of the flux is lost and to change the forms and the proportions. The methods for exploring the stray flux are described in §§ 146 and 149.

154. EXPERIMENT 7-F. —Determination of Magnetic Leakage Coefficient in Electrical Machines. —The arrangement of the apparatus is the same as in § 152; in fact, the two experiments may be performed simultaneously. Investigate the influence of the factors mentioned in § 153. At the end of the experiment increase the pole arcs by adding strips of iron on both sides of the pole-pieces; observe the resultant increase in leakage. It is not necessary to fasten the strips; they will be securely held in place by magnetic attraction. Try Goldschmidt's method of measuring the leakage coefficient with an ordinary voltmeter.

Report. Plot to exciting currents as abscissæ, values of leakage coefficient with no current in the armature. Plot to brush positions as abscissæ, values of leakage coefficient with a certain current in the armature. Show the influence of an increased pole arc. Give the results obtained by using the opposition method, with an ordinary voltmeter.

155. Distribution of Flux in an Air-Gap. —The actual distribution of flux in the air-gap of an electrical machine (Fig. 154) may be determined by inserting there a flat bismuth spiral (§ 185), or a flat coil connected to a fluxmeter. These methods are more of an academical character, and are seldom used in practice.

The method actually used with direct-current machines involves the measurement of the e.m.f. induced in the armature coils when they pass through a certain point in the air-gap. Two small brushes are mounted on a temporary brush-holder, and the distance between them is adjusted to be equal to the distance between the centers of two adjacent commutator segments. The brushes are insulated from each other, and connected to a low-reading voltmeter. When the machine is revolving, these two "pilot" brushes measure in succession the voltage at the terminals of all armature coils, when the same come into a certain position with respect to the field. In other words, they measure the voltage induced in a certain place of the field; this voltage is proportional to the flux density at the point under consideration.

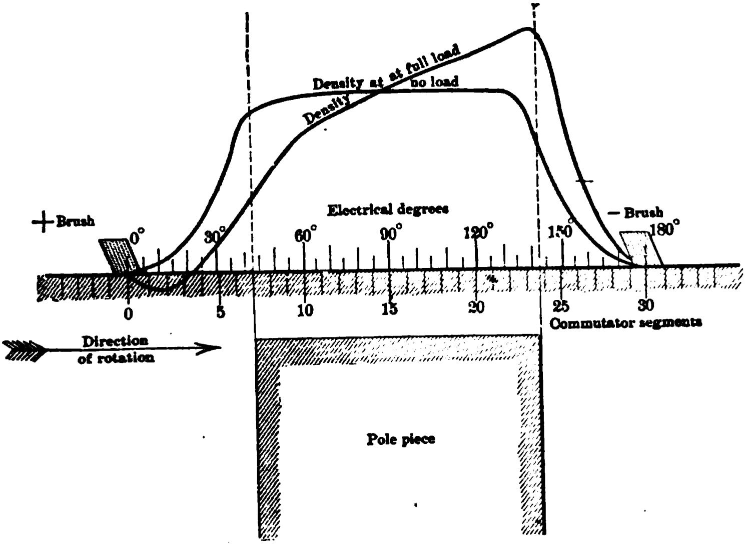

FIG. 154. Field distortion in a direct-current generator, due to the armature reaction.

By gradually moving the pilot brushes around the commutator, the distribution of the flux density may be measured from pole to pole; the results should be plotted as in Fig. 154. The same experiment, repeated with the armature loaded, gives a different distribution, because of the weakening and distorting action of the armature ampere-turns.

This method requires putting an extra brush-holder on the machine, and a divided sector for measuring angles. A simpler arrangement, sufficiently accurate for practical purposes, was described in the *Electric Club Journal* 1904, p. 484. The device consists of a template made of cardboard, with holes for voltmeter points. The holes are spaced so as to correspond to a certain number of commutator bars. The template is laid around the commutator, as close as possible, but without touching it, and is securely fastened to the brush-holder. When the machine is revolving, voltmeter points are inserted in each two adjacent holes and the voltage measured. Ordinary metal points may scratch the commutator; it is better to use hard drawing pencils.

In alternators, the distribution of magnetic flux in the air-gap may be best determined by taking a curve of induced e.m.f. in an exploring coil placed on the armature. This can be done either by the point-to-point method (§ 588) or with an oscillograph (§ 591).

156. EXPERIMENT 7-G. — Determination of Magnetic Distribution in the Air-Gap of a D.C. Machine. — The purpose of the experiment is to obtain curves similar to those shown in Fig. 154; the method is described in the preceding article. Either two movable pilot brushes may be used, or a template and voltmeter points. (a) Bring the machine to full speed, run it light as a generator at normal voltage, with the regular brushes in the neutral line. Take the magnetic distribution curve, as shown in Fig. 154, by reading volts between consecutive commutator segments and angular positions of the contacts. (b) Take similar curves with the field, first highly saturated, and then with a very low saturation. (c) Load the machine on resistances and take similar curves; if possible again have the same three terminal voltages. The brushes should be set at the point of minimum sparking. (d) Run the machine as a motor, driving a generator; take the same three curves within as wide range of field currents as possible. (e) If a machine with compensating poles is available, take curves with and without the compensating winding; also, if possible, with the compensating winding connected in the wrong direction.

Report the results in the form of curves shown in Fig. 154; explain the observed differences in the distribution of the flux.

TESTS OF LIFTING AND TRACTIVE ELECTROMAGNETS.*

157. Electromagnets are used in practice for various duties, particularly for exerting mechanical pull, or lifting weights. They are especially convenient for intermittent work, where a short stroke and quick action are required. Thus, they are used for releasing crane and elevator brakes (Fig. 496), for operating switches and starters (Fig. 484), for supporting weights carried by a crane (Fig. 155). Electromagnets are also widely used as relays for closing all kinds of auxiliary or operating circuits (see Fig. 538).

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* Figs. 156 to 160 are taken by permission from Mr. C. R. Underhill's booklet, *Facts about Electromagnets.*

Electromagnets used for heavy mechanical duty may be subdivided into two classes: lifting magnets and tractive magnets. The first (Fig. 155) are employed for merely supporting weights; the second (Figs. 156, 157, and 158) for performing certain work by the movement of a plunger. The characteristics of each class will be described separately.

158. Lifting Electromagnets.—Electromagnets of this type, Fig. 155, are used in shops and in rolling mills where large pieces of iron are regularly carried by cranes. An electromagnet, suspended from a crane in place of the usual hook, is lowered to the piece of iron to be carried, and is energized. The piece is attracted, and may be safely lifted by the crane. To release the piece, it is sufficient to open the exciting circuit of the electromagnet. The advantage of this method over the ordinary hook is that the work is performed more quickly, as no time is lost in fastening the hook, and in unhooking the piece at the end of the travel. The lifting electromagnet, shown in Fig. 155, consists of a steel casting with a circular hole in it for the exciting coil. Lines of force pass through the inside core, then through the object to be lifted, and the path is completed through the outside part of the electromagnet frame. The armature and the hook, shown in the sketch by dotted lines, are not used in regular operation; they are shown merely to indicate the way in which the electromagnet may be tested for tractive effort.

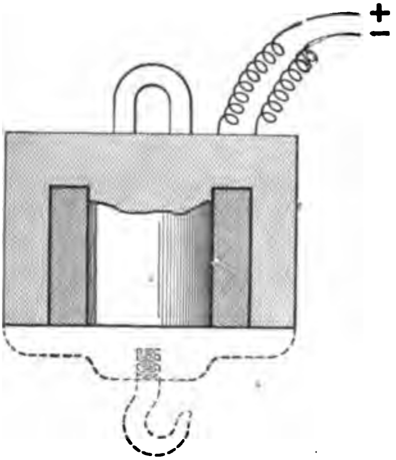


FIG. 155. A lifting electromagnet.

Lifting electromagnets are made of different shapes, according to the purpose for which they are intended to be used. Thus, in rolling mills, where large sheets are carried, lifting electromagnets are made with several short projecting poles, each provided with an exciting coil. This is done in order to support the sheet in several places simultaneously. For carrying pig-iron of irregular form, electromagnets are provided with long, projecting poles, which may be sunk into a heap of pig-iron,—the pieces stick to the poles on all sides. To make the action still more effective, the poles, or fingers, may be made movable in the vertical direction, so that each pole is sure to come in contact with the heap.

The lifting power of an electromagnet depends not only on its construction and on the exciting current, but also on the form of the piece to be lifted, and on the quality of iron in it. This is because the magnetic flux is closed through the piece to be lifted; hence the reluctance of this piece determines the value of the flux. To make the conditions definite, when testing such an electromagnet, an armature should be used with the smallest possible reluctance; such an armature is shown by dotted lines in Fig. 155. Under such conditions the electromagnet develops the maximum possible lifting power. If, however, the electromagnet is intended for a definite service, for instance, for lifting a certain kind of plates in a rolling mill, the test may be performed by using such plates, instead of an arbitrary armature.

159. EXPERIMENT 7-H. —Test of a Lifting Electromagnet.— Suspend the electromagnet (Fig. 155) from a secure place and provide an iron armature with a hook, as shown by dotted lines; weigh the armature before using it. Place the armature under the electromagnet, and gradually excite the coil until the armature is supported by the magnetic attraction. Note this critical value of the current. Place some weight on the hook, and increase the exciting current until it becomes again sufficient to support the armature. Proceed in this way until the saturation limit is reached, or until the limit of safe heating is reached. In performing this test, have under the armature a suitable support, preferably on springs, to limit the motion and to reduce the noise, when the armature is released and strikes the support. Perform the same test, varying different factors, *viz.:* (a) use a different armature; (b) interpose an air-gap, by placing a piece of paper, fiber, etc., between the magnet and the armature; (c) lift irregular objects. Before leaving the laboratory, measure the dimensions of the magnet and determine the number of turns in the exciting coil.

In the absence of a regular lifting magnet, the apparatus shown in Fig. 147 may be used for the experiment, and the influence of various factors investigated.

Report. Plot curves of pounds lifting power to amperes current as abscissæ. Figure out for a few points magnetic density B from the formula (3) in § 166. Show that this density checks within reasonable limits with that calculated from the number of exciting ampere-turns and the dimensions of the magnet; use for comparison standard B-H curves given in various textbooks and pocketbooks. Discuss the influence of the various factors on lifting power.

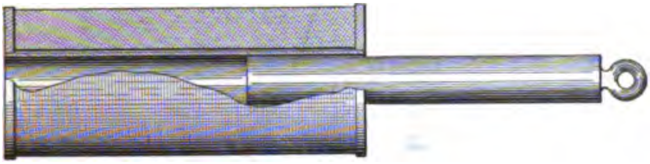


FIG. 156. A coil-and-plunger type tractive electromagnet.

160. Tractive Electromagnets.—Three common types of tractive electromagnets are shown in Figs. 156 to 158; the difference in their characteristics may be judged from the curves of pull, in Fig. 159. The electromagnet, shown in Fig. 156, consists of a coil surrounding a plunger; the curve of pull shows, that as the plunger is sucked into the coil, the pull first increases, and then again decreases. The addition of an iron frame on the outside (Fig. 157) increases the pull at the end of the stroke, as shown by the curve marked "Iron Clad." A further increase in pull is obtained by adding an inside core *C* (Fig. 158). The plunger is tapered and the core is countersunk, in order to increase the pull by reducing the reluctance of the air-gap; in many cases, however, flat-faced core and stop are sufficient. From the fact that an iron-clad electromagnet with a core gives the greatest pull does not follow that this type should be used in all cases. The selection of one or another type depends upon the character of the work for which the magnet is intended. Knowing the requirements of a particular case, the best type may be selected with reference to the curves of pull.

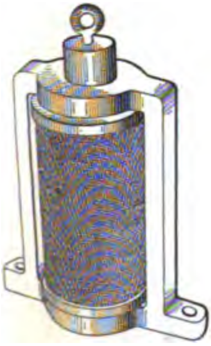


FIG. 157. An iron clad tractive electromagnet.

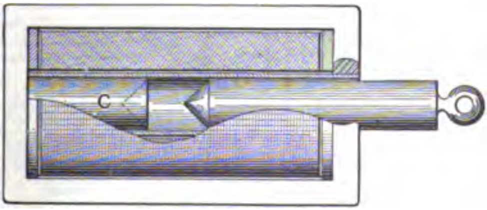


FIG. 158. An iron-clad tractive electromagnet, provided with a core C for increasing the pull at the end of the stroke.

A convenient apparatus for testing tractive electromagnets is shown in Fig. 160. The coil *C* of the electromagnet under test is connected to a D. C. supply, through the switch *S*, regulating rheostat *R*, and the ammeter *A*. The plunger *P* is suspended from the beam of a balance. The balance has fulcrums at F_1 and F_2, and two sliding weights w_1 and w_2. With this construction of the balance large pull is counterbalanced with small weights; the apparatus is thereby made sensitive and convenient to handle. The curves shown in Fig. 159 were taken by means of this device.

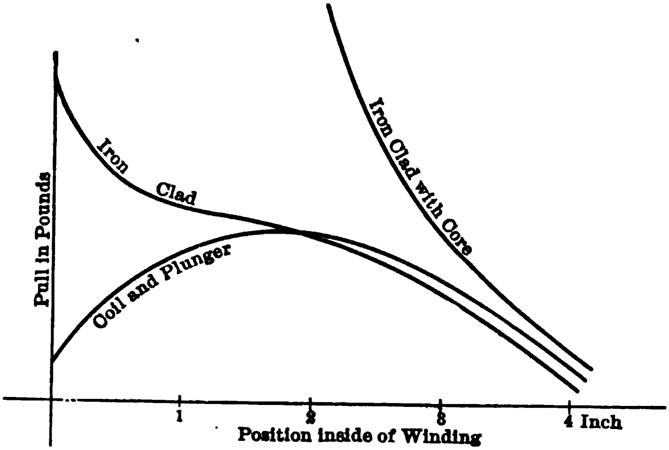


FIG. 159. Curves of comparative pull of the electromagnets shown in Figs. 156 to 158.

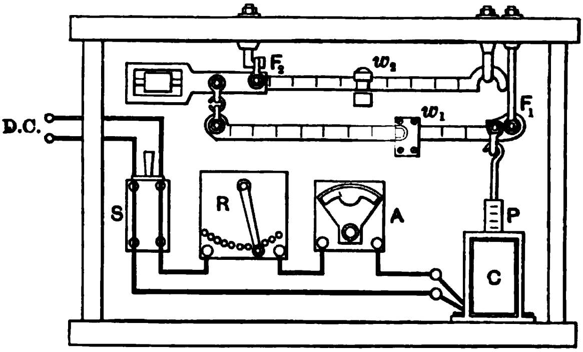


FIG. 160. An apparatus for testing tractive effort of electromagnets.

161. EXPERIMENT 7-I. —Test of a Tractive Electromagnet.—

The purpose of the experiment is to determine curves of pull (Fig. 159) of different forms of tractive electromagnets. An apparatus, similar to that in Fig. 160, is convenient for the purpose. (a) First test an electromagnet of the type shown in Fig. 156; take curves of pull with a certain position of the plunger, through a wide range of exciting currents. (b) Repeat the same test with different positions of the plunger. (c) Then take the same coil and provide it with an iron core, as in Fig. 157. Perform a similar test throughout the same range of currents and with the same positions of the plunger. (d) Finally put an inside stop *C* (Fig. 158) into the coil and test the magnet again for pull.

Report. Plot curves showing variations of the pull with the position of the plunger; also curves showing variations in pull with the exciting current. Give a theoretical justification for the general form of the curves.

CHAPTER VIII.

PERMEABILITY TESTS.

162. STEEL and iron used in the construction of electrical machinery must possess a high *permeability,* or high magnetic conductivity. This means that a required magnetic flux should be produced with the smallest possible number of exciting ampere-turns. Many devices have been elaborated for testing permeability of steel and iron, so that it becomes a rather difficult task to give a classification of the pieces of apparatus used. The principal distinction between types seems to be in the method used for measuring magnetic flux. From this point of view, the devices described below may be subdivided into those in which the flux is measured:

(1) By tractive effort;
(2) By inductive discharge through a ballistic galvanometer;
(3) By a steady magnetic or electric action;
(4) By an increase in the electrical resistance of bismuth.

Before describing the testing devices proper, it is necessary to review some points of general theory, and to indicate the manner in which results of permeability tests should be grouped for purposes of practical comparison.

163. Magnetization or B-H Curves. — The most convenient way to represent results of a permeability test on a sample of steel or iron is to plot the results in the form of a curve (Fig. 151) which gives the number of ampere-turns per unit length of the sample, necessary to produce in it a certain flux density B. The designer is immediately enabled to compute the number of ampere-turns required for the excitation of a frame of given dimensions in order to obtain a desired magnetic flux.

The older way was to plot flux densities B in iron to flux densities H which would take place if iron were removed and substituted by air. A curve of this kind shows how many times the flux has increased because of the presence of iron; in other words, it shows how many times the permeability of iron is greater than that of air. The permeability of air is arbitrarily fixed as of unity value so that the ratio $B \div H$ gives directly the permeability of iron. But the numerical value or permeability is of slight importance in practical work; what is needed is the number of ampere-turns for producing a required flux density in iron. This number must not be above a certain limit, otherwise the steel is rejected as being of an inferior magnetic quality.

The name *B-H* curves is, however, retained for curves plotted to ampere-turns as abscissæ; it simply means, that the curve shows a relation between a flux and the exciting ampere-turns which produce it. Another common name for *B-H* curves is *magnetization* curves. Both names are used below indiscriminately.

164. Hysteresis Loop. — Magnetization, or *B-H* curves (Fig. 161) should be taken with the exciting current both increasing and decreasing;

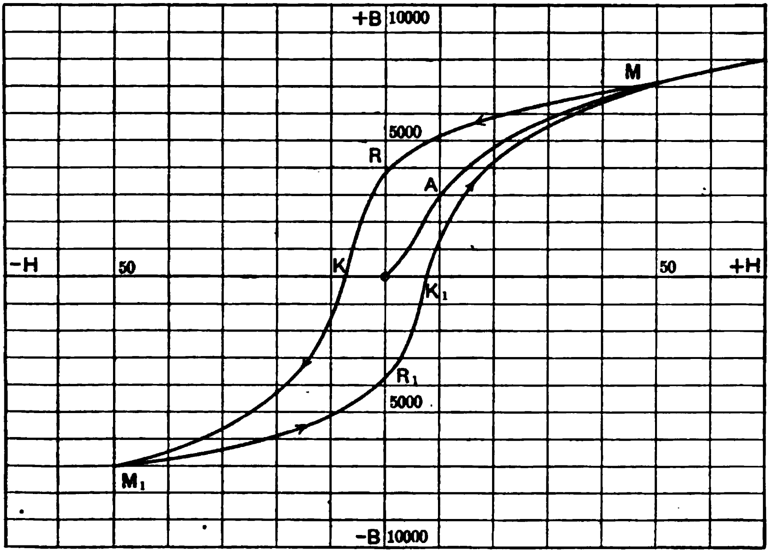


FIG. 161. Magnetization curve and hysteresis loop of a sample of iron.

also with the current reversed, for the reason that the flux density at a given excitation depends somewhat upon the previous state of magnetization in the sample. Assume first, that the sample is altogether free from magnetism. Gradually increasing the excitation gives the curve *OAM*; this is called the virgin curve (Jungfrauliche Kurve). When now the excitation is reduced again, the flux density does not decrease as rapidly as it increased, but varies according to the curve *MR*. When the exciting circuit is opened, the iron still possesses a *residual* magnetic density *OR*. Reversing the exciting current, this density is gradually decreased; the iron being deprived of its residual magnetism, when the exciting ampere-turns = *OK*. When the exciting current is further increased (in this reverse direction), the magnetization in the iron is reversed, and varies according to the part KM_1 of the curve. Decreasing the current again, and reversing it makes the densities vary according to the branch $M_1 K_1 M$ of the curve.

This peculiar phenomenon, — that two different densities are possible with the same value of exciting current, — is called *hysteresis*. It is attributed to some kind of molecular friction in iron; the root of the word signifies " to lag behind." The curve MKM_1K_1 is called the hysteresis loop. In accurate permeability tests, it is advisable to take both the virgin curve and the hysteresis loop.

A large value of residual magnetism OR is objectionable in cases where the flux is supposed to follow closely any variations of the exciting current, as — for instance — in fields of generators and motors. On the other hand, large residual magnetism is desirable in steel intended for permanent magnets, such as are used in measuring instruments, and in telephone instruments.

165. Magnetic Flux in Air. — In some of the tests, described below, it is required to figure out ampere-turns necessary for producing a certain flux density in air, instead of iron. This can be done with sufficient accuracy, only when the lines of force are parallel to each other, as for instance in a small air-gap between two large pieces of iron, or within a long solenoid. Theory and experience show, that under these conditions one *ampere-turn per* 1 *cm. length of path in air produces a flux density* H = $1\frac{1}{4}$ *lines of force* (maxwells) *per sq. cm.* (more accurately $\frac{4\pi}{10} \doteq 1.257$). Magnetic densities in air are usually denoted by H to distinguish them from those in iron, which are designated by B. Thus to produce a density H in an air-gap l cm. long (in the direction of lines of force)

$$ni = 0.8\ Hl \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

ampere-turns are required. If l is in inches, and H in lines per 1 sq. inch, the formula becomes

$$ni = 0.796 \frac{H}{(2.54)^2} \times (2.54\,l) = 0.313\ Hl \quad . \quad . \quad . \quad . \quad (1a)$$

Let it be necessary, for instance, to produce a flux of 300,000 lines in an air-gap 0.3 cm. long, and of a cross-section of 50 sq. cm. The flux density H = 300,000/50 = 6000 maxwells per sq. cm. If the air-gap were 1 cm. long, the necessary number of ampere-turns would be 0.8 × 6000 = 4800. As the air-gap is only 0.3 cm. long, the actual number is 0.3 × 4800 = 1440 ampere-turns.

A theoretical proof of the expression (1) may be found in any standard textbook on electricity and magnetism. For practical purposes it is sufficient to accept this expression as an experimental fact, which has been verified a great many times in actual practice.

TRACTIONAL METHOD.

166. Thompson Permeameter. — One of the simplest pieces of apparatus for testing iron and steel, the so-called *Thompson permeameter*, is shown in Fig. 162. It is based on the measurement of the mechanical force necessary for separating two parts of a magnetic circuit. The sample T to be tested is made in the form of a rod; it is placed within an iron yoke y, of a negligible magnetic resistance (reluctance). The closed iron circuit thus formed is magnetized by the coil C. Then the sample T is pulled out of the yoke by turning the handwheel H; the pull is read on the spring balance B.

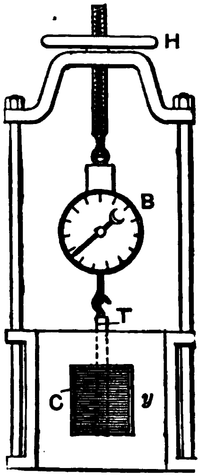


Fig. 162. Thompson permeameter.

The apparatus is intended primarily for approximate relative measurements; the better the sample (the higher its permeability) the more force is required to pull it out of the yoke, with the same magnetizing current. The device is used for such approximate tests by some electric manufacturing companies. Whenever a new lot of the material is received, samples of it are tested in the permeameter; the force necessary to pull the specimens out of the yoke must be at least equal to that previously found for the lowest acceptable grade of material, otherwise the entire consignment is rejected. A knowledge of the *absolute* value of permeability is not necessary in this case. The number of turns on the field windings of machines cannot be changed to suit the permeability of each lot, but is designed for a certain average grade of iron. The right number of ampere-turns in each individual machine is obtained by regulating its field rheostat, so as to get the desired voltage.

The permeameter may also be used for determining actual magnetization curves (Fig. 161) of samples; this is done by comparing the force necessary for pulling out the sample under test, to that required with a standard sample whose magnetization curve has been previously determined by some other method. The comparison is made on the basis of the fact, that the mechanical force of attraction between two parts of a magnetic circuit is proportional to the square of the magnetic density at the separating surface (Maxwell's law). This law may be deduced as follows: The attraction at the separating surface may be imagined as due to two equal and opposite magnetic masses. When the magnetic density increases n times, each mass increases n times. The force of attraction is proportional to the product of the masses; therefore it increases n^2 times.

Thus with the same magnetizing current, if F_1 is the spring balance reading with the sample under test, and F_2 that with the standard sample, we have

$$\left(\frac{B_1}{B_2}\right)^2 = \frac{F_1}{F_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

If magnetic density B_2 is known from the magnetization curve of the standard sample, B_1 may be calculated, and the magnetization curve for the sample under test plotted.

The magnetization curve of a sample can be obtained with the Thompson permeameter even without having a standard sample: Let B be the magnetic density in the sample (number of lines of force per sq. cm.); S the cross-section of the rod, and F the pull (in dynes). The force of attraction is proportional to the square of density and to the surface S of contact between the sample and the yoke. From theoretical considerations, if F is in dynes, the coefficient of proportionality is $1/8\pi$. Thus we have:

$$F = \frac{1}{8\pi} . \; S \; (B - H)^2,$$

where H is the number of lines of force per sq. cm. left in the air after the sample has been pulled out. $(B - H)$ is introduced into the formula instead of B, because only $(B - H)$ lines fo force are "broken" when the sample is removed. From the above formula, substituting practical units, we get

$$B = 1317 \sqrt{\frac{F}{S}} + H \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

where B and H are as before magnetic densities per sq. cm., S is the cross-section of the sample in sq. inches, and F is the pull in lbs. H is calculated from the formula (1) given in § 165:

$$H = \frac{4\pi}{10} . \frac{ni}{l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

where ni is the number of ampere-turns of the magnetizing coil, and l is its length in cm. If l is expressed in inches the formula becomes

$$H = 0.495 \cdot \frac{ni}{l} \cdot$$

The values of B and H per square inch are 6.45 times higher than those expressed per sq. centimeter.

The Thompson permeameter is adapted for quick comparative tests rather than for taking exact magnetization curves. A spring balance is not reliable enough to allow of accurate measurements, and moreover the results are vitiated by two air-gaps between the yoke and the sample. To reduce the error due to this cause the sample must fit nicely into the hole in the upper part of the yoke, and the two surface, to be pulled apart must be well planed.

Another device based on the same principle is the Ewing magnetic balance. It has a beam with a sliding weight instead of a spring balance; the specimen rests on the yoke with its cylindrical surface, instead of touching the yoke with an end, as in the Thompson permeameter. This feature does away with the planing of one end of the sample, and makes it unnecessary to have a hook on the other end; besides this, a more uniform contact is obtained. The scale is calibrated directly in flux densities by means of a standard sample, previously tested by some other method.

167. EXPERIMENT 8-A. —Magnetization Curves of Iron with Thompson Permeameter. —The purpose of the experiment is to obtain for given samples $B - H$ curves as shown in Fig. 161, or in Fig. 151. It is desired that the student test samples of cast steel, wrought iron, and cast iron, to see the difference in the magnetic properties of these materials. (a) A sample must be demagnetized before beginning the experiment. For this purpose it is put in the apparatus, and magnetized as highly as possible; then the exciting current is gradually reduced to zero, meantime being constantly reversed. Or else the sample may be magnetized with an alternating current, and gradually withdrawn from the coil. (b) First take a virgin curve OAM; read amperes excitation and lbs. tension necessary to tear the sample from the yoke. Carry the magnetizing current as high as the coil will permit. Then take a complete hysteresis loop. (c) Repeat the same test with other samples. Before leaving the laboratory, measure the cross-section of the samples tested, and the axial length of the coil; also ascertain the number of turns in the coil.

The permeameter is not a very accurate instrument, and each point should be obtained as an average of several readings. See that the sample and the yoke are nicely surfaced and that the spring balance exerts an axial pull. Another precaution, important in all permeability tests, is that once the current is raised to a certain value, it should not be reduced again, until a branch of the magnetization curve is completed. Otherwise indefinite hysteresis effects are introduced.

Report. Plot lbs. pull to exciting amperes as abscissæ, for all the samples tested. Do this before making any calculations, in order to obtain smooth curves and to eliminate possible errors of observation. Change the curve for one of the samples into a magnetization curve, shown in Fig. 151, by using formula (3). Use ampere-turns per inch as abscissæ, maxwells per square inch as ordinates. If the sample was previously tested by another method, check the results. With this curve as a standard, plot a similar curve for another sample, using formula (2).

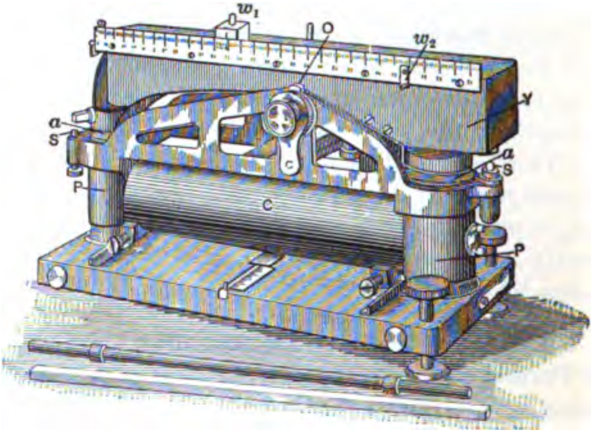


FIG. 163. Du Bois magnetic balance.

168. Du Bois Magnetic Balance. —This is also a tractional device (Figs. 163 and 164), more sensitive and accurate than the two devices described in § 166. The sample under test T is clamped between two heavy pole-pieces PP and magnetized by the coil C. Clamps are provided for accommodating either a round or a square sample; also a bundle of laminations. Two specimens are shown lying on the table, in Fig. 163. The magnetic circuit is closed through the iron yoke YY; the yoke is supported by knife-edges at O, like the beam of a balance. The shorter arm of the yoke is made of the same weight as the longer one, by weighting it with lead; the yoke is in balance when the coil C is not energized and the weights W_1 and W_2 are on zero.

When the sample is magnetized, the yoke tips over to the left, because the attraction in the left-hand air-gap acts on a longer lever. To bring the yoke in balance the weights are moved to the right. When the balance is obtained, the flux density in the sample is read directly on the scale, in maxwells per sq. cm. The movement of the yoke is limited by adjustable stops, *SS* (Fig. 163). To simplify the balancing, the stops on one side may be provided with platinum tips, and connected to a local circuit consisting of a dry cell and a bell or a galvanometer; a signal is given when the yoke touches one of the stops.

A certain flux in the sample and in the pole-pieces gives a definite force of attraction at the air-gaps *aa*. Therefore the scale of the instrument could be calibrated directly in fluxes. But as all samples are of the same cross-section, to each flux corresponds a definite density. Therefore the scale is calibrated directly in magnetic densities. The calibration is done by means of a specimen whose magnetization curve has been determined by some other method. A good feature of the instrument is that the reluctance of the air-gaps is large as compared to that of the sample; in this way small inaccuracies in the contacts between the sample and the pole-pieces are of no consequence.

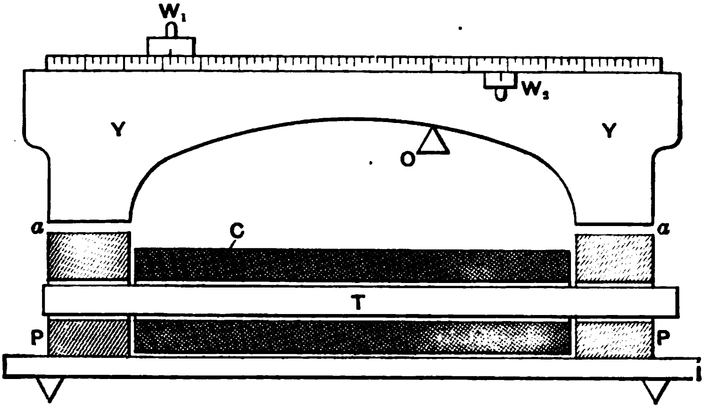


FIG. 164. Schematic representation of the Du Bois magnetic balance.

169. EXPERIMENT 8-B. —Magnetization Curves of Iron with Du Bois Balance. —The experiment is performed in the same order, as the experiment 8-A (§ 167). Before leaving the laboratory weigh the sliders, measure the length and the cross-section of the air-gaps and their horizontal distance from the knife-edge supports. Make a copy of the scale; measure the cross-section of the samples; note the number of turns in the magnetizing coil.

Report. (1) Plot curves of flux density to ampere-turns per cm. (or inch) as abscissæ (Figs. 151 and 161). If possible, use the same sheet of cross-section paper for all the samples, so as to have a direct comparison for the quality of the materials. (2) Show in a numerical example how to use these curves for determining the number of ampere-turns required to produce a given flux in a magnetic circuit. Take, for example, a circuit like the one shown in Fig. 147. The dimensions of the cores and the length of the air-gap are supposed to be given. (3) Figure out the actual pull on the yoke of the balance with a given flux (§ 166); see how closely the calculated magnetic torque checks with that of the weights.

BALLISTIC METHOD.

170. In taking magnetization curves two quantities are to be measured: Exciting ampere-turns and the flux produced by them. The chief difficulty lies in measuring the flux. It can be measured much more accurately by a discharge through a ballistic galvanometer than by tractive methods described in §§ 166 and 168. The use of the ballistic galvanometer for measuring fluxes is described in §§ 139 and 142. While this is the most accurate method for measuring fluxes, it was not until recently that devices have been perfected, such as make the method convenient for technical purposes. It may not be amiss to give here a brief history of the development of this method.

171. The Development of the Ballistic Method. —The original arrangement (Fig. 165) required samples to be made in ring form, each wound separately with exciting and secondary coils. This, of course, was a serious objection from a practical point of view. Hopkinson introduced the "divided-bar" apparatus (Fig. 166) in which the samples *TT* are made in the form of rods; these are much easier to prepare than rings. Permanent coils are used; the magnetic circuit is closed through a heavy yoke *YY*. An objection to this method is that, — due to the necessarily imperfect contacts between the yoke and the samples, — the quality of iron appears worse than it is in reality (see Fig. 151).

Ewing remedied this by testing samples with two different lengths of the magnetic circuit (Fig. 167); the resistance of the contacts is eliminated from two measurements. His method is objectionable, because it requires two identical samples, which sometimes are difficult to get. Picou, on the other hand, eliminated the influence of contacts (Fig. 169) by additional windings which compensate for the reluctance of the air-gaps. With all these improvements there still was a need for an apparatus for measurement of permeability of large castings as a whole, without preparing special samples. Dr. Drysdale's permeameter (Fig. 170) fills this demand.

Another objection to the ballistic method has been found in the ballistic galvanometer itself; in its original form, with a telescope, and with undamped swings, it was not a convenient device for practical work. Here also considerable progress has been made of late. Ballistic galvanometers of sufficient sensitiveness are now to be had, that are provided with a pointer and a scale, like ordinary voltmeters and ammeters; moreover, they are made perfectly damped electromagnetically, so that readings may be taken in quick succession. Finally the advent of the fluxmeter (§ 140) makes it possible to obtain continuous indications of fluxes instead of instantaneous deflections of a ballistic galvanometer.

The above-mentioned devices, based on the ballistic method, will now be described more in detail.

172. Ring Method. — The sample of iron or steel to be tested is turned in form of a ring *I* (Fig. 165) of a comparatively small radial breadth. If sheet steel is to be tested, separate rings are punched, assembled together, and turned down on the sides. The ring is uniformly wound with a known number of turns of wire, connected to the terminals *P* (primary). A secondary or exploring coil *E* is wound on the first coil and connected to a ballistic galvanometer. The core is excited from a source of D. C. supply, through a regulating rheostat *R* and a double-throw switch *D*. An ammeter, not shown in the figure, is connected into the primary circuit. It is thus the same arrangement as is shown in Fig. 147 and described in §§ 137 and 138, except that the magnetic circuit is perfectly closed, and no inaccuracy produced by an air-gap.

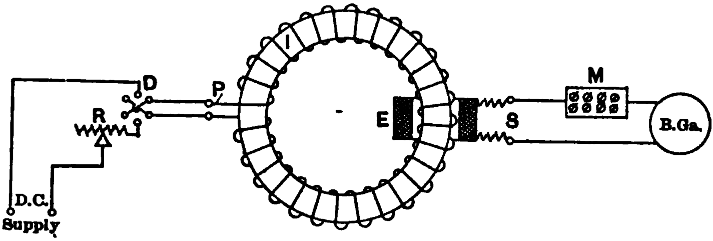


FIG. 165. Testing permeability of a sample in ring form.

The sample is thoroughly demagnetized by exciting it to as high saturation as possible, and then gradually reducing the excitation to zero, at the same time continually reversing the current. After this, the sample is magnetized again in steps, and the corresponding deflections of the galvanometer noted. This gives the virgin curve (Fig. 161); a complete hysteresis loop may also be taken in a similar way, if desired. The curves shown in Figs. 151 or 161 may be plotted by knowing the dimensions of the core, the number of turns on both windings, and the constant of the ballistic galvanometer.

In some cases it is possible to have specimens in the form of very long rods. Experience shows that a rod whose length is at least 500 times its diameter may be considered magnetically as infinitely long. This means that the same number of ampere-turns *per inch* are required in order to produce a certain flux, whether the rod is straight or made into a closed ring, without air-gap. This is not true for shorter rods on account of demagnetizing action of the ends; again, with a very long rod the reluctance of the path of lines of force back through the air is practically zero, on account of a very large cross-section of this path. For this reason the required number of ampere-turns is the same as if the rod was bent into a ring.

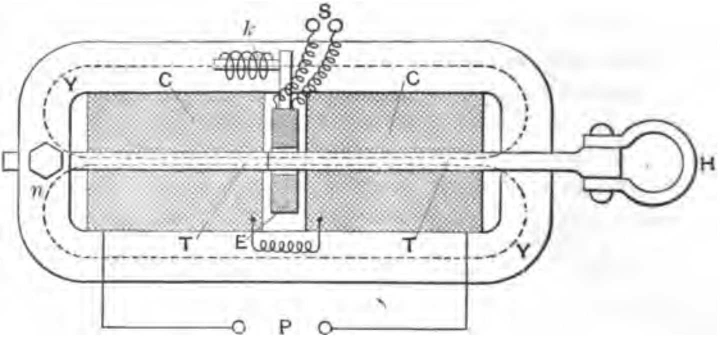


FIG. 166. Hopkinson divided-bar method for testing iron..

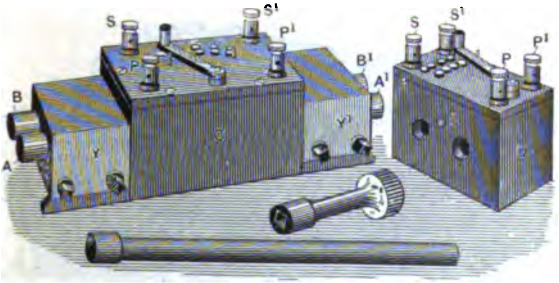
When such a rod is available for test, it is inserted in a straight coil which is slightly longer than the sample, and is tested as in Fig. 165.

173. Divided-Bar Method. — An objection to the practical use of the ring method is, that the preparation of samples is rather complicated. Hopkinson found a way out of this difficulty by making samples in the form of two straight bars. *TT* (Fig. 166). The magnetic circuit is closed through a heavy iron yoke *YY* of negligible reluctance; so that the reluctance of the circuit is practically caused by the samples alone. The left-hand bar is securely clamped to the yoke by the screw *n*. The right-hand bar is provided with a handle *H*, by means of which it may be pulled out of the yoke. The exciting or primary coil *C* is wound in two sections; it is connected to the terminals

P. The exploring coil E is inserted between the two sections and is connected to the terminals S. The electrical connections are the same as in Fig. 165.

To measure the flux produced in the samples by the coils CC, the right-hand sample is suddenly withdrawn from the yoke. This releases the coil E, and the spring k instantly snaps it out of the magnetic circuit. The change in flux embraced by this coil produces a discharge in the ballistic galvanometer, as in the ring method.

Instead of using two samples and pulling out one of them, one long sample may be used, and the ballistic discharge obtained by changing or reversing the current in CC, as in the ring method. The apparatus is fairly accurate for practical purposes, but great care must be exercised to have good magnetic contacts between the yoke and the sample.



167. Ewing double-bar method for testing iron.

174. Ewing Double-Bar Method. — To eliminate the influence of contacts and the reluctance of the yoke, Ewing suggested the apparatus shown in Fig. 167.* The magnetic circuit is formed by two test bars AA', BB', and two heavy yokes YY' made of wrought iron. Each bar is surrounded by a magnetizing coil, as in Fig. 177. A secondary coil, connected to a ballistic galvanometer, is wound on one of the exciting coils. The coils are mounted in the wooden case C; the terminals PP' are connected to a source of D. C. supply, the terminals SS' to a ballistic galvanometer. The number of turns in the secondary coil may be varied at will by the switch shown on top of the box. At the same time some resistance is automatically inserted into the galvanometer circuit, as the sections of the coil are cut out. In this way the resistance of the galvanometer circuit is kept constant, and its deflections made proportional to the number of secondary turns. The reason for varying the number of turns is to have galvanometer deflections within its best range with widely different values of the flux.

* Fig. 167 is taken by permission, from Prof. E. S. Ferry's *Practical Physics*, Vol. III.

The test is performed exactly as with the ring method. After a magnetization curve has been obtained, one of the yokes and the box *C* are removed by loosening the screws. A shorter box *D* is slipped in place, and the magnetic circuit completed again, the yokes being brought closer together. A magnetization curve is taken as before. It differs from that previously obtained because of a shorter length of the circuit and a smaller number of magnetizing turns in the box *D*. The magnetic resistance of the yokes and of the contacts is the same in both cases, and may be eliminated by comparing the two curves. Let, for instance, 850 ampère-turns be necessary in order to produce a certain flux with the box *C*, 10 inches long; let 450 ampere-turns give the same flux with the box *D*, 5 inches long. This shows that 400 ampere-turns were used in the first case for $5 \times 2 = 10$ inches of the length of the samples, so that the true ampere-turns per inch of the sample are: $400/10 = 40$. If the shorter box were not used, the reluctance of the contacts and the yokes would have to be neglected; the number of ampere-turns per inch of the sample would have to be taken as $850/20 = 42.5$, which is about 6 per cent too high. The experiment with the shorter box enables one to separate the 50 ampere-turns lost in overcoming the reluctance of the contacts and the yokes.

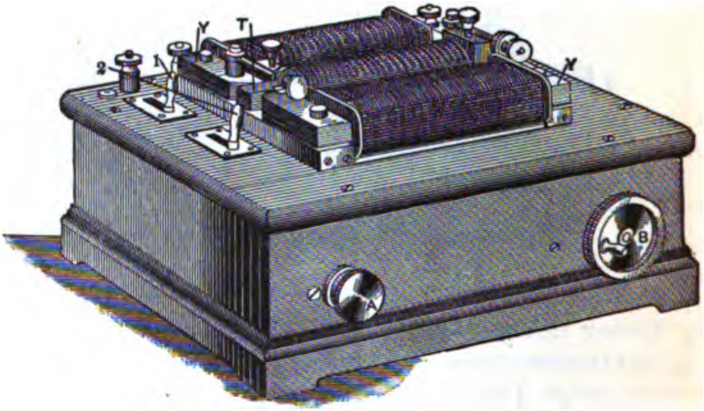


FIG. 168. The Picou permeameter.

The details of the apparatus shown in Fig. 167 are due to Prof. J. W. Esterline; in particular he deserves credit for the boxes, the method of clamping, and for the scheme of using variable secondary turns with constant resistance.

175. Picou Permeameter.—In this apparatus (Fig. 168) the disturbing influence of the contacts is compensated by additional magnetizing windings. The principle of compensation is shown in Fig. 169. The sample T under test is made rectangular, or consists of a bunch of steel laminations. It is clamped within the magnetizing coil C_3 between two U-shaped yokes YY, provided with compensating windings C_1, C_2. The winding C_3 supplies just enough ampere-turns to carry the flux through the test sample alone; the coils C_1 and C_2 provide the magnetomotive force necessary for overcoming the reluctance of the yokes and the air-gaps. A secondary coil is wound on each of the three magnetizing coils, and may be connected at will to a ballistic galvanometer.

The first step in testing a sample is to compensate for the reluctance of the yokes and the air-gaps. For this purpose the circuit of the main coil C_3 is left open; the coils C_1 and C_2 are energized so as to produce a flux through the yokes only, as shown by dotted lines in the sketch to the left. Each coil supplies the ampere-turns necessary for overcoming the reluctance of one yoke and two air-gaps. The desired value of the flux is established by trial discharges through the ballistic galvanometer. Now the current in one of the coils is reversed, so that the flux is deflected into the sample under test, as shown by dotted lines in the sketch to the right. Each of the coils, C_1 and C_2, has now to overcome the reluctance of its yoke, of two air-gaps, and in addition the reluctance of one longitudinal half of the sample under test. The flux is naturally reduced; to bring it to its former value, the middle coil C_3 is excited so as to assist the two other coils.

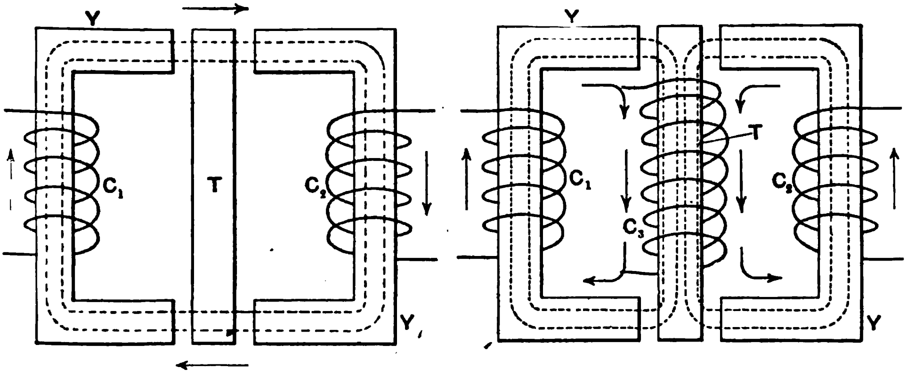


FIG. 169. Principle of compensation for the reluctance of air gaps, in the Picou permeameter.

When the former value of the flux is established in the yokes (as recognized by ballistic deflections), it is evident that the two outside coils again supply the ampere-turns necessary for the yokes and the gaps, while the middle coil carries the flux through the sample under test. Knowing the number of ampere-turns per inch in the coil C_3 and taking a ballistic discharge through the secondary coil which surrounds it, a set of data is obtained for the magnetization curve. The same test is repeated with other values of flux.

All the necessary adjustments and changes of connections are conveniently performed by means of the levers 1 and 2 and handles *A* and *B* (Fig. 168).

176. Drysdale Permeameter.—This instrument, shown in Figs. 170 to 172, is used for testing frame castings of electric generators and motors, without preparing special samples. The magnetizing and the exploring coils (Fig. 170) are wound on a plug (Fig. 171); the plug is inserted into a hole drilled at any desired place of the casting to be tested. The magnetic circuit is excited as shown by dotted lines, and a discharge taken through a ballistic galvanometer, or a fluxmeter.

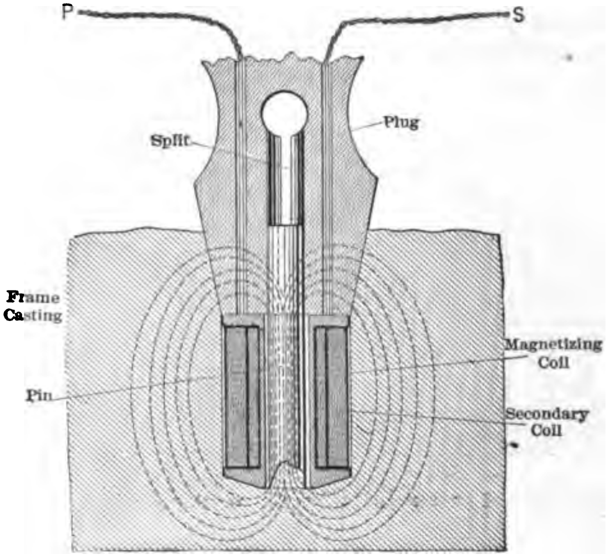


Fig. 170. **Cross-section of the Drysdale permeameter, showing the plug in the casting under test.**

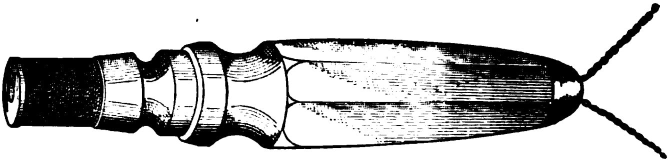


Fig. 171. **The exploring plug of the Drysdale permeameter.**

The hole in the casting has the form shown in Fig. 172. The pin left in the center constitutes the sample under test. A special hollow drill (Fig. 172) is used for making these holes, so that they will exactly fit the testing plug. It is well to test a casting in several places, in order to get its average permeability. The instrument is calibrated empirically by a sample of known magnetic properties.

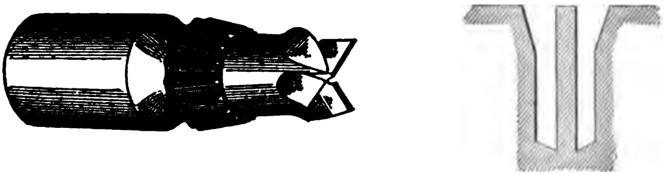


FIG. 172. The drill and the sample prepared by it, for use with the Drysdale permeameter.

177. Theory of the Ballistic Galvanometer.—The methods described in §§ 172 to 176 require the use of a ballistic galvanometer. The general features of the instrument are described in § 139; a proof will now be given, that its deflections are proportional to variations in the flux through the secondary coil. It will be proved (a) that electrical discharges through the galvanometer are proportional to variations in flux, and (b) that galvanometer deflections are proportional to electrical discharges. From these two facts will follow immediately that deflections are proportional to variations in flux.

(a) *Electrical discharges through a ballistic galvanometer are proportional to changes in flux.* Let N be an instantaneous value of a variable magnetic flux; the instantaneous e.m.f. e induced by a variation of this flux in an exploring coil that surrounds it, is, according to § 138,

$$e = s \cdot \frac{dN}{dt} \cdot 10^{-8}$$

where s is the number of turns in the coil. Let r be the total resistance of the galvanometer circuit, L its inductance; according to the equation (6), in § 103, the instantaneous current i in the galvanometer is determined by the relation

$$e = ir + L\frac{di}{dt}.$$

Substituting e from the above, gives

$$s \cdot \frac{dN}{dt} \cdot 10^{-8} = ir + L\frac{di}{dt},$$

or

$$s \cdot dN \cdot 10^{-8} = ir \cdot dt + L \cdot di.$$

Let N_1 and N_2 be the initial and the final values of the flux during the short period t of the change. Integrating the preceding equation over the period t we obtain

$$s\,(N_1 - N_2)\,10^{-8} = r\int_0^t idt + L\,(i)_0^t.$$

The integral on the right side represents the total quantity of electricity in coulombs or ampere-seconds which are discharged through the galvanometer; let this discharge be denoted by Q. The second term to the right $= 0$, because the current is zero at the beginning and at the end of the discharge period. Thus we have

$$s\,(N_1 - N_2)\,10^{-8} = rQ \quad . \; . \; . \; . \; . \; . \; . \quad (4)$$

This proves the above statement that an electrical discharge through the galvanometer is proportional to the variation in the flux.

(b) *Deflections of ballistic galvanometer are proportional to brief discharges through it.* After an electric discharge has passed through the moving coil of a galvanometer, the coil begins to move against the following forces:

(1) Torsion of the suspension wire, or the springs.
(2) Currents induced in the coil and in its aluminum frame.
(3) Air resistance.

Let the final deflection be α_1 degrees with a given discharge of Q coulombs. The force of torsion is proportional to the angle of torsion; thus the total resisting impulse of the suspension wire is proportional to

$$\int_0^{\alpha_1} \alpha\; dt.$$

The currents induced during the movement of the coil are proportional to instantaneous angular velocities $d\alpha/dt$; their impulse is proportional to

$$\int_0^{\alpha_1} \left(\frac{dx}{dt}\right) dt = \alpha_1 \quad . \; . \; . \; . \; . \; . \; . \; . \quad (5)$$

Now suppose a different discharge Q_2 to be sent through the galvanometer, such that the new angle of deflection $\alpha_2 = 2\,\alpha_1$. The time of one swing of the galvanometer is practically independent of the angle of deflection (as in the pendulum); therefore all intermediate angles α and angular velocities must be doubled. Thus the impulse of the suspension wire is doubled, and that of the induced currents. Therefore Q_2 must be $= 2\,Q_1$ in order to give a double moving impulse. This proves that electrical impulses, or discharges, and galvanometer deflections are proportional. The air resistance is not exactly proportional to angular velocity, and therefore somewhat vitiates the result; but its influence is usually negligible.

The same theory applies to the fluxmeter (§ 140), only the time of the discharge does not influence the results. This is because the instrument possesses no resisting torsion, and all of the electric impulse is absorbed by induced currents, according to expression (5).

(c) *Final formulæ.* Denoting the galvanometer constant by b, we have

$$Q = b\alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where α is the scale deflection. Consequently

$$s(N_1 - N_2)10^{-8} = br\alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Let, for instance, the constant of the galvanometer be $b = 15 \times 10^{-6}$ coulombs (ampere-seconds) per 1 cm. of scale deflection; let the resistance of the galvanometer circuit, r, equal 2000 ohms, the number of turns in the exploring coil $s = 100$. A certain flux was established through the coil, and then the exciting circuit suddenly opened, causing a galvanometer deflection $\alpha = 30$ cm. The final flux $N_2 = 0$, and we have

$$N_1 = \frac{10^8}{100} \times 15 \times 10^{-6} \times 2000 \times 30 = 900{,}000 \text{ maxwells.}$$

178. Calibration of a Ballistic Galvanometer. — Ballistic galvanometers are usually calibrated with a standard condenser, or with a standard inductance coil.

(a) Let a condenser of a known capacity of C microfarads be charged at a certain voltage e, and then discharged through the ballistic galvanometer; let the deflection be α. According to §117, the charge $Q = Ce \,.\, 10^{-6}$ coulombs, so that with reference to equation (6) we have

$$Ce \,.\, 10^{-6} = b\alpha.$$

All quantities in this equation are known except b, which can be determined.

(b) A standard coil is a long solenoid without iron, with an exploring coil placed about its central part. According to the equation (1) in § 165 the flux density in the middle part of the coil

$$H = \frac{4\pi}{10} \cdot \frac{ni}{l}.$$

If the cross-section of the coil is A sq. cm. the total flux $= AH$. The coil is excited with a known current i, and the circuit opened, producing a discharge in the ballistic galvanometer. According to the equation (7) we have

$$sAH \,.\, 10^{-8} = br\alpha,$$

from which b may be calculated.

179. EXPERIMENT 8-C. — Magnetization Curves of Iron by the Ballistic Method. — The experiment comprises the use of any of the instruments described in §§ 172 to 176. The sample is thoroughly demagnetized, and a virgin curve *OAM* (Fig. 161) taken. After this one or more hysteresis loops may be taken. The test should be performed on samples of steel, soft wrought iron, and cast iron. If the constant of the ballistic galvanometer is not known, the instrument must be calibrated as explained in the preceding article. Aside from learning the method itself, and the operation of a permeameter, it is expected that the student will make clear to himself: (a) The details of its construction, (b) the sources of error, (c) the limits of accuracy.

Report. (1) Plot the magnetization curve of one sample, using ampere-turns per inch as abscissæ and *B* per square inch as ordinates: for another sample use densities *H* in air as abscissæ and *B* per sq. cm. as ordinates; a third curve should give values of permeability $B \div H$ to densities *B* as abscissæ. In this way the student will learn the various methods of plotting magnetization curves.

(2) Illustrate in a numerical example the use of these curves for determining the number of ampere-turns in a given case, — for instance, to produce a given flux in a magnetic circuit like the one shown in Fig. 147. The dimensions of the cores and the length of the air-gap are supposed to be given.

(3) Figure out hysteresis loss by integrating one of the curves, as explained in § 203.

(4) Discuss the advantages and disadvantages of the apparatus used, sources of error, limits of accuracy, etc.

STEADY DEFLECTION METHODS.

180. Koepsel Permeameter. — In its principle the device is a millivoltmeter (Figs. 173 and 174), in which the permanent magnet is replaced by an electromagnet; the sample under test is a part of this electromagnet. The instrument consists of two heavy pole-pieces *JJ* (Fig. 173) in which is fastened the rod *P* to be tested; *S* is the magnetizing coil. The flux produced in the sample and in the pole-pieces is measured by means of a moving coil *s*, similar to those used in direct-current ammeters and voltmeters (Fig. 34). The coil is supplied with

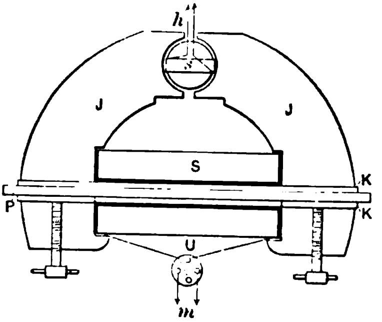


FIG. 173. A cross-section of the Koepsel permeameter.

current from a small auxiliary dry battery; deflections are indicated by a pointer on the dial (Fig. 175); the scale is calibrated directly in magnetic densities B per sq. cm.

U in Figs. 173 to 175 is a double-throw switch for reversing the current in the magnetizing coil S; W'_m in Fig. 175 is a rheostat for regulating the current in the same coil. W'_h is a rheostat for regulating the current in the moving coil; it has three handles, for coarse, medium and fine adjustment. S_1 and S_2 in Fig. 174 are compensating coils connected in series with S and opposing it magnetically. These coils are adjusted so that the instrument shows zero, when the sample is taken out; in other words, they compensate for the magnetic action of the coil S itself. In this way the apparatus is made to indicate directly the magnetic induction due to the sample under test.

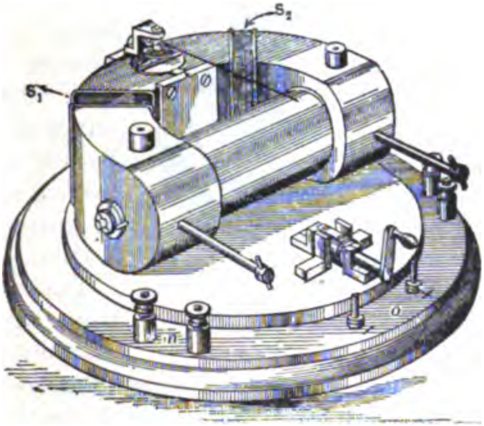


FIG. 174. The Koepsel permeameter with the cover off.

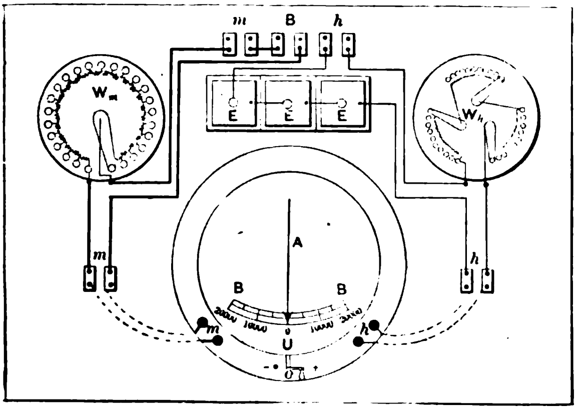


FIG. 175. The electrical connections in the Koepsel permeameter.

The apparatus is intended for taking complete magnetization curves, similar to one shown in Fig. 161. The values of B are read off directly on the scale (Fig. 176), when the current i in the moving coil is adjusted to a value given on the instrument. This current $i = k \div q$, where k is an instrument constant, and q is the cross-section (in sq. cm.) of the sample under test. The smaller this cross-section, the smaller the flux, and the larger must be the current in the moving coil, in order to produce the same deflection.

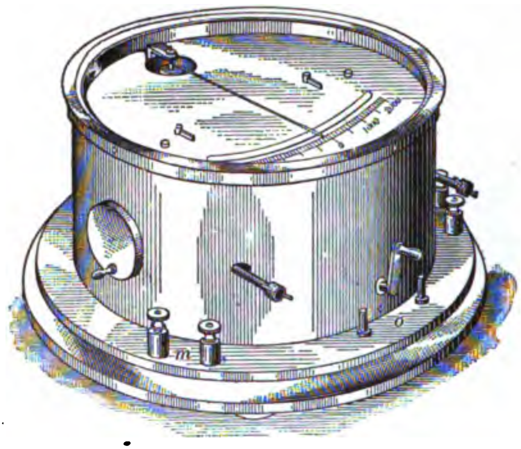


FIG. 176. The Koepsel permeameter.

The exciting ampere-turns are obtained by multiplying the current I in the coil S by its number of turns. The apparatus, as it is manufactured now, has 79.6 turns per cm., so that the exciting ampere-turns per cm. length of sample are $= 79.6\ I$. This is done in order to make the instrument direct-reading for magnetic densities H in the air (§ 165). Namely, from the formula (1),

$$H = \frac{nI}{0.796\ l} = \frac{79.6\ I}{0.796} = 100\ I.$$

There is no reason, however, why the instrument should not be made with 100 turns per cm., or per inch, so as to be direct-reading for ampere-turns per centimeter, or per inch.

The use of this apparatus is extremely simple: a sample of known cross-section is put into it, and the auxiliary current adjusted according to the formula $i = k \div q$; the value of k is given on the instrument itself. Then the main current is closed through the magnetizing coil, and simultaneous readings taken on the ammeter in the primary circuit, and on the dial of the apparatus. Ammeter readings multiplied by 79.6 give exciting ampere-turns per cm. (or, multiplied by 100, give values of H). The value of B is read off directly on the scale of the apparatus. By varying the current in S a complete B-H curve of the sample can be obtained.

Care must be taken to protect the apparatus from external magnetic fields; strong magnets, or large pieces of iron, should not be allowed near it, while readings are taken. The sample itself must have no long ends sticking out of the yoke, as they produce a stray field. The apparatus is sensitive even to terrestrial magnetism; in order to eliminate this error the device must be placed so that the line marked on the dial is in the direction " north-south." When the instrument is in this position and both currents are "on," the pointer must remain on zero, so long as there is no test sample in the apparatus.

There is in this, as in some other devices, a source of error, which is difficult to eliminate, namely, imperfect contacts between the yoke and the rod under test. The result is, that it takes more ampere-turns to produce a certain flux than it would without these unavoidable air-gaps; consequently, the apparent permeability comes out lower than it is in reality. An air-gap only 1/1000 of an inch long is equivalent in its magnetic resistance to nearly ½ inch of a good iron rod; therefore, when accurate results are required, the experimentally observed B-H curve must be properly corrected. Correction curves are usually supplied with the apparatus.

181. EXPERIMENT 8-D. —Magnetization Curves of iron with Koepsel Permeameter. —Test a steel specimen, a cast-iron specimen, and a bunch of iron laminations. Special clamps are provided with the instrument for accommodating round and rectangular samples. Set the instrument in the N-S direction; see that the needle shows no deflection, without the sample; and with both currents on. Insert the sample, and demagnetize it (§ 167); use the commutator shown in front in Fig. 174 for reversing the current. Take the curves shown in Fig. 161. Correct the data obtained, as per standardization certificate of the instrument. Investigate in how far the apparatus is sensitive to stray magnetic fields, to the influence of terrestrial magnetism, and to large iron masses in its proximity.

Report. Identical with § 179.

182. Esterline Permeameter. — The apparatus devised by Prof. J. W. Esterline for testing permeability, is essentially a small direct-current generator; the sample to be tested completes the magnetic circuit of the machine. The working principle of the device may be understood with reference to Fig. 174, if the moving coil s be replaced by a regular direct-current armature driven by a motor. The magnetizing coil S, the sample P and the pole-pieces JJ are essentially the same as in the Koepsel permeameter. The flux in the sample, instead of being measured by a deflection of the moving coil s, is measured by the voltage induced in the revolving armature. The magnetic density is figured out from the formula

$$\text{density} = \text{const.} \times \frac{\text{volts}}{\text{speed}}.$$

This follows from the fact that the voltage induced in the revolving armature is proportional to the flux and to the speed of rotation. Compensating coils are used to account for the reluctance of the air-gap and of the parts of the circuit, other than the bar under test.

In the apparatus as it is now manufactured, the exciting ampere-turns are varied by changing the number of turns in the magnetizing coil. There is no reason, however, why this should not be done by simply regulating the current in the coil by a suitable resistance, as in the Koepsel permeameter. The Esterline apparatus is substantially built, is not affected by terrestrial field, and is designed so as to be suitable for ordinary commercial work. It is claimed that an inexperienced operator, with hardly any knowledge of electricity and magnetism, can obtain accurate results after very little practice. The device is described in detail by the inventor in the *Proceedings of the American Society for Testing Materials*, VI, 1906.

183. Ewing Permeability Bridge. — An ingenious method for comparing specimens of steel and iron to a standard sample was devised by Prof. Ewing (Fig. 177). S is a standard rod whose magnetization curve is known; T is the sample under test. Both are surrounded by magnetizing coils and are clamped in heavy iron yokes $P\,P$. The magnetic circuit is closed as shown by the arrows. The standard sample is excited with a certain number of ampere-turns; the number of ampere turns on the sample under test is varied until the fluxes in both samples become equal. This moment is recognized by the pivoted-magnetic needle m returning to zero. When the fluxes in the two samples are different, some lines of force must find their path through the horns $H\,H$ and the

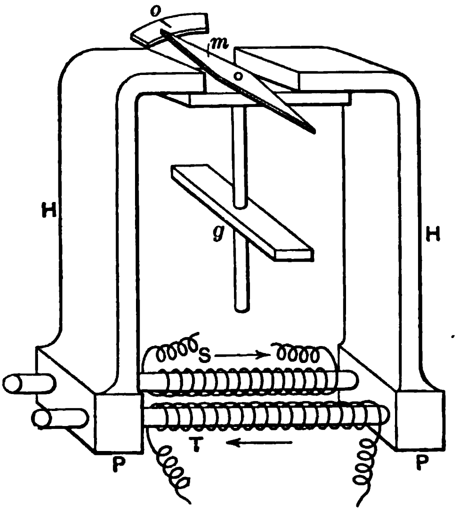


FIG. 177. The Ewing magnetic bridge.

air-gap between them, deflecting the needle. When the fluxes become equal, no stray flux passes through $H H$, and the needle returns to zero, under the influence of the permanent magnet g.

In the apparatus, as it is actually made, the number of turns around the standard sample S is fixed, that around the specimen T may be varied by a radial arm and contacts. A double row of contacts is provided, so that when a section of the coil is cut out of the circuit, an equivalent resistance is inserted in its place. This is done in order to maintain a constant current. The two magnetizing coils are connected in series, so that the ratio of active turns gives directly the ratio of ampere-turns. With this arrangement one ammeter is used instead of two.

The magnetization curve of the sample under test is plotted by changing the abscissæ of the magnetization curve of the standard sample. Let, for instance, the standard sample require 80 ampere-turns per inch at a density of 100 kilo-maxwells per sq. inch. Send through the magnetizing coils a current, such as to produce 80 ampere-turns per inch in the standard sample; adjust the number of turns of the specimen under test, until the needle m returns to zero. Let the number of turns in the standard coil be 100, that on the other coil 150. This shows that in order to obtain a density of 100 kilo-maxwells in the sample under test

$$80 \times \frac{150}{100} = 120$$

ampere-turns per inch are required. A calibration curve is usually furnished with the instrument to account for air-gaps.

The apparatus is called a "bridge," because of its resemblance to the Wheatstone bridge; the two horns $H H$ and the magnetic needle correspond to the galvanometer circuit of the bridge (§ 14).

184. EXPERIMENT 8-E. — Magnetization Curves of Iron with Ewing Permeability Bridge. — The apparatus is described in § 183. The conduct of the experiment and the requirments for the report are the same as in § 179.

185. Bismuth Spiral. — The metal bismuth has a peculiar property, that its electrical resistance increases appreciably, when it is placed in a magnetic field. Per cent increase in resistance is nearly proportional to the magnetic density of the field. The curve shown in Fig. 178 gives an idea of this increase for an average specimen of bismuth wire. This property of bismuth is utilized to some extent for measuring magnetic densities. Fine bismuth wire is made into a flat spiral wound non-inductively, the wire being doubled back on itself. The winding is held rigidly between two thin pieces of mica. Such a spiral is placed in a field, of which it is desired to measure the flux density; its ends are connected to a Wheatstone bridge. From the increase in the electrical resistance of the bismuth wire, flux density is determined by using the curve shown in Fig. 178. Bismuth spirals used for measuring flux density in air-gaps of electrical machines, stray fluxes, etc., are provided with handles and with suitable terminals. For testing permeability of iron, with bismuth spirals, an apparatus similar to that shown in Fig. 166 is used. The exploring coil E is omitted, and the spiral is inserted between the ends of the two samples TT. Bismuth has an appreciable temperature coefficient; a correction must be applied for this factor when using the apparatus.

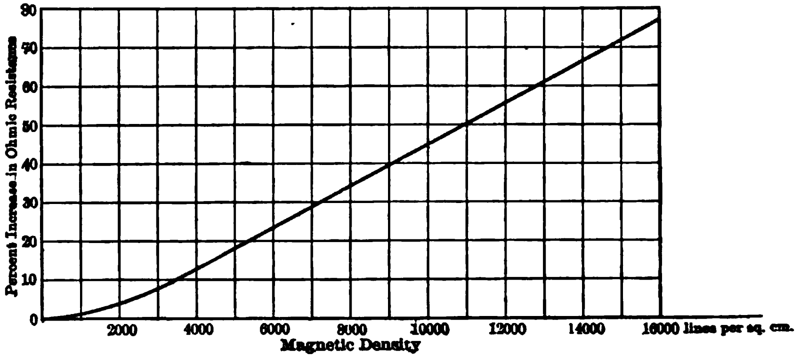


Fig. 178. **Variation of resistance of a bismuth spiral in a magnetic field of varying density.**

186. EXPERIMENT 8-F. — Magnetization Curves of Iron with Bismuth Spiral. — The method is described in § 185. The general conduct of the experiment and the requirements for the report are the same as in § 179.

CHAPTER IX.

MEASUREMENT OF CORE LOSS.

187. STEEL and iron cores used in armatures of electrical machines and in transformers are subjected to a variable magnetization. Each particle of the core is regularly magnetized in one and then in the opposite direction many times a second, according to the frequency of the supply or the speed of the machine. Under such conditions cores are appreciably heated; this heating is objectionable from two points of view:

(1) The temperature may reach a limit where it is dangerous to the insulation of the windings.

(2) The efficiency of the machine is lowered, by the amount of energy thus converted into heat.

An investigation of this heating of iron, and of the resultant loss of energy, shows that it is due to two distinct and independent causes: (1) molecular friction, or hysteresis; (2) induced eddy currents.

188. Physical Nature of Hysteresis. — The phenomenon of hysteresis is described in § 164; as is stated there, flux density in iron lags behind the exciting ampere-turns, when it is subjected to cyclic magnetization. The accompanying loss of energy is probably due to some friction among the molecules of iron, when the magnetizing force causes them to be arranged along certain lines. A certain amount of energy is necessary to take one pound of iron through one cycle of magnetization up to a certain flux density. The loss of power depends therefore upon the *number of cycles* per second, in other words, on the frequency of magnetization. This power also increases with the *maximum flux density* to which magnetization is carried.

Thus hysteresis loss $W = \eta V f . F(B)$ watts, where V is the volume of iron, f the frequency of magnetization in cycles per second, $F(B)$ a function of the magnetic density, and η a physical coefficient, which depends on the quality of iron. Dr. Steinmetz found by extensive tests that hysteresis loss increases on the average as the 1.6th power of the density B. Accordingly, for many practical purposes it is assumed that

$$W = \eta V f . B^{1.6} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

In stationary machinery, as, for instance, in transformers, the energy necessary for overcoming this loss is supplied electrically in the form of an additional component of the magnetizing current. In generators, hysteresis causes an opposing torque between the armature and the field; the energy for overcoming this torque is supplied by the prime mover. In motors the loss may be supplied from the line, or it may be considered as reducing the useful torque available on the shaft.

It is interesting to note that the counter-torque caused by hysteresis is independent of the speed of the machine. Let this torque be T foot-pounds and the speed of the machine n r.p.m. The power necessary for overcoming this torque is proportional to the product "torque times speed," so that

$$W = KnT \text{ watts},$$

where K is a numerical constant. Equating this expression to (1), we have

$$KnT = \eta Vf.B^{1\cdot6}.$$

The number of cycles of magnetization f is proportional to the speed n of the machine, so that we finally get

$$T = K'\eta VB^{1\cdot6}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where K' is another constant. This equation (2) shows that the torque caused by hysteresis does not depend on the speed of the machine.

189. Physical Nature of Eddy Currents.— Iron is not only a magnetic conductor, but also a good conductor for electric currents. Therefore, when a magnetic flux varies in iron, currents are induced in it as in any other conductor, subjected to a pulsating flux. These currents are called eddy currents, or Foucault currents, after a French physicist of that name; they constitute a pure loss of energy. To reduce these currents as far as possible, iron is subdivided into thin sheets, or laminations, insulated from each other by paint, varnish, tissue paper, etc., in order to break up the paths of eddy currents.

Like all induced currents, eddy currents are proportional to the flux density B and to the frequency f of pulsations. Thus eddy currents per cubic unit of iron are proportional to the product Bf; the corresponding loss in watts is proportional to the square of the currents (according to the expression i^2r). Thus, we have

$$\text{eddy-current loss } w = \xi\, Vf^2B^2 \text{ watts} \quad . \quad . \quad . \quad . \quad (3)$$

The physical coefficient ξ depends on the thickness of laminations, and decreases approximately as the square of the thickness. Proceeding as in § 188, we find that the torque caused by eddy currents

$$t = k' \xi VnB^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

This torque is proportional to the speed of rotation, or to the frequency of magnetization; it has been shown, that the hysteresis torque (2) is the same at all speeds. This difference in the properties of hysteresis and eddy currents is utilized in practice for separating them from each other.

190. Methods for Measuring Hysteresis and Eddy Currents. — Core loss due to hysteresis and eddy currents lowers the efficiency and increases the temperature rise in machinery. It is important, therefore, to know how to measure this loss on samples of steel used in the construction of electrical machinery. Such tests make it possible to estimate the loss of energy and the temperature rise while designing new machines; also to reject material of inferior quality, which gives an abnormal core loss. Results of core-loss tests are usually plotted in the form of the curves, shown in Fig. 179. The curves give the total loss which comprises both hysteresis and eddy currents. In most practical cases it is not required to separate the two; but should this be necessary, it can be done as is explained in § 200.

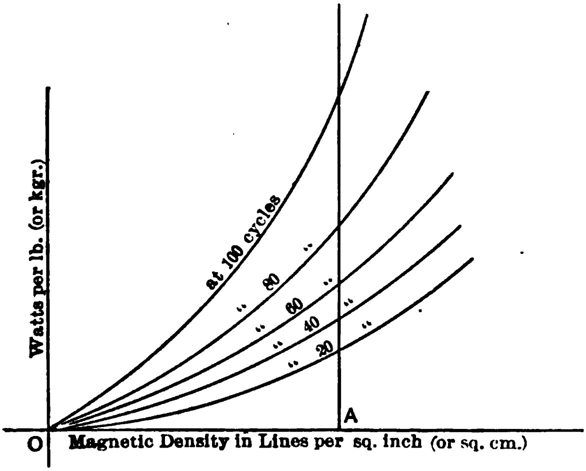


FIG. 179. Curves of iron loss at different magnetic densities and different frequencies of magnetization.

Two distinct methods for determining iron loss are chiefly used: (1) Mechanical torque method, (2) Wattmeter method. With the first method the samples to be tested are driven mechanically in a magnetic field (or vice versa), as if they were parts of the armature core of a generator. The torque produced by the core loss in the samples is measured by a dynamometer spring. With the wattmeter method samples are magnetized by an alternating current, as they would be in a transformer; the power necessary for magnetization is measured on a wattmeter.

The torque method should be used for sheet steel intended for armature cores of generators and motors; the wattmeter method is suitable for transformer iron. In practice, however, no such strict distinction is made between the two methods, because in most cases it is sufficient to know that a new lot of iron has a core loss not above a certain limit, set as the result of previous experience. It makes little difference in which way this limit is ascertained, provided the same method is used in cases to be directly compared.

A third possible method consists in integrating the area of a hysteresis loop of the sample (Fig. 161). It can be shown theoretically (see § 203) that the area of this loop is proportional to joules hysteresis loss per cu. cm. of iron, per cycle of magnetization. This method is seldom used, because it is rather lengthy; besides, it gives hysteresis loss only, and not the total core loss.

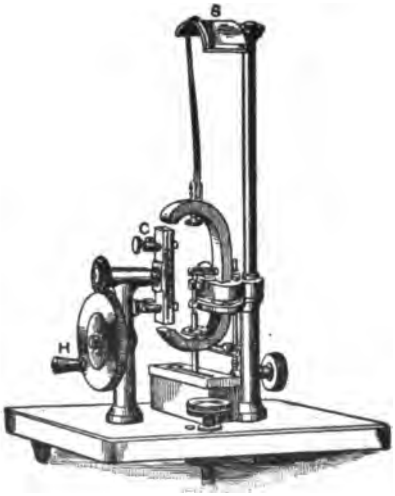


FIG. 180. The Ewing hysteresis tester.

MECHANICAL TORQUE METHOD.

191. Ewing Magnetic Tester.

—This is the simplest and the oldest device based on the principle of measuring hysteresis loss by mechanical torque; it is shown in Fig. 180. A few strips of sheet iron to be tested are clamped into a carrier *C*, which is made to revolve by turning the handle *H*. The carrier turns between the poles of a permanent magnet, which is suspended on a knife-edge. In consequence of the hysteresis torque of the specimen the magnet is deflected, and the deflection is observed by means of a pointer on the scale *S*.

The same test is made with a standard sample, whose hysteresis loss in watts per cycle has been previously determined by some other method (for instance, by integrating its B-H loop). The ratio of deflections gives the ratio of the values of hysteresis loss in the two samples.

The deflections of the pointer are practically independent of the speed of rotation; it is shown in § 188, equation (2), that hysteresis torque is independent of the frequency of magnetization. Yet, when the speed becomes rather high, the influence of eddy currents becomes noticeable, the deflection being thereby increased.

It must be noted, that the loss determined by this tester refers to a certain flux density only, and no provision is made in the apparatus for varying the density. The density within the samples is practically independent of the quality of iron tested; the value of the flux is chiefly determined by the reluctance of the two air-gaps. The instrument is adapted for rough relative tests rather than for determining absolute values of hysteresis loss.

192. EXPERIMENT 9-A. — Hysteresis Loss in Iron with Ewing Magnetic Tester. — Test a few samples of sheet steel and iron, as explained in the preceding article. Investigate the influence of the speed of rotation; make a sketch of the mechanical details of the device. Test one of the samples with laminations thoroughly insulated from each other, and then without insulation; see if the influence of eddy current makes itself perceptible in the second case. Form an opinion in regard to the accuracy of the instrument and its applicability for practical work.

193. Blondel Hysteresimeter. — This device (Figs. 181 and 182) is based on the same principle as the above-described Ewing magnetic tester, but is mechanically superior to it. It has a U-shaped permanent magnet MM which can be rotated around a vertical axis by means of the handle H. The sample sheets to be tested are made in the form of a ring R and are fastened on the support S. This support with its pivoted vertical shaft P, tends to revolve, but is retained by an opposing spiral spring, shown in the figure. When the magnet M revolves, the support with the sample turns by an angle, at which the

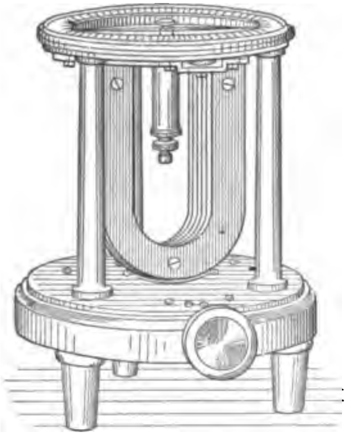


FIG. 181. The Blondel hysteresimeter.

hysteresis torque just balances the spring action. The deflection is practically independent of the speed of rotation, as in the above Ewing's apparatus, and is directly proportional to the hysteresis constant η of the sample (see § 188).

A standard sample, whose hysteresis constant has been determined by another method, is used with this instrument; the ratio of deflections given by the standard sample and the sample under test is equal to the ratio of their hysteresis constants.

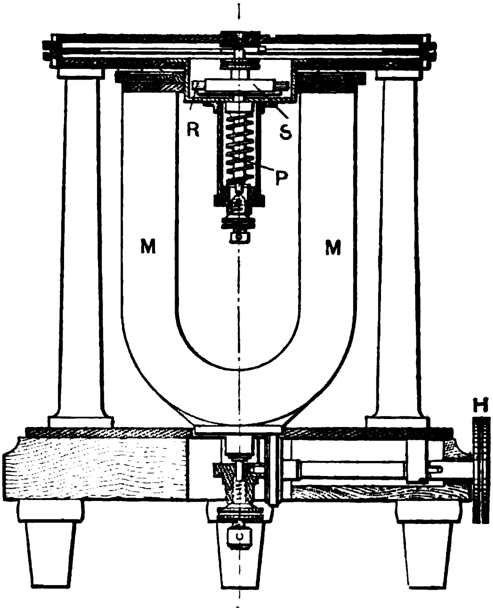


FIG. 182. A cross-section of the Blondel hysteresimeter.

The magnet *M* is selected of such a strength as to give in the sample a magnetic density *B* of about 10.000 lines per sq. cm. The reluctance of the air-gap is so large, as compared to that of the sample itself, that differences in permeability of different samples hardly affect the above value. Thus all samples are compared at a standard density of 10.000 maxwells per sq. cm.

In testing samples with this hysteresimeter it is always necessary to take readings with rotation both ways; the average of the two indications is the true zero position. This eliminates the error due to a possible previous magnetization of the sample.

To insert a sample into the apparatus, first remove the glass covering the scale, then take off the pointer and finally the support *S*. The sample is clamped to the support, after which the parts are returned to their respective places. In performing the measurement it is advisable to rotate the handle at a speed of 2 to 3 revolutions per sec.

194. EXPERIMENT 9-B. —Hysteresis Loss with Blondel Hysteresimeter. — See directions for the preceding experiment (§ 192).

195. Holden-Esterline Core-Loss Meter. — The two above-described hysteresis testers are very simple and convenient to use, but they have the following disadvantages:

(a) Being rotated by hand, the frequency is so low that samples are tested for hysteresis loss only, while total loss is usually important.

(b) Hysteresis loss is determined at one density only.

(c) Readings are merely relative, so that a standard sample is required.

These objections are remedied in a device constructed by Mr. Frank Holden. In his instrument the magnetizing field is revolved by a motor at a required high frequency; moreover, an electromagnet is substituted for the permanent magnet, so that it is possible to test samples within a wide range of magnetic densities; the instrument is made to read the core loss directly in watts. Holden hysteresis meter is described in detail in Foster's *Pocketbook*, p. 75. It was recently improved by Prof. J. W. Esterline, and in this latter form is shown in Figs. 183 and 184.

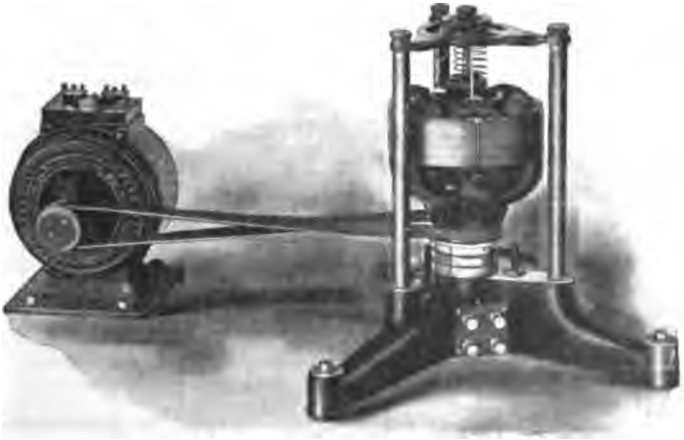


Fig. 183. Holden-Esterline core-loss meter.

The exciting field — with inwardly projecting poles, six in number — is made of stamped laminations, and mounted on a hub which can be rotated by means of a belted motor. The poles of this field are wound with exciting coils. For alternating-current work the frame is left stationary; the coils are connected to a two-phase or three- hase circuit, so as to produce a rotating magnetic field.

The sample to be tested consists of concentric rings, mounted on a brass spider, whose shaft has a pivot below, resting on a jeweled bearing. The upper end of the shaft is guided by a bearing in the dial plate. When the poles are rotated, or a rotating field produced by A. C. currents, the eddy currents and hysteresis in the sample tend to cause it to rotate also. It is, however, prevented from doing so, by a coiled spring, attached at one end to the spider which carries the sample core, at the other to a milled head on the dial plate. The pointer attached to the milled head is set at zero, when there is no twist in the spring. The angle of torsion necessary to turn the core back to zero, when the loss is taking place, is a measure of the loss.

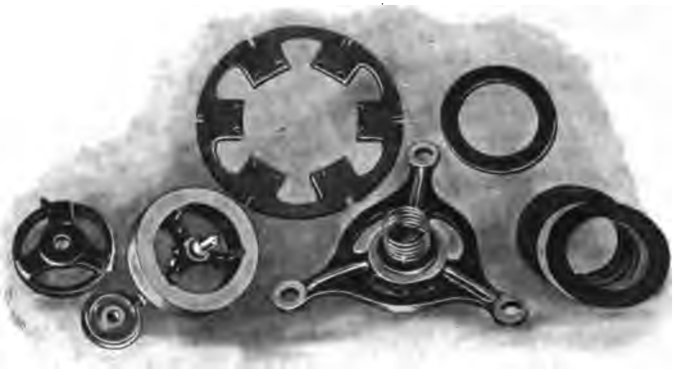


FIG. 184. Parts of the Holden-Esterline core-loss meter.

Instead of calibrating the dial in ft.-lbs., or in gram-centimeters torque, it is calibrated directly in watts loss, at a certain standard frequency, for instance, 60 cycles per sec. At lower frequencies the readings must be accordingly reduced, at higher frequencies increased. The air-gap is made of sufficient length to make the flux density in the specimen practically proportional to the exciting current. Moreover, the permeability of the sample at the usual densities has very little effect on the flux in the instrument, so that the dial can be calibrated once for all. The apparatus is well adapted for commercial work; it is strong mechanically, and not much skill or electrical knowledge is required to operate it.*

* A somewhat similar device, in which the punchings are made to revolve and the field is stationary, was described by Mr. C. E Skinner in the *Electric Club Journal*, 1904, p. 337.

196. EXPERIMENT 9-C. — Testing Iron with Esterline Core-Loss Meter. — For a description of the device see the preceding article. Test a few samples over a wide range of densities and frequencies; drive the field with a motor. Take a few readings with a revolving field produced by polyphase currents. Study the mechanical details of the apparatus; if possible, check the calibration of the dynamometer spring.

Report. Plot curves giving core loss in watts per pound of iron, at various densities and frequencies (Fig. 179). Separate hysteresis from eddy currents, as in § 200. Show the difference in the value of the loss with the field produced by polyphase currents; explain the discrepancy.

WATTMETER METHOD.

197. In the above-described core-loss testing devices the sample is subjected to "rotating hysteresis" as in generator and motor armatures. In transformers the iron core is subjected to so-called "alternating hysteresis"; in order to test samples under such conditions they are placed within coils excited with alternating current, and the magnetization loss determined by a wattmeter. The arrangement must be such, as to have a closed magnetic circuit, and at the same time not necessitate winding new magnetizing coils every time. A few practical devices of this kind are described below.

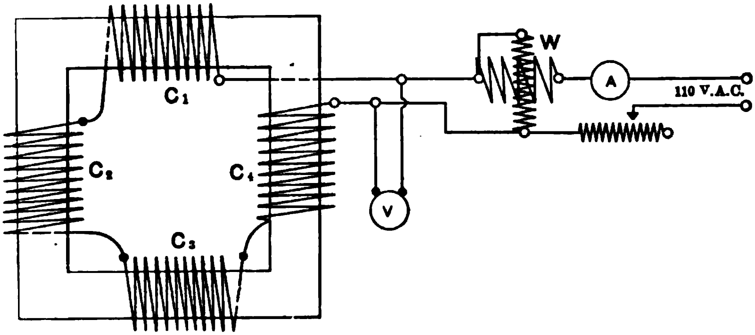


FIG. 185. Diagrammatic representation of the Epstein core-loss apparatus.

198. Types of Testing Devices. — Samples may be tested either in the form of rectangular strips, or as stamped rings; some even prefer to test whole sheets as they come from the rolling mills. The three devices described below are intended for testing iron in these three forms.

(a) *Arrangement for testing strips* (Epstein). The device shown in Fig. 185 is prescribed for core-loss tests by the German Institution of

Electrical Engineers; its details and dimensions may be found in their standardization rules (Vorschriften). The apparatus has four magnetizing coils C_1, C_2, C_3, C_4, of a rectangular cross-section; sample sheets are inserted one by one, two consecutive layers being shown in Fig. 186. The ends of the strips overlap each other, and the four corners are clamped so as to form practically a closed magnetic circuit. The coils are energized from an alternating-current supply having a wave-form as near sinusoidal as possible; watts iron loss is read directly on the wattmeter *W*. By varying the voltage at the terminals of the apparatus the induction in the iron is also varied, and a curve can be plotted giving iron loss as a function of induction.

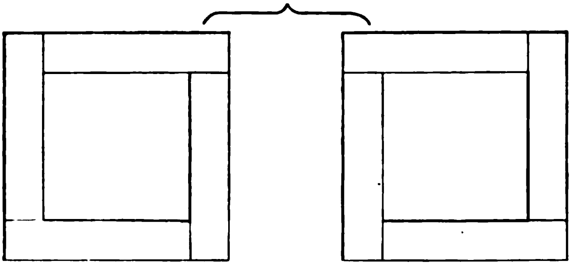


FIG. 186. Overlapping of strips in two consecutive layers, in the apparatus shown in Fig. 185.

(b) *Arrangement for testing rings* (Moellinger, Esterline). In order to avoid winding a new coil around each bunch of punchings, the turns of the winding are divided each in two halves (Fig. 187); both parts are carried by separate non-conducting shells. The punchings are placed on the lower shell and covered with the upper shell, thus form-

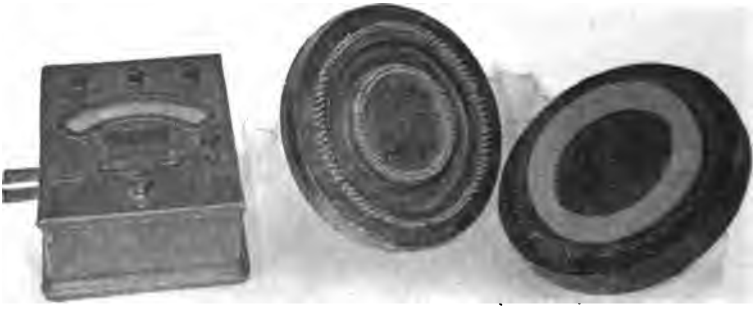


FIG. 187. Esterline apparatus for testing iron in ring-form.

ing complete magnetic and electric circuits. An alternating current of constant value is used with this device. The exciting ampere-turns are varied by changing the number of active turns in the circuit, by a dial switch. A resistance is simultaneously introduced into the circuit, so that the total resistance remains constant. In this way the correction, i^2r, for the copper loss in the winding remains constant. The wattmeter may be adjusted to read zero when a normal current is flowing through the instrument, without iron core; in this case no correction whatever is necessary.

(c) *Arrangement for testing whole sheets* (Richter). A narrow magnetizing coil is wound along the periphery of a barrel (Fig. 188), and whole sheets are shoved into it through the slot seen on top. The ends of the sheets are made to overlap each other, so as to form a closed magnetic circuit. The core loss at a definite voltage and frequency is determined by a wattmeter as in the two above-described devices. It is claimed that small samples, made from the same sheet, may show differences in core loss of over 10 per cent. Therefore, it is supposed to be more reliable to test complete sheets; besides, no time is lost in making samples.

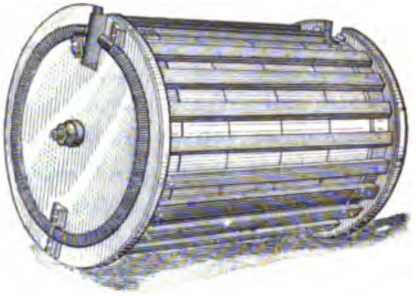


FIG. 188. The Richter drum for testing whole sheets of iron.

Instead of testing whole sheets, some manufacturing companies actually make transformer stampings of the material to be tested, build a regular transformer and determine its core loss.

199. Relation between Flux Density and Voltage. — The results of a wattmeter test on core loss are usually plotted in the form of curves shown in Fig. 179; the curves give watts loss to magnetic densities as abscissæ. These densities are figured out from the applied voltage E, number of turns s in the magnetizing coils, and the cross-section of the core under test. According to the fundamental equation of alternating-current apparatus the counter-e.m.f. e induced by the flux in the magnetizing coils is

$$e = \frac{2\pi}{\sqrt{2}} . sfN . 10^{-8} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (5)$$

(see proof below), where N is the maximum value (amplitude) of the magnetic flux. Substituting in this formula AB in place of N, where

A is the net cross-section of the iron and B the density, also substituting for $2\pi \div \sqrt{2}$ its value 4.44, we get

$$e = 4.44\, sfAB \,.\, 10^{-8} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (6)$$

In this formula all the quantities can be measured directly, except the induction B and the counter-e.m.f. e. The latter is equal to the applied voltage E, less the ohmic drop in the magnetizing coils, see formulæ (7) and (8) below. Thus the only unknown quantity is B, which can thus be calculated.

Formula (5) is deduced as follows: The instantaneous value of the induced e.m.f., according to the fundamental law of induction (§ 138), is proportional to the rate of change in flux and to the number of turns connected in series; or

$$e' = s\frac{dN'}{dt} \,.\, 10^{-8},$$

where e' and N' are instantaneous values of counter-e.m.f. and flux. If the applied voltage varies according to the sine law, the flux must also follow the same law, so that

$$N' = N \operatorname{Sin} 2\pi\, ft.$$

Substituting we find

$$e' = 2\pi fs N \operatorname{Cos} 2\pi ft \,.\, 10^{-8}.$$

The counter-e.m.f. reaches its maximum at the moments, when $ft = 0, 1, 2, 3$, etc.; its value

$$\text{max. } e = 2\pi fs N \,.\, 10^{-8}.$$

Hence the effective value of the voltage

$$e = \frac{2\pi}{\sqrt{2}} sfN \,.\, 10^{-8},$$

which is identical with the formula (5).

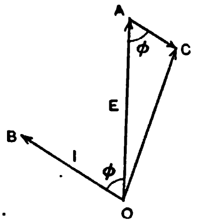


FIG. 189. Correction for the ohmic drop in the magnetizing winding.

In figuring out e from the applied voltage E it must be remembered that the ohmic drop IR has to be subtracted from E geometrically as shown in Fig. 189. OA is the vector of the applied voltage E, OB is the current I, which lags behind by an angle ϕ. This angle is known since

$$\operatorname{Cos} \phi = \frac{W}{EI},$$

where W is the wattmeter reading. Subtracting $AC = IR$ parallel to OB we get the counter-e.m.f. $OC = e$, which being substituted in formula (5) permits the calculation of B. Instead of actually constructing the diagram, e can be determined analytically. Thus, from the triangle OAC we have, according to the familiar trigonometrical formula:

$$e^2 = E^2 + I^2R^2 - 2\,EIR \operatorname{Cos} \phi;$$

substituting W in place of $EI \operatorname{Cos} \phi$ gives:

$$e^2 = E^2 + I^2R^2 - 2\,RW \quad . \; . \; . \; . \; . \; . \; . \; . \quad (7)$$

In practice the power factor Cos ϕ is quite low; therefore the angle BOC, or ACO, may be assumed = 90 degrees. The triangle OCA becomes a right one, and we simply have

$$e^2 = E^2 - I^2R^2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (8)$$

The correction in the form (7) must be used when ohmic drop is considerable; expression (8) is accurate enough for many practical purposes.

200. Analysis of Loss Curves. — The curves of total loss plotted as in Fig. 179 may be analyzed in two respects: (a) Hysteresis loss may be separated from eddy currents; (b) The laws may be determined according to which both losses vary with magnetic density.

(a) *Separation of hysteresis from eddy currents* is made on the basis of the fact that with a given density B hysteresis loss varies as the frequency f, while eddy-current loss varies as the square of the frequency; see equations (1) and (3) in §§ 188 and 189. We may take a value of density B, such as OA in Fig. 179, and plot the values of total loss to frequencies as abscissæ, — line OD in Fig. 190. If OC is drawn

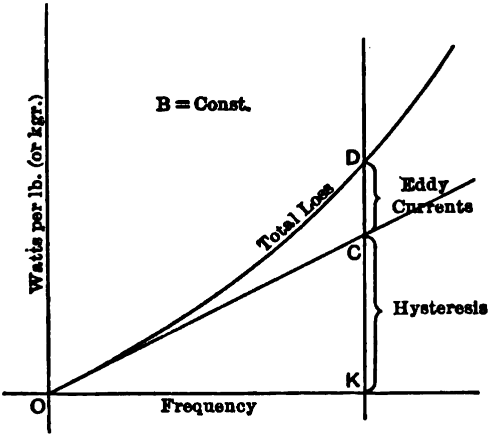


FIG. 190. Separation of hysteresis from Eddy currents.

tangent to OD near the origin O, the ordinates of OC represent watts hysteresis loss; the residue between OC and OD gives the eddy-current loss. It will be seen that the first increases directly as the frequency, the second as the square of frequency (the curve being a parabola). At low frequencies eddy-current loss is negligible, and practically all of the loss is due to hysteresis; this is the basis for the above construction. The same construction may be repeated for other values of B, and hysteresis loss separated from eddy currents throughout the entire range of the curves in Fig. 179.

(b) *The exponent, according to which hysteresis loss varies with the flux density,* is determined in the following way: Assume the exponent of B in the formula (1), § 188, to be unknown. The formula may be written then in the form

$$W = aB^n$$

where a is a constant with which we are not at present concerned. Taking logarithms of both sides of this equation, we get

$$\log W = \log a + n \,.\, \log B \quad . \;.\; .\; .\; . \quad (9)$$

This is the equation of a straight line with respect to $\log W$ and $\log B$; the unknown exponent n is equal numerically to the trigonometric tangent, which this line makes with an axis of abscissæ (Fig. 191). Thus, in order to determine n at a certain frequency, eddy-current loss must first be separated from the corresponding curve in Fig. 179. The rest is then replotted to a logarithmic scale, as in Fig. 191. It must give nearly a straight line; the slope of this line gives the exponent n.

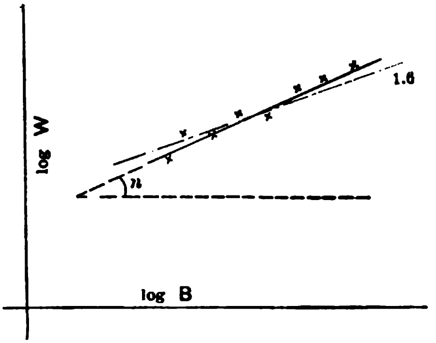


FIG. 191. Determination of the exponent according to which iron loss varies with the magnetic density.

To plot this line, use the slide-rule scale instead of the ordinary one, and you will get logarithmic abscissæ and ordinates without any calculation. Do not attempt to mark the origin; with the logarithmic scale it is at minus infinity. All that is needed is the slope n of the curve; this slope is independent of the position of the origin.

In some cases it may be desired to accept the exponent $n = 1.6$ as correct, and merely to find the value of a, in other words, of the constant η in formula (1). In this case a line, having the slope of 1.6, is drawn as closely as possible through the experimental points (Fig. 191). The coefficient a is then calculated for this line from the equation (9).

In a similar way it may be checked that eddy-current loss increases as the square of magnetic density. Total core loss increases according to a power between 1.6 and 2. The corresponding exponent may be found by plotting the curves, shown in Fig. 179, to a logarithmetic scale, without first separating hysteresis from eddy currents.

201. Magnetizing Current from Wattmeter Tests. —When performing a core-loss test by the wattmeter method, the data for the magnetization curve of the sample (Fig. 151) are incidentally determined. Let the wattless component of the current, in a test like the one shown in Fig. 185, be i_0. Let the number of magnetizing turns be s, and the length of the magnetic circuit l cm. The number of ampere-turns per centimeter length is a function of the density B; or

$$\frac{i_0 \sqrt{2} \, . \, s}{l} = F\,(B) \quad . \; . \; . \; . \; . \; . \; . \qquad (10)$$

The factor $\sqrt{2}$ is introduced because the ammeter measures the effective value of the current, while the density calculated from the formula (6) corresponds to the amplitude of the magnetic flux.

The value of magnetizing current is needed in practice for the determination of the no-load current of transformers, for which the iron under test is intended to be used. It is interesting to note, that the necessary excitation per pound of iron, at a certain density and frequency, may be given in "wattless" watts, instead of amperes. Namely, the cross-section A of the iron circuit, which enters in the formula (6), may be represented as $V \div l$, where V is the volume of the bunch of laminations. Thus we get

$$e = 4.44\, sf \, . \, \frac{V}{l} B \, . \, 10^{-8}.$$

Multiplying both sides of this equation by the magnetizing current i_0 we obtain

$$\text{magnetizing watts} = ei_0 = 4.44\, f \left(\frac{si_0}{l}\right) V \, . \, B \, . \, 10^{-8},$$

or,

$$\frac{\text{magnetizing watts}}{\text{per unit volume}} = \text{Const.} \times \left(\frac{si_0}{l}\right) . \, B \, . \, f$$

Substituting si_0/l from (10), and changing the constant in the ratio of the specific weight of iron, we get:

$$\frac{\text{magnetizing watts}}{\text{per pound}} = \text{Const.} \times f \times \text{function of } B.$$

This equation shows, that it is possible to plot an experimental curve, which gives magnetizing watts, per pound of sample at a standard frequency, as a function of magnetic density B.

This method of representing magnetizing current is very convenient in designing transformers, because the current can be figured immediately from the dimensions of the core. Or else, the dimensions of the core may be determined under the condition, that magnetizing current should not exceed a certain per cent of the full-load current.

202. EXPERIMENT 9-D. — Determination of Iron Loss by the Wattmeter Method. — Any of the devices described in § 198 may be used; the method is explained in § 199. Arrange the electrical connections as in Fig. 185; begin the test, if possible, with B as high as 12,000 per sq. cm., and gradually reduce the voltage and the density down to zero. Read volts, amperes and watts. Repeat the same test with other frequencies, taking care to have as wide a range of frequencies as possible. Make a comparative test of iron loss with and without insulation between the laminations, in order to see the influence of eddy currents. Perform this test preferably at a high frequency, where the influence of eddy currents is more noticeable. Before leaving the laboratory measure the dimensions of the magnetic circuit, and count the number of turns in the magnetizing coils. Measure the resistance of the windings; weigh the iron tested.

Report. (1) Plot curves of total loss as in Fig. 179, and separate hysteresis from eddy currents as in Fig. 190. Before doing this, wattmeter readings must be corrected for copper loss; values of magnetic density B are calculated as in § 199. (2) Determine for one of the curves, the exponent according to which the total loss, or the hysteresis loss alone, varies with the flux density (§ 200*b*). (3) Show by an example how to determine exciting ampere-turns per cubic inch at a given density. (4) Show how to plot a curve of magnetizing watts per pound of iron, at a given frequency (§ 201), and explain how to use this curve for figuring out weight of iron in a transformer. The given conditions are: in a transformer of P kilowatts the magnetizing current should not be higher than p per cent of full load current, and the core loss not greater than q watts per pound. (Give a numerical example.)

HYSTERESIS LOSS BY INTEGRATING B-H CURVE.

203. Hysteresis loss in joules or watt-seconds, per cycle, per cubic unit of iron, is proportional to the area of the hysteresis loop (Fig. 161). The magnitude of the loss depends on the maximum magnetic density corresponding to the points M and M_1.

The proof is as follows: While iron is undergoing a cycle of magnetization, e.m.f.'s are induced in the exciting winding. The instantaneous energy supplied by the exciting circuit, or returned to it, is *ei.dt*, where i and e are instantaneous values of the exciting current and of the induced voltage; *dt* is an infinitesimal element of time. The total energy supplied during one complete cycle is

$$P = \int ie.dt \text{ joules.}$$

According to the fundamental law of induction we have

$$e = s\,\frac{dN}{dt}\,10^{-8},$$

where s is the number of exciting turns, N the instantaneous value of the flux. Substituting this value of e, we get

$$P = \int (si)\,dN\,.\,10^{-8}.$$

The loss per cubic unit of the sample under test is

$$\frac{P}{Al} = \int \left(\frac{si}{l}\right) \cdot \left(\frac{dN}{A}\right) \cdot 10^{-8} \text{ joules,}$$

where A is the cross-section, l the length of the magnetic circuit. Referring to Fig. 179, the expression si/l represents magnetizing ampere-turns per unit length, or the abscissæ to which the hysteresis loop is plotted; $dN/A = dB$ is the differential of an ordinate of the same curve. Thus the above integral is reduced to the familiar formula of analytical geometry,

$$\int xdy,$$

which represents the area of a curve. If the sample undergoes f cycles of magnetization per second, we have hysteresis loss in watts per cubic unit $= f \times$ (area of hyst. loop) $\times\ 10^{-8}$. This method is very seldom used in practice for determining hysteresis loss, because:

(1) It is tedious, since it requires a complete magnetization curve to be taken and integrated for each point on the hysteresis loss curve.

(2) The iron is undergoing the magnetizing process slowly, while in actual machines it is magnetized and demagnetized many times per second. There are indications that hysteresis loss is somewhat different in the two cases.

(3) Eddy-current is not taken into account; in many practical cases the designer is interested to know not the hysteresis loss alone, but the total iron loss.

Nevertheless, the method is interesting from a theoretical and historical point of view; it is desired that the student integrate a loop in connection with one of the reports on the experiments 8-C to 8-F.

CHAPTER X.

PHOTOMETRY OF INCANDESCENT LAMPS.

204. THE two important requirements of a good incandescent lamp are: high efficiency and long life. These requirements are to some extent contradictory, since high efficiency implies heating the filament to a high degree of incandescence; while, the higher the temperature, the shorter the life of the filament. Much effort has been devoted to obtaining higher degrees of incandescence than is possible with carbon, without unduly shortening the life of the lamp. The outcome of this has been the introduction of the carbon filament "graphitized" by a special process. Incandescent lamps provided with such filaments are known in the trade as metallized filament, or "Gem" lamps. Simultaneously with this, materials other than carbon have been tried; so far tungsten, tantalum and osmium filament lamps have been developed into commercial form.* To sum up the progress made in the last few years it is sufficient to state, that, — while the ordinary carbon filament lamp consumes at least 3.1 watts per candle-power and has an average life of not over 800 hours, — the tungsten filament lamp consumes only 1.25 watts per candle-power, has a life of about 1000 hours, and gives a better quality of light.

Instruments for measurement of *candle-power*, or luminous intensity of lamps, are called *photometers*; the science of measuring intensity and distribution of light is generally called photometry.

205. Principles of the Photometer. —The photometer, most widely used in practice for measuring candle-power of incandescent lamps, is illustrated schematically in Fig. 192; its construction may be understood by a consideration of Figs. 193 and 194. The lamp L_1 to be tested, and a standard lamp L_2 whose candle-power is known, are placed at the ends of a horizontal bar. This bar, or the photometer bench, as it is called, usually has a length of 100 inches or 250 cm. The lamps are brought up to their normal degree of incandescence, and a screen is placed between them. This screen is moved back and forth along the bar until it appears equally illuminated on both sides. According to the fundamental law of optics, the intensity of illumination decreases inversely as the square of the distance. Thus we have, when the intensity of illumination is the same on both sides:

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* The Nernst lamp, with its filaments made of rare earths, also belongs to the class of incandescent lamps.

$$J_1 \div J_2 = a^2 \div b^2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (A)$$

where J_1 and J_2 denote the candle-power of the lamps in the horizontal direction, and a and b are their distances from the screen. If J_2 is known, J_1 can be calculated.

In order to understand clearly the meaning of this formula, assume first, that both lamps are of the same candle-power; it is evident then that they must be placed at equal distances from the screen in order to give an equal illumination. Now suppose that one of the lamps gives four times more light than the other; it must then be placed at a double distance from the screen in order to give the same illumination. If the lamp is nine times stronger, its distance from the screen must be three times greater than that of the other lamp. Conversely, knowing the ratio of distances at which two lamps give an equal illumination of the screen, the ratio of their candle-power may be calculated from formula (A).

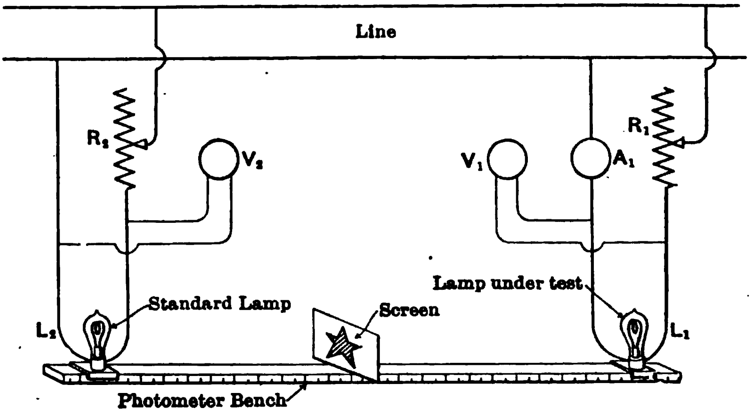

FIG. 192. Arrangement of parts in a photometer.

The photometer screen, introduced in its simplest form by Bunsen, consists of a sheet of white paper, the middle part of which is made transparent by means of paraffine or stearine. The transparent spot is usually given the form of a star (Fig. 192), or a circle (Fig. 195). Such a " grease-spot," when viewed by transmitted light, appears bright against a dark background, as shown in Fig. 195 to the right. On the contrary, when looked at by reflected light, the spot appears

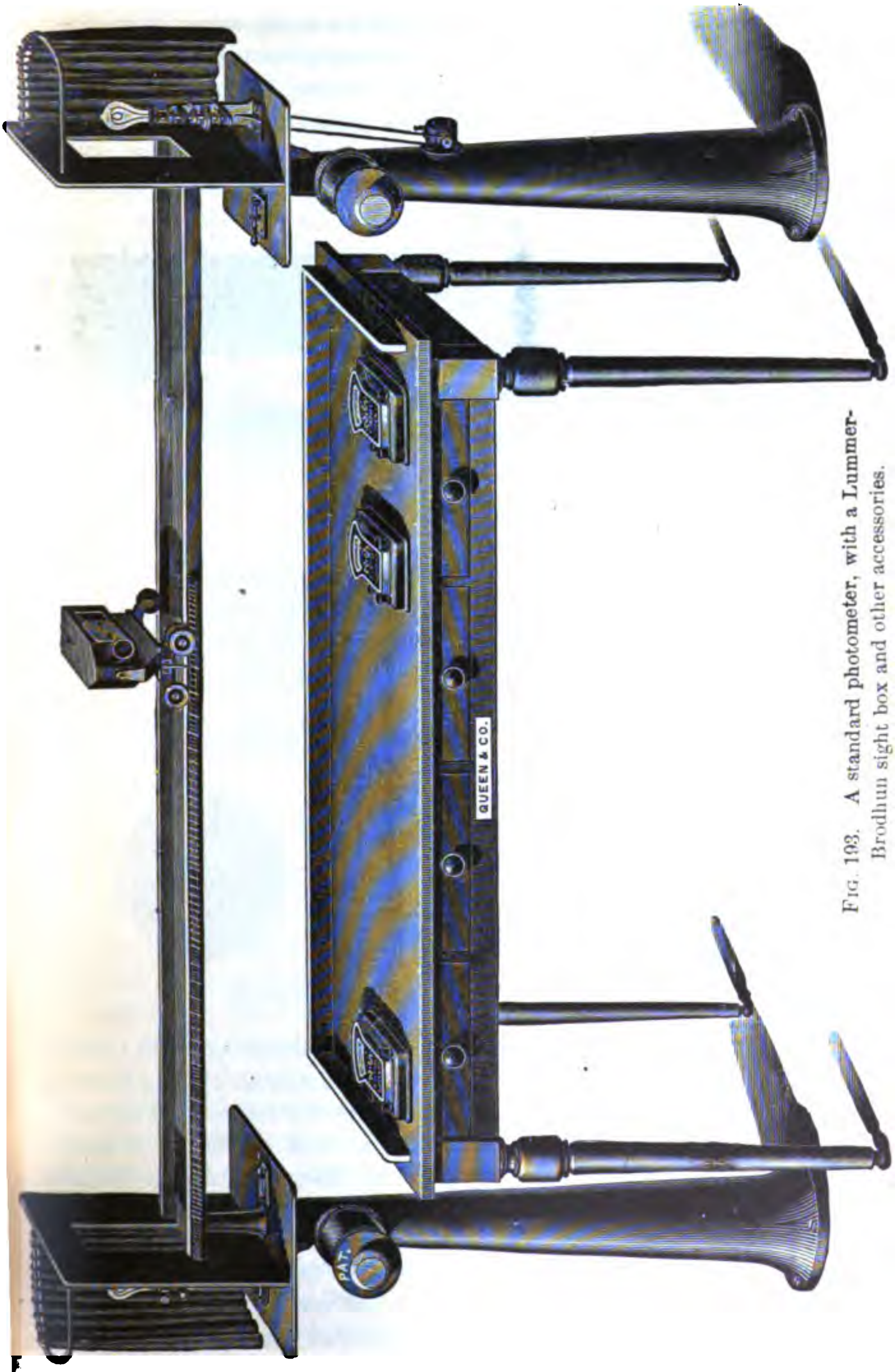

FIG. 193. A standard photometer, with a Lummer-Brodhun sight box and other accessories.

darker than the rest of the paper, as is shown to the left. In a photometer the screen is illuminated simultaneously from both sides; the side illuminated stronger appears as shown to the left; the side which receives less light looks as is shown to the right. When illumination is the same on both sides, the grease-spot should disappear, as shown by the middle circle in Fig. 195. In reality, it does not disappear altogether, but a point is found where it is least visible, or where the contrast between the spot and the rest of the screen is the same on both sides. This position is that of an equal illumination; here the distances from both lamps are measured, and the luminous intensity of the lamp under test is calculated from formula (A).

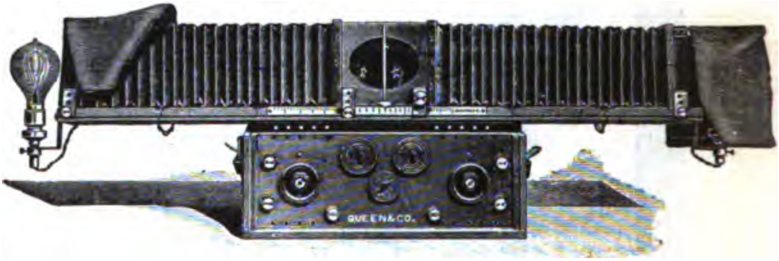

FIG. 194. A portable photometer.

206. Construction of a Photometer. — The screen described above is placed in a "sight-box" (Fig. 196) which excludes all light, except that coming directly from the lamps under comparison. The box has openings for the rays of light coming from the two lamps, and an opening in front through which the observer may look. The box is shown with part of its walls removed, in order to illustrate its construction. It is provided with two mirrors, so that the observer sees both sides of the screen simultaneously. This makes the adjustment much quicker and more accurate, than if he had to look at the sides of the screen in succession. The sight-box is mounted on a rolling carriage (Fig. 193) or simply slides along the photometer bench (Fig. 194).

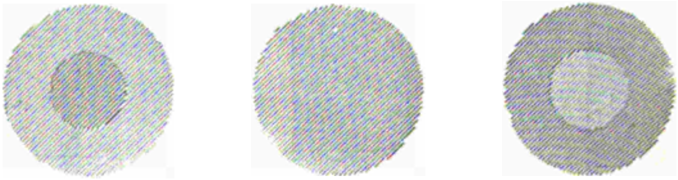

FIG. 195. Images visible on a photometer screen.

Of recent years, the so-called Leeson disk is coming into use, in place of the original Bunsen grease-spot. A star is cut out of a rather heavy paper, and is pasted between two sheets of thin paper. When viewed by transmitted light, the star naturally appears darker than the rest of the paper. When viewed by reflected light, the star looks lighter than the background, because it reflects more light. The setting is the same as with the original Bunsen photometer. The sight-box shown in Fig. 196 is provided with a Leeson disk; in the position illustrated, the screen is too near the lamp, to the left, making the action of that lamp preponderant. The sight-box should be moved to the right, until the two images in the mirror become identical, though the star does not disappear altogether with any setting.

The necessary electrical connections are shown in Fig. 192. Each lamp is provided with a rheostat and a voltmeter. The latter must be read very carefully, as the candle-power of the lamp depends essentially on the voltage at its terminals. An ammeter is provided in the circuit of the lamp under test, in order to be able to measure its power consumption. The rheostats are shown in Fig. 193 mounted on the columns which support the photometer bench; the measuring instruments are placed on the table.

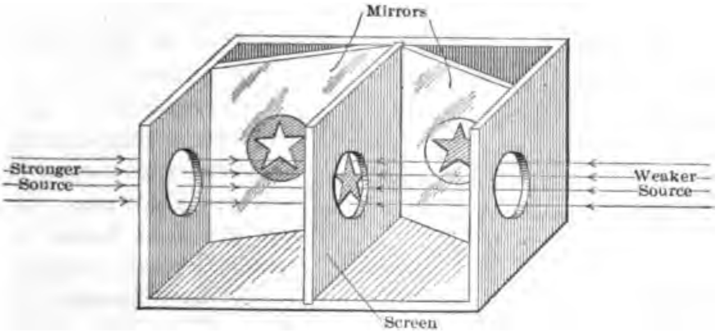

FIG. 196. A Bunsen sight-box.

In testing an incandescent lamp, it is usually rotated about a vertical axis, because the amount of light given in different vertical planes is different. The result thus obtained is called the *mean horizontal candle-power;* it is generally accepted in practice as the *rating* of an incandescent lamp. The lamp may be rotated either by hand or by an electric motor; a rotator is shown in Fig. 193 to the right; a universal rotator is illustrated separately in Fig. 197. The lamp should be spun at a speed of about 180 r.p.m.: at lower speeds, flickering is noticeable on the screen; with higher speeds, the filament is deformed by centrifugal force and may break.

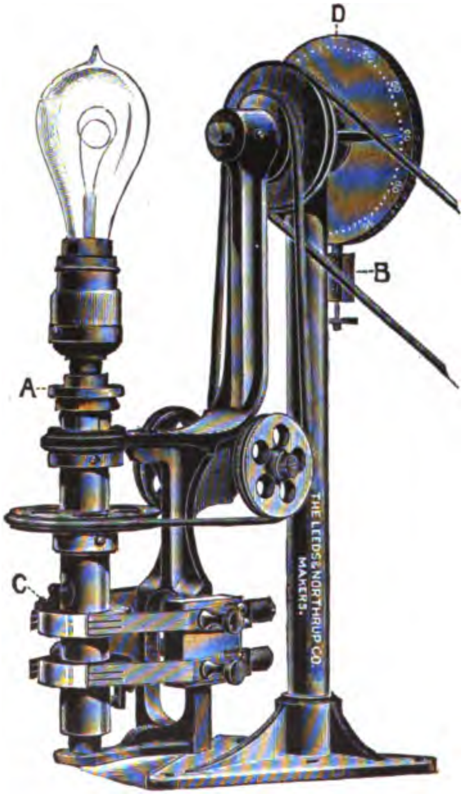

FIG. 197. The Willyoung universal rotator.

207. Standard Lamps. — The practical unit of luminous intensity in this country is the standard candle. Whatever its original luminous value, and the specifications for preparing such a standard candle, at present it has become merely a fictitious unit; strange to say, it is defined as 100/88 of the German unit. The German unit of candle power, or the "hefner," is represented by a special lamp which burns a liquid called "amyl acetate." The dimensions of the lamp, the height of the flame, etc., are exactly specified, and the standard is reproducible with a considerable degree of accuracy. The name "hefner" was given in honor of the noted German electrician V. Hefner-Alteneck who developed the amyl-acetate lamp.

The amyl-acetate lamp is used at present as the primary standard only, for calibrating secondary standards. As such, well-seasoned incandescent lamps are used most exclusively. An incandescent lamp, selected to be standardized, is first seasoned by burning — say 100 hours, so that its candle-power becomes constant. Then it is compared on a photometer (as in Fig. 193) with an amyl-acetate lamp, or with another incandescent lamp previously calibrated by means of it. The result of standardization is given in a form like this: the lamp gives at 106.7 volts 16 candle-power in the horizontal plane, and in a direction perpendicular to the plane of the filament; the current consumption is 0.523 ampere. The lamp must be used as a standard under these conditions only; it may be relied upon as long as the current remains the same at the stated voltage. When working for a long time, it is advisable to standardize temporarily other lamps, and to check them from time to time against the first standard. It is customary also to have standard lamps in sets of three and to check them from time to time against each other.

One of the inaccuracies in measuring candle-power is caused by differences in color of the light given by the standard lamp and the lamp under test. The more pronounced the difference in color, the more difficult it becomes to find the right position of the screen, such that the contrast is the same on both sides. It is advisable, therefore, to have several sets of standard lamps, each set calibrated at a different degree of incandescence. In testing a lamp, a standard lamp is selected, the color of which is the nearest to that of the lamp under test.

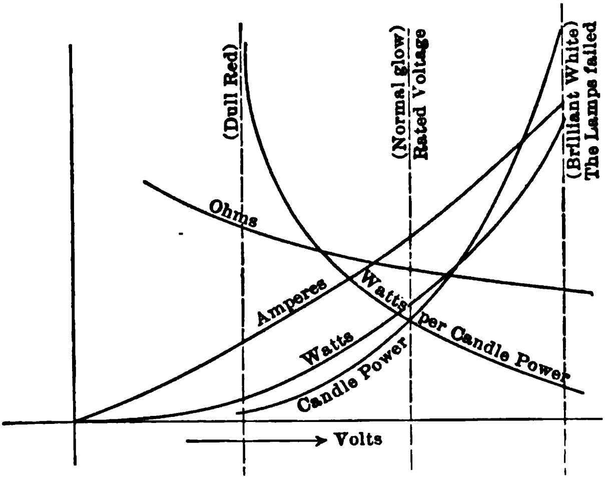

FIG. 198. Electric and photometric characteristics of an incandescent lamp, with varying voltage.

208. EXPERIMENT 10-A. — Mean Horizontal Candle-Power and Efficiency of an Incandescent Lamp, with Varying voltage. — The apparatus is arranged as in Figs. 192 and 193. The standard lamp is used during the test at its rated voltage; the voltage of the lamp under test is varied in steps, from the point where the lamp just begins to glow, up to the voltage at which it burns out (Fig. 198). At each step the position of photometric balance of the sight-box is read, also volts and amperes. In setting the sight-box, do not move it back and forth too long, trying to find the most accurate position of balance. This tires the eyes, and the accuracy of setting is impaired rather than increased. Two photometer readings must be taken with each voltage, the screen being reversed for the second reading. This is necessary because the two sides of the screen may not be identical. The average of the two settings is taken as the true reading. The lamp should be rotated in vertical position during the test (Fig. 197), at a speed of about 180 r.p.m.

All outside light must, of course, be excluded, and the window-shades drawn. A small lamp operated with a push-button is convenient for reading the scale. Up to a very recent time, it has been considered necessary to have the walls of a photometer room painted black, to keep down stray reflected light. Mr. E. P. Hyde * showed conclusively, however, that this precaution is not necessary, if vertical screens of black velvet are placed on the photometer bench, in such positions as to make it impossible for reflected light to get into the sight-box.

When reading the ammeters, have the voltmeter circuit open, so as not to read the current flowing through it; otherwise a correction must be made, as in § 2. When reading volts, mark also the quality of light of the lamp under test: dull red, yellow, white, etc.

Report. Plot to terminal volts as abscissæ, the curves shown in Fig. 198. Candle-power is figured out from sight-box settings, by means of the formula (A). Let, for instance, the bench be 300 cm. long between the centers of the lamps, and let the balance be obtained at the division 120, counting from the standard 16 candle-power lamp. The distance from the lamp under test is (300–120) cm.; as the candle-power is inversely proportional to the square of the distances, we have for the candle-power of the lamp under test:

$$16\left(\frac{300-120}{120}\right)^2 = 16 \times 2.25 = 36.$$

Tables are available which give the ratios of distances squared for various settings of the screen, so that the calculation is much simplified. Some photometers have a scale calibrated in candle-power (Fig. 194); the scale is direct-reading if a 16 candle-power standard lamp is used for comparison; otherwise the reading must be correspondingly increased or reduced.

Plot first the curves of candle-power and amperes; amperes multiplied by volts will give the watt-curve. After this the curve giving watts per candle-power may be plotted. Ohms are calculated as a ratio of volts to amperes. State in how far the difference in color of the two lamps affected the accuracy of photometer readings.

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* See *Bulletins of Bureau of Standards*, Vol. 1, No. 3, p. 417.

209. Life and Efficiency of Incandescent Lamps.—An inspection of the curves in Fig. 198 shows that the efficiency of an incandescent lamp increases with the voltage at its terminals; in other words, watts input per candle-power decreases. It may be stated here, that the latter expression is called *the specific consumption of the lamp;* some prefer to use its reciprocal (candle-power per watt) which is called the *efficiency of the lamp.* The use of the term "efficiency" is to be considered as special in this case, and should not be confused with per cent efficiency, as ordinarily applied to machinery.

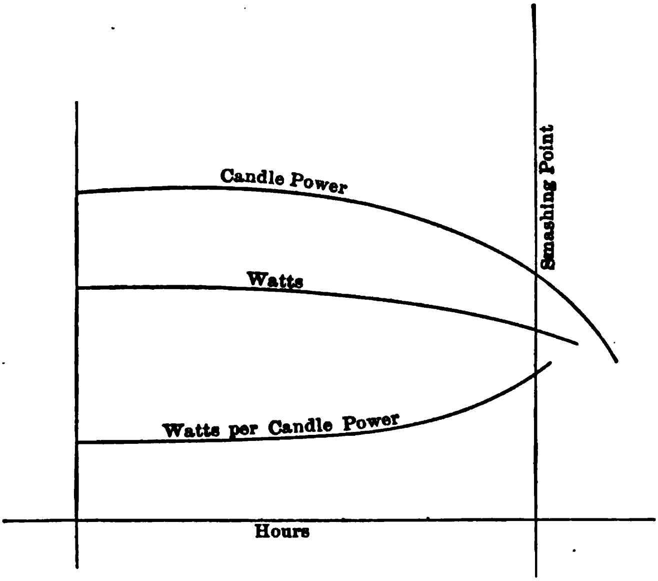

FIG. 199. Decrease of candle-power of an incandescent lamp with hours of service.

It is desirable to use lamps at as high an efficiency as possible, provided their life is not thereby shortened beyond a certain limit. For each class of incandescent lamps, there is a limit, above which an increase in efficiency is more than outweighed by a shortened life, causing an additional expense in renewals. The *useful* life of an ordinary carbon-filament lamp is considerably shorter than its *total* life (until the filament is burned out). This is illustrated in Fig. 199. After a certain number of hours of burning, the candle-power of the lamp begins to drop, chiefly on account of the bulb becoming blackened on the inside. The filament also becomes thinner, its resistance increases, and the power taken by the lamp decreases. But the decrease in candle-power is much more pronounced than the decrease in power consumption; the result is, that the specific consumption, or watts per candle-power, is gradually increasing. At a certain point, say where the candle-power is reduced to about 80 per cent of the original, it is usually cheaper to discard the lamp and to buy a new one, than to pay for an increased specific consumption. This point is called the "smashing" point of the lamp. Methods have been developed for replacing filaments, so that lamps are no longer "smashed" but sold to factories making a specialty of renewing them. To renew the filament, the bulb is opened at the tip and the old filament taken out; then the bulb is washed out, and a new filament inserted in place and soldered to the leads. The air is again exhausted, and the tip sealed.

Ordinary carbon-filament lamps may be had with a specific consumption of from 3 to 4 watts per mean horizontal candle-power. The value of the specific consumption is selected according to the purpose for which the lamp is to be used, and with regard to the cost of power. Low-efficiency lamps have a longer life, are less sensitive to voltage fluctuations, but consume more power, and give a yellowish light. High-efficiency lamps require a very good voltage regulation, or their life is much shortened; they are more economical, and give a light more closely resembling daylight. The *tungsten* lamp, which seems to be the incandescent lamp of the future, not only has a much lower power consumption than the carbon-filament lamp, but it preserves its candle-power better until the filament is burned out. In addition to this, the tungsten lamp is less sensitive to voltage fluctuations, because the resistance of tungsten increases with temperature.

Lamps are tested and rated in factories according to their voltage, candle-power, and specific consumption. It is not sufficient to determine that a lamp gives 16 candle-power at 110 volts; its specific consumption per candle-power must be near a desired figure, say 3.5 watts; if it is different, the lamp is allotted to another class. For such measurements, involving a simultaneous determination of candle-power and of specific consumption, Messrs. Hyde and Brooks have developed an ingenious "Efficiency Meter for Incandescent Lamps." The arrangement consists in the combination of an ordinary photometer and an indicating wattmeter. A variable resistance is connected into the potential circuit of the wattmeter, and is regulated automatically by the movement of the sight-box carriage. The adjustment is such, that with a standard 16 candle-power lamp, the wattmeter reads directly the specific consumption of the lamp under test when photometric balance is obtained. At the same time the candle-power of the lamp is read on the photometer. The device will be of convenience in places where a large number of lamps are regularly tested, rated and assorted. For details of this apparatus see *Bulletins of the Bureau of Standards*, Vol. 2, No. 1, p. 45.

210. EXPERIMENT 10-B. —Influence of the Blackening of the Bulb on Candle-Power and Efficiency of Incandescent Lamps.— The useful candle-power of incandescent lamps decreases with time, due to the bulb being gradually blackened on the inside by a deposit of the material of the filament. It would not be expedient to test in the laboratory lamps blackened during the regular service, as it takes several hundred hours to produce an appreciable effect. To show the student the blackening effect, it may be obtained in a few minutes by subjecting the lamp to an over-potential. Take a few new lamps and determine their candle-power at the rated voltage, as in § 209. Then subject each of them to various voltages above the normal and for different periods of time. When the current is turned off, the blackening is noticeable by holding a sheet of white paper behind the lamp. Get different degrees of blackening, and test lamps again for their candle-power and power consumption. This will give an idea of what takes place in a prolonged regular service at the normal voltage.

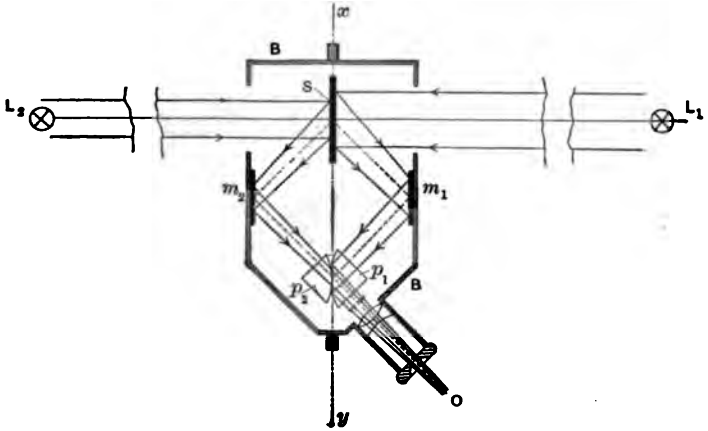

FIG. 200. The path of rays in a Lummer-Brodhun sight-box.

211. The Lummer-Brodhun Sight-Box. —This sight-box is shown on the carriage in Fig. 193; a horizontal cross-section of the same is shown in Fig. 200. It was expected by the inventors that a greater accuracy could be obtained with a purely optical combination than with a grease-spot. Many practical photometricians claim, however, that nearly as good results, at least for ordinary purposes, are obtained with an ordinary Bunsen sight-box, especially if provided with a Leeson disk. At the same time, the Bunsen sight-box is considerably less expensive, and does not so much tire the observer's eyes, because both eyes are used simultaneously. As, however, the Lummer-Brodhun sight-box is used to a considerable extent for photometrical work of precision, and when the colors of the two lamps under comparison are different, a description of this sight-box seems not to be out of place here.

Two lamps under comparison, L_1 and L_2 (Fig. 200), illuminate an opaque diffusing screen S, whose coefficient of absorption is as low as possible. The light diffused from the two sides of this screen is reflected by the mirrors m_1 and m_2 and reaches the observer's eye at O through the prisms p_1 and p_2. In the better grade of instruments, totally reflecting prisms are used in place of the mirrors m_1 and m_2. The prisms p_1 and p_2 are made of such a shape, that the observer sees the figure shown in Fig. 195. The inner circle corresponds to the light transmitted from the left side of the screen S; the outside circle is caused by the illumination of the right side of S, the rays undergoing a total reflection from the hypothenuse surface of p_1. The sight-box BB is moved along the bench until the difference in illumination disappears, as in the middle circle, Fig. 195; or at least is reduced to a minimum. To take into account a possible inequality in the diffusing power of the two sides of the screen S, the box may be turned by 180 degrees about the axis x-y, and a second reading taken.

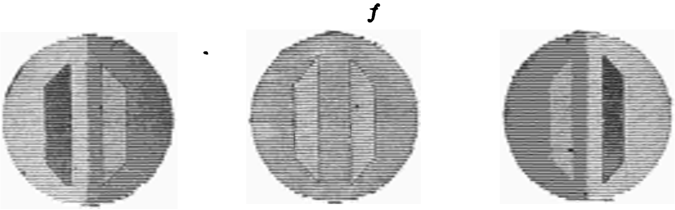

FIG. 201. Images visible in a Lummer-Brodhun sight-box, with a contrast attachment.

The accuracy of setting is increased by using the so-called *composite sight field* (Fig. 201). The trapezoids visible in the field are caused by corresponding figures being cut out on the hypothenuse surface of the prism p_2. The left semicircle and the right trapezoid show the illumination of the left side of the screen S; the rest of the field is illuminated by the light diffused from the right side of the same screen. The sight-box is moved along the bench until the field becomes as nearly as possible uniform.

When the lights to be compared are of different color, for instance, an incandescent lamp and a Nernst lamp, the figures in the sight field do not disappear with any setting. In such cases, the difference is artificially increased, by applying the so-called *contrast principle.* This is done by interposing thin strips of glass so as to slightly darken one-half of the beam of light from each source. The strips are placed on the prisms p_1 and p_2, on the sides exposed to the light. Under such conditions, the composite field appears as is shown in Fig. 201, all four parts being of different shade. When the illumination is balanced, the field appears as in the middle figure; the trapezoids do not disappear, but the *contrast* between them and the background is the same on both sides. The dividing line usually does not disappear altogether; in setting the screen, the observer should try to obtain an equal contrast on both sides, and not pay any attention to the dividing line. Lights of different color can be compared more accurately by this method than with a uniform sight field.

212. Flicker Photometer.—The contrast principle described above offers but a partial remedy in cases where there is a considerable difference in color between two lights to be compared. Thus most people would not be at all able to compare the greenish light of a mercury vapor lamp with a yellowish incandescent standard. Professor O. N. Rood was the first to point out that the accuracy of photometric comparison is much increased by examining the two surfaces of the screen in rapid succession, instead of simultaneously, as in the photometers described above. Photometers built on this principle are called *flicker* photometers; the principle may be understood with reference to Fig. 202. Let m_1 and m_2 be two identical mirrors mounted on the shaft s and rotated by the handle h. The mirror m_1 is mounted so as to give the observer the image of the lamp L_1; the mirror m_2 (when in the dotted position) gives the image of the lamp L_2. When viewing the revolving mirrors through a translucent screen, the observer receives alternate impressions from both lamps. When the speed of rotation is very low, he sees intermittent light; at very high speed the light appears continuous, because of the so-called persistence of vision. Between these two limits, there is a range of speed at which the light appears to flicker, without being completely extinguished. When the mirrors are rotated at this speed, the flickering is more pronounced, the greater the difference in the luminous intensities of the lamps under comparison. As the intensities become nearer each other, the flickering sensation gradually disappears. Experiment shows, that the effect is to a large extent independent of the difference in color between the two lights.*

Various flicker photometers have been constructed on this principle: one of the latest is that by Franz Schmidt and Haensch (*Zeitschrift für Instrumentenkunde,* 1906, p. 249). The optical system in this photometer is such that when the revolving part is at rest, the field of vision appears as shown to the left in Fig. 195. Turning one of the prisms of the system by 180 degrees changes the field of vision to that shown to the right; the lamp which gave the inside circle is now made to illuminate the outside circle. When the prism is revolved at the critical speed of flickering, the circles exchange their shades constantly, and a pronounced phenomenon of flickering is produced. The sight-box is moved until flickering disappears, and the whole field looks uniform, as in the central figure. It is important to have the right speed of rotation, particularly with lights of markedly different colors. The speed must be such, that when the setting is approximately correct, the field of vision appears in a uniform mixture of color of the two sources, and flickering is caused only by differences in the intensity of illumination. Then the final setting is made by reducing this flickering to a minimum.

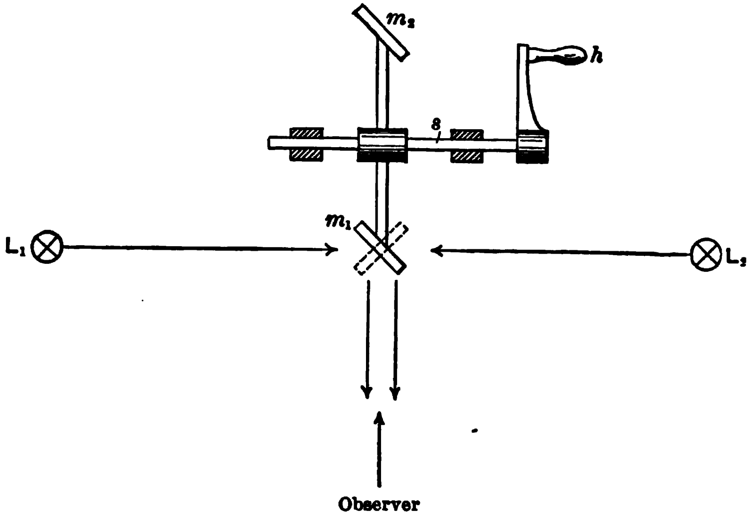

FIG. 202. Essential parts of the flicker photometer.

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* In actual flicker photometers, diffusing screens are used in place of mirrors.

It is claimed that with a good flicker photometer, lights widely different in color may be compared with nearly the same accuracy that lamps of equal color are compared with ordinary photometers.

213. EXPERIMENT 10-C. — Photometric Comparison of Lights of Different Color. — There is difficulty in comparing two lights of different color with an ordinary Bunsen photometer screen, since the screen does not appear equally illuminated on both sides in any position of the sight-box, — with the result that different observers are apt to give entirely different settings. Better results are obtained with the contrast photometer (§ 211), and still better with the flicker photometer (§ 212). The purpose of this experiment is to show the student the accuracy obtainable in various cases.

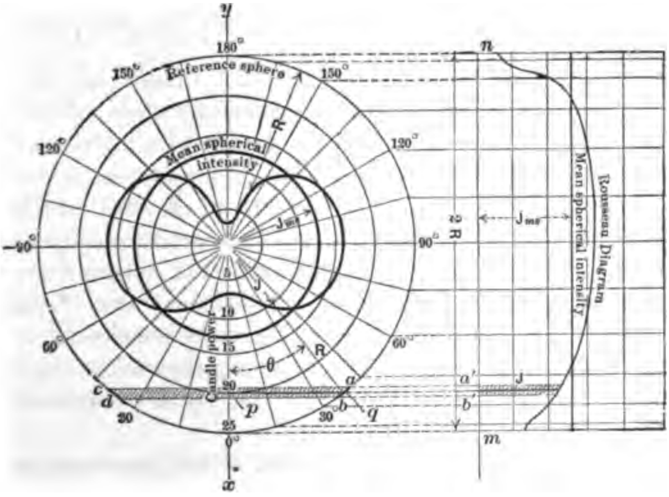

FIG. 208. Photometric curve of an incandescent lamp and the corresponding Rousseau diagram.

Take two incandescent lamps, bring them up to such voltages as to get nearly the same color of light; compare the lamps by means of an ordinary Bunsen screen, and determine per cent accuracy of setting. Now take two incandescent lamps having a markedly different color (different degree of incandescence) and determine the accuracy with which they can be compared with the same photometer. After this compare lamps still more different, — for instance, a Nernst lamp and an incandescent lamp. If desired, a greater difference in color may be obtained by taking in succession an arc lamp, a mercury vapor lamp, etc., and comparing them with an incandescent lamp. In each case pay particular attention to the accuracy with which the setting can be made. Now repeat the same tests with a Lummer-Brodhun contrast photometer, and finally with a flicker photometer. You will find that considerably more accurate settings are possible with these devices.

DISTRIBUTION OF LIGHT AND MEAN SPHERICAL INTENSITY.

214. An incandescent lamp gives different illumination in different directions. Differences are particularly pronounced in the vertical plane (Fig. 203). The radii of the oval curve give luminous intensities in the corresponding directions, the horizontal radius being equal to 16 candle-power. The form of the curve depends essentially on the form of the filament of the lamp.

Such a curve of distribution of light around a lamp may be obtained with an ordinary photometer, by having the lamp mounted on a universal rotator, such as shown in Fig. 197. The rotator is driven by a belt from an electric motor. The axis of rotation may be set at any desired angle with the vertical, in steps of 5 degrees. In this way the mean candle-power may be determined in any desired direction; the results plotted as radii give the curve shown in Fig. 203. Angles are read on the dial *D*. A positive pin-clutch *B* catches the frame of the rotator at each desired angle. The rotator has an additional pair of brushes *C* for voltmeter leads; in this way voltage is measured at the terminals of the revolving lamp, independent of the contact resistance of the main brushes.

215. Mean Spherical Candle-Power.—An examination of the distribution curve in Fig. 203 shows that a lamp rated at 16 candle-power, in reality gives considerably less than 16 candles in directions other than horizontal. In many cases a lamp is intended to throw useful light downward; it will be seen that a lamp with a nominal rating of 16 candle-power gives only between 6 and 7 candle-power in the vertical direction (unless it is provided with a reflector). This fact is appreciated more and more by illuminating engineers, and a feeling is growing that lamps should be rated on the basis of their *mean spherical candle-power*, and not mean horizontal, or maximum candle-power. The mean spherical candle-power is indicated in Fig. 203 by a circle; it will be seen, that it is only 13 candle-power, instead of 16 candle-power.

The ratio of the mean spherical to the mean horizontal candle-power is called the *spherical reduction factor* of the lamp. In the case shown, the spherical reduction factor is equal to

$$\frac{13}{16} = 0.81.$$

Knowing the spherical reduction factor for a certain type of filament, the mean spherical candle-power can be calculated from the observed horizontal candle-power; this makes unnecessary taking a complete distribution curve.

Mean spherical candle-power may be determined in two ways:

(1) By calculation, from the distribution curve.
(2) By means of integrating photometers.

These two methods will now be considered more in detail.

216. Rousseau Diagram. —The mean spherical candle-power is calculated in the following way from the distribution curve shown in Fig. 203. Let J be the luminous intensity or the candle-power of the lamp in a certain direction. Imagine a sphere of radius R around the lamp and let ds be an infinitesimal element of the surface of the sphere. We then have

$$4\pi R^2 . J_{ms} = \int J . ds \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where $4\pi R^2$ is the surface of the "reference sphere," and J_{ms} is the unknown mean spherical candle-power of the lamp. The equation (1) expresses the condition that the actual "flux of light" is the same as if the intensity were uniform in all directions and were equal to J_{ms}. When the distribution curve is obtained with a rotator, each intensity J at an angle θ is the same for an infinitesimal zone of the reference sphere. The area of the zone is equal to its circumference times the width; thus

$$ds = 2\pi R \operatorname{Sin} \theta . R\, d\theta.$$

Substituting in (1) we get

$$4\pi R^2 . J_{ms} = \int_0^\pi J . 2\pi\, R \operatorname{Sin} \theta . R\, d\theta,$$

or

$$J_{ms} = \tfrac{1}{2} \int_0^\pi J \operatorname{Sin} \theta . d\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The mean spherical intensity J_{ms} may be calculated from this expression by dividing the distribution curve into small parts, say corresponding to 5 degrees, and summing up elementary products $J . \operatorname{Sin} \theta$. Let 180 degrees be divided into n parts; $d\theta$ must be replaced by a finite increment $\Delta\theta = \pi \div n$, and summation used for integration. We obtain:

$$J_{ms} = \frac{\pi}{2n} \sum_0^n J \operatorname{Sin} \theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The expression (2) may also be integrated by means of a planimeter, if the distribution curve be replotted in rectangular coördinates, as shown in Fig. 203, to the right; this is the so-called Rousseau diagram. A vector, such as J, is produced to its intersection with the reference sphere; the point thus obtained is projected on the vertical axis of abscissæ of the Rousseau diagram, and the same J plotted as the ordinate. The mean ordinate of the Rousseau diagram gives directly the mean spherical intensity of the lamp. This can be proved as follows: The abscissæ of the new curve, counted from the middle point (latitude of 90 degrees), are equal to R . Cos θ; the ordinates represent the values of J. Thus, the area of the curve is

$$\int_0^{\pi} J \,.\, d\,(R \operatorname{Cos} \theta) = -\, R \int_0^{\pi} J \operatorname{Sin} \theta \,.\, d\theta.$$

This expression, save for the factor $2R$, is the same as formula (2). Thus, in order to find the mean spherical intensity, the area of the Rousseau diagram must be divided by its length $2R$. The process is evidently the same as in finding the mean ordinate of a curve; this proves, that mean spherical intensity is represented by the mean ordinate of Rousseau diagram.

217. EXPERIMENT 10-D. —Mean Spherical Intensity of an Incandescent Lamp from its Curve of Distribution of Light.— The lamp to be tested is mounted on a universal rotator (Fig. 197) and compared to a standard lamp by means of the photometer shown in Fig. 193. At first the lamp is rotated about a vertical axis; then the position of the axis is changed every five or ten degrees throughout the range of 180 degrees. After this, the rotator must be stopped, the lamp brought in its original position, and the luminous intensity determined in various meridians. As such, select the plane of the filament, the plane perpendicular to it, and a few intermediate vertical planes.

The same lamp should be tested with the bulb frosted. Frosting is done by dipping the lamp while warm into the so-called frosting compound (can be obtained from lamp-dealers). This gives only a temporary frosting, but is sufficient for the purpose; the compound can be washed off with alcohol. Permanent frosting is obtained by sand-blast or by dipping the bulb in hydro-fluoric acid. The same experiment may be repeated with a lamp having a decidedly different form of filament; also with tantalum, tungsten and Nernst lamps.

Report. Plot curves of distribution of mean luminous intensity in the vertical plane, as in Fig. 203; also curves of distribution of candle-power in the horizontal plane. Figure out the mean spherical intensity for one of the lamps by means of the Rousseau diagram; for another lamp by using the formula (3). Calculate the corresponding values of spherical reduction factor. Discuss differences in the distribution of light, due to differences in the form of filaments.

INTEGRATING PHOTOMETERS.

218. The above procedure for determining the mean spherical candle-power, by taking a curve of distribution of light and integrating it, has an obvious disadvantage of being rather tedious for practical purposes. Considerable effort has been devoted, therefore, of recent years, to the development of "integrating photometers," by means of which the mean spherical intenrity of a lamp is determined in one reading. Of the various types of such photometers, two are here described, which are particularly suitable for testing incandescent lamps. Some other types are described in § 239 in connection with photometering arc lamps.

219. Matthews' Integrating Photometer. — The photometer is shown schematically in Fig. 204, in elevation and in plan. The rays from the lamp L_1 under test are reflected by means of two sets of mirrors *mm* on an ordinary Bunsen or Lummer-Brodhun photometer screen. The screen is illuminated on the other side by the standard lamp L_2. The mirrors being placed along a semicircle, the screen is illuminated simultaneously by the rays of the lamp L_1 coming from different directions. The photometer screen is stationary; the standard lamp is moved along the bench until a photometric balance is obtained. The lamp under test is rotated on its vertical axis as in § 206. The setting gives directly the mean spherical intensity of the lamp under test, according to the formula

$$J_{ms} = \text{Const.}\ \frac{J_2}{D^2},$$

where J_2 is the horizontal intensity of the standard lamp, and D the distance between the standard lamp and the screen. The constant is determined experimentally by using a lamp L_1 whose mean spherical intensity has been previously determined by integration (§ 217).

The following is the proof that the illumination on the left side of the photometer screen is proportional to the mean spherical intensity of the lamp L_1. According to equation (3) the mean spherical intensity consists of elementary products J. Sin θ, the mean illumination J of various zones being reduced, or "weighed" in proportion to the areas of the corresponding zones. This is exactly the reduction of illumination obtained with the disposition of mirrors in Fig. 204. The light from the mirror at a latitude of $(90° - \theta)$ is incident on the screen at an angle of $(90° - \theta)$. According to the so-called Lambert's law of optics, the illumination on a diffusing surface varies as the cosine of the angle of incidence. Therefore the illumination produced on the screen by the above mirror, is proportional to $J \cdot \text{Cos}\ (90° - \theta)$, or to $J \cdot \text{Sin}\ \theta$. Thus, the total amount of illumination from all the mirrors is proportional to $\sum J \cdot \text{Sin}\ \theta$, or, according to equation (3), is proportional to the mean spherical intensity of the lamp.

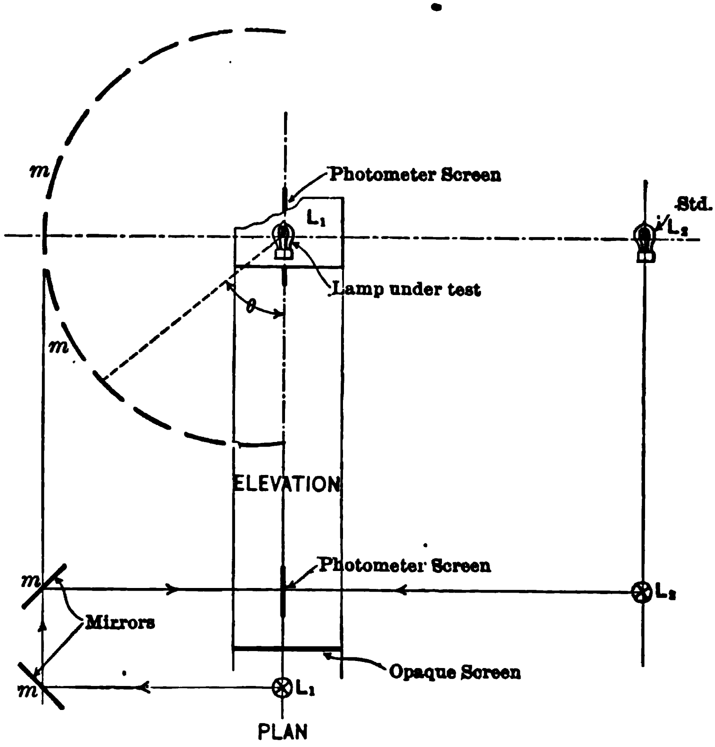

Fig. 204. The principle of the Matthews integrating photometer for incandescent lamps.

220. Ulbricht's Integrating Photometer. — The principle on which this photometer is built (Fig. 205) was discovered independently by Blondel in France and Ulbricht in Germany.* The photometer consists of a large opaque sphere painted inside in diffusing white. The lamp L_1 under test is placed somewhere within the sphere (not necessarily in the center); a small opening, covered with a translucent screen p_1 is provided in the sphere. This screen is illuminated by the diffusedly reflected light of the whole sphere, but is protected from the lamp by a small opaque screen q. The illumination of p is proportional to the mean spherical intensity of the lamp L_1, and is balanced against a standard lamp L_2, as in any ordinary photometer. The same formula is used which is given above for the Matthews' photometer. The constant of the instrument is determined experimentally by placing in the sphere a lamp whose mean spherical candle-power has been determined by integration, as in § 217.

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* *Bulletins de la Société Internationale des Electriciens,* 1904, p. 687. *Elektrotechnische Zeitschrift,* 1905, pp. 512 and 1047; 1906, p. 50.

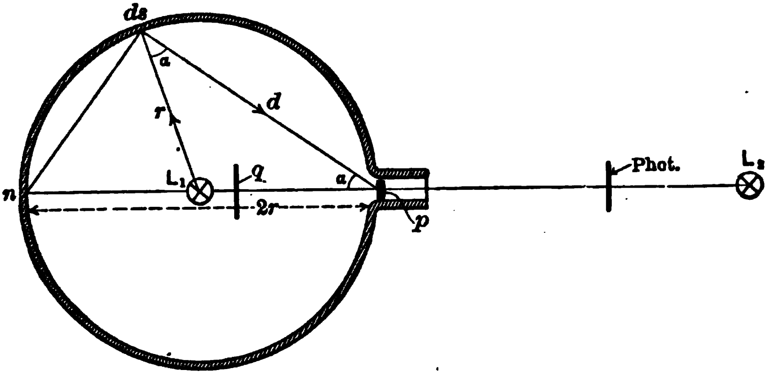

FIG. 205. General arrangement of the Ulbricht integrating photometer.

We give here a proof that the illumination of the screen p is proportional to the mean spherical intensity of the lamp L_1. Assume first, for the sake of simplicity, that L_1 is placed in the center of the sphere, and consider the illumination of an element ds of the surface of the sphere. Its normal illumination is proportional to J/r^2, where J is the candle-power of the lamp in the direction under consideration, and r is the radius of the sphere. The element ds illuminates the screen p; according to Lambert's law of diffused light, mentioned above, the quantity of light emitted towards p is $(J \div r^2) . ds . \mathrm{Cos}\,\alpha$. This light is incident on p at an angle α; hence the normal illumination of p is proportional to

$$\frac{J\,\mathrm{Cos}\,\alpha\,.\,ds}{r} \cdot \frac{\mathrm{Cos}\,\alpha}{l^2} \text{ or proportional to } \frac{J\,ds}{4\,r^4},$$

because $l = 2\,r\,.\,\mathrm{Cos}\,\alpha$, from the triangle $pn\,ds$. The total normal illumination of p caused by the whole sphere is thus proportional to

$$\frac{1}{4\,r^4}\int_0^{\pi^4} J\,ds = \text{Const.} \times \text{mean spherical intensity;}$$

see equation (1) in § 216. This proves that the illumination of the screen p is proportional to the mean spherical intensity of the lamp L_1.

The same is true, when the lamp L_1 is not in the center of the sphere. Let j be the intensity of *normal* illumination of the element ds. The normal illumination produced by this element on the screen p is, as before, proportional to

$$j\,.\,\mathrm{Cos}\,\alpha\,.\,ds\,\frac{\mathrm{Cos}\,\alpha}{l^2} = \frac{j\,ds}{4\,r^4}$$

so that the total illumination of p is proportional to

$$\int j\,.\,ds.$$

This expression is evidently proportional to the total light emitted by the lamp L_1, because this lamp is the only source of illumination of the sphere. The integral does not depend on the position of L_1. The result is not changed by the fact that part of the light is absorbed by the walls, or is reflected several times before it reaches p. This merely changes the constant of the instrument, which must be determined experimentally.

221. Mean Hemispherical Intensity. — In some cases the only useful light of a lamp is that thrown below the horizontal plane passing through its center; this is true, for instance, in street lighting. Lamps used for such purposes should be rated on the basis of their mean *hemispherical* intensity, taking into account only the light emitted below the horizontal plane. Denoting this intensity by J_{mhs} we obtain, in a way similar to that in which formula (2) is deduced,

$$J_{mhs} = \int_0^{\frac{\pi}{2}} J\,\mathrm{Sin}\,\theta\,.\,d\theta;$$

Also the expression

$$J_{mhs} = \frac{\pi}{2\,m}\sum_0^n J\sin\theta,$$

corresponding to formula (3); m is the number of parts in which 90 degrees is subdivided.

Mean hemispherical intensity may be calculated either by using this last formula, or the lower half of the Rousseau diagram (Fig. 203). It can also be measured directly by either of the integrating photometers described above. If Matthews' photometer is used, the mirrors in the upper quadrant are covered or removed. In using the Ulbricht photometer, a horizontal segment of the sphere is removed, or painted black to make it inefficient; the lamp is placed with its center in the horizontal plane separating the upper segment from the rest of the sphere. The illumination of the diffusing part of the sphere is then produced only by the light given out below this plane; therefore, the amount of light received by the screen *p* is proportional to the mean hemispherical intensity of the lamp.

222. EXPERIMENT 10-E. — Determination of Mean Spherical Intensity of Incandescent Lamps with an Integrating Photometer. — Either the Matthews' or the Ulbricht photometer may be used (§§ 219 and 220). If possible, take measurements with the same lamps that were tested in experiment 10-D (§ 217). Calibrate the photometer with one of these lamps, and with the constant thus obtained check the mean spherical intensity of the other lamps. If Matthews' photometer is used, check the curve of distribution of luminous intensity of one of the lamps, by using two mirrors at a time, all other mirrors being covered with black cloth. Devise a method for checking the angular position of the mirrors, and for determining their coefficient of absorption. If an Ulbrecht photometer is used, investigate in how far the theoretical conclusion is true, — that the illumination of the screen is independent of the position of the lamp. For one of the lamps, determine the mean hemispherical intensity, and check it with that calculated from the Rousseau diagram.

CHAPTER XI.

ARC LAMPS.

223. THE physical phenomenon of an electric arc between two carbons is so commonly known, that it seems hardly necessary to go into a detailed explanation of it.

Carbons are shown schematically in Fig. 206; the sketch to the left represents a direct-current arc; that to the right an alternating-current arc. In the direct-current arc the upper (positive) carbon forms a "crater," from which light is thrown down, in the direction where it is mostly needed. With the alternating-current arc no such crater is formed, and the light is distributed fairly equally in all directions, a large amount of it being thereby wasted. Moreover, a direct-current arc gives, with the same watts input, considerably more light than an alternating-current one. This makes the direct-current arc lamp superior to the alternating-current lamp, the more so since the latter makes an unpleasant humming noise, which may often be objectionable, especially in indoor use.

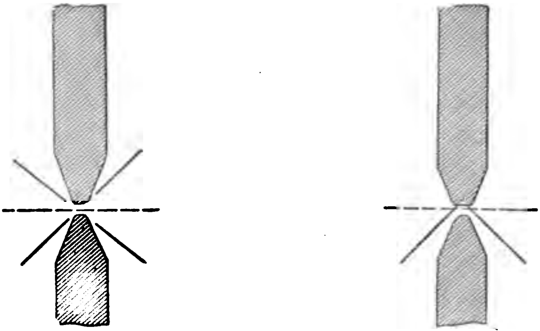

FIG. 206. A direct-current arc, with crater, to the left. An alternating-current arc, without a crater, to the right.

Nevertheless, the advantages of *energy distribution* by means of alternating currents are so great that alternating arcs are coming more and more into use. This is especially true in application to street lighting; while it is an easy matter to produce voltages up to, say, 10,000 volts by means of alternators and step-up transformers, continuous-current machines for such voltages have serious drawbacks because of commutator troubles, and are gradually going out of use. Thus, it must be understood that while direct-current arcs are themselves preferable, alternating-current arcs afford better current distribution over considerable areas.

224. Types of Arc Lamps.—Arc lamps used at present can be subdivided into direct-current lamps and alternating-current lamps; also into lamps intended for being connected in series, and those to be used in parallel. This gives four commercial types of lamps: series direct-current, multiple direct-current, series alternating-current, and multiple alternating-current. Multiple lamps are used for indoor lighting, and in other cases where lamps are connected to a low-tension network. Series lamps are used for street lighting, where as many as 100 lamps can be connected in series on a special high-tension circuit, with a very small expense of line copper.

Practically all arc lamps in use at present have so-called inclosed arcs, — the carbons are surrounded by a small globe which practically excludes the air. The advantage of such an arrangement is that the rate of consumption of the carbons is much lower. While an open arc lamp requires carbons to be renewed, say, every 10 hours, or even more frequently, inclosed lamps will burn as long as 150 hours or longer. This means a considerable saving in trimming service and carbons. The light of an inclosed arc lamp is much softer and steadier; the fire risk in indoor use is less, because the small globe surrounding the carbons prevents the falling of sparks.

The desire to increase the luminous efficiency of arc lamps led to the development of so-called "flaming" arcs. In these lamps the two carbons are placed vertically side by side at a small angle to each other, and a fan-shaped arc is formed downward; no inclosing (inner) globe is used. The carbons are impregnated with salts which give a soft yellow or reddish color and produce a flaming effect, thereby greatly increasing the efficiency of the lamp. Another new lamp is the "magnetite" lamp invented by Dr. Steinmetz. It is a direct-current arc in which magnetic oxide of iron, or magnetite (Fe_3O_4), is used as negative electrode, copper as positive; the arc is formed by the magnetite vapor. This lamp gives a brilliant white light, and its efficiency compares favorably with that of ordinary inclosed arcs. The flaming arcs and the magnetite lamp present marked advantages, although thus far they have been used to a limited extent only.

225. Regulating Mechanism of Multiple Lamps.—The arc must be adjusted to a definite voltage and current, in order to give a certain amount of light and to be steady and noiseless. Therefore, arc lamps are always provided with a regulating mechanism, which automatically feeds the carbons as they burn out, and so keeps the distance between them constant. *The mechanism of multiple lamps needs to regulate for constant current only,* because the voltage between the wires is kept constant from the power house. On the other hand, *the mechanism of a series lamp must regulate for constant voltage only,* since the current is kept constant by means of special regulating devices in the power house.

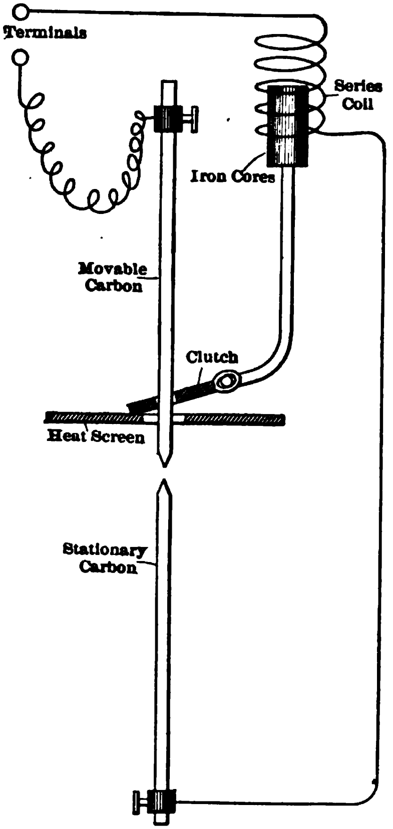

FIG. 207. Mechanism of a multiple arc lamp (series-coil control).

The regulation of *multiple lamps* (sometimes called constant-potential lamps, since they burn on constant-potential circuits) is usually brought about by an electromagnet or solenoid, connected directly in series with the arc (Fig. 207). This solenoid operates a clutch and feeds the upper carbon when required. The current from the lower terminal flows through the upper carbon, the lower carbon, then through the regulating coil and back to the upper terminal. The lower carbon is stationary, the upper is fed down by gravity when it is released by the clutch.

When current is off, the regulating coil does not support the upper carbon, and it rests on the lower carbon, keeping the arc circuit closed. The power being switched on, a current passes through the lamp and energizes the coil; the coil pulls up the upper carbon and starts the arc. The lamp is so regulated, that when the distance between the carbons, and the current through the lamp, are correct, the pull of the coil just balances the weight of the upper carbon with its carriage and other moving parts. When the upper carbon is being burned, the arc becomes longer, its resistance increases, and less current flows through the coil. The coil can no longer support the upper carbon, and it is lowered until, — due to the decreased resistance (shortened arc), — sufficient current again flows through the coil to support the carbon. This occurring several times, the clutch reaches the lower limit of its travel where it strikes against the stop. The carbon is thereby released and slides down, until the current rises to its normal value. Then the plunger is pulled up again, the clutch grips the carbon and prevents its further motion. In this way the upper carbon is fed gradually in proportion as it is burned, and the coil regulates automatically to maintain constant current.

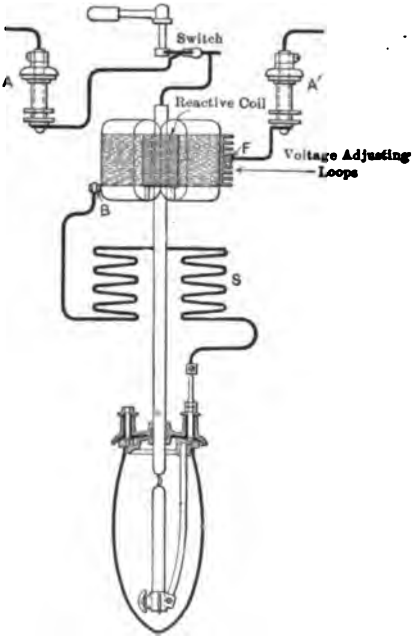

Fig. 208. The Fort Wayne multiple alternating-current arc lamp.

A resistance is always connected in series with a lamp operated in multiple, the purpose of the resistance being to take up the surplus of the line voltage. An ordinary inclosed lamp does not require more than 75 volts at its terminals, while the usual line voltage is 110 volts; the extra 35 volts must be absorbed in a "ballast" resistance. This resistance also makes the arc steadier and reduces the first inrush of current when the lamp is started.

Alternating-current lamps have a similar construction (Fig. 208), except that the plunger — operated by the coil *S*, — must be laminated to prevent eddy currents in it. A reactance coil is sometimes used in place of a ballast resistance. Such a coil is shown on top of Fig. 208; it has several taps *F* in order that the same lamp may be used with different voltages and different frequencies. An advantage of using a reactive or choke coil instead of a resistance is that the coil consumes much less power. The disadvantage of the reactive coil is that it lowers the power factor of the system, and makes the voltage regulation of the alternator unsatisfactory.

226. EXPERIMENT 11-A.—Operation of Multiple Arc Lamps.— Thoroughly inspect the lamp available for study and operation, making clear to yourself the office of the various parts. Connect it in series with an ample regulating resistance and apply current. Adjust the current and the voltage until the lamp gives a good steady light. While adjusting the lamp, study the arc by throwing its enlarged image — by means of a lens — upon a screen (or a white wall). When observing the arc itself, use dark-colored or smoked glass to protect the eyes.

Try various adjustments of the parts of the lamp: — the counterweight, the dash-pot, the resistance, etc.; find the limits of volts and amperes between which the lamp gives a satisfactory operation. Determine the exact values of current and voltage at which, in your opinion, the lamp gives the best service. It is well to have a short-circuiting switch around the ammeter to protect it when starting the arc; the lamp takes much more current when the carbons are in contact than in regular operation. Repeat similar tests with a few other arc lamps, using direct and alternating current.

Some time must elapse before the arc becomes steady; therefore, do not take readings too soon after the current has been switched on for the first time. This is especially true for inclosed lamps, because the oxygen within the globe must first be used up and a definite gas mixture and temperature established. To see this more clearly, take off the inner globe and start the lamp in the open air; observe the difference in volts, amperes, and in the quality and general appearance of the light. Before doing this do not fail to introduce more resistance into the circuit; do not keep the current on with an open arc more than a few minutes, since the lamp under such conditions takes an excessive current.

An important point upon which the satisfactory operation of arc lamps depends is *the quality of carbons* used; the best lamp will give a poor and unsteady light with inferior carbons. Make a few simple tests on carbons, as described in Foster's " Pocket Book," pp. 396–397.

Report. Give a short description of the lamps tested; illustrate their construction with rough sketches; describe the tests; give numerical results, and state any peculiarities observed.

227. Regulating Mechanism of Series Arc Lamps. — When several arc lamps are connected in series, as in street lighting, the same current must of necessity flow through all of them. Therefore, the current strength must not be regulated by each lamp independently, but must be kept constant at the distributing point, — the regulating mechanism of each lamp having merely to adjust the distance between the carbons so as to maintain the proper voltage at their terminals. Consider, for instance, the case of ten lamps connected in series, each rated to consume 6.6 amperes at 70 volts. The total pressure at the terminals of the circuit must be 700 volts; this does not, however, necessarily imply that each lamp would take one tenth of it, or 70 volts. The carbons in one of the lamps may stick together, reducing the voltage drop across this lamp almost to zero; while the next lamp may draw such a long arc as to require nearly 140 volts at its terminals. The average voltage still would be 70 volts per lamp. To prevent this, the regulating coils in the lamp itself (Fig. 209) are so adjusted that they automatically maintain the right voltage at the lamp terminals.

Thus, it must be clearly understood, that when lamps are operated in parallel, their regulating mechanism is made to maintain constant current, the voltage being maintained constant from the power house. When the lamps are operated in series, their regulating mechanism must maintain a constant voltage, the current being maintained constant at the power house.

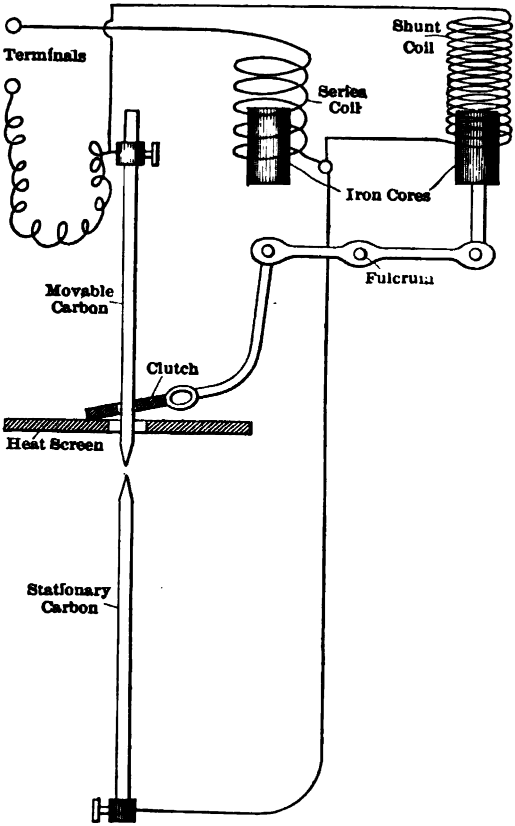

FIG. 209. Mechanism of a differential arc lamp for series connection (series- and shunt-coil control).

Regulation for constant voltage is obtained by adding a second coil to the mechanism of the lamp shown in Fig. 207. This coil (Fig. 209) is shunted across the arc, and opposes the action of the series coil; for this reason such lamps are sometimes called differential lamps. When the power is off, the upper carbon rests on the lower. When the circuit is closed, the series coil is fully energized and pulls the carbons apart, starting the arc. The carbons being originally close together, the voltage across the arc is low, and therefore the action of the shunt coil weak. When the carbons are brought up to the right distance, the action of the two coils and of the weight of the moving part just balance each other. When the carbons are sufficiently burned out, the resistance of the arc increases, the voltage across it rises, and the action of the shunt coil becomes stronger than that of the series coil. The shunt coil pulls up its plunger and lowers the carbon. Should the carbons on the contrary come too close together, the action of the series coil becomes stronger than that of the shunt coil, and the former pulls the carbons apart.

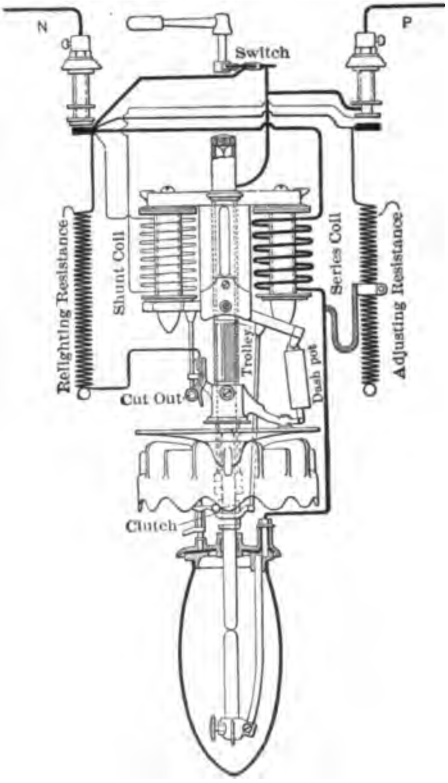

Fig. 210. The Fort Wayne differential arc lamp.

228. Example of a Series Arc Lamp. — A series enclosed arc lamp for direct current is shown in Fig. 210. *P* is the positive terminal, *N* the negative; the main circuit is from the positive binding post to the carbon tube, trolley, upper carbon holder and carbons, through the series coil and the adjusting resistance in parallel with it, to the negative binding post. The shunt circuit is from the positive binding post, through the shunt coil to the negative binding post. When not in operation, the lamp is short-circuited between *N* and *P* by the switch shown on top, the current then passing directly to other lamps in the same circuit.

Opening the switch by turning it puts the lamp mechanism in circuit. The carbons are normally together when the current is off. Turning the current on energizes the series magnet. This attracts the armature carrying the clutch which grips and raises the upper carbon and establishes an arc. The adjusting resistance in parallel with the series coil permits more or less current to pass through the series coil, thus strengthening or weakening its effects on the armature. Counter-balancing the effect of the series magnet on the armature is the shunt magnet, which is affected by the entire difference of potential existing between the lamp terminals. The action of these two coils is the same as in Fig. 209. The too rapid movement of the armature is prevented by the air dash-pot connected to one end.

If the series magnet draws too long an arc, or the carbon fails to feed, the increased resistance of the arc causes the shunt magnet to become abnormally energized, so that it closes the cut-out contacts and shunts the current around the lamp through the re-lighting coil. This action usually causes the carbons to drop together, after which the series magnet is again automatically cut into circuit and the arc re-established. If, however, the upper holder fails to feed because of the upper carbon being burned out, the shunt magnet holds the cut-out contact permanently closed. The lamp is thus replaced by a dead resistance, and the conditions of burning of the other lamps in the same circuit remain intact.

The adjusting resistance not only permits the adjustment of the lamp voltage, or of the distance between the carbons within certain limits, but also compensates for the influence of temperature. The resistance of the shunt coil becomes considerably higher after the lamp has been in operation for an hour or two, than when the lamp is first started; consequently the current in the shunt coil decreases, and the shunt coil becomes too weak to counterbalance the action of the series coil. But the series coil is shunted by the adjusting resistance made of a material having a low temperature coefficient, — for instance, German silver; therefore, as the series coil becomes heated, more and more current flows through the resistance, instead of through the coil, and the pull of the series coil is also decreased. With a correct design it is possible to maintain the same regulation of the hot lamp as when it is cold.

229. EXPERIMENT 11-B. — Operation of Series Arc Lamps. — The conduct of the experiment is similar to that of Experiment 11-A (§ 226). The performance of a series arc lamp may be studied more satisfactorily on a constant-current circuit than on an ordinary constant-potential circuit. Constant current may be obtained by means of either a suitable generator (§ 231) or an arc-light transformer (§ 233). Both machines are intended for rather high voltages, necessary with several lamps in series; for study it is advisable to have only two or three lamps in series, replacing the rest of the lamps by some ballast resistance. The operation of the lamps is much steadier under such conditions.

230. Means for Maintaining Constant Current. — It is explained in § 227 that when arc lamps are operated in series, current must be maintained constant from the power house. The first series arc lamps on the market were built for direct current, and special constant-current generators were used to operate them. Of these the Thomson-Houston, the Brush and the Wood arc-light machines have been the most popular. With the advent of the alternating-current arc lamp these machines have been gradually abandoned; regulation of alternating-current circuits being accomplished far more simply by constant-current transformers (Fig. 211).

Dr. Steinmetz, in connection with his magnetite lamp (§ 224), proposed to use *mercury vapor rectifiers* in connection with direct-current arc-lamp circuits. The magnetite lamp is essentially a direct-current lamp; a rectifier is placed between the lamps and the secondary of the constant-current transformer. In this way, the lamps are fed with high-tension pulsating unidirectional currents, while the generation, transmission and regulation are accomplished with all the simplicity inherent to alternating currents. This system apparently has good chances for success, although the mercury rectifier has not yet had a sufficient practical test as to its reliability.

231. Thomson-Houston Arc Light Machine. — Although this type of constant-current machine has been much used in direct-current arc lighting, the machine is no longer manufactured. Many of them are still, however, in regular operation. These machines have been built for voltages up to several thousand volts, and for outputs of from 6 to 10 amperes. The Thomson-Houston machine is a series-wound bipolar generator. The armature consists of three coils connected together on one end; the other ends are connected to three commutator segments; the arrangement resembles the star connection of a three-phase system. In view of the small number of commutator segments, the current generated is a unidirectional pulsating current rather than direct current. The machine is provided with two positive and two negative brushes, so that the circuit is not opened when the brush passes from one commutator segment to the next. The voltage of the machine is regulated by changing the distance between two brushes of the same polarity. This is done automatically by a solenoid mounted on the machine itself, and energized by the main current. The solenoid operates a rocker arm on which two of the brushes are mounted.

This solenoid, or the *regulator*, as it is called, is brought into action by a relay *controller* usually placed on the wall near the machine. The controller keeps the regulator winding short-circuited as long as the current is below normal. Should the current rise above normal, the controller removes the short-circuit on the regulator, and the latter increases the distance separating the brushes of the machine; this reduces its voltage and consequently the current. With such an arrangement, it is possible to keep the current constant within wide limits of voltages, — in other words, with a varying number of lamps in the circuit.

Some other features of the machine are: (a) the regulator is provided with a dash-pot to prevent jerky action; (b) an air-blast is directed on the brushes to reduce inevitable sparking; (c) the series field winding is shunted by a field rheostat for hand regulation; (d) a non-inductive resistance is connected across the relay contacts to absorb the inductive kick, when the regulator circuit is opened or closed.

232. EXPERIMENT 11-C. — Characteristics of an Arc-Light Machine. — An arc-light machine, such as described in the preceding article, may be conveniently loaded, for a study of its characteristic performance, on a rheostat consisting of, say, 20 resistances in series, each approximately equivalent to an arc lamp. Each resistance is provided with a short-circuiting switch, as in the case of a regular series arc lamp (Fig. 210). Begin the test on open circuit, when both amperes and volts are zero; then close the circuit on a high resistance, and gradually reduce the number of lamps, or equivalent resistances, until the machine is short-circuited. The voltage of the machine is nearly proportional to the number of lamps in the circuit; the current remains constant within a wide range of voltages, down almost to a short-circuit of the machine.

Read volts and amperes; note the number of sections of the rheostat in series. Having finished this run, investigate the limits within which the characteristics of the machine may be changed by (a) shunting the series field; (b) adjusting the brushes; (c) the controller; (d) the regulator, and (e) the dash-pot. Test how quickly the machine regulates with sudden changes in resistance, which are apt to occur in an arc-lamp circuit.

In working with this machine, the student must avoid touching any live parts of the circuit, since the voltages involved may give him an unpleasant shock.

Report. Plot volts and amperes to ohms resistance in the external circuit as abscissæ, or — what is practically the same — to the number of lamps in series. Give the results of various adjustments made on the machine, and its behavior with sudden changes in resistance.

233. Constant-Current Transformer. — Commercial alternators supplying current for light and power are essentially *constant-potential* machines, while *constant current* is required for supplying series arc lamps. The usual apparatus, which converts constant-voltage energy into constant-current energy, and thus allows arc-light circuits to be fed from ordinary alternators, is called a constant-current transformer; sometimes also tub transformer, or arc-light transformer.

A constant-current transformer (Fig. 211) consists, like any ordinary transformer, of two windings placed on an iron core, thus acting inductively upon each other. The essential feature of the transformer is that its secondary coil is movable, while in ordinary constant-potential transformers both coils are stationary. The stationary primary winding is connected to the alternator circuit; the movable secondary coil is connected into the arc-lamp circuit. When the arc-lamp circuit is open, the secondary coil rests on the primary, since only part of its weight is balanced by the counterweight. Closing the arc circuit causes secondary currents to flow in the movable coil, and it is repelled upward. The farther it is repelled, the lower becomes the electromotive force induced in it, this being due to the magnetic leakage between the two coils, which increases as the distance between the coils increases. The counterweight is so adjusted, that with any position of the movable coil, the secondary current in it remains practically constant. When the number of lamps, or the resistance of the circuit, decreases, the current tends to increase; this increases the repulsion between the coils, and the movable coil is repelled farther upward. The use-

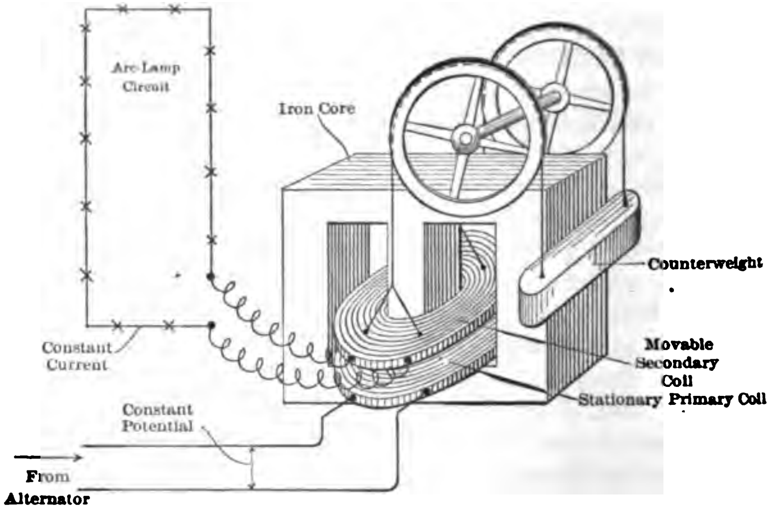

FIG. 211. An arc-light transformer, for transformation of constant-potential energy into constant-current energy.

ful flux is thereby reduced, consequently the induced current again decreases.

Large arc-light transformers are provided with two stationary and two movable coils, mounted on the same core. One set of coils is independent of the other, and the secondaries are connected to different arc-lamp circuits. Each movable coil is provided with a separate counterweight.

234. EXPERIMENT 11-D. —Operation of a Constant-Current Transformer. — Constant-current transformers are used in connection with series alternating-current arc lighting, and are described in the preceding article. Small transformers of this kind, suitable for only 6 arc lamps in series, are now manufactured. They are very convenient for studying characteristics, without handling voltages above 500 volts. Connections are made as in Fig. 211; the same rheostat, equivalent to arc lamps, may be used, as described in § 232. Have an ammeter, a voltmeter, and a wattmeter in both the primary and the secondary circuits.

(a) Begin the test with the secondary circuit open; then close it on a high resistance, and gradually reduce the resistance by equal steps to zero. Take readings, at each step, on all the instruments, and note the positions of the secondary coil by means of a scale marked on the core.

(b) Determine the limits of current within which the transformer (with suitable counterweight) is capable of regulating for constant current; for instance, with a transformer rated at 6.6 amperes try the regulation at 5 amperes and at 8 amperes.

(c) Note the effect of the secondary load being somewhat inductive, as is the case with actual arc lamps.

(d) Determine how promptly the transformer acts with sudden changes of resistance.

(e) Measure voltages induced in the movable coil in various positions, with the secondary circuit open; also when the secondary circuit is closed on a constant resistance. Do this by moving the counterweight by hand; be careful not to overload the windings by bringing them too close together.

(f) Determine for several values of the load, the phase relation between the primary and the secondary volts; also between the corresponding amperes.

The relation between the voltages can be found as follows: Let the primary terminals of the transformer be denoted by A_1 and B_1; the secondary terminals by A_2 and B_2. Connect B_1 and B_2 together; measure the voltages A_1-B_1, A_2-B_2, and A_1-A_2. The vectors of these voltages form a triangle from which the phase relation may be determined.

The phase relation between the currents can be determined by using temporarily a common wire for one side of the primary and of the secondary circuits, and connecting an ammeter in this line. Read primary amperes, secondary amperes, and common amperes. This again gives a triangle of vectors, from which the phase relation may be determined. The phase relation between the primary volts and the corresponding current is obtained from the wattmeter reading; so also for the secondary quantities. As a check on the relations thus obtained, connect the wattmeter so as to read primary volts and secondary amperes, and then vice versa.

When working with this transformer, the student should be careful not to touch any live parts of the secondary circuit, as he might receive a shock due to several hundred volts.

Report. Plot to ohms resistance in the secondary circuit as abscissæ (or to its equivalent, the number of lamps in series): primary and secondary volts, amperes, watts, power factor, positions of the secondary coil. Give the results of tests enumerated under (b), (c), (d), and (e). Plot vector diagrams, as required in (f), and explain the relations obtained between the currents and the voltages.

ARC-LAMP PHOTOMETRY.

235. The principles of photometry, or of the art of measuring luminous intensities of lamps, are explained in Chapter X, and the reader is supposed now to be familiar with them. In arc lamps, as in incandescent lamps, the mean horizontal intensity, or the mean intensity measured in any other direction, affords a simple method for rating lamps; but it is not sufficient for a judgment of the total amount of light which a lamp is capable of giving. The trend of opinion among engineers is more and more towards rating arc lamps on the basis of their mean spherical (§ 215) or mean hemispherical (§ 221) candle-power. This is done either by taking a curve of distribution of luminous intensity (Fig. 203) or by means of integrating photometers (§ 239).

The principal points of difference between photometering incandescent lamps and arc lamps are as follows:

(1) Arc lamps are much more powerful sources of light than incandescent standards to which they are compared; therefore, for measurement, the luminous intensity of the arc must usually be reduced in a known ratio.

(2) Since it is not practicable to spin an arc lamp in order to obtain its mean candle-power, the distribution of light is obtained by means of suitable mirrors.

(3) The light of an arc lamp is not quite steady; the arc flickers, runs around the edge of the carbons, the distance between the carbons varies, etc.

(4) Differences in color (§ 213) are more pronounced than with incandescent lamps.

The methods, by which these difficulties are taken care of, are described below.

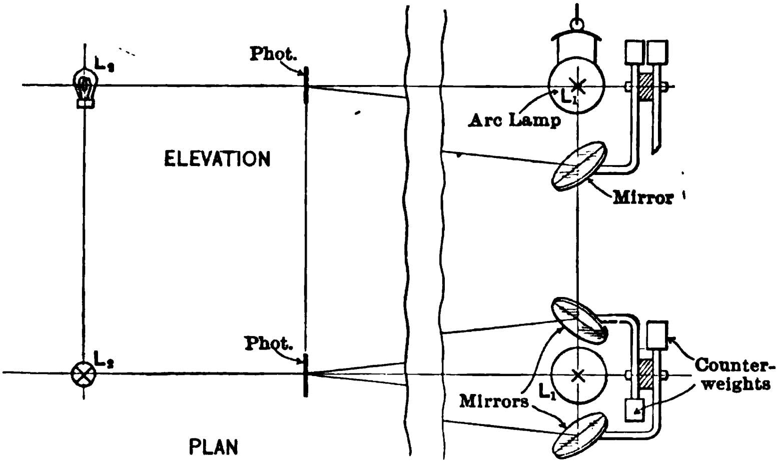

Fig. 212. An arrangement of mirrors for photometering arc lamps.

236. Distribution of Light about an Arc Lamp. — Prof. Matthews' arrangement is shown in Fig. 212, in plan and elevation. It was devised for the extensive tests which he has performed on arc lamps for the National Electric Light Association. The light from the lamp under test is reflected in any desired direction by two movable mirrors and directed upon the photometer screen. The lamp itself is screened from the photometer by a piece of black cardboard, or velvet (not shown in the sketch). With two mirrors, fluctuations in illumination of the photometer screen, caused by an unsteady arc, are considerably reduced, especially those due to the arc running around the edge of the carbon. Either the Bunsen, the Lummer-Brodhun, or a flicker photometer may be used; the illumination on both sides of the photometer screen is balanced by moving the standard lamp L_2. With two mirrors, the illumination of the screen is doubled, so that the result must be divided by two. It must also be divided by the cosine of the angle at which the rays are incident on the screen, and by the coefficient of absorption of the mirrors. It seems simplest to find an empirical coefficient for all these effects combined, by placing at L_1 a large incandescent lamp, and photometering it with and without the mirrors. Instead of moving the mirrors (to determine the intensity of illumination in various directions) and measuring their angles, it is more convenient to have a complete set of mirrors as in Fig. 213 and uncover them two by two.

An arc lamp has to be placed at quite a considerable distance from the photometer screen, in order to be balanced against an incandescent lamp. Where this is not practicable, the light from the arc lamp is reduced in a known ratio. A convenient method is to interpose a revolving sectored disk between the arc lamp and the photometer screen. The disk must be rotated at a speed sufficiently high not to produce flickering; the light is intercepted in proportion to the area of the open and opaque parts (Talbot's law). If the disk consist, for instance, of six sectors, each having an opening of 40 degrees and the opaque part of 20 degrees, two thirds of the light passes through. Suppose the photometer reading to be 400 candle-power; then the true candle-power of the arc is $400 \times 3/2 = 600$ candle-power. The method is not applicable with alternating-current arc lamps, because the light of the lamp itself is fluctuating, and would be intercepted at moments of different intensity. In this case translucent screens are used, or dispersing lenses, to reduce the illumination of the screen; they are calibrated by photometering an incandescent lamp through them and without them.

237a. EXPERIMENT 11-E. — Distribution of Light about a Direct-Current Arc Lamp. — The apparatus is arranged as in Fig. 22. Before beginning the experiment proper, determine the constant of the mirrors by measuring the horizontal candle-power of a large incandescent lamp, say 100 candle-power, with the mirrors, and without them. Investigate first the distribution of light around an inclosed arc lamp, as giving the most steady light; use a clear inner globe and no outer globe. Let the lamp burn a sufficient time for the light to become steady; measure the light intensities every five or ten degrees, throughout 180 degrees. Use, if necessary, a rotating sectored disk to reduce the illumination of the photometer screen. Replace the clear globe with a ground-glass one, and again take the distribution curve. Perform the same measurements when the arc lamp is provided with an opalescent outside globe; also with a diffusing reflector, etc. Investigate the distribution of light around an open arc, a flaming (luminous) arc, a magnetite lamp, etc.

Report. Plot curves of distribution of luminous intensity, as in Fig. 203. Figure out the mean spherical intensity from one of the curves, with the aid of the Rousseau diagram (Fig. 203); for another curve by using formula (3) in § 216. Calculate from the same curves the mean hemispherical intensities (§ 221). Give the ratios of the mean spherical and hemispherical candle-power to the mean horizontal candle-power (reduction factors). Discuss the causes of difference in the shape of the photometric curves.

237b. EXPERIMENT 11-F. — Distribution of Light about an Alternating-Current Arc Lamp. — For instructions, see the preceding experiment.

238. Arc Lamp Integrating Photometers. — The Ulbricht spherical photometer, described in § 220, may be used for measuring the mean spherical or hemispherical intensity of arc lamps, provided the diameter of the sphere is large enough. Blondel considers a diameter of about 10 feet as sufficient for the purpose. Two other practical forms of arc lamp integrating photometers are shown in Figs. 213 and 214.

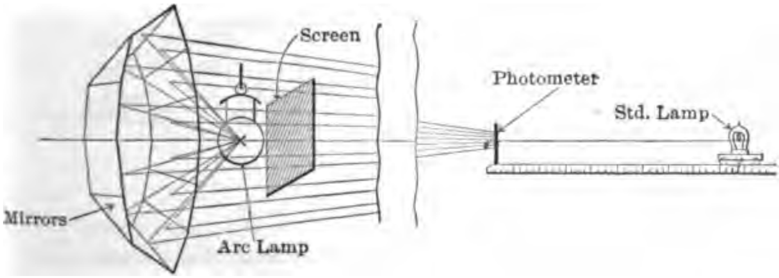

FIG. 213. An integrating photometer for arc lamps.

The photometer shown in Fig. 213 is a development of the photometer represented in Fig. 212. Instead of moving the two mirrors so as to obtain the distribution of light in steps, a set of stationary mirrors is arranged in the form of a truncated pyramid; the light from the lamp is reflected on the photometer screen simultaneously from all the mirrors. The direct illumination of the screen by the lamp is prevented by an opaque shield or screen. Such an arrangement gives an illumination of the photometer screen proportional to $\sum J$, where J is the luminous intensity in a certain direction. However, the mean spherical intensity, according to equation (3) in § 216, is proportional to $\sum J.$ Sin θ and not to $\sum J.$ Therefore, the light from each pair of mirrors must be reduced, or "weighed" in proportion to its Sin θ, or the area of the zone to which the mirrors correspond (Fig. 203). This is done in practice in three different ways:

(1) Mirrors are smoked to a different degree, so as to reduce their reflecting power in the desired proportion (Matthews).

(2) An opaque screen, with apertures proportional to Sin θ, is interposed between the lamp and the mirrors (Blondel).

(3) Mirrors themselves are spaced so as to correspond to zones of equal areas, being spread farther apart at the poles and crowded near the equator (Russell and Leonard).

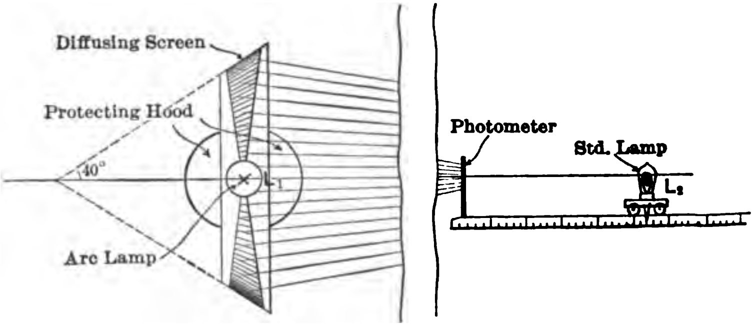

FIG. 214. The Blondel integrating photometer for arc lamps.

Whichever method is used, it is advisable to calibrate the photometer experimentally, by a lamp of which the mean spherical candlepower is known.

A modification of the above photometer, devised by Blondel, is shown in Fig. 214. A white diffusing surface is used instead of mirrors; this surface is painted on the inside of a truncated cone. The rays, diffusedly reflected from it, fall on a photometric screen, and the illumination is balanced against a standard lamp. The rays in different directions are "weighed" in the ratio of Sin θ by making the screen the widest in the equatorial plane of the lamp, and gradually reducing the width to the poles. The lamp is protected by a hood, so that no light falls on the photometric screen except that reflected from the diffusing surface. It must be noted, that the left side of Fig. 214 represents the plan of the device, while the right side is a side elevation.

239. EXPERIMENT 11-G. —Determination of Mean Spherical and Hemispherical Intensities of Arc Lamps with an Integrating Photometer. — A few types of integrating photometers adapted for use with arc lamps are described in § 238. First calibrate the photometer to be used, with a source of light of which the mean spherical intensity is known. Then test in the photometer direct- and alternating-current arcs, both open and enclosed, investigating also the influence of globes and reflectors. Determine the specific power consumption per mean spherical and mean hemispherical candle-power. If possible, the same lamps should be tested as in §§ 237a and 237b, in order to check the results.

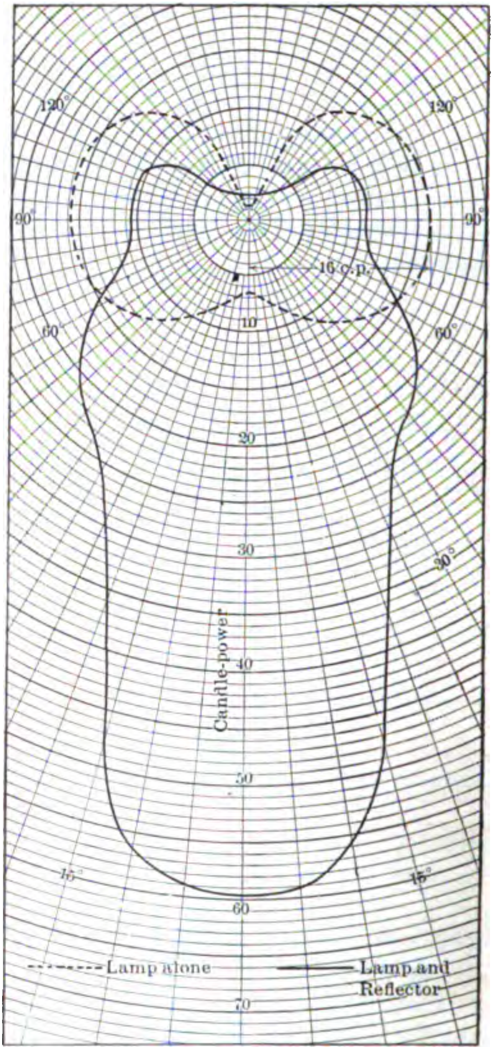

FIG. 215. The photometric curve of the Holophane reflector shown in Fig. 216.

CHAPTER XII.

INTERIOR ILLUMINATION.

240. Use of Reflectors, Shades and Globes. — The natural distribution of light about an incandescent lamp (Fig. 203) may be considerably improved by the use of proper reflectors and globes. The photometric curves shown in Fig. 215 illustrate this point; the dotted curve gives the distribution of light about a bare 16 candle-power lamp; the elongated curve shows the distribution of light when the same lamp is provided with a reflector shown in Fig. 216. The intensity of light in the vertical direction is increased from 7 to 60 candle-power; a considerable increase in illumination is also obtained in other direc-

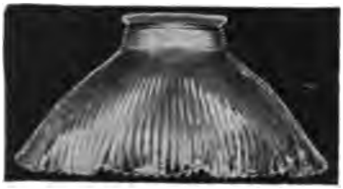

Fig. 216. Holophane concentrating reflector.

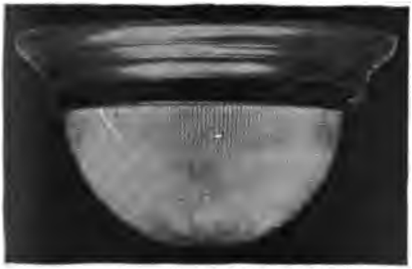

Fig. 217. Ceiling bowl.

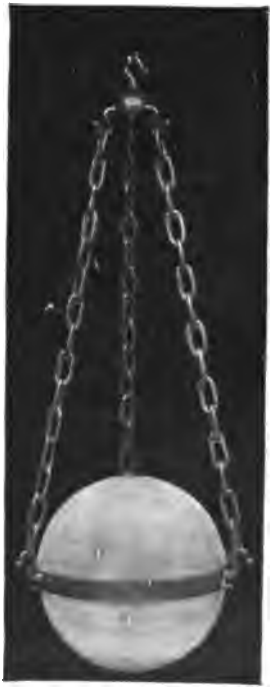

Fig. 218. Two-piece sphere supported by chains.

tions up to a latitude of 60 degrees. Beyond this, the illumination is reduced. This, however, is no objection if the light is needed chiefly under the lamp, say for the illumination of a table. The gain in this case is evident, since it would take at least three or four bare lamps to produce the same illumination on the table, and much light would be wasted where it is not needed.

Different distribution of light, as required in various practical cases, is obtained by a proper design of globes and reflectors. Three different kinds of distribution of light are shown in Fig. 219; they are obtained with various glass globes of the type shown in the upper right-hand corner of the sketch. The globe has the same general form in all cases: its outer surface is made into series of small prisms. By properly shaping these prisms, light is thrown in any desired direction. The globe shown in the upper left-hand corner may be properly used when maximum light must be thrown downward, for instance, in chandeliers over reading-tables. The one shown under it is suitable for general illumination, the distribution of light being much more uniform. The globe in the lower right-hand corner throws maximum light at from 10 degrees to 15 degrees below the horizontal, and is designed to light large areas, streets, etc.

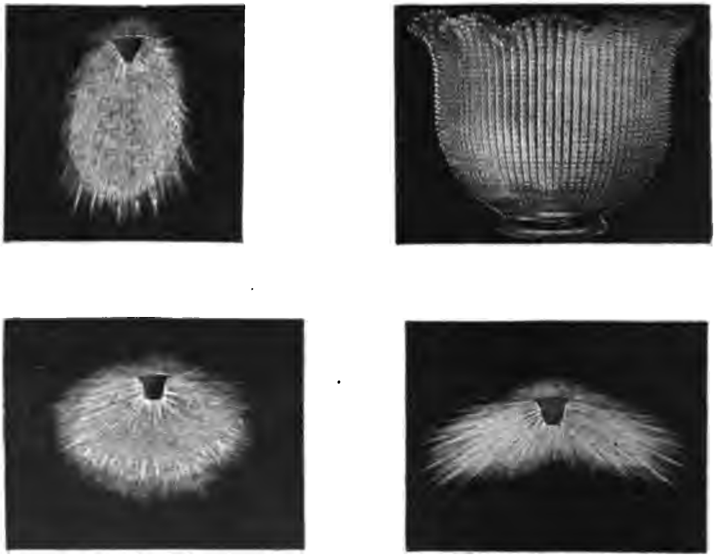

FIG. 219. One of the forms of Holophane prismatic-glass globes, and illuminating effects produced by properly shaping the external prisms.

A great variety of shades, globes and reflectors are used in connection with incandescent lamps, the determining factors in the selection being: the purpose of the illumination, artistic requirements and the price. The most common reflectors and globes are shown in Figs. 220 and 221. The globes and reflectors in Fig. 220 are made of opal glass, and according to their form are called dome, cone, bell reflectors, flat reflectors, etc. Those with grooves on the surface are distinguished by the word "fluted." Thus, the third reflector in the upper row, counting from the left, is known in the trade as the "fluted opal cone reflector." The shades shown in Fig. 221 are usually made of sheet iron, sometimes of cardboard. The two upper ones are called cone shades, the lower one between them is a flat shade, that to the right a parabolic shade (or reflector). Metal shades are used in factories, drafting-rooms, etc., where light must be concentrated on a limited surface; general illumination is usually supplied by other lamps near the ceiling. Glass reflectors allow the concentration of light, and at the same time let sufficient light through for general illumination.

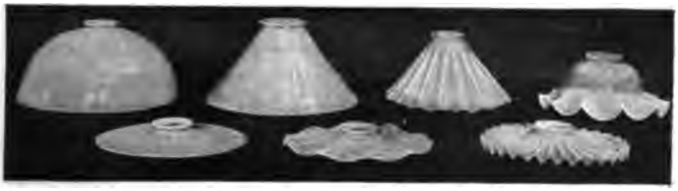

FIG. 220. Some common types of opal-glass globes and reflectors.

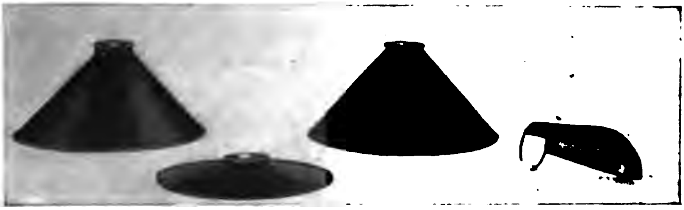

FIG. 221. Metal shades and reflectors.

241. Holophane Prismatic-Glass Globes and Reflectors. — Some of the disadvantages of the above-described globes and reflectors are: (a) they absorb a considerable amount of light; (b) light is not sufficiently diffused; (c) illumination can be concentrated in desired directions to a limited extent only. These defects are eliminated to a considerable degree in the so-called "Holophane" prismatic-glass globes and reflectors, shown in Figs. 216 to 219.

The name "Holophane" was taken from two Greek words meaning "all-light." The underlying principles, on which these globes are built, have been developed by Messrs. Blondel and Psaroudaki in France. Holophane *globes* are made of high quality pressed glass, and have both internal and external surfaces made into series of small prisms of different shapes and sizes. The function of the internal prisms (Fig. 222) is to diffuse the light given out by the lamp. Each ray, such as *ab*, is broken up into at least two rays *bcd* and *befg*, so that the eye, following back each component, is unable to see the actual, concentrated source of light, but the whole globe becomes a source of diffused light. The external prisms (Fig. 223) have in general four faces; through some of these faces light passes with a simple refraction. On others it suffers a total reflection and goes out through one of the other faces. It is absolutely essential that the prisms be accurately designed; otherwise some of the light is apt to be thrown back into the globe, thereby decreasing the efficiency.

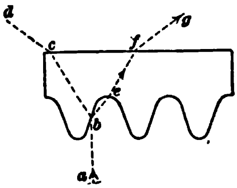

FIG. 222. Internal prisms on Holophane globes.

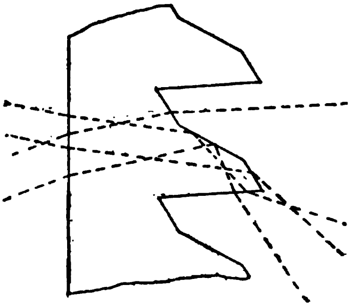

FIG. 223. External prisms on Holophane globes.

Holophane *reflectors* (Fig. 216) are based on a somewhat different principle; they are particularly adapted for throwing light downward, or if desired, at an angle of about 45 degrees, at the same time giving sufficient general illumination. They are smooth on the inside, and have a series of right-angle prisms on the outside. These prisms offer to the light a total reflecting surface; it is thus possible to throw the light strongly below the horizontal, with an extremely small loss from absorption. At the same time, some of the light passes through the top and bottom of each prism, thus allowing sufficient light to illuminate the ceiling.

Holophane globes and reflectors are made in a great many types to suit various purposes, and different artistic requirements. For most of them photometric curves, like the one in Fig. 215, are known, so that it is possible to select in each case the most suitable.

242. EXPERIMENT 12-A. — Influence of Globes, Shades, and Reflectors on the Distribution of Light about an Incandescent Lamp. — Place a bare incandescent lamp in a photometer, and obtain its curve of distribution of light, as in § 217. Then repeat the same test with representative types of globes, shades and reflectors described in §§ 240 and 241. It is also advisable to take photometric curves of one and the same globe with lamps having markedly different photometric curves. If an integrating photometer is available, absorption due to a globe may be determined directly, by measuring the mean spherical candle-power with and without the globe.

Report. Plot photometric curves of the globes and reflectors tested, as in Fig. 215. Figure out the mean spherical intensity (§§ 215 and 216) from a few representative curves, and compare it to the mean spherical intensity of the bare lamp. The ratio will give per cent absorption of light due to the globes. Give your opinion as to the relative quality of different types tested, and the cases where they should be used.*

243. Foot-Candle as a Unit of Illumination. — If an 8 candle-power lamp be placed two feet above a desk, the lamp gives a certain illumination on a small sheet of paper placed directly under it on the desk. A 32 candle-power lamp placed four feet above the desk, or a 2 candle-power lamp at a distance of one foot from the desk, will give practically the same illumination of the sheet of paper. The intensity of illumination may be understood here, for the time being, as the facility with which some print may be read on the piece of paper. Thus, the intensity of illumination of a surface depends on the candle-power of the lamp and on its distance from the surface. *Intensity of illumination produced by a source of one candle-power at a distance of one foot from it, on a small surface perpendicular to the ray, is called the foot-candle.* According to the fundamental law of optics, illumination decreases inversely as the square of the distance: therefore, it takes a 4 candle-power lamp to produce the same illumination at a distance of 2 feet, 9 candle-power at 3 feet, 16 candle-power at 4 feet, etc. In this way illumination produced by a source of light of known luminous intensity can be expressed numerically in foot-candles. Take, for instance, a 16 candle-power lamp with a Holophane reflector, shown in Figs. 215 and 216; let this lamp be placed at a distance of 4 feet above a desk. The vertical intensity of light is about 60 candle-power; therefore, if the lamp were placed at a distance of one foot above the desk, it would produce an illumination of 60 foot-candles. The actual distance being 4 feet, the illumination is reduced $4^2 = 16$ times, or

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* In case of concentrating reflectors, the law of inverse squares does not hold, and consequently the photometric curve varies for different lengths of bars: the mean spherical candle-power obtained by the Rousseau Diagram is apt to depart widely from the truth. For example, a mirror reflector, which has a diamond surface effect, has been found to have an efficiency of 98 per cent, whereas the true efficiency cannot be much over 80 per cent, owing to the fact that the light rays must pass through two layers of glass, and also that silvering used in this reflector does not have a greater reflecting power than 85 per cent.

$$\text{intensity of illumination} = \frac{60}{4^2} = 3.75 \text{ foot-candles.}$$

In general, if J is the luminous intensity of a lamp, in candle-power, and d the distance in feet, from a small perpendicular surface illuminated by the lamp (or the distance from lamp to surface illumined), the normal intensity of illumination $= \frac{J}{d^2}$ (in foot-candles).

244. Calculation of Illumination. — The above formula gives the intensity of illumination when the surface to be illuminated is perpendicular to the direction of the rays of light. If the surface is not perpendicular, the illumination is reduced as the cosine of the angle θ of deviation from the normal. This follows from the fact that the number of rays of light per unit surface is reduced in this proportion, for obvious geometrical reasons. Thus, in general,

$$\text{intensity of illumination} = \frac{J}{d^2} \, . \operatorname{Cos} \theta \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

In practice, illumination is usually calculated for the floor, or for a horizontal plane a few feet above the floor. For such cases the above formula may be modified as follows: Let h be the vertical distance between the center of the lamp and the horizontal plane for which the illumination is to be determined; we have $h = d \, . \operatorname{Cos} \theta$. Substituting the value of d into the above formula, we find

$$\text{horizontal intensity of illumination} = \frac{J}{h^2} \operatorname{Cos}^3 \theta \text{ (in foot-candles).} \quad (2)$$

Take, as an example, the lamp with a reflector, shown in Fig. 215, and assume it to be placed 8 feet above the floor. Let it be required to find the horizontal illumination in a plane 3 feet above the floor, at a point such that the rays from the lamp make an angle of 30 degrees with the vertical. From the photometric curve we find that the luminous intensity in this direction $J = 26$ candle-power; the horizontal illumination is equal to

$$\frac{26}{(8-3)^2} \text{Cos}^3\, 30° = 1.04 \times 0.649 = 0.675 \text{ foot-candles.}$$

The normal, or maximum illumination at the same point is

$$\frac{26}{(8-3)^2} \text{Cos}^2\, 30° = 0.78 \text{ foot-candles.}$$

This latter illumination could be obtained, for instance, upon a book at the point under consideration, if held in the most advantageous position with respect to the lamp. The horizontal illumination directly under the lamp is

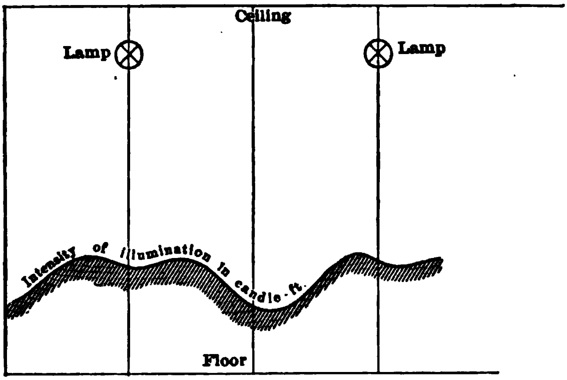

FIG. 224. Sample of an illumination curve.

$$\frac{60}{(8-3)^2} = 2.4 \text{ foot-candles.}$$

245. Illumination Curves. — If there is more than one lamp in the room, the illumination produced by each one is calculated separately, and the results added from point to point. Thus, the room shown in Fig. 224 is illuminated by rows of lamps placed near the ceiling. The height of all the lamps above the floor being the same, the horizontal illumination at any point, according to the expression (2), is represented by the formula

$$\text{horizontal illumination} = \frac{1}{h^2}\sum J \,\text{Cos}^3\,\theta \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2a)$$

The ordinates of the curve shown in the sketch give the intensities of illumination: the points of maximum illumination lie right below the lamps, points of minimum illumination between the lamps.

The illumination curves are different in different vertical planes of the room: therefore, it is customary to supplement them by curves showing distribution of illumination in the horizontal plane. These latter curves connect points of equal illumination in the room, and may be properly called "isophotal" curves, by analogy with "isothermal" curves.

In planning the illumination of a room, the designer assumes certain intensities of illumination, according to the requirements of the room, and places the lamps tentatively, until the desired intensities are approximately obtained. At the same time lamps must be placed so as to satisfy the artistic requirements and the convenience of wiring and control.

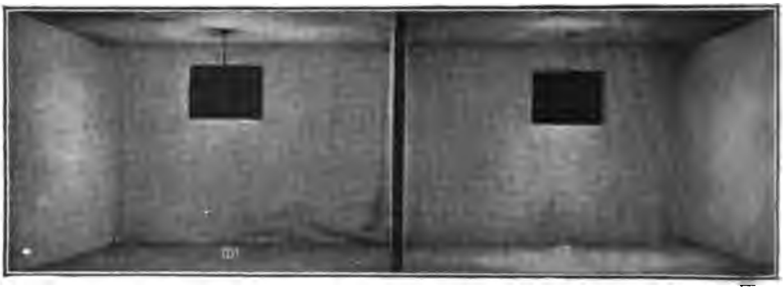

Fig. 225. Two dark rooms for demonstrating relative advantages of various globes and reflectors.

The actual values of illumination are somewhat modified by reflection from walls and ceiling. It is not possible at the present state of the art to give the necessary correction, the more so that the influence of reflected light varies greatly with the position of the lamps, the color and the state of surface of the walls, and the proportions of the room. Professional illuminating engineers assume in each case some empirical correction on the basis of their previous experience.

246. EXPERIMENT 12-B. — Demonstration Room Tests. — The foot-candles illumination at various points of a room may be measured with sufficient accuracy by photometers described below. It is desired, however, that before undertaking such accurate measurements, the student should observe the most important effects of illumination in their practical, physiological effect upon the eye. Simple demonstration tests are conveniently performed by using two small identical rooms

(Fig. 225) built side by side, within a larger dark room. The rooms may conveniently have a floor area of 4½ by 4½ feet, and be about 3 feet high; they must be separated from each other by a partition. The rooms are open in front, so that the effect of illumination may be observed or photographed.

A bare incandescent lamp is placed in one room, an identical lamp provided with a shade or a globe is placed in the other room. The lamps themselves are shaded from the observer by a piece of black cardboard. The difference in illumination is clearly noticeable in the figure.* To make it more marked, a piece of paper is put on the floor, with letters of different size, such as are used in oculists' offices. To obtain a lasting record, the two rooms are photographed on one plate. By changing the location and the size of the lamps, by putting different globes and reflectors, by changing the color and the state of surface of the walls and the ceiling, the student will get a practical conception of the effect of these factors on illumination.

In deriving conclusions from the photographs, care should be taken to compare the two sides of the same photograph only, and not the absolute intensities of illumination in different photographs. The latter may show different relative illumination than is actually the case, because of differences in the plates themselves, and of the peculiar conditions of illumination when pictures are taken.

MEASUREMENT OF ILLUMINATION.

247. The problem of illumination consists (a) in determining the necessary number of lamps and their candle-power; (b) in properly locating the lamps in the room; (c) in selecting suitable reflectors or globes. Lighting of small rooms, or places where the requirements are not strict, is designed on the basis of simple every-day experience. Lighting of large public halls, theaters, churches, etc., requires a considerable study in each individual case. Designing an efficient illumination is in many respects an art, which could hardly be discussed here. There are, however, certain fundamental principles which admit of numerical calculations, and which can be verified experimentally. Some of these experiments and tests are described below.

Illumination in foot-candles is measured by photometers of somewhat different construction than those described in Chapter X. The "illumination" photometers must be portable and self-contained so as to be placed at the points where it is desired to determine the illumination.

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* Fig. 225 is taken from Cravath and Lansingh's book, entitled *Practical Illumination*, by courtesy of the McGraw Publishing Company.

The classical Weber photometer (§ 250) has been used for such tests for many years. Attention is here called to a convenient little instrument, recently devised for the same purpose by Mr. J. T. Marshall of the General Electric Company.

248. The Marshall Luminometer. — The instrument is shown diagrammatically in Fig. 226. It is based on the fact that the resistance of an incandescent lamp changes with its candle-power, so that once calibrated, the candle-power may be determined by simply measuring the resistance of the lamp. A low candle-power, low-voltage tungsten lamp is used in the luminometer, because of the constancy of its candle-power with time, and also because it has an appreciable temperature coefficient. The tungsten lamp illuminates a Bunsen screen (§ 205) placed at a constant distance from it, and constituting a part of the instrument itself. This illumination is balanced by the illumination of the other side of the screen, it being placed at the point and in the direction where it is desired to measure the illumination. The balance is obtained by varying the voltage of the tungsten lamp by the rheostat shown in the sketch.

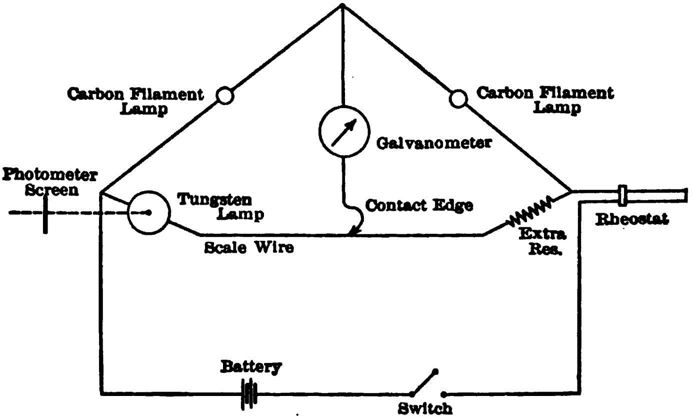

FIG. 226. The Marshall luminometer.

The resistance of the tungsten lamp is measured by an arrangement similar to an ordinary slide-wire Wheatstone bridge (Fig. 11). Two high-resistance carbon filament lamps are used as ratio resistances; the scale is calibrated direct in foot-candles. The calibration is done by presenting the screen successively to different known foot-candles of illumination.

To determine the illumination at a certain point, the instrument is set so that the photometric screen is at the required point, and in the desired plane, say horizontal. The rheostat in series with the battery is adjusted until an equal illumination is obtained on both sides of the screen. Then the exploring contact edge on one of the galvanometer leads is moved along the slide-wire until the galvanometer returns to zero. Foot-candles illumination are read directly on the scale at this point. All the parts of the instrument, including the battery, the galvanometer and the rheostat are mounted in a box, so that the instrument is absolutely self-contained, and may be used in any location.

249. EXPERIMENT 12-C. — Calibration and Use of the Marshall Luminometer. — The device (Fig. 226) is calibrated by means of a standard incandescent lamp. The lamp is placed at definite distances from the screen, say, 2, 4, 6, etc., feet, and a balance is obtained, as explained above. Let, for instance, a 16 candle-power lamp be placed at a distance of 2 feet from the luminometer screen; the illumination of the screen is $16 \div (2)^2 = 4$ foot-candles. The point of the scale at which the galvanometer balance is obtained, must be marked 4. Having calibrated the instrument, it may be used for a study of distribution of illumination in a room (Fig. 224). In this particular experiment it is sufficient to investigate the simplest case, of a room lighted by one lamp in the center. Measurements are conveniently made in a horizontal plane, about 3 feet above the floor. Take first a bare lamp and determine horizontal intensities of illumination, from a point under the lamp, to one of the walls. Repeat the same measurements with the lamp fixed at different heights, and also provided in succession with two or three different kinds of globes and reflectors. Use preferably a lamp and a globe of which the photometric curves are known (Fig. 215). If possible, this experiment should be performed in two identical rooms, in one of which the walls and the ceiling are painted dull black, while in the other they offer light-reflecting surfaces. The same effect can be obtained by temporarily covering the walls and the ceiling with black cloth.

Report. (1) Give the calibration curve of the luminometer. (2) Plot curves of horizontal illumination. (3) Check a few points with the theoretical illumination, calculated as in § 244. (4) Discuss the influence of the walls and ceiling.

250. The Weber Photometer. — This photometer, like the above-described luminometer, is primarily intended for measuring illumination, though it can also be used for measuring luminous intensities of primary sources of light. The instrument is shown diagrammatically in Fig. 227; a perspective view is given in Fig. 228. We shall describe it first as it is applied for measurement of the luminous intensity of a lamp, and then will show how to use it for measurements of secondary illumination. The lamp L_1, whose candle-power it is desired to determine, illuminates the opal-glass screen a_1. The standard benzine lamp L_2* illuminates another opal-glass plate a_2. The intensities of illumination of the two plates are compared from c, through the Lummer-Brodhun prisms p (§ 211). The plate a_2 is moved along the tube T_2 until a photometric balance is obtained (Figs. 195 and 201).

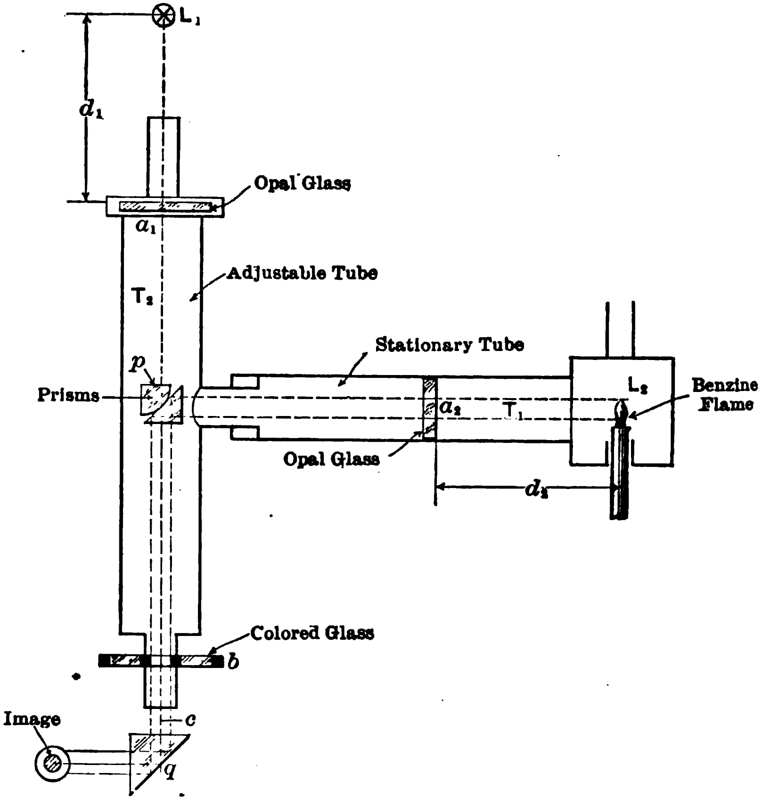

FIG. 227. The schematic arrangement of Weber's photometer.

The illumination of the plate a_1 is $J_1 \div d_1^2$, where J_1 is the candle-power of the lamp L_1 and d_1 its distance from the plate a_1. Only part of this light passes through the plate, so that the above expression must be multiplied by a constant C_1. A similar expression may be written

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* The original benzine lamp may be conveniently replaced in some cases by a small low-voltage tungsten or carbon incandescent lamp supplied with current from a portable battery, as in the above-described Marshall luminometer.

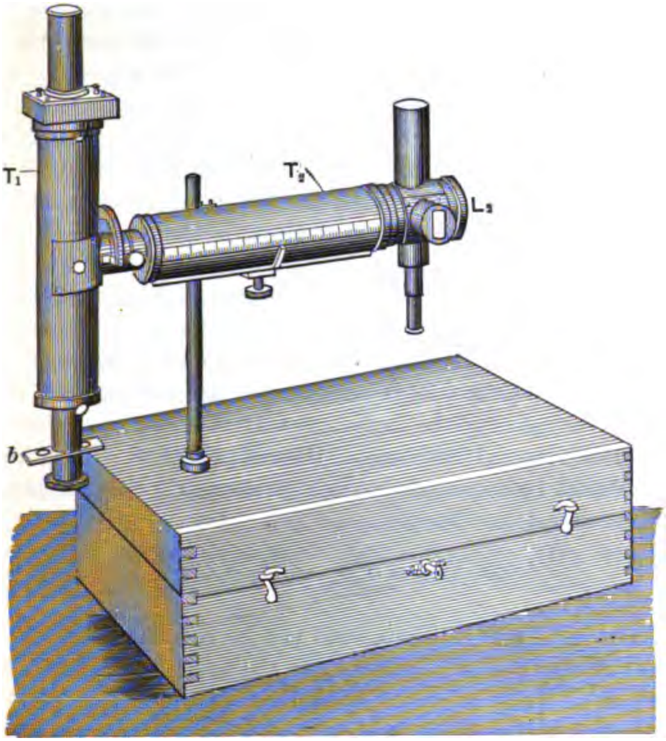

FIG. 228. Weber's photometer.

for the plate a_2; when the observer sees the two plates equally illuminated, we have

$$C_1 \cdot \frac{J_1}{d_1^2} = C_2 \cdot \frac{J_2}{d_2^2},$$

or

$$J_1 = C\left(\frac{d_1}{d_2}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The constant C comprises the candle-power of the benzine flame, and the constants C_1 and C_2 of both opal plates. The constant C is determined empirically, by using a standard lamp in place of L_1. A few plates a_1, of different translucency, are supplied with the instrument, for use with sources of different intensity.

The tube T_1 can be pointed in any desired direction and fastened by a clamping device. The angle is indicated on the divided sector seen in Fig. 228. When T_1 is pointed upward, it would be impracticable to take observations at c; a prism q is used in this case, and the observer's eye is placed at the point marked "Image." The eye-piece b is provided with three circular openings: one is covered with a piece of red glass, another with a piece of green glass; the middle opening is left blank. Colored glass is used when the lights under comparison are of different tints. Then they are compared, once with red glass and then with the green glass. Tables are given with the instrument from which the true ratio of lights may be calculated from the two settings. A flicker photometer (§ 212) affords, however, a more reliable means for comparing lights of different color. The plate a_2 is moved by a rack and pinion; the operating knob is seen in Fig. 228 under the tube T_2; distances d_2 from the center of the benzine flame are read on the horizontal scale.

The same photometer may be used for the determination of secondary illumination, by placing the instrument so that the opal-glass plate a_1 is at the desired point and in the desired direction. The small tube above a_1 should be removed. When photometric balance is obtained, we have according to (3),

$$\text{illumination of } a_1 = \frac{J_1}{{d_1}^2} = C\left(\frac{1}{d_2}\right)^2 \quad . \quad . \quad . \quad . \quad (4)$$

so that knowing C, the illumination can be calculated in foot-candles.

If it is not convenient to place a_1 in the desired location, a diffusing white screen (usually supplied with the instrument) is placed there. The plate a_1 is removed altogether, and the illumination of the white screen is compared to that of a_2. The formula is again,

$$\text{illumination of the white screen} = C'\left(\frac{1}{d_2}\right)^2 \quad . \quad . \quad . \quad (5)$$

where C_1 is another constant: it is determined by illuminating the white screen with a standard lamp at a known distance.

For a more detailed description of the Weber photometer, and instructions for its use, see Stine, "Photometric Measurements," p. 78.

251. EXPERIMENT 12-D. —Calibration and Use of the Weber Photometer.—The photometer (Figs. 227 and 228) is calibrated by means of a standard incandescent lamp. The lamp is placed at known distances $d_1 = 1, 2, 3$, etc., feet from the upper glass plate a_1, and photometric balance is obtained. All quantities in the expressions (3) and (4) being known, the constant C may be calculated. Calibrate in this way a few plates supplied with the instrument. Before using the photometer, see that the benzine lamp is adjusted according to the directions given by the manufacturer, especially that the height of the flame is correct. Have the lamp burning for 10 or 15 minutes before beginning the measurements. Determine the constants of the instrument when a small incandescent lamp is used in place of the benzine flame. After this, determine the constant C' of the white screen supplied with the instrument, and the limits within which this constant is independent of the distance of the screen, and of the angle at which the photometer is placed relative to the screen. Take a few readings with the red and the green glasses, and see to what extent the settings are different from those obtained with the blank aperture.

Having calibrated the instrument, it may be used for exploring the illumination of a room, in the same way as in § 249. As another application, determine the distribution of illumination produced in a large room by daylight. Also measure the illumination on a street lighted with arc lamps. The report is similar to that described in § 249.

252. EXPERIMENT 12-E. — Determination of Standard Values (Norms) of Illumination. — When designing the illumination of a room, some basis must be had for determining the number and the size of lamps and their location. This is done by assuming so and so many candles per square foot of the floor, or per cubic foot of the room. Or else values are given in terms of foot-candles of illumination. Careful illuminating engineers usually check their designs by means of all three types of units. The values vary according to the size and the use of the room, quality of the illumination desired, etc. The experiment described below is intended to develop the judgment of the student in this respect.

The tests should be performed in the evening, in a medium-sized room. The wiring must be planned, so that it is easy to vary the position and the number of lamps. Assume the room to be intended for a certain purpose, say for lectures: place a few lamps with reflectors or globes in the most efficient way. Vary their number and distribution, gradually passing through the degrees of illumination, which may be called: dull, medium, good, bright, and brilliant. Determine in each case candle-power per square foot and per cubic foot, also measure the intensity of illumination in foot-candles, as in §§ 249 and 250. In performing these tests, keep in mind that the purpose is not to light the room actually, but to get standards (norms) for design; therefore use your judgment in modifying the norms where the conditions in the room are different from those actually met with in practice.

After this, determine in the same room the norms for various other purposes: for lighting a parlor, a dining-hall, a library, a store, a drafting-room, an office, a workshop, etc. In each case determine the limits of illumination that might be characterized as dull, good, and brilliant. Also distinguish between a *general* illumination and the illumination for a *specific purpose*, such as lighting a dining-room table, a desk, a counter, etc.

In performing this experiment the student must rely entirely on his own judgment and try to develop it in the matters of illumination. Care should be taken to rest one's eyes from time to time, and avoid quick changes in illumination, which may bias the judgment.

253. EXPERIMENT 12-F. — Exercises in Lighting Rooms. — The exercises specified below, are intended for a practical application of the principles of illumination treated in §§ 240 to 252. Begin with a rather small room, suitable for use as a living-room, a parlor, a dining-room, or a small office. Place a few incandescent lamps, as you think best, for the desired quantity and distribution of light. Use the norms of illumination determined as in § 252, or taken from a hand-book. Measure the actual illumination at a few points by means of the Marshall luminometer, or the Weber photometer, and compare it to the calculated intensity of illumination. Try a few different arrangements of lamps, with various shades and globes; compare critically the results obtained, in regard to efficiency, convenience, and the artistic appearance. Work in one room until a satisfactory solution of the problem is found. Then take a larger room, suitable for public gatherings, or to be used as a restaurant, a workshop, etc., and again work out the problem of its illumination. Good practical suggestions for lighting rooms may be found in the second part of a book on *Practical Illumination* by Cravath and Lansingh, beginning with Chapter XVI.

Report. Make drawings of the rooms, showing the most efficient location for the lamps; mark the kind of lamps and reflectors used. Give the measured illumination, in the form of curves shown in Fig. 224, as well as isophotal curves. Check the illumination by means of photometric curves (§ 244); compare the actual amount of light used with the values deduced in § 252.

254. EXPERIMENT 12-G. — Effect of Color of Illuminants. — This experiment is intended to be a demonstration, rather than an accurate test, the effect of color being purely subjective. Accurate investigations on the effect of color belong in the domain of spectrophotometry, which so far has not been applied for practical purposes. The question of color is of importance in dry-goods stores, in places where chemical reactions are watched by their color; also in luxurious residences, picture galleries, etc. Take a few pieces of a fabric, or a few bottles of some colored chemical, say red, and place them side by side in inclosures, each illuminated by a different kind of light. As such may be taken: a kerosene lamp, a gas burner with and without mantle, an acetylene burner, a Nernst lamp, and a few incandescent lamps with carbon, tantalum and tungsten filaments. If possible, use also an enclosed arc lamp, a luminous (flaming) arc, and a mercury-vapor lamp. Observe the differences in color of the test pieces; note the effect of a white and of a black background. Try to have the intensity of illumination as far as possible the same in all cases, by interposing opal screens, or smoked glass. Compare in this way samples of different color — yellow, green, blue, etc. — and see which color is distorted the most with each kind of illuminant. Describe your findings as definitely as possible.

CHAPTER XIII.

TRANSMISSION LINES.

255. ELECTRIC power is usually generated in places more or less remote from where it is used; therefore a *transmission* line is a necessary link between the generating station and the current-consuming devices. Where current is consumed in many places, as, for instance, in the various houses of a town, wires supplying power to separate houses constitute the *distributing* network, to which the transmission line is connected at suitable places. Strictly speaking, there is no marked distinction between a transmission line and a distributing line; the subdivision is introduced here merely to indicate simple lines with power consumption at the end only (transmission lines), and complicated networks of conductors with power tapped off at different places (distributing lines). A transmission line may be compared to a trunk railway, delivering goods at the end only; while distributing lines are analogous to a system of local branched railways, delivering goods at a great many places within a comparatively small area. Electrical properties of distributing lines are considered in Chapter XIV.

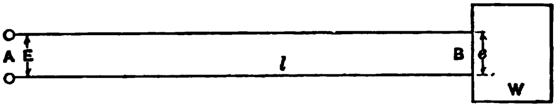

FIG. 229. A simple diagram of electric power transmission.

256. Formula for Voltage Drop. — The problem of electrical relations in a direct-current transmission line, in its simplest aspect, may be represented thus: W kw. (Fig. 229) are to be delivered at B, which is l miles distant from a power house situated at A; the pressure at B must equal e volts. The pressure E at A must be somewhat higher, to compensate for the loss in the line. It is required to compute the cross-section of the line conductors. This cross-section depends on the loss allowed in the line: the larger the permissible loss or drop of pressure, the smaller and less expensive becomes the line. There are, however, several limitations as to the allowable loss in the line; these are discussed in § 257.

Generally speaking, the problem is solved by merely applying Ohm's law to the given case; but for practical purposes this general law can be reduced to more specific expressions, which bring into prominence the relations between the voltage drop and the cross-section of the line. Let the resistance of a wire 1 foot long and having a cross-section of one circular mil be b ohms; then the resistance of a single wire l feet long and of a cross-section of q circular mils will be

$$\frac{bl}{q}$$

The line current $i = W \div e$, so that total voltage drop in the line is

$$\frac{2\,bli}{q} \text{ or } \frac{2\,bl\,W}{qe}$$

Let p be per cent drop, or the ratio of the voltage drop in the line to the receiver voltage e. From the above formula it follows that

$$p = \frac{200\,bl\,W}{qe^2}.$$

Two unknown quantities usually enter into the practical problems: the cross-section q of the wire, and the permissible voltage drop p. From the above formula the product

$$pq = \frac{200\,bl\,W}{e^2} = \frac{200\,bli}{e} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

can be figured out for any given case; the selection of the proper values for p and q is left to the sound judgement of the designer. In this selection he is guided by the three considerations discussed in the next article: voltage regulation, energy loss, and the limit of temperature rise. Here, as in most practical problems, an infinite variety of solutions are possible; engineering skill, feeling, and previous experience have to supply the lack of exact mathematical relations.

257. Practical Limits of Voltage Drop. — The principal considerations to be kept in mind when selecting the per cent drop p in the formula (1) are:

(1) *Voltage regulation at the receiver end.* The higher the loss in the line the more difficult it becomes to maintain constant voltage at B when the load varies. The drop in the line is proportional to the load, so that at full load the voltage at B may be considerably lower than that at A, while at light loads the two voltages are approximately equal to each other. As the switchboard attendant in the power house

cannot always follow the fluctuations of the load and adjust the voltage, the pressure at B varies with the load. If the voltage drop and fluctuations of pressure exceed a certain limit, the line gives a poor service, particularly for incandescent lamps.

The per cent difference in voltage (at the receiving end of the line) at full load and at no load is called *the regulation of the line.* Suppose the voltage at the receiving end to be 200 volts at full load, and assume the voltage drop in the line itself to be 14 volts; the generator voltage is then 214 volts. When the load is thrown off and the same generator voltage kept as before, the voltage at the receiving end rises to 214 volts. By definition, the regulation of the line is

$$\frac{214 - 200}{200} = 7 \text{ per cent.}$$

(2) *Energy loss in the line.* The higher the loss allowed in the line the smaller is the original investment in the line copper; but this is counter-balanced by a continual loss of energy in the line. If the price of coal is high, it may be cheaper in the end to use a large cross-section of conductors, in order not to lose too much energy as I^2R heat in the line. If cheap and abundant water-power is available, a smaller cross-section may be more economical. The most economical cross-section and loss in the line must be determined in each case separately, according to the local conditions.

(3) *Temperature rise in conductors.* Having determined the cross-section of the conductors so as to satisfy the two above-named conditions, it still remains to determine if the conductor selected can carry the current without excessive heating. The smaller the cross-section the less current the conductor can carry safely. The safe carrying capacity of conductors has been determined by experiments; the results are given in various engineering pocket-books, in Fire Underwriters' Rules, etc. If the conductor selected is too small for the current, one of the next larger sizes of wire must be taken, which can stand the current without excessive heating.

258. Factors Influencing Voltage Drop in Transmission Lines.—Formula (1) shows that the voltage drop p depends on five variables:

(1) Load W;
(2) Length l of the line;
(3) Its cross-section q;
(4) Transmission voltage e;
(5) Specific resistance b of the material.

The influence of each of these factors should be studied separately. The first two factors are considered in Experiment 13-A; the next two in Experiment 13-B. This subdivision is made because voltage drop *increases* with the load and with the length of the line (Fig. 231), and *decreases* with the increase in cross-section and the voltage (Fig. 232). The influence of the material may be investigated in connection with either experiment.

In alternating-current transmission lines, voltage drop depends, in addition, upon

(6) Power factor of the load;
(7) Inductance and capacity of the line itself.

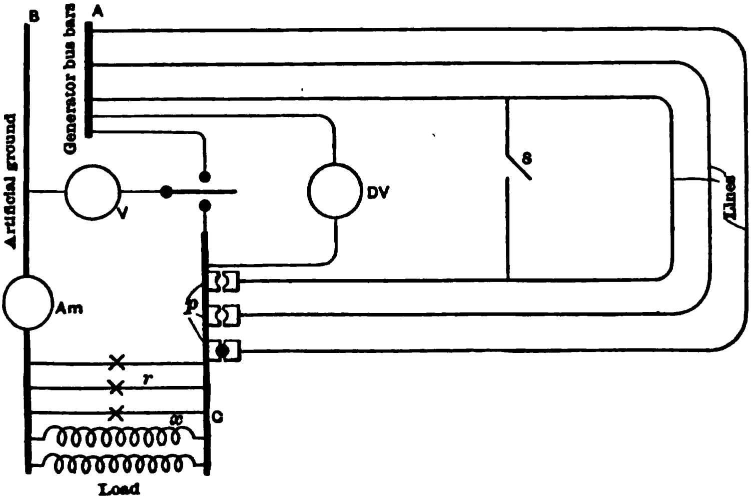

FIG. 230. A laboratory arrangement for studying electrical relations in transmission lines.

The influence of these two factors is studied in Experiments 13-D, 13-E, and 13-F.

259. Experimental Lines. — The practical import and the physical meaning of the deductions made in §§ 256 and 257 can be best studied on a few experimental lines strung in the laboratory. The materials used in actual lines are copper and aluminum. These should be replaced in the laboratory lines by iron and German silver wires which have a considerably higher resistance; this is necessary in order to obtain a sufficiently large voltage drop on comparatively short distances.

A convenient arrangement of experimental lines is shown in Fig. 230. The lines form a loop, so that the generating and the receiving end are brought together. The drop along the line is read directly on the low-scale voltmeter *DV*. One of the line wires is replaced by a bus-bar of negligible resistance (artificial ground), in order to simplify the measurements. Any desired number of lines may be provided between the generator and the receiver bus-bars, and each may be used by inserting the plug in one of the receptacles *p*. A voltmeter *V* is provided for measuring the pressure at the two ends of the line. The ammeter *Am* in the return bus-bar measures the line current. An indicating wattmeter may be connected into the circuit, if desired; its current winding must be in series with the ammeter, and the potential winding either across the load, or across the generator bus-bars.

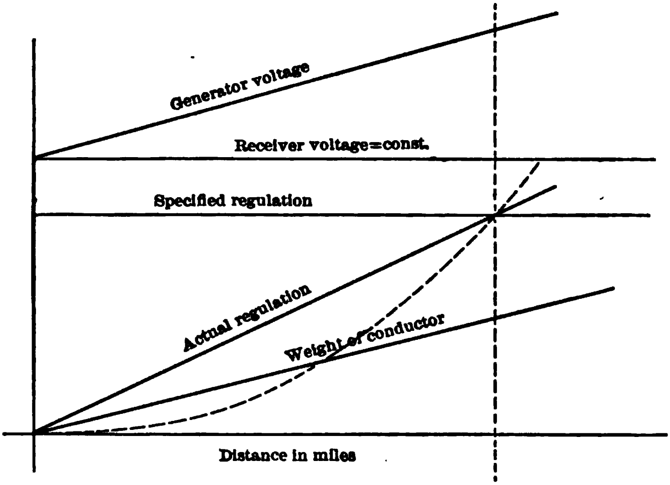

FIG. 231. Influence of distance of transmission on voltage regulation of a transmission line.

The voltage of the supply is regulated either by the field rheostat of the generator, or by a rheostat in series with the line. A short-circuiting switch *S* is provided for changing the length of one of the lines, if desired.

The influence of the seven factors enumerated in § 258 may be conveniently studied on this model.

260. Experiment 13-A. — Influence of Load and of Distance of Transmission on Voltage Regulation of a Line. — The purpose of the experiment is to verify some of the relations expressed by formula (1) in § 256. The experimental lines may be conveniently arranged as in Fig. 230; some of the results are plotted in Fig. 231.

The voltage at the receiver end is kept constant; the load is varied from zero to a practicable maximum. Read amperes, volts at both ends, and the voltage drop along the line on the low-scale voltmeter *DV*. Take two or three other lines of the same material and same cross-section, but of a different length; repeat the same test. Finally perform a similar test on a line of different material. Before leaving the laboratory measure the cross-section and the lengths of the lines experimented upon; ascertain the material of which they are made.

Report. Plot, for each line, to amperes load as abscissæ: voltages at the generator and the receiver ends; also, to a larger scale, the difference of these voltages, from the readings on the low-scale voltmeter. Plot per cent regulation as explained in § 257. Combine these curve sheets into one, as shown in Fig. 231, for the largest load used during the experiment. Take per cent regulation of the longest line and figure out the weights of shorter lines, which would give the same per cent regulation. The result is a parabola, shown by dotted lines. Explain that the weight, or the cost of the line conductor increases as the square of its length, with the same load and same per cent regulation. For one of the lines figure out *b* from formula (1); check the value obtained with that given for the same material in engineering pocket-books.

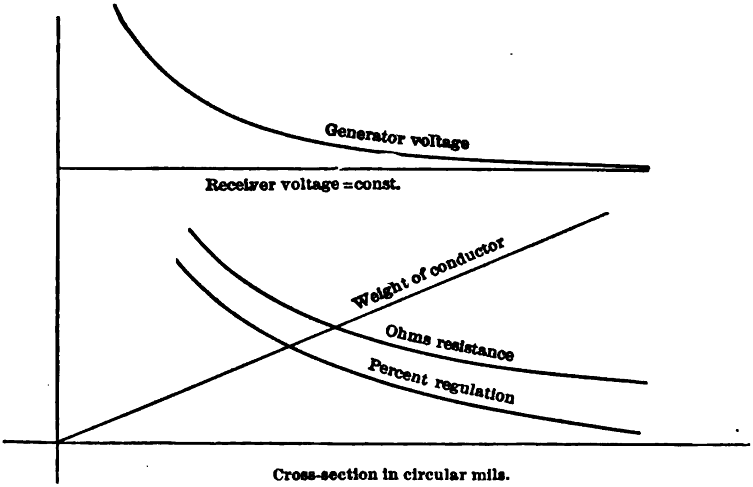

FIG. 232. Influence of cross-section of a transmission line on its voltage-regulation.

261. EXPERIMENT 13-B. — Influence of the Transmission Voltage and of the Cross-Section of a Line on its Regulation. — This experiment is similar to the preceding one, save that the transmission voltage and the cross-section of the line are varied, while the load and the length of the line are kept constant.

Of the several experimental lines of the same length (Fig. 230), take the one having the smallest cross-section, and put on a certain load at the lowest practicable voltage. Read volts, amperes and voltage drop. Gradually increase the voltage at the generating end; at the same time reduce the line current, so as to keep kilowatts load constant. Repeat the same experiment on two or three lines of different cross-section and material. A greater variety of cross-sections is obtained by using two or more lines in parallel. Before leaving the laboratory, measure the cross-sections and the lengths of the lines experimented upon, and inquire about the material of which they are made.

Report. Plot to receiver voltages as abscissæ: generator voltage, line drop, per cent regulation (§ 275), line current. Verify, that the product pe^2 is constant for each curve, according to formula (1).

Combine the curve sheets into one for a certain value of the receiver voltage, as shown in Fig. 232. Add curves of weight and resistance of the line. Check the value of b obtained from the test data with the value given for the same material in engineering pocket-books.

262. Branched Transmission Lines. — In many cases power generated in a power house has to be transmitted to several localities. If independent transmission lines are used for each locality, the problem is solved for each transmission line separately. Sometimes, however, it is more economical to use a common transmission line for at least a part of the way (Fig. 233), and then divide it into several branches. Fig. 4 represents a case where power generated at A is transmitted by a common line to D, and thence branched to towns B and C.

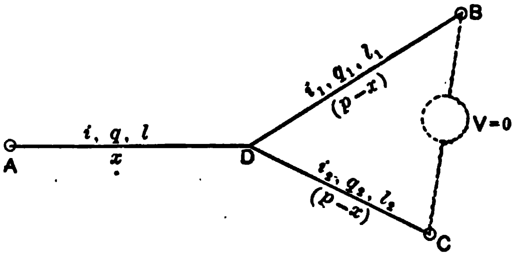

FIG. 233. Branched transmission line.

The problem here is to determine the cross-sections of the three parts of the transmission line, so that the weight of copper be a *minimum*, for a given total drop of voltage between A and C, or A and B. Assume as before the voltage at B (and at C as well) to be e, and the total drop between A and B to be p per cent, as before. The only unknown quantity is the per cent drop x between A and D; if this be known, the per cent drop $(p - x)$ in the branches DB and DC will also be known; each conductor can then be calculated separately by the fundamental formula (1) given above. The drop x can be calculated as follows: On the basis of formula (1) we have:

$$xq = \frac{200\ ibl}{e} \quad \dots\dots\dots\dots \quad (2)$$

also

$$(p - x)\ q_1 = \frac{200\ i_1bl_1}{e};$$

and

$$(p - x)\ q_2 = \frac{200\ i_2bl_2}{e}; \quad \dots\dots \quad (3)$$

The total weight of the line conductors is proportional to

$$P = ql + q_1l_1 + q_2l_2;$$

substituting the values for q, q_1, q_2 from the above equations, we obtain:

$$\frac{P}{200} = \frac{ibl^2}{ex} + \frac{i_1bl_1^2}{e\ (p - x)} + \frac{i_2bl_2^2}{e\ (p - x)}.$$

The weight is a minimum, when $\frac{dP}{dx} = 0$, or

$$-\frac{il^2}{x^2} + \frac{i_1l_1^2}{(p - x)^2} + \frac{i_2l_2^2}{(p - x)^2} = 0,$$

whence

$$\left(\frac{x}{p - x}\right)^2 = \frac{il^2}{i_1l_1^2 + i_2l_2^2}.$$

This equation gives

$$\frac{x}{p - x} = \sqrt{\frac{il^2}{i_1l_1^2 + i_2l_2^2}} = a;$$

and finally

$$x = p \cdot \frac{a}{1 + a} \quad \dots\dots\dots\dots \quad (4)$$

where a is introduced for the sake of brevity. The drop x is calculated from this equation; the cross-sections of the three parts of the line are figured out from formulæ (2) and (3).

In checking these relations experimentally it is absolutely essential that the drop $A - B$ be the same as $A - C$, since this condition was presupposed to be fulfilled in deducing the rule of copper economy. In order to be sure that this is actually fulfilled during the test, it is advisable to have a low-reading voltmeter connected between B and C (Fig. 233); the load is adjusted so that this voltmeter reads zero. Or else, a low-resistance wire can be connected between B and C, so as to equalize the voltages.

263. EXPERIMENT 13-C. — Rule of Minimum Copper in Branched Transmission Lines. — The experiment is intended to illustrate the relations derived in the preceding article. Several experimental lines, or ordinary resistances of uniform material, should be provided in the laboratory, and connected as in Fig. 233.

(a) Take a combination of three lines, and by trials so adjust the load in both branches that the condition (4) is approximately fulfilled. Vary the load in the two branches so, that the total drop p remains constant. Read volts, amperes and drops p and x. Note the value of q and l in the three branches. (b) Take another set of readings, keeping the total current i constant, but varying i_1 and i_2. (c) Repeat the same experiment under different conditions, viz., different lengths, different cross-sections of the lines, etc.

Report. Show from the first series of readings, that when (4) is fulfilled, maximum amount of power is delivered through a given line with the same drop p. From the second set of readings prove, that a given amount of power is delivered with the minimum drop in the line, when (4) is fulfilled.

ALTERNATING-CURRENT TRANSMISSION LINES.

264. The electrical relations deduced in § 256 for direct-current lines are modified in alternating-current lines by the influence of inductance and capacity, either in the load or in the line itself.* These factors will be taken up in detail in the following articles. All deductions that are made there, and the experiments specified, are for single-phase lines. The results may be applied to two- and three-phase lines, keeping in mind the following relations, deduced in Chapter XX.

(a) A *two-phase line* can be considered as a combination of two independent single-phase lines, AA' and BB', each carrying one half of the load (Fig. 234). The cross-section of each conductor is only one half of what it would be for a single-phase line; but as there are four conductors instead of two, the total weight of the line is the same as with the single-phase line.

(b) A *three-phase line* can be considered as a combination of two lines, say AB and CB (Fig. 235), the conductor B being their common return wire. Therefore A and C need only have one half the cross-section of a single-phase line carrying the same total load W. The wire B must have the same cross-section as either A or C, because (in a three-phase system) the return current is equal to each of the line currents, and any of the three wires can be considered as a return wire. From this, it follows that total weight of the conductors in a three-phase line is but 75 per cent of the weight of an *equivalent* single-phase line.

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* The following treatment presupposes on the part of the reader a knowledge of the fundamental relations established in Chapters V and VI.

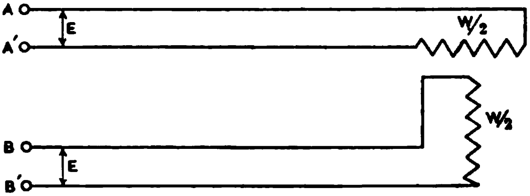

FIG. 234. A two-phase power transmission.

Thus, whether you have a three-phase or a two-phase line, figure the cross-section of the conductors, as if it were a single-phase line having the same total load W, the same distance of transmission and the same permissible voltage drop. Then, for polyphase transmission use wires of *one half* cross-section, — four wires for the two-phase line, three wires for the three-phase line (see also § 443).

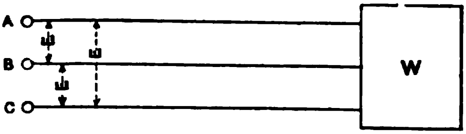

FIG. 235. A three-phase power transmission.

265. Influence of the Power Factor of the Load. — Assume first that the line has ohmic resistance only, but no inductance or capacity; the load, however, is partly inductive (§ 96). The relations expressed by the formula (1) for direct current may be represented here by the triangle OAB (Fig. 236). Let OB be the vector of the receiver voltage e, OC that of the current i, lagging behind the voltage by an angle ϕ, which depends on the properties of the load. The voltage drop ir in the line can be represented by a line BA parallel to the direction of the current vector OC; r is the total ohmic resistance of the line. The geometrical sum of OB and BA is equal to OA, and represents the voltage E in the power house. It will be seen from this diagram that the voltage drop, AB, in the line is subtracted *geometrically* from the station voltage E, instead of algebraically, as in the case of direct currents. From the triangle OAB, we have

$$E^2 = e^2 + i^2r^2 + 2\,ire \text{ Cos. } \phi.$$

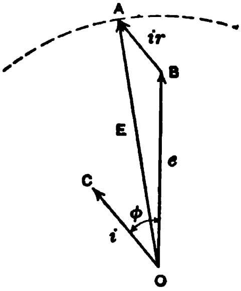

FIG. 236. Voltage drop in a transmission line, at a partly inductive load.

This formula, or the diagram, Fig. 236, takes the place of the simpler relations for direct-current lines. Suppose, for instance, that it is desired to figure the cross-section of a transmission line, under the condition that the power W is consumed at a given power factor $\operatorname{Cos} \phi$ and at a given voltage e; the voltage drop in the line must not exceed p per cent of e. First determine the line current $i = \frac{W}{e \operatorname{Cos} \phi}$; then construct OB and OC, and from O strike an arc of a circle with a radius $E = (1 + p / 100)\, e$. Draw the line BA parallel to OC; the point A of its intersection with the arc gives the length $BA = ir$. Knowing i, the resistance r of the line can thus be determined, and the cross-section of the conductor found from the familiar formula

$$r = \frac{2bl}{q}.$$

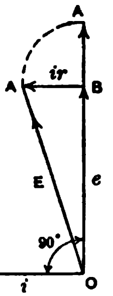

FIG. 237. Voltage drop in a transmission line at a totally inductive load.

It will be seen from Fig. 236 that in case of an inductive load, per cent voltage drop $AB \div OB$ and per cent regulation $\frac{(OA - OB)}{OB}$ are different. Take, for instance, the extreme case of a purely inductive load (Fig. 237), and suppose that the line drop BA is equal to 30 per cent of OB. Then the voltage E in the power house

$$\begin{aligned} OA &= \sqrt{e^2 + (0.30\, e)^2} \\ &= e\sqrt{1.09} = 1.044\, e; \end{aligned}$$

in other words, the difference between E and e, or the regulation, is 4.4 per cent, while the line drop is 30 per cent. If the load were non-inductive the voltage E should be 30 per cent higher than e, as shown by the vector OA'.

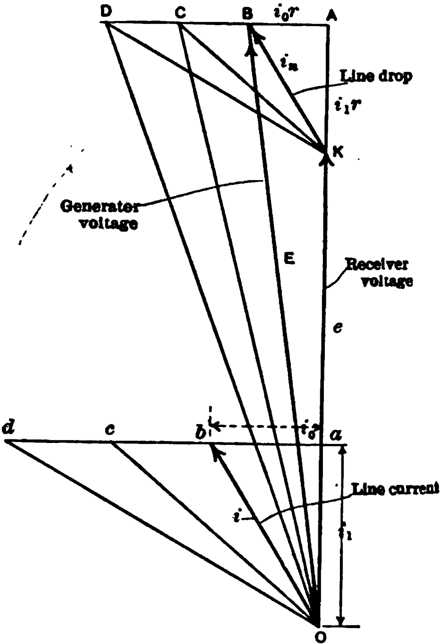

FIG. 238. Influence of the power-factor of the load on the voltage drop in a transmission line.

This should not be understood, however, in the sense that the lower the power factor the more favorable are the conditions for transmitting power. On the contrary, a lower power factor means a higher current *with the same power transmitted;* therefore a larger line drop and a larger I^2R loss; this case is taken up in the next article.

266. Constant Power Transmitted at Different Values of Power Factor. — A diagram corresponding to this case is shown in Fig. 238: the curves in Fig. 239 illustrate the result. $Ob = i$ again represents the line current. It consists of the working (or power) component $Oa = i_1$, which is in phase with the voltage $OK = e$ at the terminals of the load, and of the wattless component $ab = i_0$ perpendicular to OK. The vector $KB = ir$ of the voltage drop in the line is parallel to i; OB represents the generator voltage E.

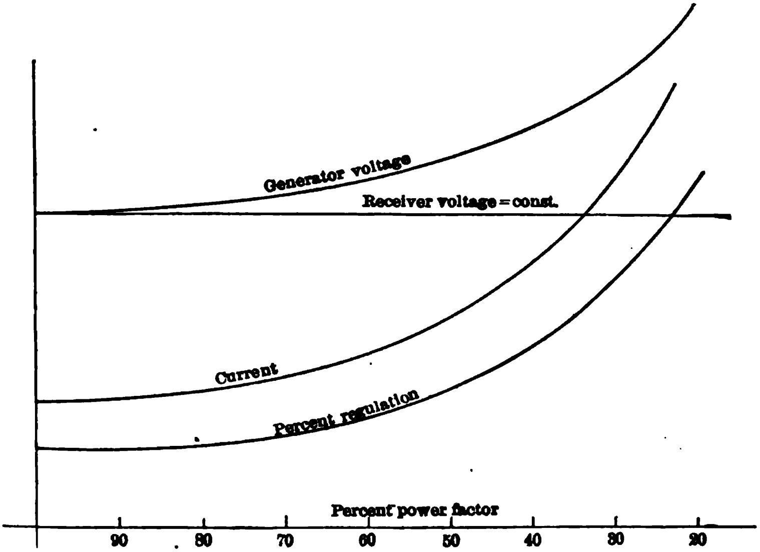

FIG. 239. Curves showing the effect of a varying power factor of the load on the regulation of a transmission line.

If the power delivered to the load is constant, the working component $Oa = i_1$ is constant. Changes in total current i are caused by variations in the wattless component, which becomes, successively, ac, ad, . . as the power factor of the load decreases. The corresponding voltage drop in the line also increases, and assumes the values KC, KD. The generator voltage becomes OC, OD, . . . At a power factor of 100 per cent, total current i is equal to the working component i_1; line drop is represented by KA, the generator voltage $= OA$.

The values of OB, OK and Ob are plotted in the form of curves in Fig. 239; the regulation curve gives per cent increase of OB over OK.

267. EXPERIMENT 13-D. — Influence of the Power Factor of the Load on Voltage Regulation of a Transmission Line. — The experiment may be arranged as in Fig. 230, or as in Fig. 240; in the latter figure the reactance x in the line should be omitted. Iron wires should not be used for this experiment, as they give an appreciable reactance effect, which destroys the value of the results. Use copper or German silver wires.

(a) Apply a certain load at as low a power factor as possible. Read volts at both ends of the line, and, if possible, the voltage drop along the line itself; also load amperes and watts. *Keep watts constant,* and gradually reduce the wattless current by decreasing the reactance of the load connected across the line, until a power factor of 100 per cent is reached. This condition is practically obtained when the load consists of incandescent lamps only. Repeat a similar run with a different resistance of the line, and then with a different load.

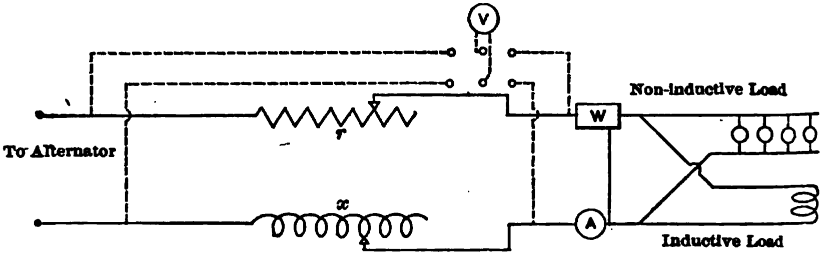

FIG. 240. Diagram of connections for studying the effect of inductance and of resistance in the transmission line and in the load.

(b) Now reëstablish the first conditions and vary the load *so as to keep total amperes constant,* gradually increasing watts. Repeat a similar run with a different resistance of the line, and then with a different load.

Report. Plot curves, as in Fig. 239; check some of the results by constructing diagrams, similar to those shown in Figs. 236 and 238.

268. Influence of Inductance in the Line. — Inductance in transmission lines is caused by the magnetic flux embraced within the loop formed by line conductors. The influence of this inductance is noticeable on long, high-tension lines with considerable distance between the wires. Inductance and ohmic resistance are distributed along the line; but for experimental purposes it is more convenient to have them separated, as in Fig. 240.

The voltage relations, shown in Fig. 236, must be modified into those in Fig. 241, in order to take into account the inductance of the line.

$OK = e$ is the receiver voltage, i is the line current, lagging behind it by an angle ϕ, which depends on the properties of the load. The drop KL in the line consists of the ohmic drop $KM = ir$, parallel to i, and of the inductive drop $ML = ix$ perpendicular to i. The generator voltage E is represented by the vector OL.

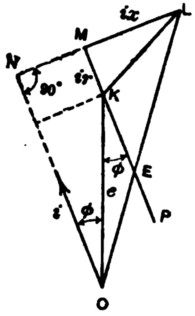

FIG. 241. Vector diagram corresponding to the connections in Fig. 240.

With lines having but a few per cent drop, the triangle KLM becomes too small to allow the difference between E and e to be measured with a sufficient accuracy. In such cases it is preferable to figure out E analytically. By completing the triangle OLN we have:

$$E^2 = (e \operatorname{Cos} \phi + ir)^2 + (e \operatorname{Sin} \phi + ix)^2 \quad . \quad . \quad . \quad . \quad (5)$$

from which E can be calculated.

It will be seen from Fig. 241 that per cent regulation, or the difference between E and e, depends not only on the current, but also on the power factor of the load. It may be shown by constructing a few diagrams, that a line with a comparatively high inductance may regulate satisfactorily on non-inductive loads, but give a poor regulation on inductive loads, and vice versa (see Fig. 237).

269. EXPERIMENT 13-E. — Voltage Drop in Lines having a Considerable Inductance. — (a) Begin the experiment (Fig. 240) with a line having negligible ohmic resistance, but a considerable reactance. The latter is conveniently provided for laboratory purposes by a coil of wire, with a movable iron core inside. Take a highly inductive load and adjust the conditions so as to have a difference of from 10 to 15 per cent between the generator voltage and the load voltage. Keep the current constant, and gradually raise the power factor of the load by increasing the non-inductive portion and decreasing the inductive portion of the load. Read the voltages and the voltage drop, amperes, and watts, until the load becomes entirely non-inductive. (b) Repeat the same experiment with a line having ohmic resistance only and giving the same drop. (c) Finally test a line possessing both ohmic and inductive resistances in about equal proportion.

Report. Plot the results to power factor of the load as abscissæ (compare Fig. 239). Construct a few diagrams, as in Fig. 241, for low values of power factor, and check by means of them the results obtained at higher values of power factor. Explain the difference in regulation obtained with the lines having large and small inductive drop, at different values of power factor.

270. Influence of Capacity in Transmission Lines.* — Let *I* in Fig. 242 represent the generator end, *II* the receiver end of a transmission line, and *C* its electrostatic capacity. This capacity in reality is uniformly distributed along the line, but, for the sake of simplicity, may be assumed to be concentrated at one point. The generator has to supply not only the load current, but also a leading wattless component, or charging current. This capacity effect takes place independently of the load. There are cases known on long high-tension transmission lines, where this capacity effect amounts to over 1000 apparent kilowatts; this means that a large generator has to run at practically its full ampere output, without supplying any actual load at the receiver end.

Inductance, as a rule, is an undesirable element of the load, because it produces a lagging wattless current and lowers the power factor.

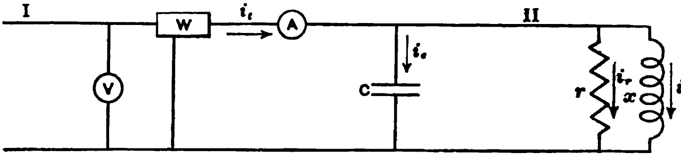

FIG. 242. Capacity, inductance, and resistance in parallel.

However, in the case under consideration, inductance *improves* the power factor at the generating station, since the lagging component compensates for the leading capacity current (§ 132).

This principle also furnishes a practical means of obtaining this compensating effect when the line is comparatively short and the voltage not very high, but the load is highly inductive (motors and transformers running at light load). Then the capacity effect is small, while at the same time the line carries considerable lagging currents. The power factor can be improved by connecting across the line an apparatus which draws a leading current, and consequently neutralizes the lagging component. Ordinary condensers are too expensive, and not reliable enough on high voltages, to be used for such a purpose. A large over-excited synchronous motor (§ 470) is quite often used for correcting low power factor on a transmission line. It takes in a leading current, and thus compensates for the lagging currents of the rest of the system. The generating station has to supply the power component of the current only. For further details in regard to current and voltage resonance, see §§ 131 to 136.

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* For the fundamental properties of electrostatic capacity the reader is referred to Chapter VI.

271. EXPERIMENT 13-F. — Influence of Capacity in Transmission Lines. — (a) Take a line, as in Fig. 242 (or Fig. 230), possessing ohmic resistance only; connect some capacity across it at the middle point between the generator and the load. Select a load at a power factor, say between 60 and 80 per cent. Gradually increase the capacity, keeping the load constant. Read load current, capacity current and total current; generator volts, load volts, and the volts across the condensers. When varying the load be careful not to obtain conditions which might give a dangerous current or voltage resonance. Repeat a similar run with the same load current, but at a power factor of 100 per cent; then at a power factor as low as obtainable.

Adjust conditions so that the influence of capacity is particularly noticeable, and distribute the same capacity in two, three or four points along the line, instead of having it concentrated at one point. See how this affects the regulation of the line.

(b) Perform a similar experiment with a line having inductance, instead of resistance. Produce voltage resonance (§ 134) under conditions which may occur on actual transmission lines. In doing this be sure to have the line protected by a circuit-breaker or a fuse.

Report the observed relations in the form of curves, and explain them by means of vector diagrams.

VOLTMETER LINE-DROP COMPENSATORS.

272. It is desirable in any system of electrical transmission to be able to read, on a voltmeter placed in the generating station, the actual voltage at the point of consumption of energy. The simplest and the oldest device for this purpose is a pair of *pilot wires* between the point of consumption and the power house. A voltmeter situated in the power station, and connected to these wires, reads the voltage at the farther end of the line, and the generator pressure is regulated accordingly.

As the distance of transmission increases, the cost of pilot wires becomes prohibitive. Devices are used in such cases, that enable the operator in a power house to read the voltage at the receiving end of the line, without pilot wires. Some types of line-drop compensating devices are described below.

273. Direct-Current Compensator. — A simple arrangement of this kind is shown in Fig. 243. The station voltmeter, in addition to its regular winding, has a compensating winding connected in series with the line. This series winding opposes the action of the potential winding, so that the voltmeter shows less, the heavier the line is loaded. The decrease in reading is proportional to the line current; the drop in the line is also proportional to the line current. Therefore, by suitably adjusting the number of turns on the compensating winding, or by providing a shunt around it, the instrument may be made to indicate the voltage at the receiving end of the line, at any load.

Such a compensating winding may be adapted to almost any commercial type of direct- or alternating-current voltmeter; but the principle is not quite correct on alternating-current lines, because the drop depends in this case not only on the value of the current, but also on the power factor of the load. The arrangement described below takes this factor into account, and is therefore preferable on alternating-current lines.

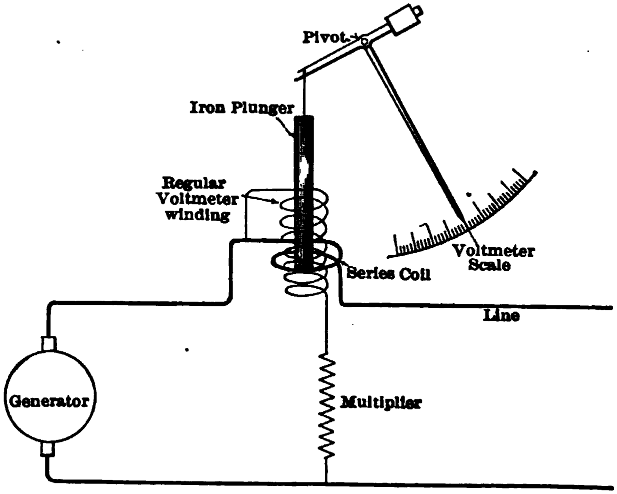

FIG. 243. A voltmeter compensated for line drop, and intended primarily for direct-current circuits.

274. Mershon Alternating-Current Line-Drop Compensator.— The total voltage drop in a transmission line is caused by its resistance and reactance, and is proportional to the current. Therefore, by inserting a certain resistance R and inductance L into the voltmeter circuit (Fig. 244), and making the voltage drop across these proportional to the line current, conditions are created in the *local* circuit, similar to those in the actual transmission line. Consequently, voltmeter readings may be reduced by the amount of the line drop. With this arrangement, the voltmeter, though placed in the power house, automatically reads the voltage at the receiving end of the line, at any value of the current and of the power factor. *V* is an ordinary station voltmeter connected to the line through the potential transformer *P*; if the voltage is sufficiently low, this transformer may be omitted. The resistance *R* and the inductance *L* are connected in such a way that the current passing through them is at any moment proportional to and in phase with the line current. This is obtained by the use of two series transformers, *C* and *K*, the first of which carries in its primary the whole line current. A part of the circuit *R-L* is introduced into the voltmeter circuit, as shown in the figure. The points, where the connections are made, are adjustable, so as to be able to compensate for a definite per cent reactive and ohmic drop.

The voltage at the voltmeter terminals is a geometrical sum of the generator voltage, as supplied by the transformer *P*, and of the voltage

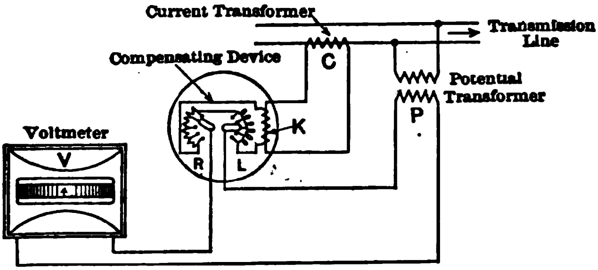

FIG. 244. The Mershon arrangement for compensating a voltmeter for line drop; the method can be used on alternating-current circuits only.

drop across the compensator. The latter voltage is proportional to the line current, and has the same inductive and ohmic components as the actual line drop, provided that the compensator is adjusted correctly. Therefore, the voltmeter shows the generator voltage, less the drop in the line; in other words, the voltage at the receiver end.

The purpose of the series transformer *C* is to make possible the use of the same compensator on lines having widely different current output. The ratio of turns of this transformer is selected so as to have about 4 amperes in the secondary circuit at full load. The purpose of the transformer *K* is twofold: it permits of giving the resistance *R* and the reactance *L* values most suitable for the voltmeter circuit, and insulates the voltmeter circuit from that of the transformer *C*. In this way it is possible to use the transformer *C* not only for the compensator, but also for the ammeter and the series coil of the wattmeter.

275. EXPERIMENT 13-G. — Calibrating Line-Drop Compensators. — (a) Connect a compensated voltmeter to a direct-current line, as shown in Fig. 243. Load the line and calibrate the device by actually reading the voltage at both ends of the line. The adjustment is conveniently made by a shunt around the compensating coil. Adjust the instrument on a heavy load, and see if it reads correctly on partial loads.

(b) Connect the line to an alternating-current supply, and see if the adjustment of the device is still correct with non-inductive loads. If not, adjust it to read correctly; determine per cent error at other values of power factor. After this, take a line having both ohmic and inductive resistances. Adjust the compensator to read correctly at full load and at a power factor of about 80 per cent. Calibrate the instrument within practical values of power factor, say from 50 to 100 per cent, and from half load to load and a quarter.

(c) Connect the compensator shown in Fig. 244 and check its calibration separately for ohmic drop and for inductive drop; then try it on a line having both ohmic and inductive resistances. Compare its indications with voltmeter readings taken at the receiving end of the line, at different values of load and of power factor.

PREDETERMINATION OF REGULATION OF TRANSMISSION LINES.

276. In many cases it is difficult, or even impossible, actually to measure the voltage drop or regulation of a transmission line at its rated load. A method has been devised which permits the calculation of the regulation of a line at any load and power factor, from a simple short-circuit test.

With the same notations as in § 268 the voltage relations at a certain load are represented by Fig. 241. The unknown part of the diagram is the triangle KML, or, with a given current i, the unknown quantities are: the ohmic resistance r and the reactance x of the line. Both may be determined experimentally in the following way: short-circuit the line at the points C and D (Fig. 245) and connect the points A and B to a source of direct current. Read volts and amperes; the ratio will give the ohmic resistance r of the line. Now apply an alternating current, instead of the direct current; again measure the current and the

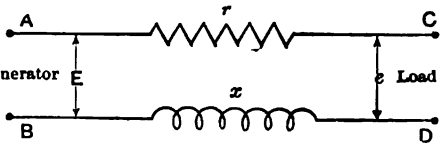

FIG. 245. Determination of impedance of a transmission line from a short-circuit test.

voltage. The ratio will give the impedance of the line. From the data thus obtained, the triangle of drop, shown in Fig. 246, may be constructed, and the reactance x calculated. The values of r and x are then used in constructing the diagram, Fig. 241, for any desired conditions of load.

277. Kapp's Diagram. — It is not necessary to construct the diagram, shown in Fig. 241, for each value of power factor or current. The triangle MKL remains the same at any value of the power factor as long as the current remains constant. If the generator voltage OL is constant, the point O travels on a semicircle having L for its center. This gives the so-called Kapp's diagram, shown in Fig. 326; it is the same for transformers, as for transmission lines, and with some limitations is even applicable to alternators.

The triangle MKL is denoted there by OKC; the generator voltage is represented by EC. OI is the direction of the current vector; the receiver voltage is $= EO$. With non-inductive load the receiver voltage is represented by the vector OE_r; at purely inductive load the receiver voltage is OE_x. Should the line draw a leading current, because of its capacity, or of an over-excited synchronous motor, the receiver voltage becomes higher than the generator voltage. The theoretical limit of a pure capacity load corresponds to the vector OE_c. This rise in voltage is explained by a partial resonance between the reactance of the line and the capacity of the load (see § 134).

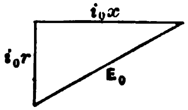

FIG. 246. Triangle of voltages from a short-circuit test.

For other values of the current, the triangle OKC is proportionally reduced in size; the remainder of the diagram is unchanged If the receiver voltage e, and not the generator voltage E, is maintained constant a semicircle is drawn from the point O as a center. Generator voltages are in this case represented by variable vectors from the point C.

Charts and tables have been constructed, which give numerical values of the quantities entering into diagram, Fig. 241. With these tables and charts, voltage drop and regulation can be obtained directly, without constructing a diagram. See articles on the subject in the *Electric Journal*, March and April, 1907; also F. G. Baum's booklet, *Alternating-Current Calculating Device.*

278. Analytical Calculation of Voltage Drop. —When voltage drop in a line is slight, the difference $E - e$ cannot be obtained with sufficient accuracy from a diagram. It is better in this case, to calculate it analytically. Denoting the angle NOL in Fig. 241 by α we have

$$\left\{ \begin{aligned} E \operatorname{Cos} \alpha &= e \operatorname{Cos} \phi + ir; \\ E \operatorname{Sin} \alpha &= e \operatorname{Sin} \phi + ix. \end{aligned} \right.$$

For a given current i and a phase angle ϕ the angle α may be eliminated from these two equations, and the resultant equation solved for e, or E, whichever is unknown. The solution is simple, when E is unknown; both equations are squared and added together. This gives

$$E^2 = (e \operatorname{Cos} \phi + ir)^2 + (e \operatorname{Sin} \phi + ix)^2 \quad . \quad . \quad (5)$$

which is identical with the formula (5) given in § 268. But when e is unknown, the solution becomes more complicated. The following approximate method of solving it is sufficiently accurate for practical purposes, except when the drop in the line is excessive.

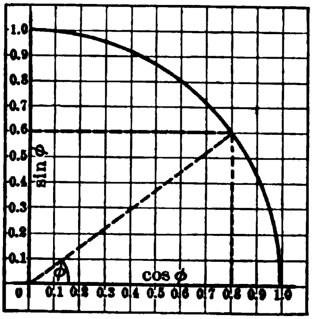

FIG. 247. A diagram giving the values of Sin ϕ and of the angle ϕ for any value of power-factor Cos ϕ.

The difference $E - e$ (Fig. 241) is obtained by striking an arc of a circle with OK as a radius, from O as a center. Instead of this circle, draw a perpendicular from K to its intersection with OL; when the triangle KLM is small, this perpendicular practically coincides with the arc of the circle.

Then we get from the diagram,

$$E - e = ir \operatorname{Cos} \phi + ix \operatorname{Sin} \phi,$$

from which the unknown e may be calculated. Let, for instance, the ohmic drop ir be 5 per cent of E, the inductive drop $ix = 7$ per cent of E. To determine the total line drop at a power factor of 80 per cent, we have (Fig. 247): Cos $\phi = 0.80$; Sin $\phi = 0.60$, and

$$E - e = 5 \times 0.80 + 7 \times 0.60 = 8.2 \text{ per cent.}$$

In other words, the arithmetical difference between the generator voltage E and the receiver voltage e is $= 8.2$ per cent of E. The actual voltage drop in the line

$$KL = \sqrt{5^2 + 7^2} = 8.6 \text{ per cent of } E.$$

279. EXPERIMENT 13-H. — Predetermination of Regulation of a Transmission Line. — The method is explained in § 276. Short-circuit the terminals at one end of an experimental line containing some ohmic and inductive resistance (Fig. 240). Send a direct current through the line, gradually increasing it to the safe limit: read volts and amperes. Repeat the same experiment with alternating current. This comprises all the necessary experimental data.

In order to have a check on the predetermined regulation, it is well to take a few readings with the line actually loaded, preferably with considerable currents, in order to have a large drop. Take a curve of voltage drop, at a current corresponding to 50 per cent overload, between as wide limits of power factor as possible.

Report: From the short-circuit test figure out the resistance and the reactance of the line (Fig. 246). Construct Kapp's diagram for a current for which the regulation was determined experimentally; check a few calculated voltages with those taken experimentally. Figure out a few points by using the charts mentioned at the end of § 277, and the formulæ deduced in § 278. See how closely Kapp's diagram checks with them.

CHAPTER XIV.

DISTRIBUTING LINES.

280. It was explained in § 255 that the principal distinction between a distributing line and a transmission line is, that the latter is loaded at the end only, while a distributing line has power led off at different points (Fig. 248). Only one side of the line (say positive) is shown in this and in the following two sketches; the relations in the negative, or the return wire are exactly the same.

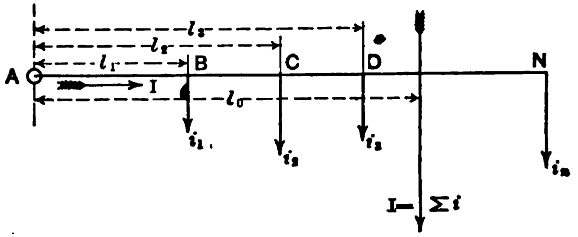

Fig. 248. A distributing line fed from one end.

The principal types of distributing lines met with in practice are:

(1) Lines fed from one end (Fig. 248).
(2) Lines fed from both ends (Fig. 249).
(3) Ring lines (Fig. 250).

A ring-conductor is evidently but a special case of a line fed from two ends. City networks are usually laid out so that street conductors are fed, if possible, from at least two points, as shown in Fig. 249. In case of an accident to one of the feeders the customers may still be

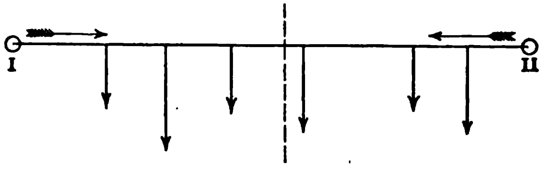

Fig. 249. A distributing line fed from both ends.

temporarily supplied from the other feeder, even though the service be not as satisfactory, because of a greater voltage drop.

281. Voltage Drop with Distributed Load.—The voltage drop in a line fed from one end (Fig. 248) may be calculated on the basis of formula (1) given in § 256. With the same notation as there we have for the drop to the farthest consumer N:

$$\frac{pe}{100} = \frac{I.b.2l_1}{q} + \frac{(I - i_1).b.2\,(l_2 - l_1)}{q} + \frac{(I - i_1 - i_2).b.2\,(l_3 - l_2)}{q} + \dots\dots$$

The first term on the right side of this equation represents the voltage drop across AB, the second across BC, etc. In this formula i_1, i_2, i_3, etc., represent the currents taken by various consumers; the current

$$I = i_1 + i_2 + i_3 + \dots\dots$$

is the total current supplied from the feeder. After simple mathematical transformations the above formula is reduced to

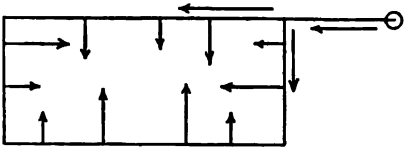

FIG. 250. A ring line.

$$\frac{pe}{100} = \frac{2b}{q}\,(i_1l_1 + i_2l_2 + i_3l_3 + \dots)$$

of finally to

$$pq = \frac{200\,b\,\Sigma\,il}{e} \quad \dots\dots\dots\dots\dots \quad (5)$$

This formula is perfectly analogous to the expression (1) in § 256, and like it is used for determining the cross-section of a conductor, when the load is given; or for determining the load that a given conductor can carry, under the condition that the voltage drop should not exceed a certain limit.

The expression (5) shows that the voltage drop in a conductor, loaded at several places, depends not only on amperes load, but on *the position of the load* as well. A small load situated far from the feeder A may affect the line drop quite as much as a comparatively heavy current taken near the feeder. In other words, *voltage drop with distributed load is proportional, not to amperes, but to ampere-feet.*

It is customary in practice to measure load by the number of 16 candle-power lamps, instead of amperes. To show the application of formula (5), suppose that it is required to figure the sizes of street conductors in a city, the maximum drop allowed being 2 per cent, and the pressure 100 volts. For copper conductors b = about 10 ohms; a 100-volt 16 candle-power lamp consumes about 0.5 ampere, and we have

$$q = \frac{200 \times 10 \times 0.5 \Sigma Ll}{100 \, . \, 2},$$

or

$$q = 5 \Sigma Ll,$$

where number of lamps L is introduced instead of amperes i. Knowing the number of lamps to be connected at different places along a certain conductor, its cross-section in circular mils can be easily found from this formula.

The expression Σil may be represented in the form $Il_0 = l_0 \Sigma i$, where l_0 is the distance from A to the "center of gravity" of the load (Fig. 248), considering i_1, i_2, i_3, etc., as if they were mechanical forces, applied at the distances l_1, l_2, l_3, etc.; I is the total load of the line. With this substitution, formula (5) becomes:

$$pq = \frac{200 b . Il}{e} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (6)$$

282. Experimental Lines.

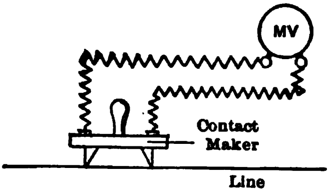

FIG. 251. A contact-maker for measuring currents in a wire.

282. Experimental Lines. — Experimental lines, for studying voltage and current relations with distributed load, are preferably made of German silver, manganin, or any other material of a comparatively high specific resistance, and a low temperature coefficient. Incandescent lamps of 4, 8, 16, and 32 candle-power may be conveniently used as a load. It is simpler and preferable to have only one side of the line made of high-resistance material, the return wire being replaced by a heavy copper wire or a bar of a negligible resistance, as in Fig. 230.

It would not be feasible to have ammeters or even ammeter shunts connected in different places of the line, as this would change the uniform resistance of the line per unit length. It is better to use parts of the line itself as ammeter shunts, the drop being taken by a contact-maker, shown in Fig. 251. The milli-voltmeter MV, connected to the knife-edges of the contact-maker, measures a voltage drop which is proportional to the line current, and thus, if the proper constant be applied, measures the current itself. The distance between the contacts may be made adjustable, so as to give this constant a simple value.

Two voltmeters should be used, one for keeping the terminal voltage constant, another with a three-volt scale for measuring the voltage drop along the line. One double-scale voltmeter may be substituted for the two instruments, and a double-throw switch provided for using either scale. The return wire of the experimental line should be made inaccessible, so that the student could not, by mistake, touch it with one of the voltmeter leads connected to the three-volt scale.

283. EXPERIMENT 14-A. — Voltage Drop in Distributing Lines Fed from one End. — The arrangement of an experimental line is described in § 282; the formula for drop is given in § 281. (1) Connect to the line a certain distributed load, as in Fig. 248, and trace out the voltage drop along the line with the three-volt voltmeter. Using the contact-maker, measure the currents taken by separate consumers; as a check measure the currents in various parts of the line itself. Repeat the same experiment with a different distribution of load. (2) Connect to the line a consumer near the source of power, and another one as far from it as possible. Measure the voltage which both of these consumers get, as other lamps are gradually connected to the line. This should illustrate the fact that the consumer at the end of the line is affected by a fluctuating load much more than the consumer at the beginning of the line. (3) Investigate the influence of the factor $\Sigma\, il$ on voltage drop to the consumer at the end of the line. Connect to the line a certain number of lamps near the feeding end, and then the same number of lamps near the opposite end of the line. Measure, in both cases, the voltage at the terminals of the last consumer. It will be found that the same number of lamps affect his voltage differently in the two cases. Now change the number of lamps in one of the groups until he gets the same drop in both cases. Verify, that in this case the number of ampere-feet ($\Sigma\, il$) is the same in both groups of lamps. (4) Before leaving the laboratory, measure the resistance of the line wire per foot of its length, using the drop-of-potential method. Make a sketch of the line with distances between the connections to the consumers; measure the distance between the knife-edges of the contact-maker.

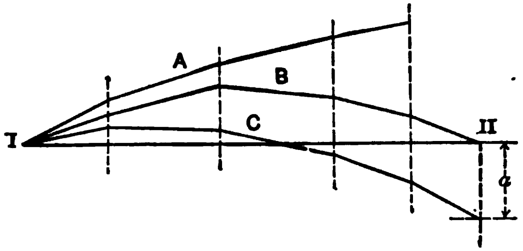

FIG. 252. Voltage drop in lines fed from one end and from both ends.

Report. Plot to conductor lengths as abscissæ voltage drop as shown by the curve *A* in Fig. 252. *I* is the feeding-point corresponding to the point *A* in Fig. 248; the dotted lines indicate the places of connections to the customers. Check the observed results by calculating the drop from the known values of the line resistance and the currents. Give and explain the results showing that voltage fluctuations are most pronounced at the consumers remote from the feeding-point. Show from the observed data that the voltage drop is the same with different distribution of the load, as long as Σil has the same value.

284. Line Fed from Both Ends. — When a conductor is fed from two ends (Figs. 249 and 250) there is always a point of division, on one side of which the customers are supplied with current from one feeder, on the other side from the other feeder. The position of this point depends on the instantaneous load, and is always such that Σil is the same on each side of the point of division. This follows immediately from formula (5), because the conductor may be cut in two at the point of division of the load, and each half considered as a line fed from one end. The condition must then be fulfilled that the drop from both feeding points is the same; the drop being proportional to the value of Σil, we have

$$(\Sigma il)_{\text{I}} = (\Sigma il)_{\text{II}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

In figuring out the cross-section of a line fed from two ends, all the consumers are supposed to be connected simultaneously, and the point of division determined under these most unfavorable conditions. Then the conductor is assumed cut in two at this point and the cross-section figured out from formula (5).

In some cases the pressure is different at the two feeding-points, because the feeders themselves may have different cross-sections, different lengths, and carry different loads. In such cases, the feeder with a higher voltage carries a proportionately larger part of the load. The above rule, that the point of division of the load is determined by the expression (7), is only true when the same voltage is maintained at *I* and *II*. If the voltage at *II* is *a* per cent higher, we have, according to the formula (5),

$$p = \frac{200\,b\,(\Sigma il)_{\text{I}}}{qe},$$

and

$$a + p = \frac{200\,b\,(\Sigma il)_{\text{II}}}{qe}.$$

Subtracting one equation from the other we get

$$a = \frac{200\,b}{qe}\left[(\Sigma\, il)_{\mathrm{I}} - (\Sigma\, il)_{\mathrm{II}}\right] \quad . \quad . \quad . \quad . \quad . \quad (8)$$

When $a = 0$, this is reduced to the former condition $(\Sigma\, il)_{\mathrm{I}} = (\Sigma\, il)_{\mathrm{II}}$ Formula (8) determines the position of the point of division of the load, with a given difference of potential a per cent between the feeders. By inserting some resistance in one of the feeders, part of its load can be transferred to the other feeder, and vice versa.

The ring-line, Fig. 250, is evidently identical, from an electrical standpoint, with a line fed at the same potential from both ends.

285. EXPERIMENT 14-B.—Voltage Drop in Distributing Lines Fed from Both Ends.—The arrangement is similar to that used in Experiment 14-A, except that feeders are connected from the source of supply to both ends of the line, and rheostats are provided for regulating each feeder independently.

(1) Load the line, and take the distribution of voltage drop with only one feeder connected to it (Curve *A*, Fig. 252). (2) Connect the other feeder and see in how far the conditions are improved, and what part of the load is taken by the second feeder (Curve *B*). Keep both feeders at the same potential and determine the point of division of the load. Measure volts and amperes at separate consumers, in order to be able to check the results by calculation. (3) Raise the potential of one of the feeders slightly, and find the new distribution of currents and voltages; also the new point of division of the load (Curve *C*). (4) Repeat the same experiment with a larger difference of potential at the two feeders. (5) Before leaving the laboratory, measure the resistance of the line wire per foot of its length, using the drop-of-potential method. Make a sketch of the line, with distances between the connections to the customers; measure the distance between the knife-edges of the contact-maker.

Report. (1) Plot to conductor lengths as abscissæ, voltage drop as shown in Fig. 252. Curve *A* refers to a conductor fed from one end, *B* — fed from both ends at the same voltage, and *C* — when the feeder at *II* has a voltage *a* per cent higher than that at *I*. (2) Check the observed drop by calculation, from the resistances and currents. (3) Show that the point of division of the load, as found during the experiment, satisfies the condition expressed by formula (7), when the two feeding-points are at the same potential, and that its position is in accordance with formula (8), when the potential is different.

INSULATION MEASUREMENTS AND LOCATION OF FAULTS.*

286. All parts of an electric circuit must be either thoroughly insulated from the ground or carefully grounded. The return wire of a trolley circuit and the middle wire of a three-wire system are examples of grounded parts of the circuit; most other parts have to be thoroughly insulated from the ground. One accidental ground connection may not be objectionable, at least on low-voltage circuits, unless there is a regularly grounded wire; in this latter case a second ground means a short circuit.

However, since even one ground connection means that the insulation is becoming defective, action should be taken before the fault has assumed such proportions as to make necessary the shutting down of the station, or at least cutting out the damaged part of the circuit. Any apparatus which indicates a ground connection, or defective insulation, is called a *ground detector*. On high-tension circuits static ground detectors are used (Figs. 253 and 254); on low-tension circuits ordinary incandescent lamps or a voltmeter are preferable (Fig. 255), since the static instrument in its usual form requires a considerable voltage in order to give a noticeable deflection.

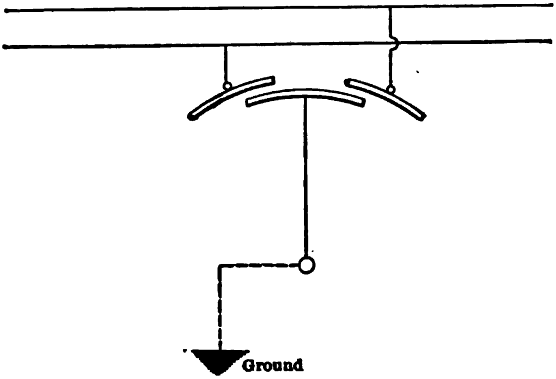

FIG. 253. A static ground-detector.

287. Static Ground Detectors. — The ground detector, shown in Fig. 253, consists of two stationary metal pieces connected to the wires of the circuit, and of a movable vane connected to the ground. So long as the line is perfectly insulated from the ground, the vane remains in its middle position, equal static charges being induced on both sides. When one of the line wires becomes grounded, it can induce no more charge, the vane itself being connected to the ground; therefore the vane is attracted toward the other side, and the deflection is shown on the scale. A ground does not necessarily mean a direct metallic connection to the ground; it may be merely a leak through the insulation, having a resistance of several thousand ohms. The deflection of the pointer depends on this "resistance to the ground," and the scale of the instrument may be calibrated in ohms, or any other arbitrary units.

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* The methods described below apply equally well to transmission lines and to distributing lines.

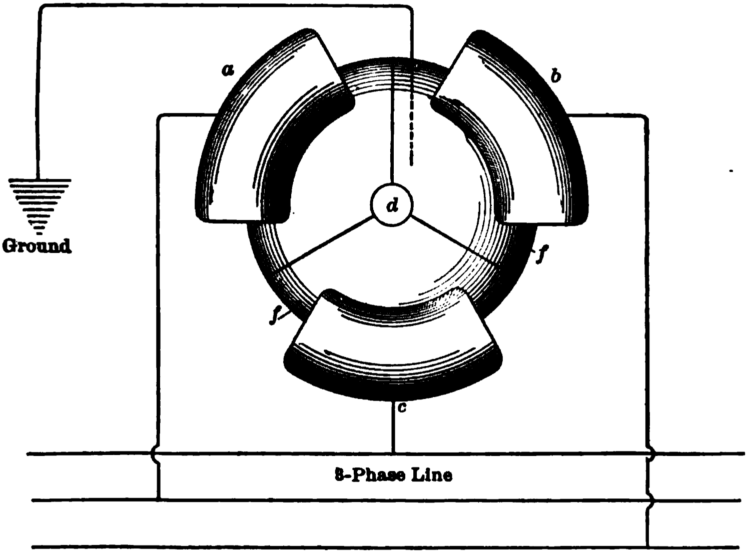

FIG. 254. A three-phase ground-detector.

On three-phase circuits a ground detector is used, shown in Fig. 254; it operates on the same principle as the one described above, but has three stationary parts, instead of two, according to the number of line wires. The movable part is pivoted so that it can move in all directions around a point. As in the single-phase ground detector, the vane moves *from* the grounded wire.

288. Ground-Detector Lamps and Voltmeters. — The low-tension ground detector, shown on the left of Fig. 255, consists of ordinary incandescent lamps connected across the line; the middle point is thoroughly grounded. A sufficient number of lamps is taken so that they just glow red when there is no ground. Should a dead ground occur on one of the wires, one half of the lamps go out, while the other half glow bright. Any fault on one side of the line is shown by the lamps on the other side glowing brighter. This arrangement is very simple and reliable, but does not allow of any quantitative determination of the resistance of the fault. This is remedied by the addition of a voltmeter, as shown in the same sketch to the right. Knowing the resistance of the instrument, the resistance to the ground can be calculated from the voltmeter indication. Or else, the voltmeter can be calibrated directly in ohms, instead of volts.

289. EXPERIMENT 14-C. — Calibration of Ground Detectors. — The experiment consists of a study of ground-detecting devices described in §§ 287 and 288. The following separate measurements should be performed:

(1) Connect ground-detector lamps, as in Fig. 255, and observe their action, when one of the line wires has a fault, or a dead ground. Calibrate the lamps qualitatively, so as to be able to judge regarding the magnitude of a fault. For instance, when the insulation resistance of a fault is about 2500 ohms, the difference in brightness of the two lamps is just noticeable in a rather dark place. The difference in brightness is easily detected at 1500 ohms; one lamp apparently goes out at 200 ohms, etc. Investigate the influence of the number of lamps in series on sensitiveness of the indications.

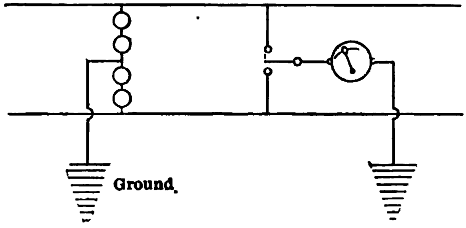

FIG. 255. Ground-detecting lamps and voltmeter.

(2) Calibrate an ordinary voltmeter (Fig. 255) for ohms leakage resistance to the ground. This should be done in a low-voltage circuit (not more than 500 volts). Connect the voltmeter across the line and gradually increase the resistance in series with it, by means of a calibrated multiplier. The calibration curve is that between the deflections and the resistances in the multiplier. No actual ground is used in calibration, but the electrical conditions are the same as in Fig. 255.

(3) Calibrate a static ground detector (Fig. 253) for ohms resistance to the ground. The necessary voltage, say 2200 volts, may be obtained by an ordinary voltmeter transformer. For calibration use high-resistance carbon rods, the resistance of which has been previously determined by some other method, — for instance, by the Wheatstone bridge. Connect the stationary blades of the ground detector to the

line, and place the carbon rods between one of the line wires and the moving element. The observed deflections, plotted to resistances of the rods, constitute the calibration curve of the ground detector. Here again, no actual ground need be used in the calibration; however, the electrical connections are identical with those obtaining when a fault exists on one of the line wires, and the moving element is connected to the ground.

290. Insulation Measurements on Dead Lines. — Insulation measurements may be conveniently performed, if the line can be disconnected from the source of power during the test. A separate source of power should be used in this case, the voltage of which must be at least as high as that to which the line is subjected in regular operation.

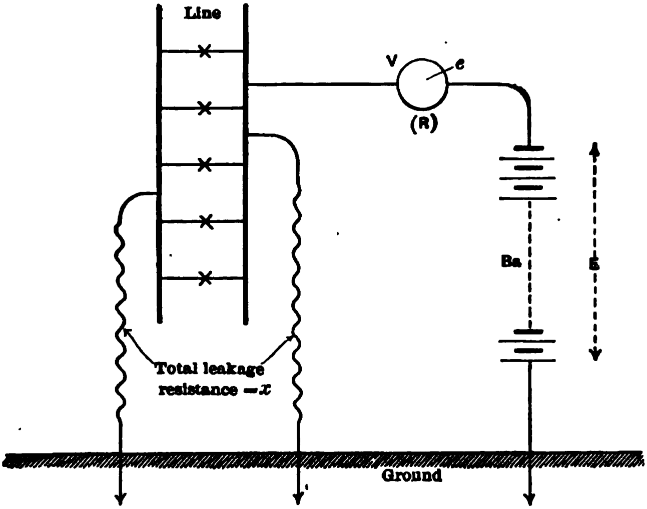

FIG. 256. Measuring insulation of "dead" lines.

Sometimes the regular source of power, to which the line is later connected, is used for tests. The connections for insulation measurements are shown in Fig. 256: a voltmeter of a known resistance R is connected first across the source of voltage Ba, and then between the source and the line, as shown in the sketch. Let the first reading be E, the second e. Then, if the resistance of the fault is x we have,

$$\frac{R + x}{R} = \frac{E}{e},$$

whence

$$x = R\frac{E - e}{e} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

The voltmeter in both cases measures the voltage of the source of power, save that in the first case it is connected to it directly, in the second case through the unknown resistance x of the fault, which thus acts as a multiplier. The deflections of the voltmeter are inversely proportional to the total resistance in the voltmeter circuit. This gives the above equation, from which the resistance to the ground may be calculated. For values of insulation resistance, prescribed by the Fire Underwriters for complete installations, see section 66 of the " National Electrical Code."

291. Evershed Ohmmeter. — There are instruments on the market which give the resistance of a fault directly in ohms, and supply their own source of power. A popular instrument of this kind is the Evershed ohmmeter, shown in Fig. 257. The box to the right contains a small D. C. generator which is operated by hand through a gearing. The generator may be wound for any voltage up to 1000 volts. The box to the left contains a special galvanometer with a scale calibrated in megohms. The connections shown in the sketch are those for measuring the insulation between two cables. If the insulation is to be measured between a cable and the ground, one of the terminals to the left is connected to the cable conductor, the other to its lead sheathing.

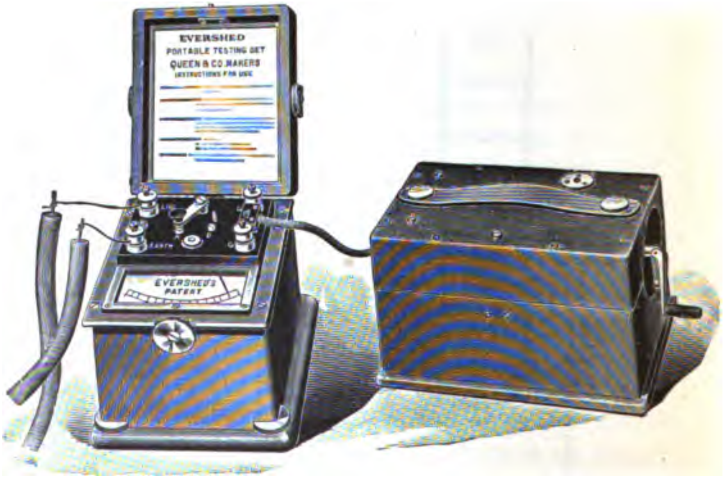

FIG. 257. The Evershed ohmmeter.

The construction of the galvanometer is shown in Fig. 258. The moving part consists of a soft-iron needle n with a pointer attached to it. The coils cc are connected in series with the generator circuit, the coil pp across the unknown resistance X. The magnetizing action of the coils *cc* tends to set the needle in the position corresponding to zero of the scale. The action of the coil *pp* tends to set the needle in the position of "infinite resistance." When currents flow through all the coils, the needle assumes an intermediate position which depends on the ratio of the magnetizing actions of *pp* and *cc*. But the current in *pp* is proportional to the pressure at the terminals of the unknown resistance; the current in *cc* is equal to that flowing through the unknown resistance. Therefore, the ratio of the actions of the two coils is a measure of the unknown resistance (voltage ÷ current).

The scale is calibrated empirically by known resistances. The proper speed of rotation for the generator handle is about 60 revolutions per minute. The needle is made astatic, so that the indications of the instrument are independent of the direction of the test current and of terrestrial magnetism.

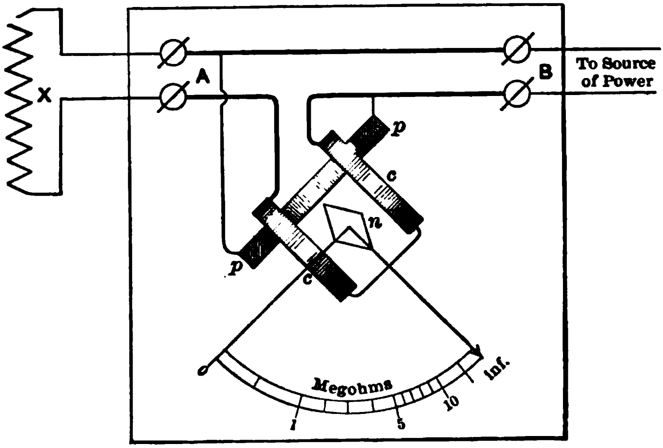

FIG. 258. The operating parts of the Evershed ohmmeter.

The Sage ohmmeter, described in § 19, and the portable bridge, shown in Figs. 23 and 24, may also be used for measuring insulation resistances of lines. It must be kept in mind, however, that these devices use low-voltage batteries; in some cases a fault does not show up unless a sufficiently high voltage is applied. Therefore, the voltmeter method (§ 290) is preferable, unless an ohmmeter like Evershed's is available, with which comparatively high voltages may be used.

292. Insulation Measurements on Live Lines. — On low-voltage lines accurate determinations of insulation resistance are possible even when the line is in use. The principle of the method is the same as in the ground-detector voltmeter (Fig. 255). The relations are shown more in detail in Fig. 259. A voltmeter of a known resistance R, shows a voltage e_p when connected between the positive wire and the ground; it shows e_n between the negative wire and the ground. The unknown insulation resistances between the line wires and the ground are denoted by x_p and x_n .

Assume first that a fault exists on the positive side only. Then $x_n =$ infinity, and $e_p = 0$. If the voltmeter reads a voltage E across the line, we have the same relation as in § 290:

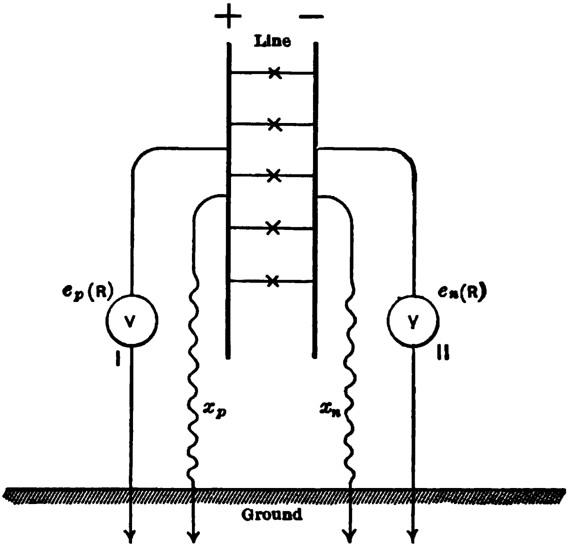

FIG. 259. Insulation measurements on "live" lines.

$$x_p = R\frac{E - e_n}{e_n} \quad \ldots \ldots \ldots \quad (10)$$

The path of the leakage current is from the positive side of the line, through the fault x_p, the ground and the voltmeter e_n to the negative side of the line.

The relations become more complicated when the insulation is defective on both sides, so that the voltmeter gives appreciable deflections in position *I* and in position *II*. When the voltmeter is connected between the positive wire and the ground, the leakage current flows through x_p and the voltmeter in parallel with it, through the ground and back through x_n to the negative line wire. The total resistance of the leakage circuit is thus

$$\frac{Rx_p}{R+x_p}+x_n,$$

and the leakage current

$$i=\frac{E}{\dfrac{Rx_p}{R+x_p}+x_n}.$$

Of this current only the part

$$i'=\frac{ix_p}{R+x_p}$$

flows through the voltmeter, the current being divided in the inverse ratio of the resistances. On the other hand,

$$i'=\frac{e_p}{R},$$

because the voltmeter reading is e_p and the resistance of the instrument is R. Eliminating the currents i and i' from the above three equations, we get

$$\frac{e_p}{Rx_p}=\frac{E}{x_nR+x_n\,x_p+x_p\,R} \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

When the voltmeter is connected between the negative wire and the ground, we get by analogy

$$\frac{e_n}{Rx_n}=\frac{E}{x_pR+x_p\,x_n+x_n\,R} \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

or, from the last two equations, the simple result

$$\frac{e_p}{x_p}=\frac{e_n}{x_n}.$$

Eliminating x_n by means of this equation from (11), we finally obtain

$$x_p=R\,\frac{E-e_n-e_p}{e_n} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

and by analogy

$$x_n=R\,\frac{E-e_n-e_p}{e_p} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

Using these expressions, the unknown resistances to the ground are calculated from the voltmeter readings e_p and e_n. If the voltmeter shows 0 on the positive side, equation (13) is reduced to (10). Equation (14) shows that in this case there is no fault on the negative side (x_n = infin.).

293. EXPERIMENT 14-D. — Insulation Measurements on Low-Tension Lines. — The methods are described in §§ 290 to 292. (1) Measure the insulation of a dead line by the method shown in Fig. 256. Connect the line to a gas- or water-pipe through a resistance box, and take voltmeter readings with the value of this resistance varied from zero to infinity. Determine the resistance of the voltmeter by taking a reading of some constant voltage without multiplier, and with a multiplier of known resistance. The insulation of the line under test may not be quite perfect; therefore it is better to use an *artificial* ground, for instance, a wire carefully insulated from the earth. The leakage to this artificial ground may be made to be of any desired value.

(2) If a direct-reading ohmmeter is available, calibrate it with standard resistances; then use the instrument for determining the insulation resistance of actual lines previously disconnected from the supply. Perform this test in accordance with section 66 of the *National Electrical Code.*

(3) Measure insulation resistance of a live line, as explained in § 292. As the line available for test may have a fault to the ground, use an artificial ground, so as to establish known values of leakage resistances x_p and x_n (Fig. 259), by means of calibrated resistance boxes. Apply the method to the actual measurement of the insulation between the line and the earth, without resistance boxes. Before leaving the laboratory, measure the resistance R of the voltmeter.

Report. Give the results of the test (1) and show how nearly the resistances calculated from formula (9) check with those actually used in the resistance box. Give the results of practice with the direct-reading ohmmeter. Calculate the insulation resistances from the test (3) and compare them with those actually connected between the line and the ground.

294. Locating Faults. — After a fault has been discovered by a ground detector, by measuring insulation resistance, or by some disturbance in the operation of the line, the next problem is to locate the fault, in order to remedy it. Two schemes for locating faults on long lines are widely used in practice, both being based on the familiar principle of the Wheatstone bridge. The method shown in Fig. 260 is called the Murray loop (§ 295); it is convenient for use with a slide-wire Wheatstone bridge, such as is shown in Fig. 12. With plug or dial-type bridges (Figs. 21 and 22), the Varley loop (§ 296), shown in Fig. 261, is preferable. With both methods the two ground connections d_1 and d_2 are in the battery circuit, outside the bridge proper, so that the resistance of the fault, even though considerable, does not affect the result.

Portable bridges, such as are shown in Figs. 15 and 23, intended for general commercial work, usually have a provision for the connections according to the Murray or Varley loop. Explicit instructions for such tests always accompany the bridge.

The place of a cross of one wire on another may be determined in the same way as a fault to the ground, the good wire being considered as an artificial ground.

295. Murray Loop. — The line is short-circuited on one end and connected to a slide-wire Wheatstone bridge on the other end, as shown in Fig. 260. The connections and the lettering are identical with those of a regular Wheatstone bridge (Fig. 10); the battery circuit is closed through the ground. The ratio of the lengths *m* to *n* on the slide-wire of the bridge gives directly the ratio of the distances to the fault from the points *a* and *b*. Knowing the total length of the line, the place of the fault is thus calculated with sufficient accuracy; then the fault is found by an actual inspection of the line near the place determined by calculation.

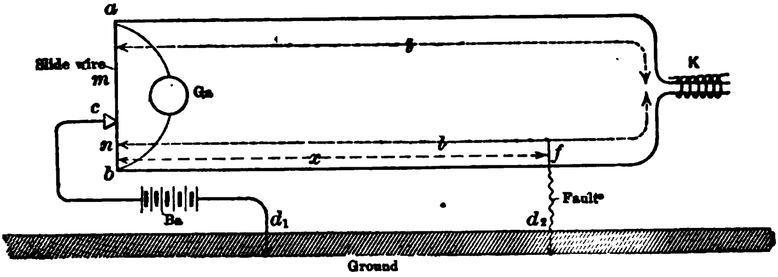

FIG. 260. The Murray loop for locating faults.

With the notations in the sketch, we have, when zero balance of the galvanometer is obtained,

$$\frac{2l - x}{x} = \frac{m}{n}$$

or

$$x = l \cdot \frac{2n}{m + n}.$$

296. Varley Loop. — Instead of varying the ratio $m \div n$ for obtaining the galvanometer balance, as is done in the Murray loop, here this ratio is left constant; but some resistance r (Fig. 261) is gradually added to the line, until the galvanometer shows zero. Then with the notations in the sketch we have

$$\frac{(2l - x)a}{r + xa} = \frac{m}{n},$$

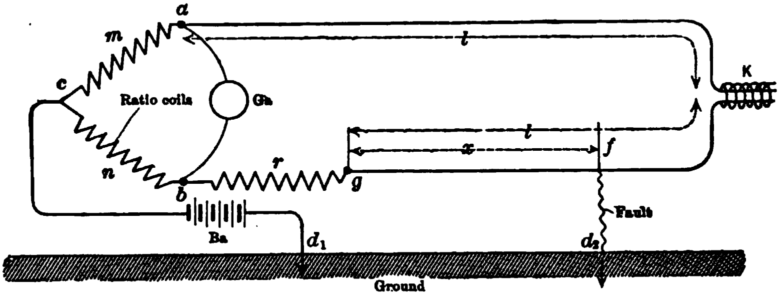

FIG. 261. The Varley loop for locating faults.

where a is the resistance of the line conductor per unit length. Solving this equation for the distance x to the fault, we get

$$x = 2l \cdot \frac{n}{m + n} - \frac{r}{a} \cdot \frac{m}{m + n}.$$

All the quantities in this formula are known, and x may be calculated.

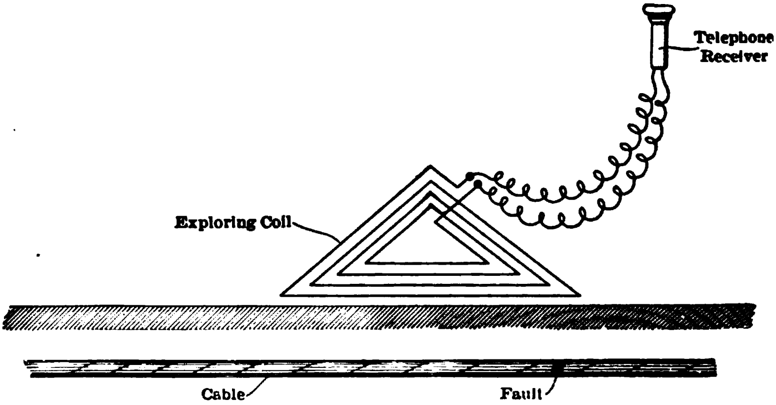

FIG. 262. An induction fault-localizer.

If a is not known, the bridge is connected to the line in the ordinary way, and the resistance of the total loop measured. Dividing it by the length $2l$ gives the resistance a per unit length. When $r = 0$, the above expression becomes identical with that deduced for the Murray method.

For arrangements used in localizing faults on three-wire systems, see an article in the *Electrical Review*, London, March 15, 1907.

297. Induction Localizers. — The devices shown in Figs. 262 and 263 are used to some extent for localizing faults in cables, using alternating current. Either a telephone receiver or an alternating-current ammeter may be used as indicators.

With the device shown in Fig. 262, alternating current is put on the defective cable, or between the cable and the ground, as the case may be, and the observer listens to the noise produced in the telephone receiver by secondary currents induced in the triangular coil. He follows the cable, and when he passes the fault, the noise either increases or decreases; this change in noise locates the fault.

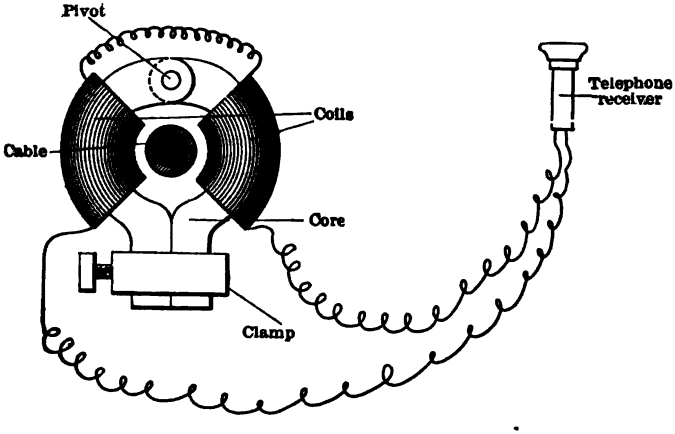

FIG. 263. A fault-localizing transformer.

The device shown in Fig. 263 is a series transformer for which the faulty cable serves as a primary; the secondary is connected to an ammeter, or to a telephone receiver. The core of the transformer consists of two parts hinged together, so that the device can be put on the cable at any place. The position of the fault is indicated by a change in the hum produced in the telephone receiver, or by a change in the ammeter current. The same device may be used for measuring currents in cables, without opening the circuit.

298. EXPERIMENT 14-E. — Locating Faults in Lines. — The experiment is intended to afford practice with the devices and methods described in §§ 294 to 297. At first, the student himself should provide faults on the lines given him, and check the position of the fault by observation and calculation. Then he should try to locate faults, provided for him by the instructor, and hidden so that they could not be located by a mere inspection of the line.

NOTE. The above described methods for locating faults are not always applicable on high-tension transmission lines, since a high voltage is often required to produce a flow of current through a partially disabled insulator. A method for quickly locating such grounds, using the line voltage, has been described by Mr. L. C. Nicholson, in a paper entitled *Location of Broken Insulators*, read before the Annual Convention of the American Institute of Electrical Engineers, in June, 1907. The method is being successfully used on some 60,000 volt lines radiating from Niagara Falls.

CHAPTER XV.

DIRECT-CURRENT GENERATORS — OPERATING FEATURES.

299. THE essential parts of a direct-current generator are shown in Fig. 264. The machine consists of a revolving part, or armature, in which e.m.f.'s are induced (generated), and of a stationary magnetic field, necessary for inducing these e.m.f.'s. In the particular case shown, the field has four poles.

The armature is driven by a prime mover whose power is thereby transformed into electrical energy, and delivered to the line through the armature terminals. The armature itself consists of a cylindrical iron core, mounted on the shaft and provided with slots on its periphery. A winding consisting of copper bars or coils is placed in these slots. The coils are properly interconnected so as to assist each other in their electrical action, and are also connected to the so-called commutator shown to the left. The commutator converts alternating voltages and currents, induced in the armature, into direct voltages and direct currents. This commutator consists of copper segments mounted on a

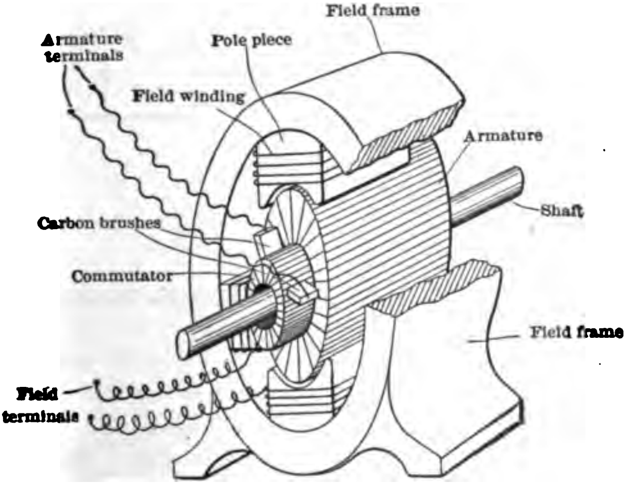

FIG. 264. The principal parts of a direct-current generator or motor.

sleeve and insulated from each other by sheets of mica. Two or more stationary carbon brushes make contact with the commutator segments and conduct direct current to the line.

The stationary frame and the poles are made of some magnetic material, such as cast iron or steel. The pole-pieces may be cast with the frame, or cast into the frame, or bolted to it. The poles are provided with windings, through which direct current is sent, necessary for exciting the magnetic field. The exciting current is usually generated by the machine itself, as is explained in the next article.

300. Self-Excitation. — The most commonly employed connections between the field winding and the armature are shown in Fig. 265. The field winding consists of many turns of a comparatively small wire, and is connected across the brushes of the machine, in other words, in parallel with the armature winding and with the current-consuming devices in the external circuit. Such generators are called *shunt-wound machines*, the field winding being "shunted" across the armature terminals. The rheostat shown in the exciting circuit is for regulating the field current, so as to maintain the proper voltage.

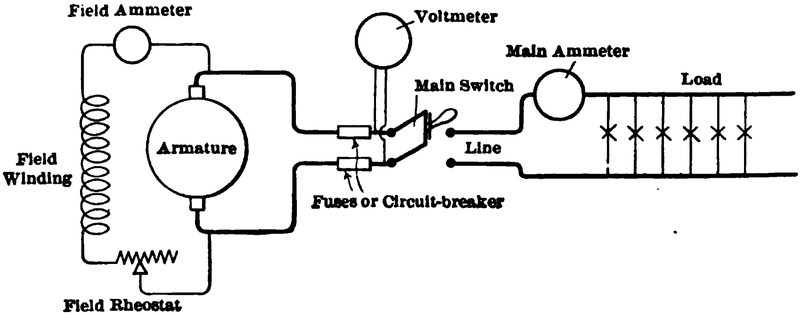

FIG. 265. Diagram of connections between a shunt-wound generator and the load.

Self-excitation is made possible by virtue of *residual* magnetism in the frame and in the pole-pieces of the machine. The machine once magnetized from an external source retains a small part of the magnetism permanently. When started, with the line circuit open, the residual magnetism induces a small current in the armature; this current flows through the field winding and strengthens the original field. This increased field induces stronger currents in the armature, which sends a stronger current through the field, etc., until the excitation reached its full value.

The voltage induced in the armature is proportional to the magnetic flux issuing from the poles (this is according to the fundamental law of induction). The flux itself depends on the value of the exciting current, so that finally the voltage of the machine depends on the exciting or field current (Fig. 266). If iron had a constant permeability, this curve would be a straight line, the magnetic flux and the induced voltage being proportional to the exciting current. In reality the permeability of iron decreases, as the flux density increases, the iron being gradually "saturated" (§ 141). This explains the shape of the curve in Fig. 266: the flux, and therefore the induced voltage, increase more slowly than the exciting current. The curve is called the no-load saturation curve, no-load characteristic, or the magnetization curve of the machine.

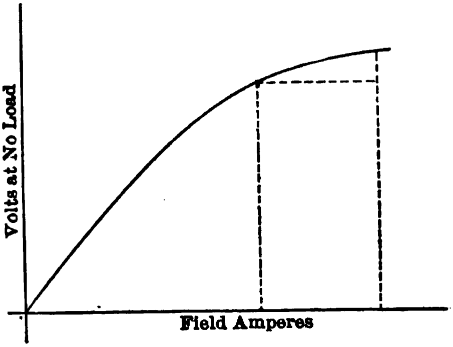

FIG. 266. No-load characteristic.

301. EXPERIMENT 15-A. — No-Load Characteristics of a Shunt-Wound Generator. — The purpose of the experiment is to determine the relation between the exciting current and the terminal voltage of a shunt-wound machine, at no load (Fig. 266). The generator (Fig. 265) is driven at its rated speed by a belted or direct-connected motor. An ammeter is inserted into the field circuit, and a voltmeter across the brushes, — the main line circuit is kept open.

(1) The readings are begun with the highest possible value of the field current, in other words, with the filed rheostat short-circuited;

the field is then reduced in steps to zero. Readings should be taken of amperes, volts and the speed of the machine. The value of the residual magnetism at the beginning of the experiment is rather indefinite; therefore it is advisable to begin the excitation at its maximum and reduce it to zero. After this, take readings with an increasing field current, in order to see the influence of residual magnetism. This is analogous to taking a hysteresis loop, shown in Fig. 161.

(2) Induced voltage is proportional to the speed of the machine, provided the field current is kept constant; this is according to the fundamental law of induction. To prove this experimentally, select a field excitation and drive the machine within as wide a range of speed as the driving motor will permit. Keep the exciting current constant by regulating the field rheostat, or excite the machine from a separate source. Repeat this run with two or three different values of field current.

(3) Finally investigate the ability of the machine to excite itself. Run it at the rated speed, and find the rheostat notch on which the field just begins to build up. Measure the corresponding field current, the final voltage, and the number of seconds of time from the closing of the field switch to the moment when the voltage reaches its final value. Repeat the same experiment with less resistance in the field rheostat.

Report. Draw a diagram of the connections used and give a short description of the machine. Plot the saturation curve, shown in Fig. 266, correcting the voltages (by direct proportion) where the speed was above or below normal. Plot curves showng proportionality of voltage to speed. Give numerical results of the experiment on self-excitation.

302. Voltage Drop and Regulation.— One of the most important requirements in practical operation of generators, supplying current for light and power, is that the terminal voltage must be nearly constant, independent of the load. This practically means, that each customer should get the same quantity of light from his lamps, and same speed from his motors, whether many or only a few other customers are using current at the same time.

This condition cannot be strictly fulfilled with a shunt-wound machine, unless the field current is regulated by the field rheostat. Without regulation the voltage drops considerably, as the load increases. This is due to the following three causes:

(*a*) Part of the induced voltage is consumed in the resistance of the armature; this drop (*ir*) is proportional to the armature current, or (practically) to the load of the machine.

(*b*) The armature currents tend to produce a magnetization opposite to that due to the shunt field winding, and in this way weaken the original field (§ 147). The voltage induced in the armature is therefore correspondingly lower.

(*c*) As the terminal voltage of the machine decreases, on account of the above two causes, the current in the shunt winding also decreases in the same proportion. This again weakens the field and reduces the voltage still further.

Curves which show the influence of the load on the voltage of a machine are generally called its characteristics (Fig. 267). They show voltage fluctuations of the machine, variation of field current, etc., with changes of load.

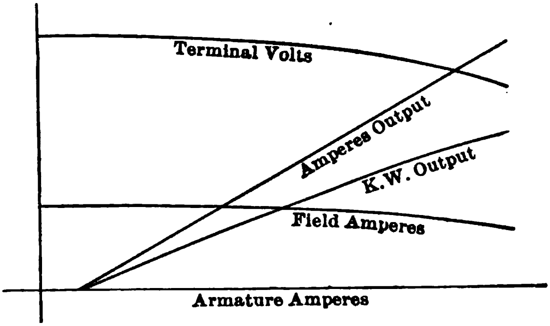

FIG. 267. Performance curves of a shunt-wound generator.

When a shunt-wound generator is running on a variable load, the operator usually regulates the voltage from time to time, by means of the field rheostat, keeping it fairly constant. In order to see what regulation can be expected under such conditions, two characteristics may be taken in the laboratory, as corresponding to two extreme cases:

(1) The machine is left *entirely without attention*, so that the voltage fluctuates freely with the load; see § 304.

(2) The switchboard attendant *regulates the field continuously*, thus keeping the voltage absolutely constant; see § 305.

In actual practice an intermediate condition exists: the attendant does not watch the voltmeter constantly, but corrects the voltage from time to time.

According to the standardization rules of the American Institute of Electrical Engineers, the regulation of self-exciting machines is defined as per cent increase in voltage, when full load is thrown off the machine, the speed and the setting of the field rheostat remaining unchanged. To illustrate, suppose the rated voltage of a shunt-wound generator to be 100 volts. Full load is put on the machine, the field rheostat being adjusted, so that the terminal voltage is 100 volts, then the main switch is opened. Let the terminal voltage, when the prime mover comes back to the normal speed, be 109 volts. According to the above definition, the regulation of the machine is 9 per cent.

303. Components of Voltage Drop. — The curve of terminal volts, shown in Fig. 267, refers to the case, when the generator is left without attention, so that all three causes of the voltage drop in the armature (§ 302) take place to their full extent. These three factors are shown separately in Fig. 268.

The ohmic drop *a* is calculated by multiplying the armature current by the resistance of the armature (including that of the brushes). The drop, due to the decrease in the field current, can be found from the no-load characteristic shown in Fig. 266. Knowing from Fig. 267 the values of the field current *Oa* at no load and *Oc* at the load under consideration, the corresponding decrease in voltage *bb'* can be read off directly on the curve. The remainder of the total observed drop is caused by the armature reaction. In most textbooks it is stated that voltage drop in shunt-wound generators is caused by *two* factors: ohmic drop in the armature, and the armature reaction. This is true, so long as the machine is separately excited, or the exciting current is kept constant. In the actual operation, however, the exciting current itself varies with the terminal voltage of the machine, so that in analyzing the voltage drop of a machine *left to itself* (without regulation), this third cause must also be considered.

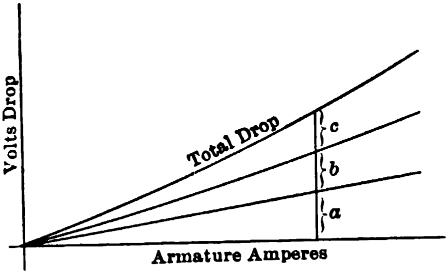

FIG. 268. The components of voltage drop in a shunt-wound generator.

When making a special study of the armature reaction, the field current can be kept constant from no load to full load, by means of the field rheostat. In this case, the voltage drop is caused by *two* factors only: ohmic drop and armature reaction. Subtracting the former, we readily find the magnitude of the armature reaction, with a given load and excitation.

304. EXPERIMENT 15-B.—Voltage Characteristics of a Shunt-Wound Generator. — The purpose of the experiment is to determine the performance curves shown in Fig. 267, and to separate the voltage drop into its components, as per Fig. 268. This corresponds to the first extreme condition of operation, mentioned in § 302. The machine is connected up as in Fig. 265, and is driven at its rated speed by a motor. The readings may be conveniently recorded on a data sheet, similar to that shown below.

	Normal Voltage at —— Load.			
	Field Amps.	Load Amps.	Volts.	Speed.
Inst. No.				
Const.				

Bring the machine up to its full speed, and load it on resistances; regulating these resistances and the field rheostat, so as to get the rated voltage of the machine at a load exceeding the rated capacity of the machine by about 25 per cent. Read amperes load, voltage of the machine, and field current; the speed should be kept as nearly constant as possible, throughout the whole test. Now reduce the load somewhat, leaving the field rheostat in the same position as before; in other words, allowing the voltage of the machine to vary at will. Read again, — volts, amperes and field current; then again reduce the load, etc., until all of the load is gradually taken off the machine.

The same test is then repeated with another setting of the field rheostat, — for instance, such that the full rated voltage takes place at 100 per cent, or at 75 per cent of full load; it is also desirable to have a third curve corresponding to such a position of the field rheostat, that the machine gives its full rated voltage at a small load, say about 25 per cent of full load or even at no load. In each case, after having properly set the rheostat, begin the run with about 25 per cent overload, and gradually reduce the load to zero.

Having finished the test, measure the ohmic resistance of the armature, with and without the resistance of the brushes; use the drop-of-potential method (§ 10). The no-load saturation curve (Fig. 266) is supposed to be known from experiment 15-A; otherwise it should be taken before leaving the laboratory.

Report. Plot the curves, shown in Fig. 267, for the three runs made. Armature amperes are obtained by adding field amperes to load amperes. Kilowatt output is calculated as a product of load amperes by terminal volts (divided by 1000). Figure per cent regulation of the machine at full load, as explained at the end of § 302. For one of the curves separate the voltage drop into its three components, as in Fig. 268.

305. EXPERIMENT 15-C. — Excitation Characteristics of a Shunt-Wound Generator.—This experiment corresponds to the second extreme condition mentioned in § 302, namely, the operator regulates the field rheostat continually so as to keep the terminal voltage constant. The purpose of the experiment is to ascertain the necessary variations in field current. The connections are the same as in Fig. 265; the same data sheet may be used as in the preceding experiment, except that the voltage, instead of the setting of the field rheostat, is now kept constant. Bring up the load and the field current of the machine so as to have the full rated voltage at a reasonable overload, say 25 per cent; then gradually reduce the load and take readings. The resistance in the field circuit must be gradually increased, so as to keep the terminal voltage constant.

Per cent variation in field current depends on the degree of saturation of the machine (§ 141) and is less with highly saturated machines. This is evident when we consider that a larger number of field ampere-turns are required with higher saturation; therefore the same number of armature demagnetizing ampere-turns constitutes a smaller per cent of the active ampere-turns, and its influence is less noticeable.

To see the influence of saturation, repeat the same test at as high a terminal voltage as it is possible to obtain at full load. Then perform a similar run at a voltage considerably below normal.

Report. Plot performance curves, as in Fig. 267. Determine per cent variation in field current, between no load and full load. Show that the regulation is better, the higher the saturation of the magnetic circuit of the machine.

306. Compound Winding. — The above two experiments show that the terminal voltage of a shunt-wound machine decreases, as the load increases. In order to keep the voltage constant, it is necessary to regulate the field rheostat, increasing the exciting current as the load increases. The same effect can be obtained automatically by placing on the pole-pieces a second winding, in series with the main circuit (Fig. 269). As the main current increases, more current flows through the series winding, adding excitation to that supplied by the shunt winding. By properly adjusting the number of turns in the series winding, the voltage at the terminals of the machine is made practically independent of the load. This additional series winding is called the *compounding winding*, and the machine is called *compound-wound.*

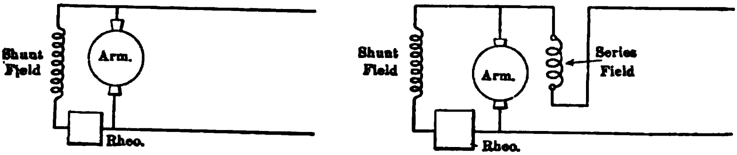

FIG. 269. Shunt-wound generator to the left, compound-wound generator to the right.

In reality the voltage curve (flat compounded) has an aspect as shown in Fig. 270. The machine compounded so as to give the same voltage at full load as at no load, gives somewhat higher voltages at partial loads. This is caused by the magnetic flux of the machine being not proportional to the number of exciting ampere-turns. Commercial machines are often designed with such a degree of saturation in the magnetic circuit that this circumstance is scarcely noticeable. In many cases this rise in voltage above normal at partial loads is of

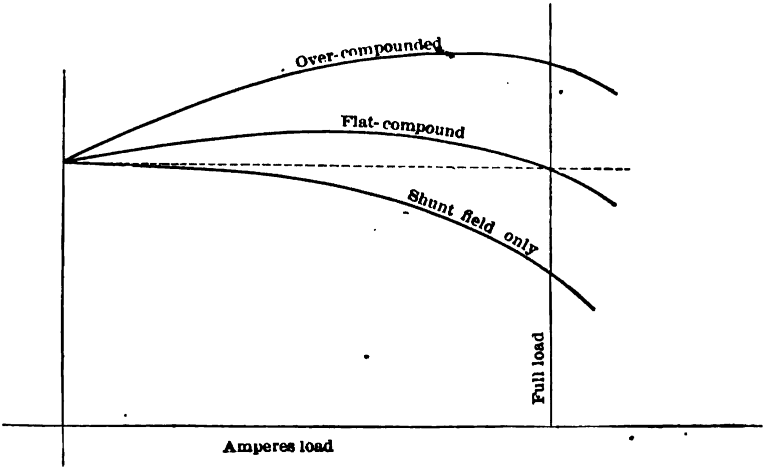

FIG. 270. Effect of series-field winding on voltage characteristics of a generator.

no consequence; if, however, an exact compounding is required, a little regulation of the field rheostat by hand becomes necessary.

By providing more turns in the series winding than is necessary to compensate for the voltage drop, the machine is *over-compounded*,—in other words, its voltage rises with the load (upper curve in Fig. 270). This is frequently done in generators supplying power for electric railways; over-compounding is there necessary to compensate for voltage drop in long feeders, and thus to give a more constant pressure on the trolley wire. Suppose, for instance, that the drop in a feeder at full load is 50 volts; then, in order to give the standard voltage of 500 volts on the trolley wire, the generator voltage has to be 550 volts. At half load the drop in the feeder is only 25 volts; so the generator voltage must be 525 volts in order to give the same pressure on the trolley wire. Finally, at very light loads the drop in the feeder is negligible, and the generator pressure must be about 500 volts. Thus, the curve of the generator pressure must be a straight line, ascending with the current.

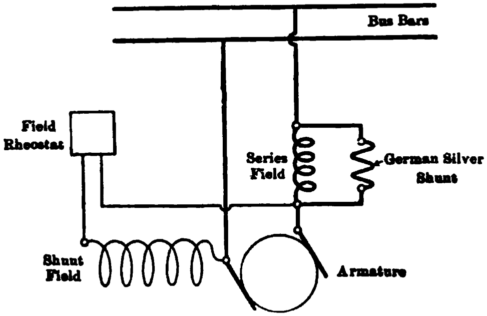

FIG. 271. Diagram of connections in a compound-wound generator with the shunt-field winding connected across the armature (short shunt).

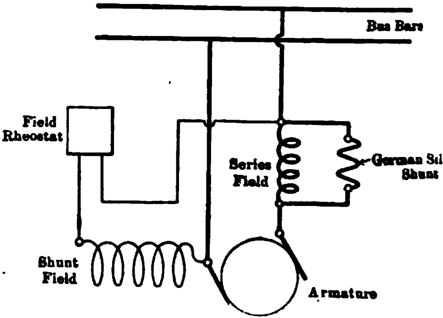

FIG. 272. Diagram of connections in a compound-wound generator with the shunt-field winding connected across the terminals of the machine (long shunt).

It is difficult to design a machine with the right number of series turns for a certain degree of compounding. The permeability of iron may be somewhat different from that admitted in the calculations, and the armature reaction cannot be predetermined with a sufficient accuracy. Moreover, different customers may desire the same type of machine with a different degree of over-compounding. Therefore, generators are usually provided with a number of series turns, sufficient for a reasonably high degree of over-compounding, and the *compound winding is shunted* to a desired degree by German silver strips (Figs. 271 and 272), so that only part of the total current flows through the series field. In this way the magnetizing action is reduced, and by varying the resistance of the shunt, any degree of compounding is obtained, without changing anything in the machine itself.

The shunt winding of compound-wound generators can be connected either across the armature (short shunt, Fig. 271), or across the terminals of the machine (long shunt, Fig. 272). There is not much difference in the performance of these two types, both of which are used in practice.

307. Number of Turns in the Series Winding. — The number of turns in series windings, necessary for flat-compounding, may be determined either by successive trials, or better, by the following experiment: Suppose the machine under test to be a 110-volt 200-ampere generator; let the machine give 110 volts at no load at a field current of 5.5 amperes. When full load is put on and the series winding is disconnected the voltage naturally drops; and in order to raise it to 110 volts the shunt field excitation must be increased to, say, 6.1 amperes. Let the total number of turns on the shunt winding be 2400; then the difference $(6.1 - 5.5) \times 2400 = 1440$ ampere-turns, represents the additional excitation, necessary for compensating the armature reaction and the ohmic drop in the armature. This number of ampere-turns is to be supplied by the series winding at the full-load current of 200 amperes; consequently, in the series winding there must be about 8 turns $(200 \times 8 = 1600$ ampere-turns). This is somewhat more than necessary, but an allowance has to be made to compensate for the ohmic drop in the compounding winding itself.

The number of turns required for any degree of over-compounding may be determined in a similar way. With the same data as before, suppose the machine to give the right voltage (110 volts) at full load, with 8 turns of compounding winding and with the same field current of 5.5 amperes as at no load. Let it be required now to put on more turns of compounding winding, in order to get 116 volts at full load. To determine this additional number of turns experimentally, strengthen the shunt field until the voltmeter reads 116 volts at the terminals of the machine. Suppose the necessary field current to be 6.2 amperes, which corresponds to an additional excitation of $(6.2 - 5.5) \times 2400 = 1680$ ampere-turns. Not all of this excitation, however, needs to be supplied by the series winding, because with the higher terminal voltage the shunt current increases automatically; in our case it will be $\frac{116}{110} \times 5.5 = 5.8$ amperes, which consequently means an increase of

(5.8 — 5.5) × 2400 = 720 ampere-turns; the rest, 1680 — 720 = 960 ampere-turns must be supplied by the series winding. With a current of 200 amperes, 5 turns will be sufficient to bring about the desired overcompounding (in addition to 8 turns put on before).

If the number of turns in the shunt winding is not known from the design data of the machine, it can be easily determined by exciting the machine separately by a known number of ampere-turns and comparing the voltage thus obtained with that given by self-excitation. For instance, if it takes 72 turns of an auxiliary winding at a current of 100 amperes to excite the machine up to, say, 50 volts, while it takes only 3 amperes to induce the same voltage by using the shunt winding, then the number of turns of the shunt winding is evidently

$$\frac{100 \times 72}{3} = 2400 \text{ turns.}$$

308. EXPERIMENT 15-D. — Exercises in Compounding Direct-Current Generators. — The purpose of the experiment is to investigate the action of the compounding winding, explained in §§ 306 and 307. The machine used for this exercise should be a shunt-wound generator; the student himself is expected to provide it with a compound winding. The experiment can be conveniently conducted as follows:

(1) Take a load characteristic of the machine, using it as a shunt generator, that is to say, without the compounding winding. Set the field rheostat so as to get the rated voltage at no load, and then keep it in this position, gradually increasing the load to, say, 25 per cent overload. Note the voltage drop with the increasing load. This run is intended to give an idea of what the regulation of the machine would be without the series winding.

(2) Set the field rheostat and the load resistances so as to get full rated load current at normal voltage. Note the necessary increase in field current.

(3) Determine the number of turns in the shunt winding. To do this, first take a magnetization curve of the machine using the shunt winding only. Then put a *known* number of turns on the field magnets of the machine, and take the same no-load characteristic, having the machine separately excited through this field. The shunt winding must in this case be kept open. The comparison of the exciting currents in both cases will permit the determination of the number of turns in the shunt winding, as explained above.

(4) From the results of (2) and (3) figure out the necessary number of series turns for flat compounding and place them on the machine.

(5) Check the number of series turns by taking a load characteristic of the machine.

(6) Determine the number of series turns for over-compounding the machine by about 10 per cent, and check the results experimentally.

(7) Shunt the series winding by a German-silver strip, and adjust it so as to again make the machine flat-compounded.

Report. (1) Plot the load characteristics (Fig. 267) with and without compounding winding; also for over-compounding, if sufficient data were taken in the laboratory.

(2) Plot no-load characteristics obtained by using the shunt winding alone, and the series turns alone; explain how the number of turns in the shunt winding was determined from these two curves, and give the numerical results.

(3) Explain how the number of turns of the series winding was predetermined for flat-compounding and for over-compounding.

(4) Give the result of applying German-silver shunts.

309. Tirrell Regulator. — The compounding winding does not give a perfect voltage regulation, nor does it correct for speed fluctuations of the prime mover; moreover, the machine must be made somewhat larger and heavier, in order to accommodate the series winding. Various attempts have been made to do away with the series winding and to regulate the voltage of the machine automatically, — for instance, by solenoids, acting on the shunt-field rheostat. An objection to such regulators is that they are rather sluggish, not following variations in voltage quickly enough.

Mr. Tirrell conceived the idea of accomplishing an automatic voltage regulation by periodically short-circuiting the field rheostat, or a portion of it (Fig. 263). This is done by light vibrating contacts with practically no inertia. The relative lengths of time, during which the rheostat is short-circuited or active, determine the average value of the field current and consequently the voltage of the machine. The leads from the field rheostat are short-circuited between the platinum tips marked "relay contacts"; a condenser is placed across the contacts for reducing sparking. The closing and opening of these contacts is done by the differentially wound electromagnet, controlled by the "main contacts." The main contacts are closed and opened by the control electromagnet connected across the main bus-bars.

The action of the regulator is as follows: Suppose the main contacts to be open; then the left-hand side only of the differential electromagnet is energized; it opens the relay contacts and cuts the field rheostat into the circuit. The terminal voltage of the machine immediately drops below normal. This reduces the current in the control electromagnet, and the spring above it closes the main contacts. Now a current flows through the right branch of the differential electromagnet and destroys its previous magnetization. The spring above closes the relay contacts and short-circuits the field rheostat. The line voltage rises above normal, the main contacts are again opened, and the operations of the two electromagnets are repeated in the same order. In reality, both contacts are continually vibrating; the periods during which the relay contacts are closed adjust themselves automatically, so as to maintain a constant voltage at the terminals of the machine.

An effect, similar to over-compounding, is produced by providing an additional series winding on the main control electromagnet. The voltage then rises as the load increases. It is not necessary to send the whole main current through this winding, but merely a small part of it. To accomplish this, a low-resistance shunt is inserted into the main circuit; the series winding of the electromagnet is connected across the terminals of this shunt, as in direct-current ammeters (Fig. 35). The series winding opposes the action of the shunt winding; therefore, with a heavy load on the line the control electromagnet becomes weaker, and the opposing spring keeps the main contacts closed for a comparatively longer time.

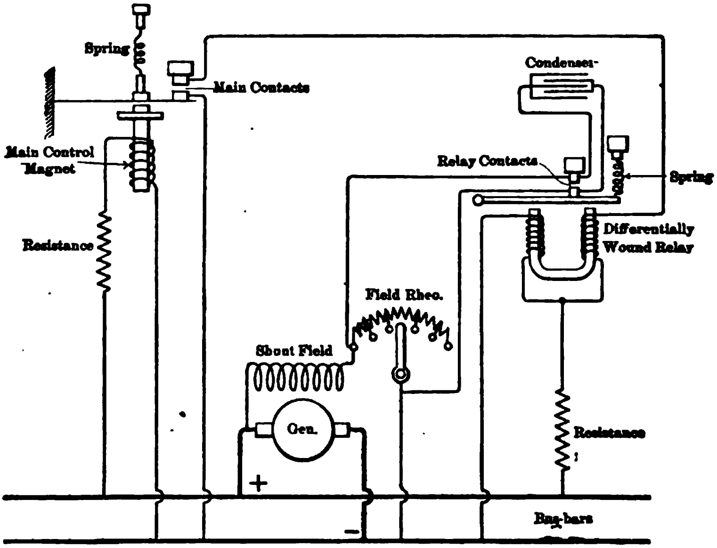

FIG. 273. Diagram of connections of a Tirrell voltage regulator for use with direct-current generators.

Fig. 274 shows the external view of a Tirrell regulator; the main relay contacts are seen to the left, the differential electromagnet to the right. The radial switch below is for connecting the regulator to the field of any of the machines working in parallel. The double-throw switches are for reversing the polarity of the spark, so as to wear the contacts equally.

310. EXPERIMENT 15-E. — Study of Tirrell Regulator. — The regulator, described in the preceding article, is connected as shown in Fig. 273. The experiment consists in observing its performance under various conditions met with in practice. Vary the load of the machine alternately gradually and suddenly; vary the speed of the driving motor, setting of the field rheostat, adjustment of the regulator springs, etc., noting how the voltage regulation is affected by these changes. Connect the compounding winding of the regulator, and observe its action. If two or more generators are available, run them in parallel, and have one of them connected to the Tirrell regulator. See if the machines distribute the fluctuating load properly under these conditions; perform this experiment with shunt-wound and with compound-wound machines. Investigate all the factors separately and report results observed.

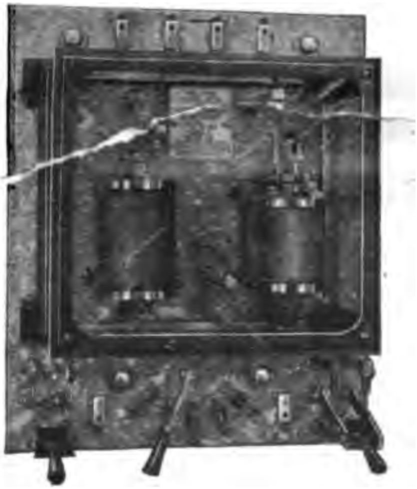

FIG. 274. Tirrell voltage regulator for direct-current generators.

OPERATION IN PARALLEL.

311. It is common practice in power houses to have more than one generator connected to the same bus-bars and operated simultaneously. This gives a more flexible arrangement than one large machine of a size sufficient to supply the heaviest load of the day. The chief advantages of having more than one machine are:

(1) The number of machines actually used during certain hours can be made to correspond closely to the demand; this permits the operation of the machines near the point of their maximum efficiency.

(2) An accident to one machine does not shut down the whole station.

In large power houses it is necessary to use several machines, simply because the total output is considerably above the capacity of the largest generator built.

With the system of distribution of electrical energy used at present (constant-potential system), machines which operate simultaneously are connected *in parallel*, so that each runs at full voltage and supplies a part of the current demand.*

Certain conditions must be fulfilled in order that two or more machines can supply power to the same bus-bars, without interfering with each other, and each take a proportionate share of the total load. When but one machine is connected to the line, the power generated can flow only into the line; when, however, two generators are connected to the same line, the current from the first machine, instead of flowing to the line, may, under certain circumstances, also flow into the second machine, driving it as motor. In this case, the second machine not only does not help the first one, but constitutes an additional useless load for it. Even if this is prevented, and both machines are working as generators sending power into the line, it does not necessarily follow that each machine takes its proper share of load, unless certain conditions are fulfilled.

These conditions are discussed in the following articles; shunt-wound machines are more easily operated in parallel than compound-wound machines and are therefore considered first.

312. Shunt-Wound Generators in Parallel.— Two identical shunt-wound machines driven by identical prime movers usually run satisfactorily in parallel, and automatically divide the load in two equal parts, for there is no reason why one machine should carry more load than the other. If it should endeavor to take more load, its voltage would drop, and the other machine, becoming electrically stronger, would automatically take up more load.

Referring to Fig. 275, suppose that machine No. 1 is supplying the load and that it becomes necessary to put the other machine in operation. Generator No. 2 is first excited so as to give the same voltage as No. 1 (and the right polarity, of course); then its main switch is closed. The first machine still continues to supply the total load, unless the excitation of the second machine be somewhat *increased* and that of the first machine *reduced*. By regulating the field rheostats of both machines, the load can be divided in any desired proportion; if the machines are identical in operation each takes its share of load, even with fluctuating current and voltage. One voltmeter only is used with any number of machines in parallel. Each machine is provided with receptacles, such as p_1 and p_2; a plug is inserted in one of the receptacles, and thus connects the voltmeter to the desired machine.

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* With distribution at a constant current, — for example, for arc lighting (§ 230), or as is proposed for certain long-distance transmission lines, — the machines, if more than one are used, must be connected *in series*, each supplying the total line current, and taking part of the total voltage. The considerations of the present article do not apply to such systems.

If it is desired to disconnect machine No. 1 and to transfer the whole load to the other machine, this should *not* be done by simply opening the main switch of the first machine. This would cause an arc at the switch, the line voltage would drop, and the second machine would experience a harmful shock, as the extra load is suddenly thrown upon it. This may not be noticeable on small laboratory generators, but is of consequence with large machines. Therefore, before opening the main switch of one of the machines its field excitation must be *lowered* and that of the other machine *increased*, until this latter carries practically the whole load. Then the main switch can be opened without disturbing the established electrical relations.

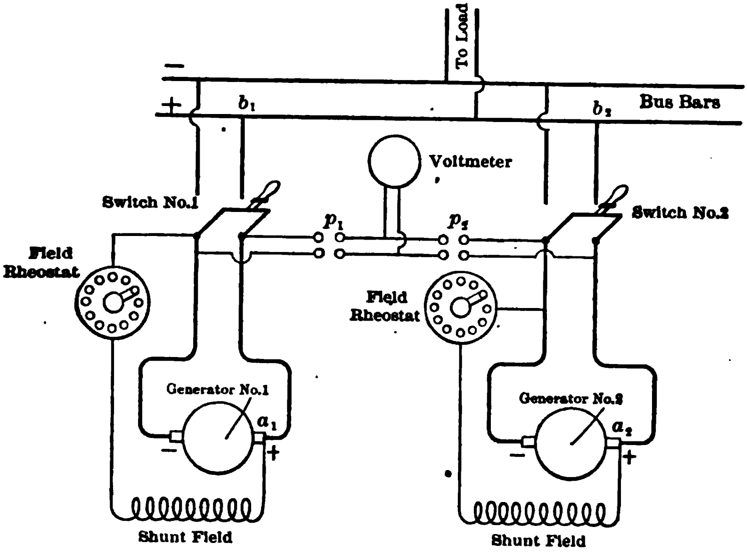

FIG. 275. Two shunt-wound generators in parallel.

By lowering the field excitation of the first generator still more, its armature current is reversed, and the machine begins to run as *a shunt motor*, driving its prime mover. This, of course, must be avoided in practice, but, should it accidentally occur, no particular harm would be done, as the machine continues to run in the same direction. Some power houses are provided with "reverse-current relays," or special circuit-breakers, which open the circuit of the machine in case the current becomes reversed.

If the two machines have different characteristics, that is to say, voltage drop in one differs from that in the other with increasing load, it may easily occur, that, while the machines divide the current properly at a certain load, the machine which has the smaller internal drop becomes relatively overloaded on heavier loads, and vice versa. Under such circumstances it becomes necessary to artificially "spoil" the characteristics of the better machine, by connecting some resistance in its armature circuit, until it gives the same per cent drop as the other machine. Then the machines will work satisfactorily under all load conditions, unless the degree of saturation in their magnetic circuits is very different.

313. EXPERIMENT 15-F. —Operating Shunt-Wound Generators in Parallel. —The student is given in the laboratory two shunt-wound generators of different type, possessing different characteristics; the machines are connected in parallel, as shown in Fig. 275, and loaded on a rheostat. Take a moderate load and practice in transferring it from one machine to the other, by regulating the field rheostats. Also start and stop one of the machines when the other is running, without disturbing the load. Weaken the field of one of the machines until the current in it is reversed and it is driven by the other machine as a motor. Try to run one or both machines in parallel with the laboratory direct-current supply.

After this, investigate whether the machines divide the current properly at all loads. Take a load nearly equal to the total capacity of the two machines, and adjust the field rheostat so that each machine takes its share of the load. Gradually reduce the load and observe whether it is all the time divided in about the same proportion. If not, introduce some resistance into the armature circuit of the machine that has the tendency to take a larger amount of the load. Adjust the resistance to the best value, and then take a complete curve from full load to no load, showing the current distribution between the two machines.

314. Compound-Wound Generators in Parallel.—Two compound-wound generators, connected to the same bus-bars, are shown in Fig. 276. The general scheme of connections is the same as in Fig. 275, except for the addition of the series windings of the machines. The series windings are provided, if necessary, with German-silver shunts

(Figs. 271 and 272). It may seem at first, that the addition of series windings should not in any way affect a satisfactory operation of the machines, and that all that has been said in regard to shunt-wound machines, is directly applicable to compound-wound generators.

Experience shows, however, that two compound-wound generators do not run stable enough in parallel, unless an *equalizing* connection E_1E_2 is established between the brushes a_1 and a_2, adjacent to the series windings.

The cause for this is as follows: Suppose first, that the switch E_1 is open so that there is no equalizing connection, and assume that for some reason one of the machines, for instance No. 2, gives a slightly higher induced e.m.f. The result would be that this machine would take a slightly larger share of the load than No. 1; therefore a larger current will flow through the compounding winding of No. 2 than through that of No. 1. The e.m.f. of No. 2 will thereby be *still more increased,* and the share of load of the other machine decreased. This will continue until one of the machines carries the whole load, while the other machine runs at no load. After this, unless the machines are provided with reverse-current circuit breakers, generator No. 2 will

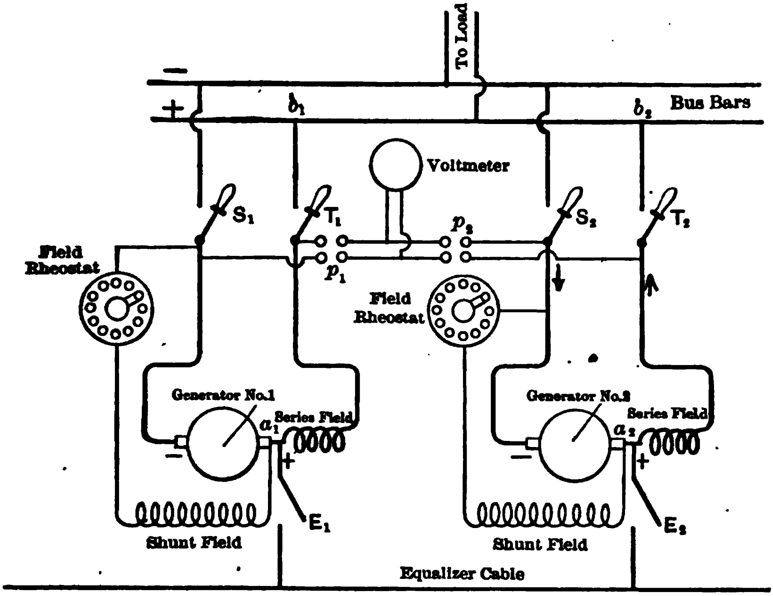

FIG. 276. Two compound-wound generators in parallel.

reverse the current in No. 1 and will drive it as a motor. Such an incident would do no harm to a shunt machine, as was mentioned above; but with a compound-wound machine the reversed current flowing through the series field weakens the total field, and the machine begins to run faster and faster, driving its prime mover. This causes the other machine to become overloaded, and it slows down. Now machine No. 1, having a higher e.m.f. because of a higher speed, causes a sudden rush of current into the line and into the other machine; the phenomenon repeats itself with the machines exchanging their relation of driver and driven.

315. Action of the Equalizer.—Whether the above description of "pumping" between two compound-wound generators is exactly correct or not, experience shows that two compound-wound machines do not work satisfactorily without an equalizer between the points a_1 and a_2; the load is distributed unevenly between the machines, from moment to moment, and the current is easily reversed. The action of the equalizer is as follows (Fig. 277): Suppose, as before, that for some reason the induced e.m.f. of machine No. 2 is higher and therefore the current supplied by it larger than that of machine No. 1. This tends to make the ohmic drop between a_2 and b_2 larger than that between a_1 and b_1; but, with an equalizing cable of a negligible resistance between a_1 and a_2, this is impossible, and thus part of the current of machine No. 2, instead of flowing from a_2 to the positive bus-bar directly through b_2, flows to the same bus-bar through the equalizer c and the series winding of machine No. 1. Therefore, the field of machine No. 2 is strengthened *less* than it would be without the equalizer; at the same time the field of the weaker machine, No. 1, is strengthened by the excess of the current of the other machine; machine No. 1 is thus helped to keep up its voltage. In short, *the equalizing connection prevents the currents in the series fields of two or more machines from differing widely from each other, however different their armature currents may be.* Therefore,

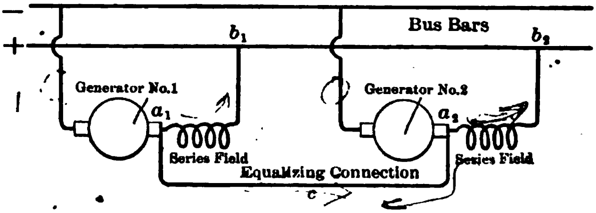

FIG. 277. Diagram illustrating the use of an equalizing connection between two compound-wound generators operating in parallel.

when connected by an equalizing bus-bar, different machines cannot have widely different voltages; cannot easily have the load disproportionately distributed, and the more remote becomes the possibility of one machine pumping power back into the other machine.

Two or more *identical* compound-wound machines, provided with an equalizer cable, as in Fig. 276, distribute the current among themselves equally well at all loads. A difficulty arises, however, when the machines have *different characteristics*, as was explained in the case of shunt-wound generators. The two conditions under which two compound-wound machines of different type can be satisfactorily operated in parallel are as follows:

(1) Both machines must have the same degree of compounding, or over-compounding, when running separately. If such is not the case, the machine which has a higher degree of compounding will take a larger share of the total output, as the load increases, because its induced e.m.f. will be higher than that of the other machine. The equalizing connection cannot remedy this fault.

(2) When each machine is delivering its proper share of load, the drop between a_1 and b_1 must be equal to that between a_2 and b_2; otherwise an equalizing current will flow through the equalizer; this would be wrong when the machines are delivering each the right current. As a general rule, the resistances a_1 b_1 and a_2 b_2 do not satisfy this requirement, and it becomes necessary to introduce some artificial resistance into one of the circuits. Then the machines divide the current properly under all conditions of load.

316. Directions for Connecting in Parallel. — Let machine No. 1 be supplying the load alone (Fig. 276), and suppose it to be required to put generator No. 2 in parallel with it. The main switches S_2 and T_2 are open, as well as the equalizer switch E_2. First, machine No. 2 is excited so as to give its full voltage; then the switches E_2 and T_2 are closed in order to cause part of the current of machine No. 1 to flow through the series field of machine No. 2. Then, after the voltage of machine No. 2 is properly adjusted to that of the bus-bars, the last switch S_2 is closed and the excitation brought up so as to distribute the load properly between the machines. In disconnecting one of the machines the switches must be opened in reverse order.

With large machines it is desirable to have three separate switches, as described above. With medium size machines two switches connecting the machine to the bus-bars can be united into one double-pole switch, though in this case the machines cannot be thrown in parallel as smoothly, because the main circuit becomes closed before the field is properly adjusted. In inexpensive isolated plants three-pole switches are sometimes used, combining all the three switches in one; with small machines a momentary rush of current is not considered harmful.

317. EXPERIMENT 15-G. — Operating Compound-Wound Generators in Parallel. — The student is given two D. C. generators of somewhat different characteristics; the machines may have been previously provided with a series winding, or the student himself may be expected to put it on. As is explained in §§ 314 to 316, three conditions are necessary in order that the machines may operate satisfactorily:

(1) An equalizing connection must be provided.

(2) Both machines must have the same degree of compounding or over-compounding when running alone.

(3) Voltage drop in a_1 b_1 must be equal to that in a_2 b_2 (Fig. 276).

First try to run the machines without any of these conditions being fulfilled, and see what occurs as the load is varied. Then introduce the necessary modifications, one by one, and observe the improvement in operation. Try to obtain a satisfactory operation at all loads, and take a curve showing the division of the load between the two machines.

The performance is considerably influenced by the resistance of the equalizer wire. The lower this resistance the better is the equalization of exciting currents and therefore the more satisfactory the operation of the machines. Take several readings at a constant load but with different resistances in the equalizing circuit, so as to observe the influence of this factor.

THREE-WIRE SYSTEM.

318. There are cases in which it is advantageous to distribute direct-current energy by three wires, instead of two (Figs. 278 to 280). The two outside wires are the main wires; the middle conductor merely takes up the unbalanced part of the load. The three-wire system is used in two cases:

(1) On lighting circuits, where the economy in copper thus obtained warrants the complication.

(2) In shops where variable-speed motors are used.

In three-wire lighting circuits the lamps are connected between each of the outside wires and the middle wire (Fig. 281). The voltage across the outside mains is about 220 volts, and the copper economy corresponds to this pressure; at the same time standard 110-volt lamps are used. Equation (1) in § 256 shows, that the weight of copper with the same power W is inversely proportional to the square of the transmission voltage. The cross-section of the middle conductor is usually not more than 50 per cent of that of the outside wires, so that even the addition of a third wire still leaves a considerable margin of economy, as compared to an ordinary 110-volt two-wire distribution.

The use of the three-wire system for variable-speed drive is illustrated in Fig. 295. The motor armature may be connected either across 110 volts, or across 220 volts, giving half speed in the first case and full speed in the second case. The same results could be obtained on the 220-volt circuit by inserting some resistance into the armature circuit; but this would be too wasteful of energy, and, moreover, the speed would fluctuate with the load.

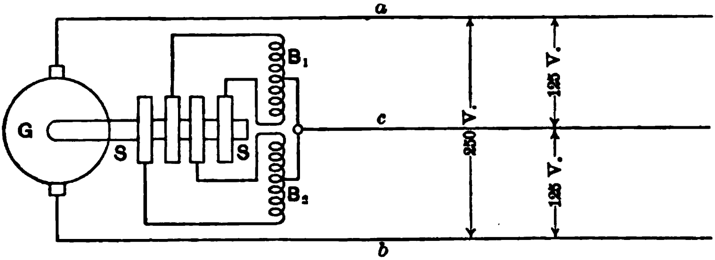

FIG. 278. Dividing line voltage in two by means of balancing coils.

The details of the three-wire system and the numerical relations are explained in the following articles.

319. Methods for Dividing Voltage. — The two outside wires in the three-wire system are connected to the terminals of the main generator; special devices are used for dividing the voltage in two, so as to obtain a point for connecting the middle conductor. The methods in common use are:

(1) Balance coils (Fig. 278).

(2) Motor-generator, or balancer set (Figs. 279 and 281).

A storage battery may also be used for dividing the voltage, as shown in Fig. 280, but a comparatively high cost and depreciation prevent their universal application for this service.

The balance coils (Fig. 278) are ordinary reactance coils, or auto-transformers, provided with taps B_1, B_2 in the center. The generator *G* has four slip-rings *SS*, connected to four equidistant points of the armature winding, as in a two-phase rotary converter (§ 505). The balance coils are connected as in the quarter-phase system, Fig. 334, and the middle wire *c* is connected to the neutral point of this polyphase system. This neutral point being symmetrically situated in regard to the outside wires *a* and *b* on the direct-current side, its potential is midway between the potentials of these wires, and thus divides the voltage in two equal parts. The energy consumption in the balance coils is small, their inductance being large as compared to their resistance.

Instead of two coils and four slip-rings, three coils and three slip-rings are sometimes used, in regular three-phase *Y*-connection (Figs. 337 and 386). The middle conductor is connected to the neutral point of *Y*. This system is particularly well adapted for use with rotary converters which are regularly provided with slip-rings. No balance coils are needed in this case, the neutral point being obtained on the main transformers.

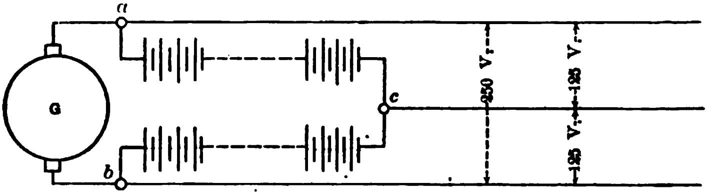

FIG. 280. Dividing line voltage in two by means of a storage battery.

The balancer set (Figs. 279 and 281) consists of two identical shunt-wound machines, B_1 and B_2, of comparatively small capacity. The armatures are connected in series electrically, and the machines run idle between the two outside wires; they must be mounted on the same shaft, or belted together. The middle wire is connected at *c* between the two armatures.

The machines being identical, excited to the same point, and run at the same speed, the voltage is naturally divided at *c* into two equal parts. When the load becomes unbalanced, one of the machines runs as generator, the other as motor; the voltages still remain approximately the same, the induced e.m.f.'s being the same; the details of distribution of currents are explained in the next article.

An advantage of balance coils over a motor-generator set is that the former are less expensive, and, being stationary, require but slight attention. On the other hand, a three-wire generator with slip-rings is more expensive than an ordinary two-wire machine. Here, as in most practical problems, the preference is given in each particular case to one or the other system, according to local conditions, and the personal views of the engineer who decides the question.

320. Effect of Unbalancing. — The distribution of currents in a three-wire system with unbalanced load is shown in Fig. 281, the voltage being divided by a motor-generator set. It is assumed that 1000 ordinary 110-volt, 16 candle-power incandescent lamps are connected on one side of the line, and 800 on the other side, each lamp consuming one half ampere. This gives 500 amperes and 400 amperes in the two outside conductors; the difference, 100 amperes, returns through the middle wire to the balancer set. The balancer set, not being connected to a prime mover, cannot create energy; it merely equalizes, or transfers it from the less loaded side to the more heavily loaded side. Therefore, if the machines were ideal, 100 amperes would divide into 50 and 50, flowing in the opposite directions, the lower machine running as motor, the upper as generator. The main generator would supply the average load of 450 amperes at 220 volts. With this distribution of currents, the condition is fulfilled at the points *a* and *b*, that the sum of the currents flowing towards a point must be equal to the sum of those flowing from the point.

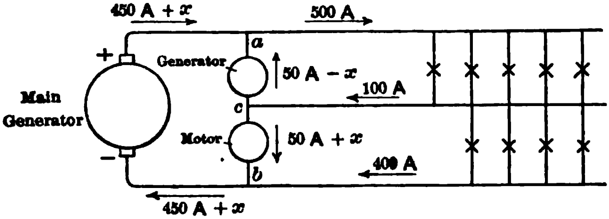

FIG. 281. Distribution of currents in a three-wire system at an unbalanced load.

In reality, the current distribution is somewhat different, because of the losses in the balancer set. A certain current x flows through the set, even when the load is perfectly balanced; this current is necessary for overcoming iron loss and friction in the two machines. Moreover, the main generator has to supply some current for the shunt fields of both machines. Superposing the current x upon the useful currents mentioned above, we get a distribution of currents shown in Fig. 281;

the current in the balancer machine, working as generator, is reduced, that in the motor increased by the amount x. The condition $\Sigma i = 0$ at the points a and b is again fulfilled.

With an unbalancing of currents, the voltages between the middle wire and the two outside conductors become somewhat different; but, with a properly designed balancer set, the difference should not be more than a very few per cent; moreover, it may be corrected to some extent by regulating the field rheostats of the balancer machines. An investigation of current relations in balance coils (Fig. 278) leads to similar results.

321. Efficiency of Balancer Sets. — Referring to Fig. 281, the efficiency of the balancer set is equal to the ratio of the output of the machine working as generator, to the input into the machine working as motor; the excitation losses of both machines should be added to the input. This is according to the standard definition of efficiency of any motor-generator set. It will be easily seen, however, that in the case of a balancer set, the efficiency, according to this definition, becomes negative, when the load is nearly balanced, because then both machines operate as motors. As unbalancing increases, the efficiency gradually rises to zero and then becomes positive. There is no physical contradiction in this result, provided that the actual conditions of the operation of the set are clearly kept in mind.

There is another way of looking at the efficiency of a balancer set, namely, from the standpoint of the three-wire system, taken as a whole. The losses in the balancer set increase the output of the main generator above the useful demand for power; the ratio of this demand to the output of the main generator may be called the efficiency of the three-wire system, with reference to the balancer set.

To illustrate the two above definitions of efficiency, assume that the no-load current of the balancer set, x, is 10 amperes (Fig. 281), and that the field circuit of each machine consumes 2 amperes, both fields being connected in parallel, across the outside conductors.

The efficiency of the motor-generator set itself, according to the first definition, is

$$\frac{(50 - 10)\,110}{(50 + 10)\,110 + (2 + 2)\,220} = 70.6\%.$$

The efficiency of the three-wire system itself, according to the second definition, is

$$\frac{500 \times 110 + 400 \times 110}{(450 + 10 + 2 + 2)\,220} = 97\%.$$

Both expressions have definite physical meaning, and both are used according to the conditions of a problem. If a greater accuracy is required, copper loss in the armatures of the balancer set, and the resulting unbalancing of voltages, must be taken into account in the above expressions for efficiency.

322. EXPERIMENT 15-H. — Study of a Symmetrical Three-Wire System. —Wire up the generator and the balancer set, as in Fig. 279, and provide variable resistances, to be used as a load. Load up one side of the line until the balancer runs as nearly as possible at its full rated capacity. Note the input and the output of the motor and generator constituting the balancer set, voltages on the loaded and unloaded sides of the line, speed of the set, currents in the three line wires, and the field current of the balancer machines. Then begin loading the other side of the line, keeping the load on the first side constant. As the unbalancing decreases, the balancer load also decreases, though the total load on the line increases. Take readings as indicated above with different degrees of unbalancing, until the load on both sides is the same, and the balancer runs idle. Increasing the load further merely reverses the conditions in the balancer; the machine which was running as motor begins to run as generator, and vice versa.

If a generator with slip-rings is available, or a rotary converter, a three-wire system with balance coils (Fig. 278) may be realized. It is not absolutely necessary to have three or four slip-rings; if only two slip-rings are available, the neutral point may be obtained with one balance coil. In fact, it is desired that the student investigate the effect of using only one balance coil instead of two. With a three-phase rotary converter, three balance coils are connected in Y, and the middle wire connected to the neutral point. The same load experiment should be performed with balance coils as with the motor-generator set. Note, that balance coils consume a magnetizing current, which is an alternating current, and cannot be detected by ordinary moving-coil ammeters (§ 37).

Report. Draw a diagram of actual connections. Plot to amperes in the middle wire: currents in the two outside wires, voltages, generator output and currents in the balancer armatures and fields; also the speed of the set. Figure out the efficiency of the balancer set and of the three-wire system, as in § 321. Give similar results for the run in which balance coils were used.

323. Unsymmetrical Three-Wire System. — When a three-wire system is used for variable-speed drive, it is not necessary to have the voltage divided in two *equal* parts; in fact, by making the system *un-*

symmetrical a wider range of speeds can be obtained. Such an unsymmetrical, or, as they are improperly called, "unbalanced" system, is shown in Fig. 282. The two machines, constituting the balancer set, are wound for different voltages, so that the total pressure of 250 volts is subdivided into 90 volts and 160 volts. With this system the armature of a motor can be connected on either 90, 160 or 250 volts, which gives a ratio of about 3 : 1 between the lowest and highest speed. If, in addition, field control is used, by which the speed of the motor is increased twice, a total speed range of 6 : 1 may be had, instead of 4 : 1, as with the ordinary symmetrical three-wire system. The disadvantages of this system as compared to the ordinary three-wire system are: more expensive wiring because motors take a heavy current at low voltage; the system is unsymmetrical and therefore more difficult to supervise and the balancer set is more expensive. The application of this system has thus far been a limited one, though it may be profitably used in large repair shops where a wide range of speed of machine tools is of prime importance.

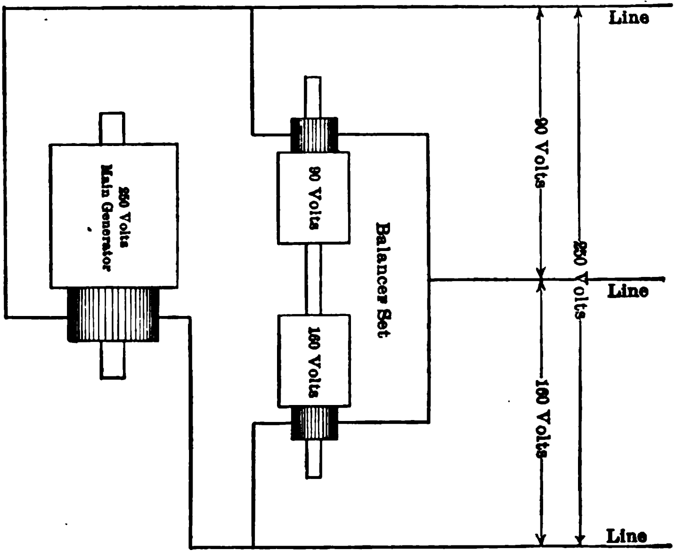

FIG. 282. The Allis-Chalmers unsymmetrical three-wire system for variable-speed drive.

324. EXPERIMENT 15-I. — Study of an Unsymmetrical Three-Wire System. — Same directions as in § 322.

325. Four-Wire System. — Some companies have gone so far as to use a *four-wire* unsymmetrical system, in order to get a still wider variation of speed of the motors. One of these systems, which has been practically applied in a few cases, is shown in Fig. 283. The balancer set consists of three direct-connected machines, *A*, *B* and *C*. Two of these machines may be combined into one two-commutator machine; this makes the set less expensive and less cumbersome. It will be seen, that with such a system, six different combinations of voltages, and therefore six different speeds, are possible, without field control. The ratio of the highest to the lowest speed is as 240 : 40 = 6 : 1. By using field control, the range of speeds may be about doubled. The complication and the expense of a four-wire unsymmetrical line are such as to justify the use of this system in exceptional cases only. However, the more expensive labor and machinery become, the greater field will be opened for such multivoltage systems, as they permit a considerable increase in the output of the tools.

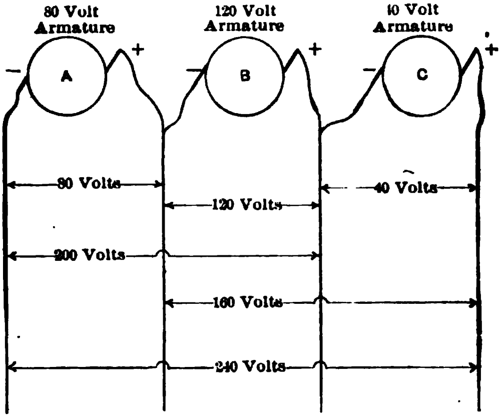

FIG. 283. The Crocker-Wheeler four-wire system for variable-speed drive.

326. EXPERIMENT 15-J. — Study of a Four-Wire System. —

Same directions as in § 322.

Note. For troubles in operation of direct-current machines and their detection, see §§ 345 to 349.

CHAPTER XVI.

DIRECT-CURRENT MOTORS — OPERATING FEATURES.

327. DIRECT-CURRENT machines, such as are shown in Fig. 264 and described in § 299, are *convertible*, that is to say, they may be operated either as generators or as motors. When such a machine is driven by a source of mechanical power (prime mover) it is operating as a generator, and delivers electrical energy. Conversely, when electrical energy is supplied to the machine, it runs as a motor, and is capable of developing mechanical energy at its shaft.

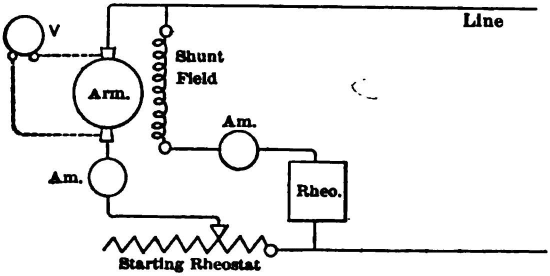

FIG. 284. Diagram of connections of a shunt-wound motor.

This convertibility is a direct result of two fundamental laws of electromagnetism; viz.: (1) when an electrical conductor, forming part of a closed circuit, is moved across a magnetic field, currents are induced in the conductor such as to oppose the motion; (2) when a current is sent through a conductor located in a magnetic field, it tends to move out of the field. In the first case the conductor acts as an element of a generator, in the second as an element of a motor.

328. Types of Motors. — Two types of direct-current motors are most commonly used in practice: shunt-wound motor (Fig. 284) and series-wound motor (Fig. 285). In the shunt-wound motor the field winding is connected directly across the constant voltage supply circuit, so that the magnetism of the motor is practically constant at all loads. In the series-wound motor, the field winding is in series with the armature circuit; as the load increases, the current taken by the motor also increases, this increased current strengthening the magnetic field of the motor.

There is a marked difference in the performance of these two types of motors, and each type has a field of application of its own. The shunt motor is essentially a constant-speed motor, while the speed of the series-wound motor increases as the load decreases. A theoretical exposition of this is given in the next article, which explains the facts observed experimentally. Shunt-wound motors are used for machine-tool drive, and for driving various machines in textile and other industries, where a constant, or nearly constant, speed is required, independent of the load. Series-wound motors are used in railway, crane and hoisting work, where a heavy starting torque is necessary, and where the motors have to be frequently started and stopped. In these cases it is desirable to have a motor which automatically slows down as the load increases, in order not to have too large fluctuations in power demand. In some special cases motors are desired, with characteristics intermediate between those of the two above-mentioned types. Such motors are provided with both series and shunt field windings, and are called compound-wound motors. They are used with elevators, also for driving planers, punches, etc.

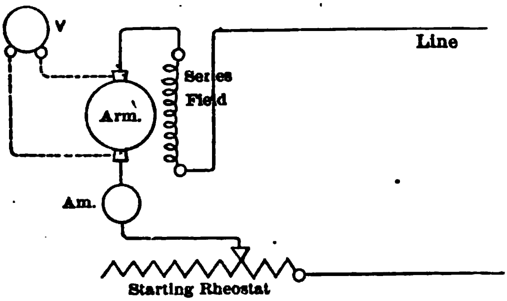

FIG. 285. Diagram of connections of a series-wound motor.

In operating electric motors, starting and regulating rheostats are required; these devices are described in Chapter XXX (" Controllers and Regulators "). It is advisable that the student perform one or two of the first experiments of that chapter before beginning the brake tests described in this chapter.

329. Field Strength and Speed. — The difference in the behavior of shunt, and series-wound motors under various loads can be explained as follows: When the armature of a motor is revolving, a counter-electromotive force is induced in it by the magnetic field of the machine

This e.m.f., e, is smaller than the applied voltage E by the amount of the ohmic drop IR in the armature and brushes, or

$$e = E - IR.$$

In commercial motors the term IR amounts to but a very few per cent of E, in order to keep the efficiency of the motor sufficiently high. Therefore *e is approximately equal to E*, or the counter-electromotive force induced in the armature is approximately equal to the applied voltage. On the other hand, this counter-e.m.f., according to the fundamental law of induction, is proportional to the magnetic flux of the machine, and to the speed at which the armature conductors cut the flux. Thus

$$e = C.\ Nn. \qquad (1)$$

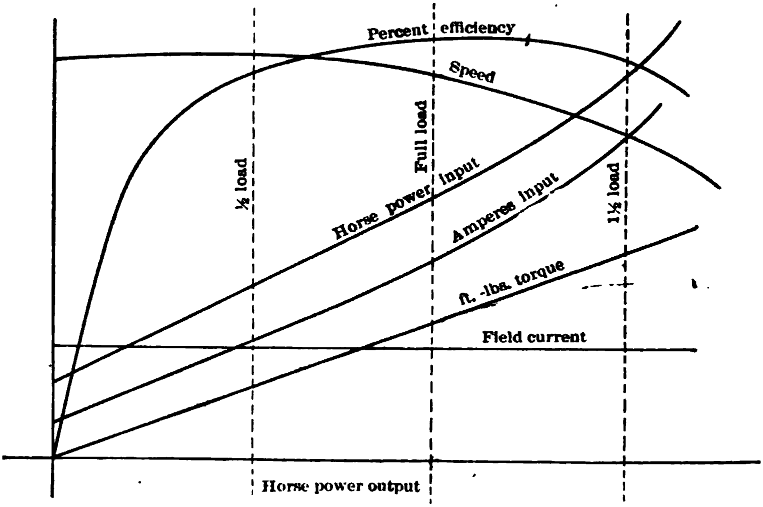

FIG. 286. Performance curves of a shunt-wound motor.

where C is a constant, N is the useful flux of the machine, and n is its speed. The e.m.f. e being approximately constant, so long as the voltage at the armature terminals is constant, the product, flux times speed, in this formula is also constant.

In a shunt-wound motor (Fig. 284) the fields are separately excited, so that the flux *is* constant, and does not depend on the load. Therefore, the speed of such motors is also approximately constant. In series-wound motors (Fig. 285) the field strength depends entirely upon the load, since the exciting winding is connected in series with the main circuit. As the load decreases and the motor begins to consume less power, a smaller current flows through the field winding, and the field is weakened. But, according to the equation (1), the weaker the field, the higher the speed of the motor. For this reason a series motor runs faster the lighter the load. A series motor should never be connected to the supply without any load; it will run away, unless a sufficient resistance is connected in its circuit.

The difference in characteristics of the shunt and the series motor is made clearer by a study of their performance curves, shown in Figs. 286 and 287, — particularly by a comparison of the speed curves.

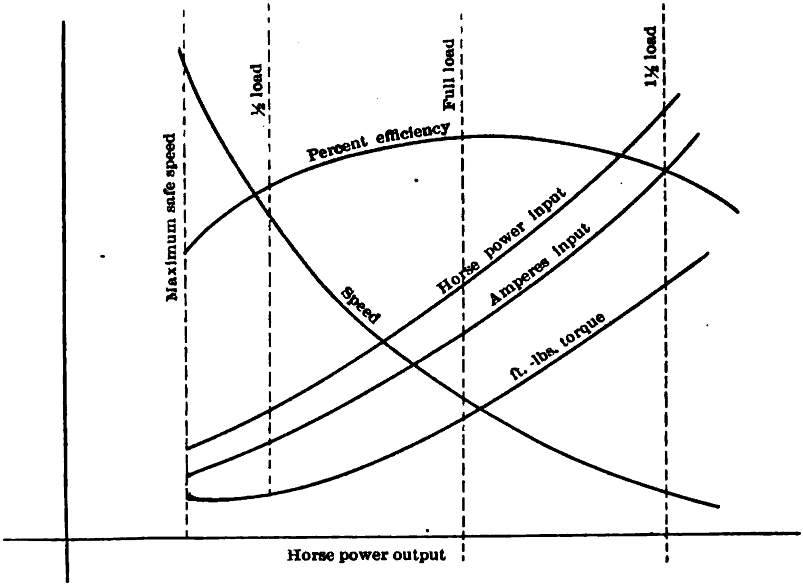

FIG. 287. Performance curves of a series-wound motor.

330. Speed Control.— It has been shown above that the speed of a *shunt-wound motor* remains approximately constant, as long as the field excitation is constant. If, however, it is desired to increase the speed, this can be done by weakening the field of the motor by means of the field rheostat. When the field current is reduced, the counter-e.m.f. is also reduced the first moment; this causes a rush of current into the armature, and an excess of power above that necessary for carrying the load. This excess of power accelerates the motor, until the counter-e.m.f. has increased sufficiently to allow but the normal current to flow through the armature.

This method of speed control is actually used, — for instance, in machine-tool drive. A limit to weakening the field is imposed by sparking at the brushes, for, with a weak field, the armature action becomes more prominent and distorts the field; the currents in the coils short-circuited under the brushes become excessive, and cause a destructive sparking at the commutator.

The value of the armature reaction depends on the position of the brushes; therefore the speed of the motor may be varied within certain limits by merely shifting the brushes. No considerable speed variation can be obtained by this means, since the motor begins to spark as soon as the brushes are shifted too far either way from the right position, but this method is convenient for adjusting a speed to an exact value, after an approximately right speed has been obtained by the regulating rheostat.

Speed of a shunt motor is lowered by reducing the voltage at the armature terminals; this follows directly from equation (1). Inserting resistance in the armature circuit is seldom done except with very small motors, the method involving considerable waste of energy. The voltage may be varied economically by having the motor connected to a three-wire (three-voltage) supply (see §§ 318 and 323).

Speed of a *series motor* may be increased also by weakening the field; this is done by providing a shunt around the field winding. This method has been used to some extent on street-car motors, where the car had to run on long level stretches. This expedient has been abandoned, because weakening the fields increases the effect of armature reaction and causes the motors to spark. *Compensated* railway motors are now coming into use, in which the armature reaction is overcome, and the method of shunting fields is coming into use again, especially on interurban roads, where considerable time can be saved by running long stretches at a high speed.

To reduce the speed of a series motor, resistances are inserted into the armature circuit. This method is used in practice with railway and crane motors, where economy is not so important because of an intermittent service, but where speed control is very essential. It is also possible to reduce the speed by shunting the armature by a resistance, so as to have a comparatively strong field with small armature currents. This method is hardly ever used in practice, except for tests (see § 370).

331. Prony Brake. — The performance curves, shown in Figs. 286 and 287, are obtained by loading the motor and measuring input, output and speed. The simplest device for loading motors is a Prony brake (Fig. 288); it is extensively used with small and medium sized motors. In the form shown in the sketch, it consists of an iron band *ab* lined with soft wood, or with heavy canvas. The band embraces the pulley *P* of the motor, and is fastened to the beam *k*, the end of which rests on the scale *S*.

When the motor revolves, friction is developed between the lining of the brake and the pulley; the power of the motor is thus converted into heat. The brake pressure is regulated by the hand-wheel *h*, and in this way any desired load is obtained. The turning moment, or the *torque*, as it is called, is measured on the scale.

In most cases it is necessary to carry away the heat developed by friction, in order to prevent burning of the brake lining. The pulley, shown in the sketch, is cooled by a stream of water from the pipe *w* (see cross-section to the left); water is thrown by centrifugal force against the inner surface of the face of the pulley; the flange prevents it from being spilled.

If *P* is the net pressure on the scale at the end of a lever *l* feet long, the pressure at the end of a lever 1 foot long is *Pl* lbs.; consequently the work in ft.-lbs., performed during one minute, is $Pl.\,2\pi n$, where *n* is the number of revolutions of the motor per minute. The same in horse-power is

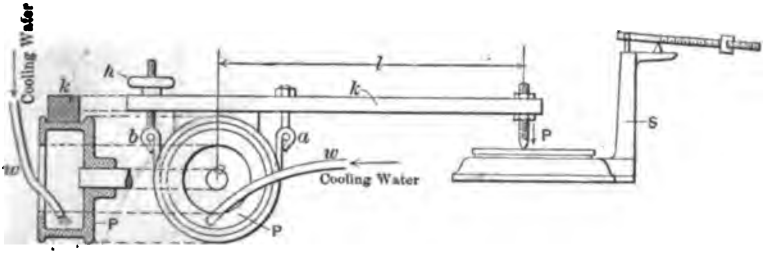

FIG. 288. A Prony brake with water-cooled pulley.

$$\frac{Pl.\,2\pi n}{33000},$$

or

$$\text{horse-power} = \frac{Pln}{5252}.$$

This is the common expression for figuring the output of a motor from a Prony brake test.

For loading very large motors the so-called water-brake is used; power is absorbed in it by driving water through paths of high resistance. The resultant friction heats up and even evaporates the water, while cold water is continually being supplied from the mains.

Eddy-current brakes are used to some extent with small motors. Such a brake usually consists of a copper disk mounted on the motor shaft, and of a set of electromagnets placed near the disk and supported from knife-edges, or ball bearings. When the motor revolves. eddy currents are induced in the disk as it cuts through the field produced by the electromagnets. The electromagnets tend to follow the disk, as in the classical Arago experiment, but are prevented by a lever which rests on a scale, as in Fig. 288; the pressure on the scale measures the torque. Load is varied by regulating the exciting current; artificial cooling of the disk is sometimes necessary. Eddy-current brakes give a steadier load than is possible with mechanical brakes.

332. Transmission Dynamometer. — An objection to the Prony brake is that load is not quite steady while speed is being measured, this limits the accuracy of the test. Another disadvantage is that a considerable amount of energy must be dissipated in a limited space. It is much more convenient to load the motor on an electric generator; a pump, a blower, etc., provided that the output of the motor can be readily measured. One method is to use a generator, the efficiency of which is known from a previous calibration. Another way is to connect the load machine to the motor under test, through a transmission dynamometer, — such as is shown in Figs. 289 and 290. The dynamometer does not absorb the power, but merely measures the torque between the driving and the driven shafts.

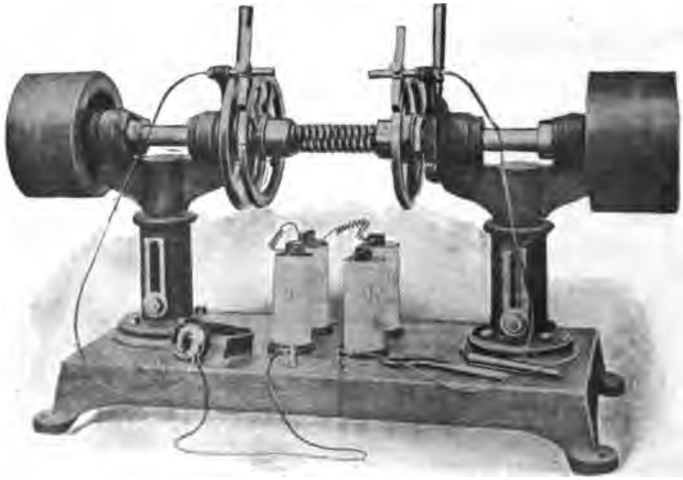

FIG. 289. A transmission dynamometer (Central Laboratory Supply Co.)

The torque is measured by the torsion spring previously calibrated in pounds. The difficulty in former dynamometers consisted in accurately measuring the angle of torsion of the spring, with the shafts revolving. This difficulty is ingeniously solved in the dynamometer, shown in the sketches, by measuring this angle electrically. An electric circuit is established through a few dry cells, a telephone receiver and the dynamometer spring, through two brushes and the contacts, — all clearly seen in Fig. 290.

When the machines are stationary, the brushes are in such a position, that when moved around together they come under the contacts simultaneously, and the circuit is closed; this is recognized by a click in the telephone. When the set is revolving, carrying a load, the spring is twisted, and the contacts are no longer closed simultaneously; the click in the telephone disappears. The left-hand brush is then shifted along the contact ring, until the click is again heard; the angle is read on the stationary index, and is evidently equal to the angle by which the spring was twisted. The power transmitted is calculated from the formula:

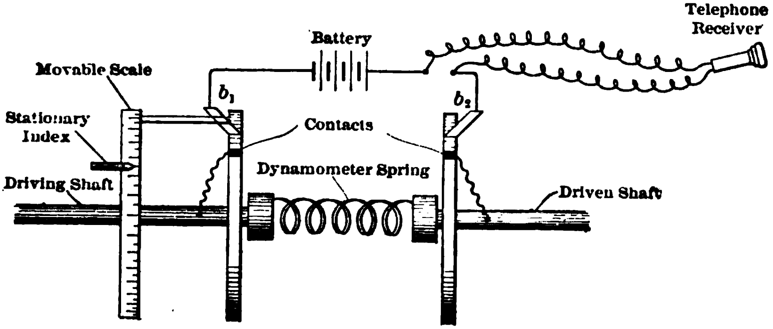

FIG. 290. Electrical connections of the dynomometer shown in Fig. 289.

$$\text{horse-power output} = \text{constant} \times \text{angle} \times (\text{r.p.m.}).$$

The spring is previously calibrated by weights, and the torsion constant calculated. Several springs of different rigidity are used with the same dynamometer, to increase its useful range. The dynamometer may be adapted to be used with machines direct-connected; or else, is provided with a pulley, and the load machine is belted to it.

333. EXPERIMENT 16-A. — Brake Test of a Shunt Motor. —

The purpose of the experiment is to obtain performance curves such as are shown in Fig. 286. The motor should be wired up, as shown in Fig. 284; one independent circuit includes the field winding with its regulating rheostat, and an ammeter; the other circuit is formed through the armature of the motor, with its starting and regulating rheostat, and an ammeter. A voltmeter V is connected across the armature, for measuring the pressure at which the machine operates. Either a Prony brake (§ 331), or a transmission dynamometer (§ 332) may be used for loading the motor.

Begin the test with the highest load, say 25 per cent overload; read armature amperes, field amperes, terminal voltage, speed, and brake load. Then gradually reduce the load to zero, taking a sufficient number of readings (from 8 to 10) for plotting curves. The field current and the terminal voltage must be kept constant throughout the whole test. The readings may be conveniently recorded on such a data sheet as is here shown.

	Arm. Amps.	Field Amps.	Volts.	R. P. M.	Torque Lbs.
Inst. No.					
Const					

Take a few points with field current 10 and 20 per cent below normal (see § 330). Also make runs with line voltages, say 10 per cent above and a like amount below normal, in order to see the influence of this factor on the performance of the motor.

Report. Plot performance curves, as shown in Fig. 286. On the same curve sheet plot the readings taken with lower field currents; also those corresponding to the various voltages of the supply. Explain the results.

334. EXPERIMENT 16-B.—Brake Test of a Series Motor.—The purpose of the experiment is to obtain performance curves for the series motor, as shown in Fig. 287. The experiment is conducted in much the same way as the preceding experiment, except that precautions should be taken to prevent the motor running away. With a shunt motor, the brake can be safely released, since the speed of the motor is practically the same at no load as when loaded. In a series motor the speed increases enormously, as soon as the load is taken off, and either the armature, the commutator, or the bearings are sure to be damaged, if the motor be allowed to run at this speed. For this reason, *always open the circuit before releasing the brake;* or at least have a sufficient resistance inserted into the circuit, to keep down the speed.

As an additional precaution against the motor running away, an *underload circuit-breaker* should be connected into the circuit; when the load is taken off, the current falls below a certain limit, and this device automatically opens the circuit. The student should not, however, rely absolutely on this circuit-breaker. It may "stick" just when it is necessary for it to act. It is best to have one man of the section stand near the main switch, and open the circuit if the motor reaches a dangerous speed.

The motor is wired up in series with starting and regulating rheostats and an ammeter. A voltmeter is connected across the motor terminals, and the voltage should be kept as nearly constant as possible. Begin the test as usual with the highest load. Take readings of amperes, volts, speed and torque. Then reduce the load by approximately equal steps, until the safe limit of the motor speed is reached.

Take a few readings with the field weakened by 10 and 20 per cent; also with the armature shunted by the same amounts (see § 330). Take a few runs with the supply voltage about 10 per cent above and below rated. The report is similar to that or the preceding experiment.

COMPOUND-WOUND MOTORS.

335. An ordinary shunt-wound motor is entirely satisfactory in most cases where an approximately constant speed is required with variable load, as, for instance, in machine-tool drive. There are, however, cases of considerable practical importance, where the characteristics of the shunt motor can be improved by providing it with an additional series winding, similar to that of compound-wound generators. Such motors are called compound-wound motors; the series winding can be arranged either so as to strengthen the field produced by the shunt winding, or to weaken it. The first is called *cumulative* compounding; the second, *differential* compounding.

336. Cumulative Compounding. — Cumulative compounding of shunt motors is used when the motor is regularly subjected to heavy but short overloads, for instance in driving punch and shears, planers, in elevator work, etc. Each time when the punch touches the sheet of metal, or the planer tool begins a new cutting stroke, there is a rush of current into the motor armature. This affects the generator, affects the lamps and other motors by producing a fluctuating voltage, and is detrimental to the motor itself, on account of excessive heating and sparking.

This rush of current into the armature could be reduced if the field were automatically strengthened each time when an overload occurs.

The reason for this is that the torque of a motor, or the attraction between the armature and the field, is proportional to the product, "field × armature current;" if the field becomes stronger when an overload occurs, the armature current does not need to be so large for the same torque. This is precisely what the cumulative compound winding accomplishes: when an increased current flows through the armature it has to flow through the series field winding, and thus automatically strengthens the field.

Suppose, for instance, that at times the motor has to overcome a torque six times larger than that at which it is running most of the time. With a constant field, as in the shunt motor, the armature current must increase *six* times. With a compound winding, *three* times the normal current may be sufficient, because the field strength increases as the current increases; this stronger field times the triple current may give six times the normal torque. For this reason motors used on widely fluctuating loads are often provided with a cumulative compound winding. Of course, such a motor slows down more than a shunt motor would do, when the load increases, but this is not objectionable in most cases.

337. EXPERIMENT 16-C. — Effect of Cumulative Compounding of Motors. — The experiment is conducted essentially as an ordinary brake test on a shunt motor (see § 333). In order to see more clearly the influence of the compound winding, it is advisable to provide different degrees of compounding, either by dividing the series winding into sections, or by connecting a variable shunt around it. The test itself should be conducted as follows:

(*a*) Begin with the heaviest load and no compounding, that is, use the shunt winding only; adjust the brake so as to have the armature current, say 25 per cent, greater than its normal rating. Read amperes, volts, speed, field current, and torque, as in a regular brake test.

(*b*) Introduce part of the series winding into the circuit, and take readings with the same brake torque as before. Increase the compounding action, with the same load torque, and read as before; repeat under new conditions, etc., taking altogether four or five sets of readings. This will illustrate the action of the compound winding at a certain torque.

(*c*) Take a lighter load and again get readings with different degrees of compounding; continue the same process down to no load. The results obtained will show the influence of compounding on the current taken by the motor, on its efficiency, and on speed variations as compared to an ordinary shunt motor. If possible, the brushes should not be shifted during the whole test, as this would change the armature reaction, and consequently the speed of the motor. Should it be necessary to move the brushes because of an excessive sparking, this must be noted in the results, and a record made of the number of commutator bars by which the brushes were shifted.

(*d*) The behavior of the motor under actual fluctuating load conditions should be observed, both with and without the series winding. A lathe is convenient for this purpose; a piece of work is put in it, of such a shape that the tool cuts only part of the revolution, thus giving the motor a fluctuating load. No *exact* measurements can be attempted under these conditions, except a special recording ammeter and a tachograph are available. In the absence of such, the current is read every few seconds, and speed observed as closely as possible with an ordinary tachometer. See also the arrangement for recording variable load, described in § 701. The results plotted to time as abscissæ will give an approximate idea of the general performance of the motor under sudden overloads, with and without compound winding.

Report. The results of the regular load test should be plotted to either torque, horse-power output, or armature amperes as abscissæ, all on the same sheet, so as to show the influence of compounding on speed, efficiency, input and output of the motor.

338. Differential Compounding. — In contra-distinction to the cases, in which cumulatively compounded motors should be used, there are cases, in which a motor is hardly ever subjected to sudden heavy over-loads, but where an absolutely *constant speed at all loads* is very essential. This is the requirement, for instance, in some textile factories, where one motor operates a large number of spinning or weaving machines. The speed of the motor should not depend on the number of machines in actual operation, for every change in speed affects the quality or the design of the product. Motors used for such purposes are usually provided with a demagnetizing, or *differential* compound winding. The reason for this is as follows:

A shunt-wound motor usually slows down a few per cent as the load increases; in order to bring up the speed to its former value, it is necessary to weaken the field of the motor by a field rheostat. In a differentially-wound compound motor the same is done automatically: When the load increases, the armature current also increases; thus a larger current flows through the series winding; as the same is connected so as to oppose the shunt-field winding, the field strength is reduced, and the motor speeds up. By properly adjusting the number of turns of the series winding, a fairly constant speed can be secured throughout the working range of the motor. By further increasing the number of turns an over-compounding is obtained, that is to say, the speed of the motor increases with the load. This, however, has so far had but little practical application.

A difficulty in obtaining an absolutely constant speed is caused by saturation in iron. The speed depends on the *flux,* while the compounding action is proportional to the number of *ampere-turns.* When the iron is far from saturation the flux *is proportional* to ampere-turns, so that theoretically a perfectly constant speed may be had at all loads; near saturation this condition does not hold, and a motor compounded so as to give the same speed at full load as at no load, runs at somewhat different speeds at partial loads. This result of saturation was observed in a generator compounded so as to give the same voltage at full load as at no load, but which gives too high a voltage at intermediate loads (Fig. 270). This effect is more noticeable the nearer the magnetic circuit approaches saturation.

A differentially-wound motor does not start as easily as an ordinary shunt motor. This is due to the starting current weakening the field and so reducing the starting torque. If an excessive starting current is allowed to flow through the series field, the action of the latter may become even stronger than that of the shunt field, and the motor will have a tendency to start in the wrong direction. Therefore, differentially-wound motors are usually provided with a switch for short-circuiting the series winding during the starting period. After a considerable speed has been attained, and the current has dropped to its normal value, this switch is opened, and the motor runs at its rated speed.

339. EXPERIMENT 16-D. — Effect of Differential Compounding of Motors. — The experiment consists in loading the motor, and observing speed variations. A steady load is obtained by means of a generator, or a blower; either of these being preferable to a Prony brake. In order to be able to determine the motor load, the generator, or the blower, must be calibrated, unless a transmission dynamometer is used (§ 332). If the load cannot be measured, refer motor speeds to amperes input, instead of horse-power output. The test should comprise several sets of readings, at various loads and with different degrees of compounding. From these readings, when plotted as curves, conclusions can be drawn as to the variation of speed with the load; also, the right number of turns in the series winding can be determined. The test should be performed with at least two values of shunt-field current, one value corresponding to the field highly saturated, another with the field considerably weaker, that is to say, at a higher speed. This will show the influence of saturation of the magnetic circuit on speed regulation.

VARIABLE-SPEED DRIVE.

340. Varying Speed by Field Control. — It was explained in § 330 that the speed of a shunt motor increases as the field excitation is weakened, — the limit being usually imposed by sparking at the brushes, due to the increased effect of armature reaction. Therefore, the efforts of designers of variable-speed motors have been constantly directed towards a possible reduction of the armature reaction. This meant, in the first place, a strong and saturated magnetic field, and a comparatively weak armature (as expressed in ampere-turns). The next step was to compensate for the armature reaction by providing an auxiliary field (Fig. 291) with an effect equal and opposite to that of the armature. The coils comprising this compensating field must evidently be connected in series with the armature, so as to give the right number of ampere-turns with varying armature currents. The field frame of a compensated motor is shown in Fig. 292. The four large poles produce the main field (shunt field), the narrow poles constitute the auxiliary compensating field. Such "inter-pole" motors easily give a speed ratio of 4 to 1 by field control, without excessive sparking.

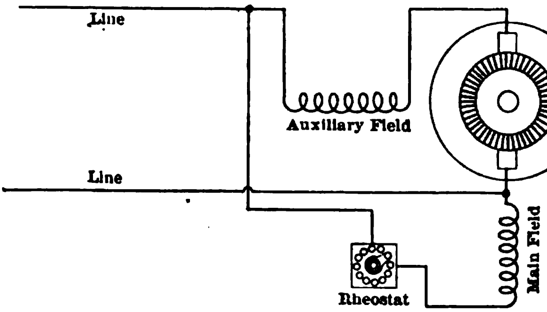

FIG. 291. A shunt-wound motor with a compensating winding for neutralizing the armature reaction.

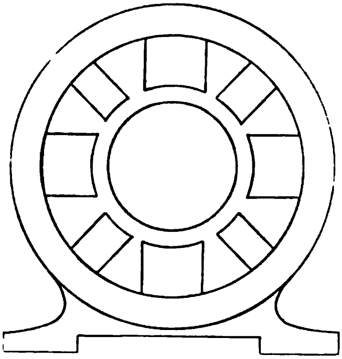

FIG. 292. A four-pole motor frame with compensating poles (inter-poles).

The inter-poles, or compensating poles, are now becoming quite popular, and have been applied not only to shunt motors, but to series-wound railway motors as well. They are also used to some extent with direct-current generators, especially those driven by steam-turbines; this is done in order to suppress sparking caused by high frequency of commutation.

The field arrangement of another type of variable-speed motor is shown in Fig. 293 (Stow motor). Instead of varying the field strength by regulating the shunt current, the field is varied in this motor by changing the length of the air-gap. The pole-piece consists of a plunger actuated by a hand-wheel. The higher the pole-piece is raised, the larger is the air-gap, and therefore the weaker is the field (with the same field current). In four-pole motors of this type all the pole-pieces are moved simultaneously by a beveled-gear transmission. The reason why the effect of armature reaction in this motor is reduced at higher speeds may be seen by a comparison of Fig. 293 to Fig. 294, which latter represents the field of an ordinary motor. In the Stow motor, as the air-gap becomes larger, the reluctance of the magnetic path for the distorting lines of force, created by the armature, also increases, thus reducing the harmful cross-magnetization flux. On the contrary, in an ordinary motor field, distortion is greatest when the field is weakest.

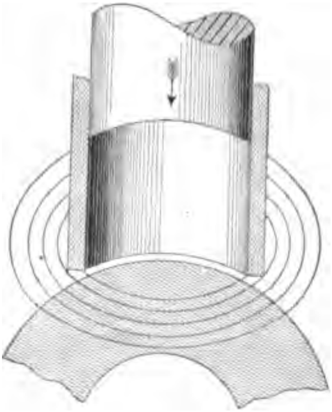

FIG. 293. Pole-piece in a Stow motor; cross-magnetization is reduced by an air column.

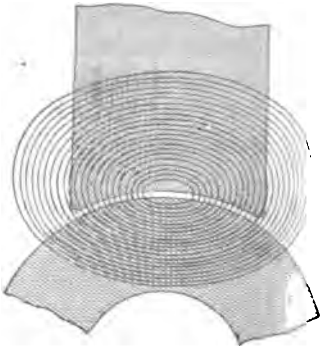

FIG. 294. Pole-piece in an ordinary motor; field distortion is the greatest when the field is the weakest.

In another type of motor, based on a similar principle (Lincoln motor), the armature is made slightly larger at one end than at the other, and can be shifted along the shaft by a hand-wheel. As it is withdrawn from the pole-pieces, the useful magnetic area decreases; moreover, the air gap increases because of the conical shape of the armature. The result is that the useful flux decreases, and the speed of the motor increases.

341. EXPERIMENT 16-E. — Study of Compensated Variable-Speed Motors. — The experiment consists in a study of the performance of a shunt motor, provided with inter-poles. A Stow, or a Lincoln motor may be studied in a similar way. Either a Prony brake or a calibrated generator may be used as a load. In order to save time and have more runs taken under different conditions, readings on partial loads can be omitted.

(*a*) Start with the heaviest load and the lowest speed possible; read input, output, speed, etc., as in a regular brake test. Reduce the load to about 50 or 75 per cent of the rated capacity of the motor, and take another set of readings. Then run the motor light, with the same field. These three runs will give sufficient information concerning the behavior of the motor with a certain field current.

(*b*) Load the motor again to its full capacity, but at a lower field current (higher speed), and repeat the three runs as before. Repeat at higher speeds, until the limit of speed is reached, beyond which it would not be safe to go, on account of the danger from centrifugal force, and destructive sparking.

Adjust the brake load so as to have the same horse-power output with different values of the field current, in order to get comparable results.*

(*c*) The speed characteristics of a compensated motor depend essentially upon the position of the brushes. Take a few runs with different settings of the brushes, and measure accurately the speed, with the motor running in both directions.

(*d*) Disconnect the compensating winding, and take load readings with the same horse-power output as before, and with the same values of field current. Weaken the field as far as the sparking limit will allow.

Report. Plot to field current as abscissæ: speed and efficiency, with and without the compensating winding. Points on the same ordinates must refer to equal values of horse-power output. Discuss the results.

342. Motors on Three-Wire Systems. — Instead of by field control, speed may be regulated by varying the voltage at the armature terminals. In most cases it would be out of the question to do this regulation by rheostatic control, as it is too wasteful of energy. However, it can be done economically on a three-wire system (Figs. 281 and 295).

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* Variable-speed motors are usually designed to give the same horse-power output at all speeds; this means that the torque must be decreased as speed is increased.

A great deal has been written by various authorities on the relative advantages of three-wire systems and field control; as the question now stands, each individual case must be decided according to local conditions. If but a few variable-speed motors are needed, compensated shunt motors with field control are more economical; where a good many motors are to be installed, it becomes a question of dollars and cents, whether (a) more expensive compensated motors should be used, with an ordinary two-wire system; or (b) an extra investment put into the third wire, a balancer set, etc., and standard shunt motors used.

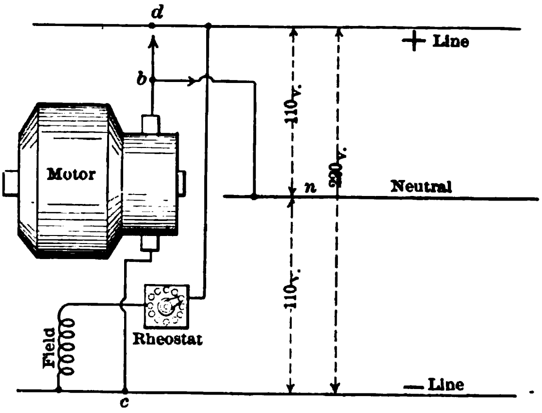

FIG. 295. Motor connected to a three-wire supply.

A shunt motor connected to a three-wire system is shown in Fig. 295. The field is permanently connected between the outside wires, but may be varied to some extent by the rheostat in its circuit. The armature may be connected at will, either on half voltage, *cbn*, or on full voltage, *cbd*. To get the lowest speed, say 400 r.p.m., the motor field is fully excited, and the armature connected across 110 volts. By gradually weakening the field, the speed can usually be doubled before difficulties in commutation begin; this gives any desired speed between 400 and 800 r.p.m. After this, the armature is switched on to the full voltage, 220 volts, and the field fully excited; this gives again 800 r.p.m.; then by weakening the field the speed can be brought up to 1600 r.p.m. This gives the same speed limits 4 to 1, as with a good compensated motor.

By using an unsymmetrical three-wire system (§ 323), or a four-wire system (§ 325) a still wider range of speed may be obtained.

Instead of a three-wire system, two motors can be used for driving the same tool or shaft; by connecting them in series and in parallel (as in a street car), half speed and full speed can be had. It is even unnecessary to have two separate motors; the two armature windings can be put on the same core and revolve in the same field. This gives the so-called "two-commutator motor" (Fig. 296). When *a* is connected to *b*, the two armature windings are in series, and the motor runs at half speed; when *a* is connected to *c*, and *b* to *d*, the two windings are in parallel, and the motor runs at full speed. The field circuit is provided with a regulating rheostat, by means of which the range of speed is further increased and intermediate speeds are made possible. Such an arrangement may be useful in special cases, but, as a rule, either the compensated motor or the three-wire system is preferred.

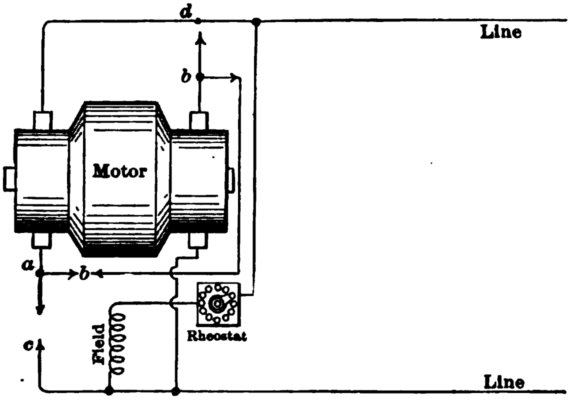

FIG. 296. A two-commutator motor: the armature windings may be connected in series or in parallel.

343. EXPERIMENT 16-F. — Operating Motors on a Symmetrical Three-Wire System.

— Provide a three-wire source of supply, as in Fig. 278, or in Fig. 279, and connect to it an ordinary shunt motor as in Fig. 295. (*a*) Load the motor to its rated capacity at the lowest speed possible, viz., strongest field and half voltage on the armature. Gradually increase the speed by weakening the field as far as sparking will allow; vary the torque accordingly, so as to keep the same horse-power output. Read armature and field amperes, volts, brake load and speed, as in a regular brake test.

(*b*) Having reached the speed limit, switch the armature over to the full voltage; start again with the strongest field and continue the test as before, keeping horse-power output constant. After each reading, release the brake before going to the next point; read input and speed at no load, so as to see per cent speed variation between no load and full load.

Report. Plot to speed as abscissæ all the data observed, also an efficiency curve.

344. EXPERIMENT 16-G. — Operating Motors on an Unsymmetrical Three-Wire System. — An unsymmetrical three-wire system is produced, as explained in § 323. The experiment is performed in a way similar to the preceding one. The armature is connected in succession across the three different voltages.

TROUBLES IN DIRECT-CURRENT MACHINES.

345. Troubles in an electrical machine may be either of mechanical or of electrical nature. Mechanical troubles, such as unbalancing, defective bearings, etc., can be comparatively easily detected; they are remedied in the same way as in any non-electrical machine. Purely electrical troubles require more special knowledge for their detection; some of the symptoms, and the detection of the more important electrical troubles, are described below.

The three parts of a direct-current machine that may give trouble of an electrical nature, are: armature, commutator, and field magnets: these will be considered separately. It is hardly possible to give detailed and complete instructions for locating troubles, since they often occur in a manner and place in which they are least expected. Sound judgment, and the instinct bred of experience, permit a trained man to locate the trouble in a few minutes, where a beginner may spend days.

346. Troubles in Field. — The principal troubles in the field circuit are: The winding may be partly or totally short-circuited, or grounded on to the frame; it may be connected in a wrong way so as to produce two adjacent poles of the same polarity; the winding may be broken (open circuit); the machine may have lost its residual magnetism, or may have it in the wrong direction. All these troubles can be detected by comparatively simple means. A short-circuit is detected by measuring the resistance of the winding with a Wheatstone bridge, or by the drop-of-potential method. If the winding is grounded to the frame, a lamp will light up between the winding and the frame; or else the ground can be detected by a Wheatstone bridge, a galvanoscope, a magneto, etc. If the ground resistance is high, it is measured with a voltmeter, as explained in § 290. One ground connection is not so bad, as long as the rest of the circuit is thoroughly insulated from the ground. But should another point touch the ground a short-circuit is produced, with all its harmful effects. An open circuit, or a loose connection, is also easily traced out by a lamp. Wrong polarity can be detected by bringing a magnetic needle near the poles. Loss of residual magnetism is usually manifested by the generator failing to excite itself, though of course a broken or a wrong connection has the same effect.

With a sufficient knowledge of the theory and performance of direct-current machines, the possible causes of a trouble may be investigated and gradually eliminated, until the actual cause is found.

347. Troubles in Armature. — There are three principal faults in the armature to be looked after: short-circuit, open circuit, and grounded winding. The usual method for locating a short-circuit consists in putting a current through the armature and measuring the voltage drop between each two adjacent commutator segments by means of a milli-voltmeter, or a low-reading voltmeter, as shown in Fig. 297. At the place of a short-circuit, as between *a* and *b*, the drop is almost zero, or at any rate much lower than between other segments. When the trouble is thus located, it remains to determine whether the short-circuit is in the winding *c* itself or between the two commutator bars. This is done by disconnecting the suspicious coil from the commutator and trying the voltage drop again between the same two commutator bars and between the ends of the coil.

A modification of this test consists in using alternating current, instead of a direct current, and applying a telephone receiver instead of a milli-voltmeter. Where the winding and the commutator are clear of short-circuit, a distinct humming is heard in the receiver; the humming ceases when passing over the short-circuited bars. An open circuit is also tested by means of a low-reading voltmeter. It will be easily seen that in this case the deflection suddenly rises when passing over the fault. A ground is detected by the same means as in the field (§ 346).

348. Commutator Troubles. — These are usually manifested in an excessive heating and sparking; sometimes the cause lies in the commutator itself, and sometimes in the armature winding, as explained above. The principal commutator troubles proper are: rough surface, a high bar, a wrong setting of the brushes, and a short-circuit between the bars. A rough surface, or a high bar, is usually detected by an inspection of the commutator as soon as the trouble has been noticed. The improper setting of the brushes is discovered by shifting the brushes until the position of minimum sparking is found. Old machines usually require shifting of the brushes with various loads because of a large armature reaction. More modern machines generally have a permanent setting from no load to full load.

A short-circuit between the bars is the most serious trouble and requires immediate attention; if the bars *a* and *b* (Fig. 297) are short-circuited they short-circuit the coil *c*. As this coil revolves in a strong magnetic field, heavy currents are produced in it, and the coil is usually burned out in a short time. The method for locating this trouble is the same as for locating a short-circuited coil in the armature and is described in the previous article.

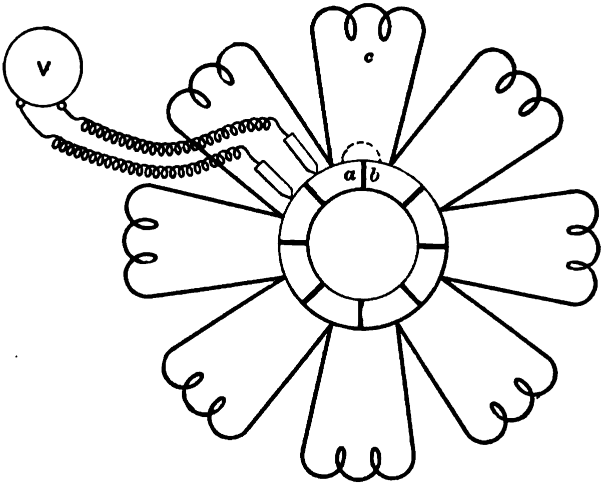

FIG. 297. Locating faults in an armature, using a voltmeter.

349. EXPERIMENT 16-H. — Locating Troubles in a Direct-Current Machine. — The best way to arrange this exercise would be for the instructor to provide several faults in a machine and then let the student locate and remedy them. The objection to this method is, however, that an inexperienced man may spend hours and even days before he finds a trouble. Therefore it is deemed advisable to let the student himself provide various faults in the machine, observe its behavior, and then perform the measurements described above for locating troubles. In this way more can be accomplished in a shorter time, and various faults studied systematically. Various troubles give different symptoms according to whether the machine is running as a generator or a motor; it is, therefore, desired that the student should run the machine both ways, whenever practicable.

(a) Begin the experiment by short-circuiting part of the field winding and operate the machine as a generator and as a motor; note the abnormal symptoms observed. Then do the same with the other troubles enumerated in § 346: make wrong connections, produce an opposite residual magnetism, etc. Report all the symptoms observed, and state how you would locate the true cause, by gradually eliminating all other possible causes.

(b) Produce in the armature the faults described in § 347 and observe the symptoms accompanying the faults. Care should be taken not to damage the machine, as these faults are usually accompanied by vicious sparking and heavy currents. The necks of two commutator bars may be provided with small screws for placing an artificial short-circuit between them. Use a small fuse wire for short-circuiting, so that it would blow out before the coil could be damaged.

It is interesting to observe the effect of two or more coils being short-circuited simultaneously; figure out how the coils can be located in this case by the drop-of-potential method.

(c) After having had sufficient practice with the above faults, the student should feel enough confidence in himself to locate some prearranged troubles, without knowing their place or nature. One man of the section, or the instructor, may be asked to arrange one or more faults in the machine, for the others to locate. This will bring more interest into the work and will make it more profitable.

CHAPTER XVII.

DIRECT-CURRENT MACHINERY — EFFICIENCY AND LOSSES.

350. A DIRECT-CURRENT machine can be operated either as a generator or as a motor. In the former case it transforms the mechanical energy of the prime mover into electrical energy available at its terminals; in the second case it converts the electrical input from the line into mechanical energy available on its shaft. In both cases this transformation of energy is necessarily accompanied by losses in the machine itself.

These losses cause the output of the machine to be less than the input; the ratio of the two is called the efficiency of the machine. Efficiency is thus indirectly a measure of the losses. By definition

$$\text{efficiency} = \frac{\text{output}}{\text{input}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

This expression holds true for either a generator or a motor. Efficiency may also be expressed in terms of output and losses thus:

$$\text{Efficiency of generator} = \frac{\text{electrical output}}{\text{electrical output} + \text{losses}}; \quad . \; . \; . \quad (2)$$

$$\text{Efficiency of motor} = \frac{\text{mechanical output}}{\text{mechanical output} + \text{losses}}. \quad . \quad (3)$$

The efficiency of a motor may also be conveniently expressed in the following form:

$$\text{Efficiency of motor} = \frac{\text{Electrical input} - \text{losses}}{\text{Electrical input}} \quad . \; . \; . \; . \; . \; . \; . \quad (4)$$

351. Direct and Indirect Methods for Determining Efficiency.— The four above expressions for efficiency are identical in principle, the efficiency in all of them being expressed by the ratio of output to input. Two experimental methods may be used for determining efficiency, according to whether it is expressed explicitly in terms of both input and output, as in the formula (1), or of only one of these and the losses, as in (2), (3) and (4). The first method is sometimes called *the direct method* for determining efficiency, the second one — *the indirect*. Both are used in practice, and there are cases in which each one is preferable.

When both input and output are to be measured directly, an actual load test is necessary: a brake is used with motors, generators are generally loaded on resistances. While the measurement of the *electrical* input or output is comparatively simple, determining the *mechanical* power requires a brake, a transmission dynamometer, or similar devices which are neither easily operated nor accurate in results, even on small machines, and hardly applicable for large machines. Moreover, the waste of power unavoidable with such tests is objectionable, and in many cases even prohibitive, because it may be quite impossible to get the necessary amount of power.

At any rate, the direct method gives only the sum total of the losses and not the separate losses. The results obtained cannot be very accurate, because the losses are determined as a difference between two large and not very different quantities — the input and the output. A small error in either of these may result in a considerable error in the value of the losses.

The *indirect* method is therefore usually employed, and by its use the segregation of the losses into the separate components is made possible. The procedure is, in brief, to run the machine under test at *no load*, and to determine the power necessary for driving it. The power is used in this case entirely for overcoming the losses, and is thus a direct measure of the sum total of the losses.

352. Losses in Direct-Current Machines. — The losses in an electric machine, whether running as a motor or as a generator, can be subdivided into three different classes:

(a) *Copper loss* (I^2R) in the armature, and in the field circuit;

(b) *Iron loss* (hysteresis and eddy currents) in the armature core;

(c) *Mechanical losses:* bearing friction, brush friction, and windage (air resistance).

The copper losses need not be determined experimentally; it is only necessary to measure the ohmic resistances of the corresponding windings; then the I^2R [illegible]s can be calculated for any desired value of the current.

The iron loss in any machine depends upon the magnetic flux of the machine and upon its speed. In shunt-wound machines the flux is constant as long as the field current is constant. The speed is also approximately the same at all loads; therefore the iron loss under these conditions is approximately constant.

The brush friction and the windage loss depend on speed only. The bearing friction is constant in a direct-connected motor, but when the motor is used for belt drive the friction depends on the tension of the belt. This increase in friction could hardly be taken into account, and it is customary not to charge it to the motor.

Thus the items (b) and (c), viz., iron loss and friction in a shunt-wound machine, can be assumed nearly constant at all loads, and having the same value as at no load. This gives a convenient method for measuring these losses; all that is necessary is to measure the amount of power put into the armature of the machine, running as a motor, at no load. This input is all converted into iron loss and friction, with the exception of a small part of it, which is necessary for supplying the I^2R loss in the armature winding and the brushes. The correction is usually negligible, but, if necessary, can be readily calculated.

There are cases where the machine under test cannot be run as a motor; for instance, if it is a 500-volt machine, and only a 110-volt source of supply is available. In such cases, the machine is belted to a small auxiliary motor and driven at the right speed and excitation, at no load. The input into the driving motor is equal to the losses in the machine, with a correction for the losses in the motor itself.

To determine the losses in the driving motor the belt is taken off, and the motor run at no load within the same limits of speed and excitation as when driving the machine under test. The input into its armature is a measure of the losses in the motor itself, and can thus be calculated and eliminated from the results.

Thus, there are two methods for determining the losses in a machine. The machine under test may be driven electrically as motor, or mechanically by an auxiliary motor. A third possible way, the so-called retardation method, is described in § 361.

353. Example in Calculating Efficiency from Losses. — Suppose that it is required to calculate the efficiency curve of a 50 horse-power 220-volt motor. It was found that at no load the motor takes 10.6 amperes (armature current), and the field current at which the motor is supposed to run is equal to 5.8 amperes. The resistance of the armature was found by measurement to be 0.0377 ohm. We have: no-load losses (neglecting copper loss) consisting of iron loss and friction $= 220 \times 10.6 = 2.33$ kilowatts; excitation loss $= 220 \times 5.8 = 1.275$ kilowatts; total losses independent of the load $= 2.33 + 1.275 = 3.605$ kilowatts. The copper loss in the armature depends upon the load; as we do not know what current the motor would take at an output of 50 horse-power, we have to find it by trials. A still better method is to construct the whole efficiency curve from no load to one and a quarter load and then take from this curve the point corresponding to full load.

An ideal 50 horse-power motor takes at full load,

$$\frac{50 \times 746}{220} = 170 \text{ amp.};$$

the real motor will probably take 190 or 200 amperes. Therefore, we construct the efficiency curve for points between, say, 100 amperes and 250 amperes; then we can be sure that the full-load point lies on this curve. Take, for instance, the 150-ampere point. Total electrical input $= (150 + 5.8) \times 220 = 34.28$ kilowatts; copper loss in the armature $= 150^2 \times 0.0377 = 0.848$ kilowatts; total loss $= 3.605 + 0.848 = 4.453$ kilowatts.

$$\text{The efficiency is } \frac{34.28 - 4.453}{34.28} = 87 \text{ per cent.}$$

$$\text{The output} = \frac{34.28 - 4.453}{0.746} = 40.1 \text{ horse-power.}$$

In this way, efficiency can be calculated for various values of output; and the efficiency corresponding to an output of 50 horse-power found from the curve plotted.

This method of figuring efficiency from losses is not quite correct, for the reason that the speed does not remain altogether constant at all loads, but may drop a few per cent between no load and full load. In order to take this into account, the no-load run is sometimes supplemented by the so-called *ampere-speed curve* giving actual values of speed of the motor, with varying input. The no-load test is then run within the limits of the speeds of the motor, and in figuring out the efficiency, the values of the iron loss and friction are taken for the speed corresponding to a given amperes input.

When the machine under test is driven mechanically, the losses in the driving motor are taken into account as follows: Suppose, for instance, that in one of the runs the input into the armature of the auxiliary motor was 5 amperes at 104 volts, or 520 watts. When the auxiliary motor was run alone (with the belt off) it took 1.3 amperes at 101 volts, with the same excitation and the same speed in both cases. The resistance of the motor armature was found by measurement to be $= 0.16$ ohm. Then the iron loss and friction in the motor amount to $131.3 — 1.3^2 \times 0.16 = 131$ watts; the iron loss and friction in the machine under test is therefore

$$520 - 131 - 5^2 \times 0.16 = 385 \text{ watts.}$$

It will be seen from these figures that the correction for I^2R is small, and in many cases may be neglected altogether.

354. EXPERIMENT 17-A. — Efficiency from Losses, Machine Driven Electrically. — (a) Connect up the machine under test, as in Fig. 284, and run it at no load and at the rated voltage.* Vary the speed within the range for which the values of efficiency are desired, by regulating the field current. (b) Determine the resistance of the armature by the drop-of-potential method (§ 10). (c) If the machine under test is intended to be used as a motor, and accurate results are required, ampere-speed curves should be taken. For these, the machine is belted to a generator or a blower and is driven at the values of the field current and armature current for which the efficiency is desired. The no-load runs are repeated for the same values of field current and speed; speed is regulated in this case by the rheostat in the armature circuit.

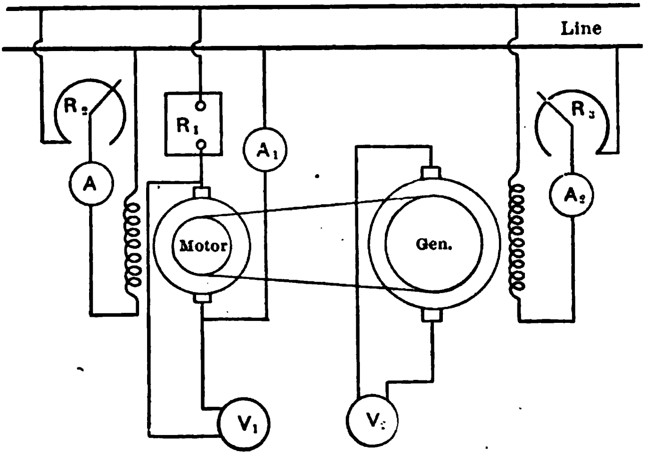

Fig. 298. Separation of losses in a direct-current machine. The machine is driven mechanically as a generator.

If the machine under test is intended to be used as a generator, no ampere-speed curves are necessary, but an excitation characteristic should be taken instead (see § 305), so that the values of the field current at different loads, but with a constant terminal voltage, may be known. Then the no-load runs are taken for the corresponding values of field current and speed, the latter being again varied by a rheostat in the armature circuit.

Report. Plot ampere-speed curves and an excitation characteristic;

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* Before beginning the test it is advisable to let the motor run idle for at least half an hour, in order to get the bearings warmed up and the friction more constant. This may not be practicable in the laboratory, because of the limited amount of time for test, but this precaution is advisable in more accurate work.

also curves of no-load losses. Figure out the resistance of the armature Plot to horse-power output as abscissæ, efficiency curves of the machine working as a motor, with two or three different values of the exciting current. Plot to kilowatt output as abscissæ an efficiency curve of the machine, working as a generator.

355. EXPERIMENT 17-B. — Efficiency from Losses, Machine Driven Mechanically. — The experiment is performed as explained at the end of § 353. The diagram of connections is shown in Fig. 298; the

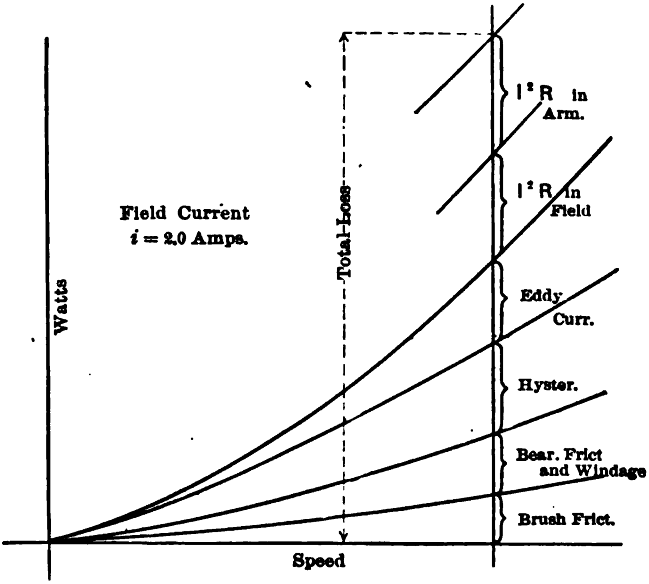

FIG. 299. **Separate losses in a direct-current machine.**

driving motor is at the left, the machine under test at the right. The fields of both machines are excited separately, so that the power loss in the field windings does not enter into calculations.

This method is usually selected when no source of power is available to drive the machine electrically, as in the preceding experiment; therefore, taking an ampere-speed curve is out of the question, and in figuring out the efficiency of the motor, speed variations with load can only be estimated. If the machine is intended to be used as a generator, it is possible to take its excitation characteristics, provided a driving motor of sufficient size (of a capacity about 10 per cent of that of the generator) is available.

Before leaving the laboratory, measure the resistance of the armature. Report requirements are similar to those for the preceding experiment.

356. Separation of Losses. — The designer of electrical machinery — and often the user — is not satisfied to know the sum total of the losses, but desires to know the separate component losses, as shown in Fig. 299. The same losses are shown in the following table.

Total loss	(1) Copper loss	Copper loss in armature; Copper loss in field.
	(2) Core loss	Hysteresis loss; Eddy current loss.
	(3) Mechanical losses	Bearing friction; Brush friction; Windage.

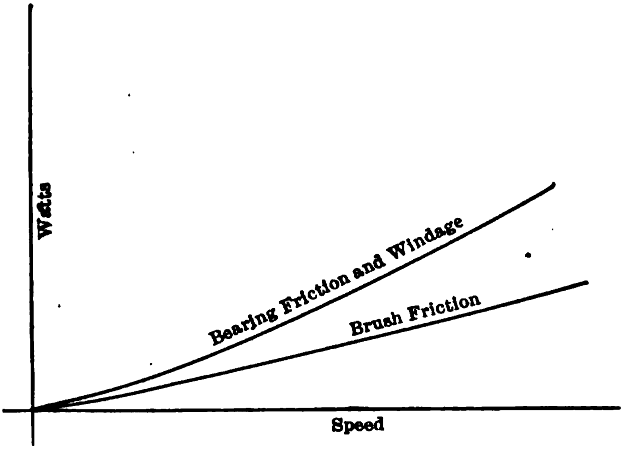

FIG. 300. Brush friction separated from bearing friction and windage.

Knowledge of the separate losses is of importance for the following reasons:

(a) The losses and their dependency on the load, speed, etc., determine the rating of the machine for different classes of service (intermittent load, variable load, constant load, etc.).

(b) The magnitude of the separate losses is a check on the quality of materials, workmanship, and design.

(c) Knowing the values of the principal losses the designer is enabled to vary the dimensions of the machine, still keeping its efficiency as high as competition may require, and preserving reasonable limits of temperature rise.

Of the three kinds of losses, only those mentioned under (2) and (3) need to be discussed here. Copper loss, I^2R, is easily calculated from the measured resistances of the windings.

357. Core Loss and Mechanical Losses. — Frictional losses (3)

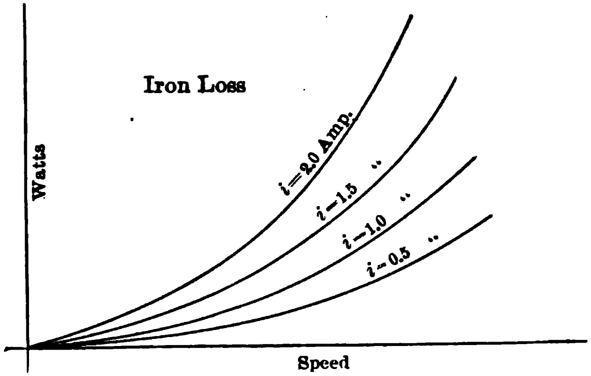

FIG. 301. Curves of iron loss as a function of speed and of exciting current.

depend only on the speed of the machine, and may be plotted as in Fig. 300. In ordinary practical testing, bearing friction is never separated from windage, and the author is not aware of any simple method

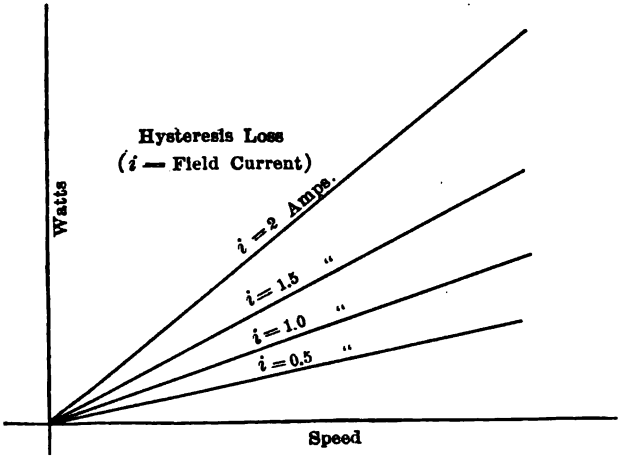

FIG. 302. Curves of [illegible] amperes, field function of speed and of exciting current.

stant at this maximum value, gra[illegible] to have the
for doing this. [illegible] proximately equal steps to its lowest possible
bearings ther[illegible] resistance into the armature circuit. The readings (w[illegible] by
means of a [illegible] ction for the armature I^2R) will give directly the data for
machine in [illegible] curve in Fig. 304.

Core loss depends on speed and on field current, and is usually represented by a set of curves, as shown in Fig. 301. The components of the core loss — hysteresis and eddy currents — are shown in Figs. 302 and 303. It is important for the manufacturer to have eddy currents separated from the hysteresis loss. A high hysteresis loss means that the iron used was of an inferior quality, or the flux density allowed was too high; high eddy-current loss indicates that the laminations are too thick, or not sufficiently insulated from each other. Thus the remedy is quite different in the two cases.

The sum of losses (2) and (3), or, as it is sometimes called, total no-load loss (Fig. 304), is determined by measuring the power necessary for driving the machine at no load. The machine may be driven either *electrically* as a motor, or *mechanically* as a generator (from an auxiliary motor), or the kinetic energy of the machine itself is utilized (*retardation method*). All these methods are used according to the local conditions, and all are applicable for separating the losses in either series, shunt- or compound-wound generators and motors; * wi some changes the same methods can be used for alternating-curr machines. The details of these three methods are given below.

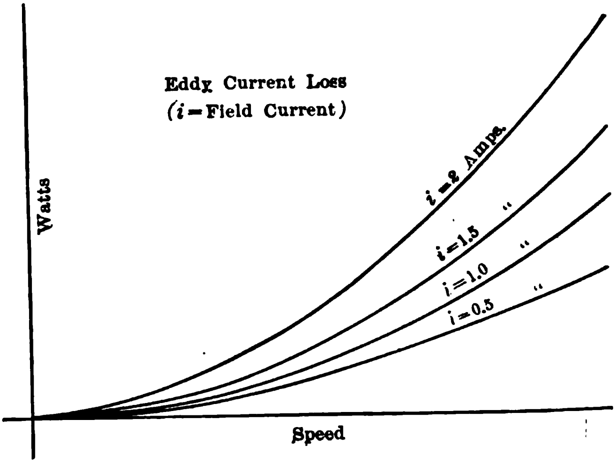

Fig. 303. Curves of eddy-current loss as a function of speed and of exciting current.

The separation of iron loss into its components (Figs. 302 and 3 is done by calculation, on the basis of the fact that hysteresis and eddy currents vary in a different way with changes in speed and field current (§ 366).

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* A special method for separating iron loss from friction in se[illegible] motors described in § 370.

358. EXPERIMENT 17-C. — Separation of Losses — Machine Driven Electrically. — The purpose of the experiment is to separate the total losses in a direct-current generator or motor into the components shown in Fig. 299. The machine under test is connected up as shown in Fig. 284 and driven electrically, as a motor, within as wide limits of field current and speed as is feasible. The machine ought first to be run for at least half an hour at a high speed in order to attain constant friction conditions. Lubrication must, of course, be maintained the same during the whole test.

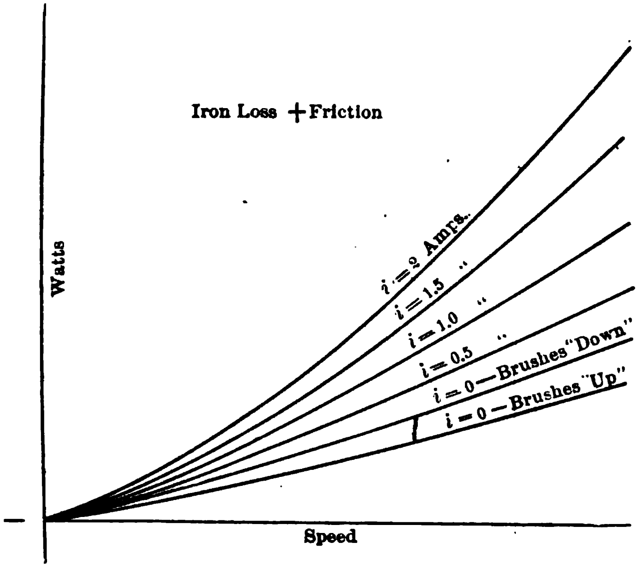

FIG. 304. Total no-load losses as a function of speed and of exciting current.

(*a*) Begin the test with the strongest field current possible and at maximum speed that can be obtained with this field current, — i.e. at full voltage. If necessary, the pressure at the armature terminals can be raised even above the rated voltage of the machine. Read armature volts and amperes, field amperes and speed. Then, *keeping field current constant at this maximum value,* gradually reduce the speed by 6 or 8 approximately equal steps to its lowest possible limit, by introducing resistance into the armature circuit. The readings (with a small correction for the armature I^2R) will give directly the data for the upper curve in Fig. 304.

(*b*) Repeat similar runs with smaller values of the field current, corresponding to the other curves on the same sheet.

The two lower curves cannot in this case be determined directly from the test, since the machine requires some appreciable field current in order to run as a motor. The friction can be determined by calculation, as is shown in the next article. Also, the separation of the brush friction from the bearing friction and windage cannot be done in this case as accurately, as when the machine is driven mechanically, from an auxiliary motor. This is because at least two of the brushes must lie on the commutator when the machine drives itself as a motor. The only possible way is to lift up *part* of the brushes and from the decrease in input to figure out what would be the reduction in power if *all* the brushes could be removed; this reduction may be assumed to be due to brush friction. It is advisable to perform this test with a low field current, since in this case the friction constitutes a larger percentage of total losses.

(*c*) Measure the resistances of the windings.

(*d*) If necessary, take an ampere-speed curve, or an excitation characteristic (see § 354).

Report. Plot curves shown in Fig. 304. Separate friction from iron loss as is explained in § 359; separate hysteresis loss from eddy currents, as shown in Fig. 308. Do this at least for the normal excitation of the machine; though preferably for all the curves shown in Fig. 301, so as to show the influence of the field current on the value of iron losses. Plot all the losses at the normal field current of the machine to speed as abscissæ (Fig. 299) and figure out the efficiency at full load and at the rated speed, for the machine running as generator and as motor.

359. Separation of Friction from Iron Loss by Calculation.— It is mentioned in the previous article that when the curves, shown in Fig. 304, are obtained by driving the machine electrically, as a motor, it is impossible to obtain friction alone from test. This is due to the fact that the machine must have some field excitation to run as a motor, so that some iron loss is necessarily present; the friction curve in this case is determined by calculation. This is done by applying the self-evident principle that *the friction loss is the limit towards which the total loss is tending* when the excitation is being reduced to zero; the speed being kept constant. In order to make use of this principle, the curves of total loss in Fig. 304 are replotted, as shown in Fig. 305, to volts at the armature terminals as abscissæ, each curve referring to a certain constant speed. With the machine running at no load, volts at the armature terminals are practically equal to the counter-e.m.f. of the machine (§ 329). These curves are then produced to the axis of ordinates as shown by dotted lines; the intercepts give the values of friction for the corresponding speed. Plotting these values of friction against speed as abscissæ (Fig. 304) the required friction curve is obtained. This method becomes clear upon consideration of the evident fact that the machine being under motion, the armature volts (counter-e.m.f.) can become zero only when the field has a zero value. With zero field there can of course be no iron loss, and the whole loss consists of friction alone.

The dotted parts of the curves in Fig. 305 are produced either empirically, or theoretically, since they are practically parts of parabolæ. The general equation of the curves is

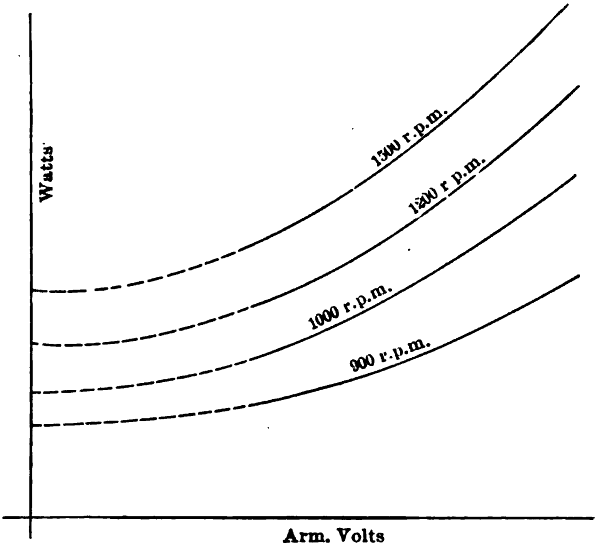

FIG. 305. Extrapolation of the total-loss curves in order to separate friction from the iron loss.

$$W = F + kE^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where W is total loss, F is the friction loss, and E — the voltage at the armature terminals; k is a constant the numerical value of which does not enter into the calculations. This equation is deduced as follows: Each curve refers to a particular speed; therefore the friction loss is constant for each curve. The hysteresis loss increases somewhat slower than the square of the flux (Steinmetz's $B^{1.6}$); eddy-current loss increases as the square of the flux. With a constant speed the flux is proportional to the voltage; thus the iron loss can be assumed as approximately proportional to E^2. Thus the above equation becomes obvious. Writing it for two points on the curve, we have

$$W_1 = F + k\,E_1^2$$
$$W_2 = F + k\,E_2^2$$

whence

$$F = \frac{W_1 - W_2\left(\frac{E_1}{E_2}\right)^2}{1 - \left(\frac{E_1}{E_2}\right)^2} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (6)$$

This value of F gives the point at which the curve meets the axis of ordinates.

360. EXPERIMENT 17-D. — Separation of Losses — Machine Driven Mechanically. — The purpose of the experiment is to separate the total losses in a direct-current generator or motor into the components shown in Fig. 299. The machine under test is belted or direct-connected to an auxiliary motor and driven at no load, at various speeds, with varying field excitation. The scheme of connections is shown in Fig. 298; both machines should be separately excited in order not to introduce into the calculations the power lost in the fields. The input IE into the armature of the driving motor represents the sum of iron loss and friction in both machines, less a small correction for I^2R loss in the motor armature itself.

(*a*) The machines should first be run for at least 30 minutes at a high speed in order to warm up the bearings and produce a constant friction. Then the machine under test is excited to its highest possible value, and the speed raised to its highest safe limit. Read all the instruments, as shown in Fig. 298, and also the speed of the set. Then gradually reduce the speed, either by increasing the motor field current or by cutting some resistance into its armature circuit;* both methods can be used simultaneously if preferred. The field current of the machine under test must be kept constant during each run, the same readings being taken for several speeds. This test corresponds to the upper curve in Fig. 304, except that the losses in the driving motor must first be subtracted from the volt-ampere input into its armature, as is explained at the end of § 353.

(*b*) The same run should be repeated with lower values of the field current in the machine under test, until a sufficient number of curves is obtained (five or six). The last curve is that with the field circuit opened and corresponds to the frictional loss alone.

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* This latter method is preferred by many, since it is simpler to correct data for the variable losses in the driving motor, if its field has been maintained constant.

(*c*) After this, the brushes should be lifted from the commutator, so as to eliminate their friction; the difference in the input with the brushes " down " and " up " gives the value of the brush friction. If the machine has several pairs of brushes it is advisable to lift them up in pairs, to check the fact that the friction is reduced in proportion to the number of brushes removed.

(*d*) Finally the belt should be taken off and the driving motor run alone at the same speeds and with the same values of exciting current as before, in order to determine its own losses. The loss in the belt can be only roughly estimated. The arrangement must be such as to make this loss as small as possible. Under normal conditions it may be assumed to be equal to 2 per cent of the power transmitted.

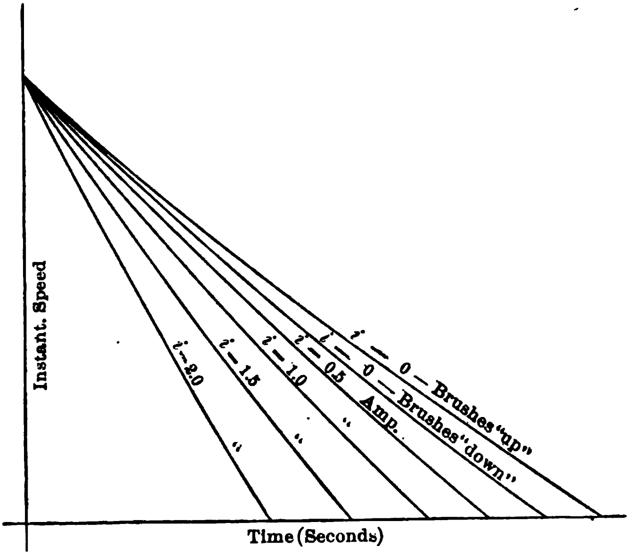

FIG. 306. Retardation or time-speed curves for various values of the field current.

(*e*) Measure armature resistances of both machines, and if necessary take ampere-speed curves, or an excitation characteristic (see Exp. 17-A, § 354).

Report. The requirements are the same as in § 358, except that the iron loss is separated from the total loss directly, by using the experimentally determined friction curve.

RETARDATION METHOD.

361. — Instead of driving a machine electrically or mechanically for separating the losses, it may be brought up to the highest safe speed, and the power shut off; the machine then gradually slows down to a standstill, overcoming the losses by its stored kinetic energy. While it is slowing down, instantaneous values of speed are read every few seconds. Such runs are made with various values of field current, and the results plotted as speed-time or *retardation* curves (Fig. 306).

The form of these curves depends on the relative magnitude of iron loss and the friction, and on their variation with speed. A method is given in § 363 for converting these curves, by calculation, into those shown in Fig. 304; subsequently the losses can be separated, as already explained.

This method is more applicable to large machines, in which the stored energy of the armature is sufficient to maintain the rotation during a comparatively long time (several minutes), so that instantaneous speeds can be taken with considerable accuracy. If the machine has been brought up to speed mechanically, by an auxiliary motor, the belt is thrown off; if it was speeded up electrically, the armature circuit is broken.

362. Measuring Instantaneous Speeds. — The principal difficulty in performing a retardation test lies in the accurate measurement of speed; otherwise the method is simple and reliable, especially with large machines. Several methods are used for measuring instantaneous speeds:

(1) A small magneto, belted to the machine and connected to a sensitive voltmeter, is probably the best device for the purpose unless a special recording tachometer is available.

(2) An ordinary speed-counter may also be used to record total number of revolutions from the moment at which the connection to the external power was broken. This speed-counter is read every two or three seconds without stopping it; the difference of each two consecutive readings gives the total number of revolutions during these two or three seconds. From these the average speed corresponding to the middle of this time period can be determined.

(3) Still another method is to leave the voltmeter across the armature terminals, and to read volts every two or three seconds. If the field current is kept absolutely constant, the volts are exactly proportional to the speed. Before breaking the connections to the source of external power, the coefficient between the speed and the voltage must be determined, so as to be able to convert the observed time-voltage

curve into a time-speed curve. This method is not very applicable for the run with the fields opened, unless a low-reading voltmeter is used, the speed being measured by the voltage induced by the residual magnetism. If this is not practicable, one of the two other above-mentioned methods for measuring speed can be used for this run.

(4) It is important to measure accurately the speed during the first part of the retardation curves, just after the power is shut off, because the values of the losses are usually desired at these speeds. Sumpner's differential voltmeter method is convenient for the purpose. The machine is speeded up as a motor, and the armature circuit is opened on one side of the line only, by a single-pole switch. A low-scale direct-current voltmeter is connected across the switch terminals and shows practically zero, as long as the switch is closed. When the switch is opened and the motor slows down, the voltmeter measures the difference between the line voltage and the e.m.f. induced in the motor.

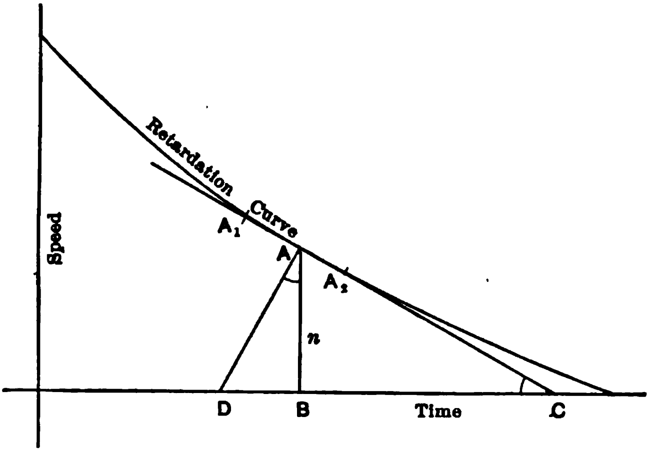

FIG. 307. Analysis of a retardation curve.

The line voltage being constant, and the induced e.m.f. proportional to instantaneous speeds, voltmeter readings are proportional to the drop in speed. When the end of the scale is reached, the switch is closed, and the motor speeded up again for the next retardation run.

363. Theory of the Retardation Method.—The energy stored in the armature at a certain speed, $n = BA$ (Fig. 307), is $\frac{1}{2} Ga^2$, where G is the moment of inertia of the revolving part, and a is its angular velocity. The latter is proportional to the speed n expressed in r.p.m., so that the stored energy can be also represented in the form $\frac{1}{2} Cn^2$, where C is a certain constant proportional to the moment of inertia of the revolving part.

If W is the value of the total losses (iron loss and friction) corresponding to this speed, we have the relation

$$-d\left(\frac{Cn^2}{2}\right)=W.\,dt. \qquad (7)$$

which expresses, that the decrease in kinetic energy during an infinitesimal interval of time dt is equal to the work performed by the armature in overcoming the losses W during the same interval. Differentiating, we obtain:

$$-Cndn = W\,.\,dt$$

or

$$W = -C.\frac{ndn}{dt} \qquad (8)$$

It is shown below that ndn/dt is numerically equal to the length of the *subnormal* BD (Fig. 307) to the curve showing the relation of n to t, which is the retardation curve for the machine tested. Thus we obtain the relation:

$$W = C.\,BD \qquad (9)$$

or, *instantaneous total loss* W *is proportional to the length of the subnormal to the retardation curve.*

Proof that $-ndn/dt = BD$. The ratio $-dn/dt$ is the trigonometrical tangent of the angle ACB, the line AC being tangent to the retardation curve at the point A. If AD is normal to the curve, the angle DAB is equal to the angle ACB. From the triangle DAB we have

$$DB = AB\,.\tan DAB = -n.\,\frac{dn}{dt}$$

which proves the above proposition. The sign "minus" is necessary because dn is negative (speed decreases with time), while DB is positive.

The shape of the revolving part of an electric machine is too complicated to allow its moment of inertia G to be calculated with sufficient accuracy; therefore G, or the constant C which is proportional to it, is eliminated by performing an additional experiment, as is explained in the next article.

Thus curves shown in Fig. 306 are reduced to those in Fig. 304 by drawing normals and measuring the corresponding subnormals, which, to a certain scale, represent iron loss + friction. Some error is introduced by the uncertainty of the exact direction of the normal, but with reasonable care, reliable results are obtained. The simplest way of drawing normals is to take two points, A_1 and A_2 (Fig. 307) at small equal distances from A and to draw AD perpendicular to A_1A_2. Some prefer analytical methods, of calculating W, to drawing tangents and normals. Writing the equation (7) for the finite period of time between the points A_1 and A_2, we get

$$\tfrac{1}{2}\, C(n_1^2 - n_2^2) = W\,(t_2 - t_1) \quad . \; . \; . \; . \; . \; . \; . \; . \quad (10)$$

from which W may be calculated and assumed to represent the losses at the point A, for the moment of time $\tfrac{1}{2}(t_1 + t_2)$.

After the retardation curves, shown in Fig. 306, have been reduced to those in Fig. 304, the separation of hysteresis from eddy currents is done as in §§ 366 to 368.

364. Elimination of Moment of Inertia of Revolving Part. — The constant C, which enters into the preceding formulæ, and which is proportional to the moment of inertia of the revolving part of the machine, may be eliminated by two methods:

(1) The value of the loss W is determined for one point, by an independent method;

(2) The amount of loss W is changed by a known amount.

According to the first method the machine under test is driven electrically or mechanically at a certain speed n, and with a certain field current, the value of total loss W being determined from the amount of power input into the armature. Then the subnormal DB is measured at a point corresponding to the same speed n and the same field excitation. This gives directly the scale for W, since $W = C \times BD$. If, for instance, W was determined to be 600 watts, and $DB = 2.5$ cm., the scale is $600 \div 2.5$, or 240 watts loss per 1 cm. length of subnormal.

According to the second method, an extra retardation run is taken with a known additional load w put on the machine. Then we have

$$\left.\begin{aligned} W &= C \,.\, BD \\ W + w &= C \,.\, B_1D_1 \end{aligned}\right\}$$

where B_1D_1 is the subnormal to the new retardation curve. Eliminating W, we get:

$$C = \frac{w}{B_1D_1 - BD} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (11)$$

The load w may be obtained, either by closing the armature on a resistance and keeping the output constant with a wattmeter; or by applying a Prony brake with a definite torque. With either method w can be kept constant through a limited range of speed only. This is sufficient, however, as only one point on the curve is necessary. Sumpner's differential method for measuring speed (§ 362) is quite convenient for the purpose.

365. EXPERIMENT 17-E.—Separation of Losses by the Retardation Method.— (*a*) Have the machine wired so as to be able to bring it up to speed either electrically or mechanically; run the machine for about thirty minutes, to attain steady conditions of lubrication. Bring it up to the highest safe speed, measure this speed, and then suddenly open the circuit (both armature and field), allowing the machine to slow down to a standstill. Measure instantaneous retardation speeds, by one of the methods described in § 362. Repeat the run several times until you have learned to read speeds quickly; a metronome is of assistance in this work.

(*b*) Bring the machine up to speed again, and simultaneously with opening the circuit lift up the brushes, taking a retardation curve without brush friction. Repeat this run several times until you get reliable results.

The armature circuit can be most conveniently opened by a circuit-breaker; in breaking the field circuit special precautions must be taken in view of the high inductance of the field windings. It is well to provide a special *field-discharge* switch, which, in opening the main circuit closes the field winding upon itself, through a suitable resistance. In this way an easy path is offered for the inductive kick, and the danger of breaking down the insulation is minimized.

(*c*) After this make several retardation runs, leaving the field circuit closed, and opening only the armature circuit. The value of the field current at which the machine is slowing down does not necessarily need to be the same as in speeding up. Before starting the machine mark on the field rheostat the notch on which you desire to keep the current during retardation; speed up the machine at any convenient field current, and simultaneously with breaking the armature connection. quickly move the field rheostat handle to the position marked. Make runs with five or six values of the exciting current.

(*d*) To determine the constant C of the revolving part use either method described in § 364 With the first method, take a total-loss curve by driving the machine as a motor and measuring the input into the armature. Take this curve very carefully; keep the field current at exactly the same value as in one of the retardation curves, and vary speed within as wide limits as possible by varying the impressed voltage. With the second method connect a comparatively high resistance across the armature terminals, and have a wattmeter in the circuit, or at least an ammeter and a voltmeter, for keeping watts delivered to this resistance constant. Take two special retardation runs with the same field current; keep the resistance closed in one run and open in the other. Read the speeds only as far as the power in the resistance can be kept constant.

(*e*) Before leaving the laboratory measure the resistance of the armature winding.

Report. Plot the observed retardation curves (Fig. 306), and calculate C as in § 364. Convert the retardation curves into the loss curves shown in Fig. 304, according to the directions given in § 363. The rest of the requirements are the same as in § 358.

SEPARATION OF HYSTERESIS AND EDDY-CURRENT LOSSES.

366. The curves shown in Fig. 304 give total losses and friction loss for various speeds and field currents. Subtracting friction loss from total loss gives core losses (hysteresis and eddy currents) for various speeds and values of the exciting current (Fig. 307). The next step is to separate hysteresis loss from eddy currents. This is done on the basis of the fact that *with a given excitation, hysteresis loss is proportional to the speed, while eddy-current loss increases as the square of the speed.* The reason for this is as follows (see also §§ 188 and 189):

Hysteresis loss per second is proportional to the number of cycles of magnetization of the armature iron, and this number of cycles is equal to the number of times each piece of the armature passes by the poles of the machine, or, in other words, it is proportional to its speed. Eddy currents are induced currents, and, as such, are proportional to the speed of the machine; but the energy loss is proportional to the square of the current (I^2R); therefore, eddy-current loss is proportional to the square of the speed.

Thus, at a certain speed of n revolutions per minute,

$$\text{total iron loss } P = Hn + Fn^2 \quad . \quad . \quad . \quad . \quad (12)$$

where H and F are two constants characterizing hysteresis and eddy- (Foucault) current losses respectively.

This equation can be used for the separation of the losses in two ways: purely analytical and graphico-analytical; both solutions are given below.

367. Analytical Separation. — Select one of the curves in Fig. 301 and apply the above equation (12) to two points taken sufficiently far apart and corresponding to two values of speed, n_1 and n_2:

$$P_1 = Hn_1 + Fn_1^2 ;$$
$$P_2 = Hn_2 + Fn_2^2 .$$

These equations can be easily solved for H and F. After this is done the straight line Hn representing the hysteresis loss (Fig. 302) can be plotted, as well as the parabola Fn^2, giving the eddy-current loss (Fig.

303). The same procedure is then repeated for other curves in Fig. 301, and in this way hysteresis separated from eddy currents, for all conditions under which the machine may operate.

The disadvantage of this purely analytical method is that in selecting two points on a curve an assumption is made that these points are absolutely correct; in other words, that any two other points on the same curve would give exactly the same values for H and F. In reality, the values of H and F vary somewhat for different points selected on a curve. This means a tedious process of calculating H and F for several combinations of points and taking average values.

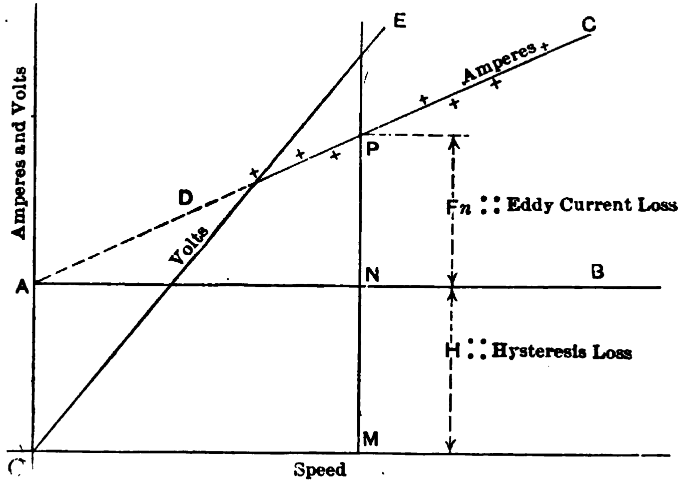

FIG. 308. Graphico-analytical method for separating hysteresis from eddy currents.

The graphico-analytical method permits of taking simultaneously into account as many points on a curve as desired; in this manner the most probable values of the separated losses are obtained in a simpler way.

368. Graphico-Analytical Separation. — Equation (12) may be written in the form

$$P = IE = IKn = Hn + Fn^2,$$

where K is a constant, since with a constant excitation the e.m.f. E of the motor is proportional to its speed; I is that part of the total armature current which is spent in overcoming hysteresis and eddy currents. Dividing both sides of this equation by n, we get

$$KI = H + Fn.$$

This is the equation of a straight line between the current I and the speed n. A line DC of this kind is shown in Fig. 308. By producing it to the axis of ordinates, a horizontal line AB is obtained, which subdivides the ordinates of DC into the constant part $MN = H$, corresponding to hysteresis loss, and the part $NP = Fn$, proportional to the speed, and corresponding to eddy currents.

Each of these being multiplied by the voltage corresponding to the same speed and the same field current (ordinates of the line OE) gives the corresponding loss in watts. This completes the separation of magnetic losses.

An example will make this graphico-analytical method clearer. Let the iron-loss curve corresponding to 2 amperes field current (Fig. 301) be determined by the data given in the first two columns in the table below; the third column gives the corresponding armature volts, as shown by the curve OE in Fig. 308. Dividing watts by volts gives amperes iron loss (column 4); these values plotted against the speed give the straight line DC. This line is produced to the axis of ordinates, and the part OA corresponding to hysteresis loss is found to be equal to 3 amperes. Multiplying this by the volts given in column 2, watts hysteresis loss is obtained (column 5). This loss, subtracted from total iron loss (column 2), gives eddy-current loss. (column 6). The data thus obtained are plotted as in Figs. 302 and 303.

1	2	3	4	5	6
Speed (r. p. m.)	Iron Loss Watts.	Arm. Volts.	Current Amps.	Hysteresis Watts.	Eddy Current Watts.
1800	1875	250	7.5	750	1125
1440	1320	200	6.6	600	720
1080	855	150	5.7	450	405
720	480	100	4.8	300	180
360	195	50	3.9	150	45
			Field current = 2 amp.		

369. Effect of Armature Reaction on Iron Loss. — Armature reaction (§ 147) affects not only the speed of a motor, or the generator voltage, but the iron loss as well. The armature reacts on the field so as to weaken or to strengthen it, and moreover distorts it, making it stronger at one end of the pole-piece and weaker at the opposite end. Thus, with the same field current, the actual magnetic flux is different in a loaded machine from that at no load; this naturally affects the iron loss, which depends on the flux density in different parts of the magnetic circuit. The practical result is that the actual efficiency of a machine under load is somewhat lower than that calculated from the losses.

A good deal has been written on the subject of the increase of iron loss with the load (*zusätzliche Verluste*, additional losses), but as no simple practical rules are available for taking this additional iron loss into account, it is usually left without consideration. The error committed is very small unless the machine is of a very poor design, and the armature reaction has abnormal proportions. Compensating poles for neutralizing armature reaction (Fig. 292) are coming more and more into use; in such machines the iron loss is the same at no load as with a load, and no correction is necessary.

370. Separation of Losses in Series Motors. — The determination of iron loss and friction in shunt-wound motors is simplified by the fact that the speed, with a given field excitation, is nearly constant, and the losses are approximately the same at no load as at any other load. It is entirely different with a series motor: here the speed at no load is many times higher than at full load; moreover, the field strength, and consequently the iron loss, depend on the speed and on the load, and these vary within wide limits. Therefore, the results obtained by running a series motor at no load are of no use at any other load.

This can be remedied by separately exciting the motor and varying its field strength independently of the speed, which latter is regulated by a rheostat in the armature circuit. By this method the series motor is really converted into a shunt motor, and iron loss and friction can be determined as explained above. The method can be somewhat modified, so as to make it more adapted for series motor, as follows: Two series of runs are made at no load; one for determining iron loss and friction *together;* another in which iron loss is negligible; in this way iron loss is separated from the frictional losses.

For the first series of runs the armature of the motor is connected in series with the fields as in actual operation, but is shunted around by a variable resistance. A suitable regulating rheostat is also inserted into the main circuit. In this way the field and the armature currents can be regulated at will either together or separately. The motor is run at no load, so that the input into the armature represents, as before, the sum of iron loss and friction. Having a resistance in parallel with the armature permits of having a strong field even with a small armature current; in this way the speed of the series motor is kept within reasonable limits at no load even with the normal voltage.

These losses depend on two independent variables: field current and speed. As the speed with a given field is proportional to the voltage at the armature terminals, it may be said that *iron loss and friction depend on the field current and the voltage at the armature terminals.* In order to plot curves, one of the variables must be kept constant during each run. It is convenient to keep the armature voltage constant for each curve, and to vary the field current by means of the two above-mentioned rheostats. Several curves should be taken within as wide limits of armature voltages and field currents as possible.

The second series of run is intended to give a curve of friction alone; this is done by simply connecting the armature and the field of the motor in series and running the machine at no load. The terminal voltage must be kept sufficiently low; otherwise the motor will run away. The motor takes very little current when running at no load; therefore its field is so weak that iron loss can be neglected and the whole input into the armature assumed to be equal to the friction loss. Note that this is not so with shunt motors, because there the field has its full value at no load as well as at any other load, and iron loss is not negligible under any circumstances.

Subtracting the values of friction obtained from this curve from the ordinates of the curves previously taken, curves can be plotted giving iron loss separately as a function of speed and field excitation.

The first series of runs gives not only the total losses, but also ampere-speed curves corresponding to various terminal voltages. For the latter curves, the current in the field winding is determining, since part of the armature current is shunted around. The voltages during the test are measured across the armature; in order to refer the curves to terminal voltages, due allowance must be made for the ohmic drop in the armature and the field.

For some further details of this method and sample curves, see *Electric Club Journal* 1904, pp. 170–174.

371. EXPERIMENT 17-F. — Efficiency of a Series Motor from its Losses. — The machine is wired up as a regular series motor (Fig. 285) and a variable resistance is shunted across the brushes. The resistance shown in the diagram in series with the field is used for regulating the main current. Have an ammeter in the main circuit, another in the armature circuit, and a voltmeter across the brushes. Run the motor at a high speed for at least 30 minutes to warm up the bearings.

(*a*) For the first series of runs (iron loss + friction) shunt the armature so that the greater part of the current passes around it. Keep the field current large enough so that the motor will not run away, and gradually cut the resistance out of the main circuit, if possible altogether. Adjust the conditions so as to have full rated voltage or more across the terminals of the motor, and a current in the field of a value about 25 per cent above the rated current of the motor. Read field amperes, armature amperes, volts across the armature and speed.

Gradually reduce the current in the shunting resistance, at the same time regulating the main rheostat to keep a constant voltage across the brushes. Reduce the currents until the upper safe limit of speed is attained. Repeat similar runs with the terminal voltage equal to 80 per cent, 60 per cent, etc., of the rated voltage.

(*b*) For the second series of runs (friction alone) remove the shunting resistance, and run the motor light throughout the possible range of speeds. A considerable resistance must be kept all the time in the main circuit to prevent the motor from running away. Take readings of amperes, volts and speed.

(*c*) Measure the resistances of the armature and of the field windings.

Report. (1) Plot watts input into the armature (iron loss + friction) to speed as abscissæ, each curve referring to a constant voltage.

(2) Plot, on the same curve sheet, field amperes *vs.* speed, each curve for a constant voltage.

(3) Plot, on the same sheet, watts friction loss to speed as abscissæ.

(4) Subtract friction loss from the curves (1), and plot curves of iron loss separately to armature volts as abscissæ, each curve for constant amperes.

(5) Correct one of the curves (2) to represent the true ampere-speed curve at the rated voltage.

(6) Show, with a numerical example, how to use the above curves for calculating efficiency, torque and output of the motor at a certain current and at the rated voltage.

CHAPTER XVIII.

DIRECT-CURRENT MACHINERY — OPPOSITION RUNS.

372. TESTS for determining the efficiency, voltage regulation, and temperature rise in electric generators and motors, without the expenditure of much energy, may be conveniently performed when two machines of about the same size are available. The machines are then loaded to their full capacity *on each other,* no power being wasted in outside resistances, or in a Prony brake. The machines are connected mechanically and electrically (Figs. 310 and 311), and driven so that one machine acts as generator, the other as motor. The motor drives the generator, while the latter supplies electric power back to the motor. This is called the opposition or the "pumping-back" method of testing machines.

If the machines were ideal — without losses — the set would be self-contained and would require no power from the outside to drive it. In reality, the losses in both machines must be supplied either by an auxiliary motor, a booster, or by connecting the set to a source of supply. The power supplied to the set from outside covers only the losses in both machines, and herein lies the economy of the method. Suppose, for instance, that two 500-kilowatt generators are to be tested at full load. Assuming the efficiency of each machine to be 90 per cent the shop line would have to supply $500 \div (0.90)^2 = 618$ kilowatts if one machine were to drive the other in the ordinary way, the second machine being loaded on resistances. When running the same two machines in opposition, the line has to supply only 10 per cent of the 500 + 500 kilowatts, or only 100 kilowatts. If a small driving motor or a booster is used (Fig. 310), the losses in these machines must also be supplied from the line; but even then the power demand is several times less than with an ordinary load test.

Electrical conditions in a generator or a motor, run in opposition with another machine, may be adjusted so as to have full-load current and the rated voltage. Therefore the opposition, or the pumping-back method is used for determining efficiency, regulation and temperature rise of the machine, as these require running it under full-load conditions.

373. Mechanical Analogy to Opposition Runs. — The following analogy (Fig. 309) may make clearer the underlying principle of all opposition methods. Let it be required to determine the efficiency of a large crane or hoist under full-load conditions, when the circumstances are such, that there is not enough power to drive it at full load, but it happens that another identical hoist is available. The two hoisting drums are then belted, as shown in the sketch, so that when the hoist No. 1 lifts its weight P_1, the hoist No. 2 lowers its weight P_2. The weights are equal and correspond to the rated capacity of the hoists; the whole system is mechanically balanced. Without friction, a very small force would produce a movement in either direction. In reality quite a considerable force is necessary to overcome the friction in both machines, and to start the movement.

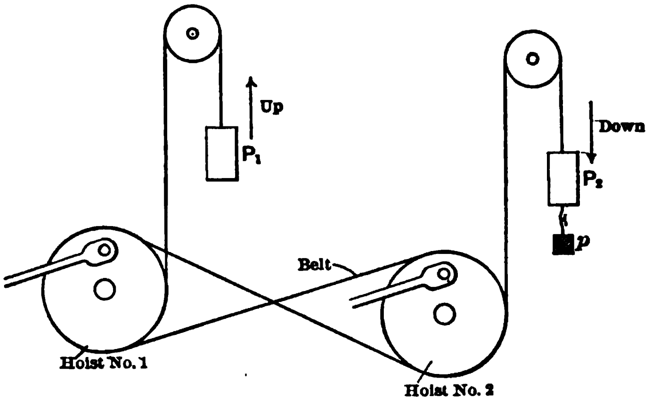

FIG. 309. A mechanical analogy illustrating the electrical opposition methods.

This extra power may be applied either as a driving force on the shaft, or as an extra weight P added to the load of one of the hoists. The first solution is analogous to the auxiliary motor shown in Fig. 310, the second solution corresponds to power being supplied from the line, as in Fig. 311. In either case, efficiency is calculated from the ratio of the power supplied from the outside, to the load of the machines. Suppose, for instance, that each of the large weights on the hoists is 100 tons, and that an additional weight of 20 tons is required to move the system at its rated speed. This corresponds to a friction loss of 10 tons per hoist; the average load is 110 tons, consequently the efficiency of either hoist is

$$\frac{110-10}{110} = 91 \text{ per cent.}$$

374. Classification of Opposition Methods. — The methods by which two electrical machines may be run in opposition are usually classified according to the way in which the losses are supplied. Copper loss may be supplied electrically or mechanically, and iron loss + friction may also be supplied either electrically or mechanically. This gives the four combinations shown in the following table:

	Iron loss and friction supplied mechanically	Iron loss and friction supplied electrically
Copper loss supplied electrically	(1) Blondel	(3) Kapp — Parallel Potier — Series Hutchison — Series and Parallel
Copper loss supplied mechanically	(2) Hopkinson	(4) Impracticable (?)

Blondel's method (1) is theoretically the most rational, because electrical loss is supplied electrically, while mechanical losses are supplied mechanically; its disadvantage is that it requires a booster set and an auxiliary motor (Fig. 310).

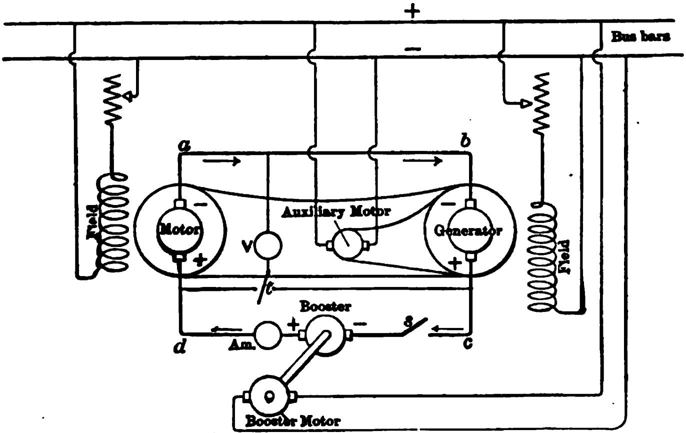

FIG. 310. Blondel's opposition method for testing two direct-current machines.

Omitting the booster set and supplying all the losses by the auxiliary motor gives Hopkinson's method (2); this method is as good as Blondel's for determination of temperature run and regulation, but gives only approximate results for efficiency.

On the other hand, omitting the auxiliary motor in the Blondel scheme gives the three methods marked (3) in the table (see Fig. 311). All the losses are supplied here electrically, either by a booster or directly from the line, or by both methods. The absolutely correct method among these three is that of Hutchison, in which copper loss is supplied by the booster and iron loss + friction from the line.

Kapp omits the booster and supplies all the losses from the line; Potier does not use the line and supplies all the losses by the booster.

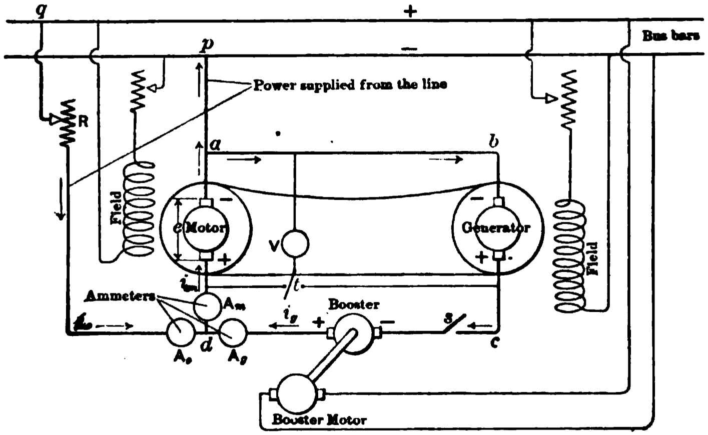

Fig. 311. Hutchison's opposition method for testing two direct-current machines.

These two methods are simpler than Hutchison's and just as good for temperature run and regulation, but give only approximate results for efficiency.

The method (4) has never been worked out in practice, for it does not seem rational to supply electrical losses mechanically, and vice versa. It may be possible, however, that a simple and satisfactory arrangement of this kind could be devised.

All the above methods will now be described more in detail.

375. Losses Supplied Electrically and Mechanically. — We shall begin with Blondel's method, as it is more correct theoretically, and moreover all other methods may be deduced from it. The connections are shown in Fig. 310. Two identical machines under test are belted or coupled together; their armatures are connected electrically in opposition. The fields should be excited separately, from the line, whenever possible. A small auxiliary motor is belted to one of the machines, and supplies the iron loss and friction of the set. A booster is connected in series with the armatures and generates the voltage necessary for overcoming the resistance drop in the two armatures. The booster is shown in the diagram driven by a separate motor.

The set is started and brought up to speed by the auxiliary motor with the switch *s* open. Then the fields of both machines are excited to the required value, and when the voltmeter *V* gives the same reading on both sides, the switch *s* is closed. Very little current flows between the two machines, the armatures being in opposition. By exciting the booster so as to add its voltage to that of the generator, any desired current may be made to circulate between the two machines. If there should be sparking on the commutators, the brushes should be shifted forward on the generator and backward on the motor, by equal amounts.

By regulating the field rheostats of the two machines, the speed of the auxiliary motor, and the booster voltage, any desired conditions of current, voltage, and speed of the set under test can be established. It is easily seen that the booster supplies the copper loss only, when both machines are equally excited; this is because the electromagnetic torque between the fields and the armatures is the same in the two machines, so that no excess of power is left on the belt for overcoming mechanical losses. On the other hand, the induced e.m.f.'s being also equal, the current is made to circulate only by the voltage supplied by the booster. Therefore, the copper loss is supplied entirely by the booster; consequently, iron loss and friction are supplied by the auxiliary motor.

376. Details of Blondel's Method.—Suppose that the machine under test (Fig. 310) is intended to be used in regular service as a generator. Then its speed and voltage during the test are adjusted to conform with the data on the name-plate, the load current being taken a little larger to account for the field current which during the test is supplied separately, but which, in regular operation, must be supplied by the armature itself. The test may be performed either for determining the efficiency, regulation, or temperature rise of the machine. We shall consider these three cases separately.

(*a*) *Efficiency.* The other machine must in this case be identical with the machine under test, or else its efficiency must be known. Assuming it to be identical, its fields must be excited to the same degree, in order to have the same iron loss in both machines; the current is adjusted to a proper value by the booster. Let *P* be the watts power supplied by the auxiliary motor, *I* the current circulating between the machines, E the rated voltage of the generator, i the exciting current during the test, and e the voltage across the booster armature.

We have then:

useful output of the generator $= E\ (I - i)$;
iron loss + friction in each machine $= \frac{1}{2}\ P$;
copper loss in each machine $= \frac{1}{2}\ eI$;
excitation loss in each machine $= Ei$;

the latter includes the loss in the field rheostat.

$$\text{The efficiency of the generator} = \frac{E\ (I - i)}{EI + \frac{1}{2}P + \frac{1}{2}\ eI}.$$

The output of the auxiliary motor is determined from its input and losses (see latter part of § 353); all other quantities entering into this expression are measured directly during the test.

It is left to the reader to develop a similar efficiency formula for the case when the machines under test are intended to be regularly used as motors. The exact input corresponding to the rated horse-power output can in this case be determined only by trials; it is advisable to take readings within wide limits of input.

(b) *Regulation.* The adjustment is the same as before, but the machines do not need to be alike. Having obtained full-load conditions on the machine under test, reduce the booster voltage, or increase that of the other machine until the circulating current is reduced to zero, or, more correctly, to the value of the field current. This corresponds to throwing the load off the machine, as required for the regulation test (see § 302). Measure the terminal voltage under these conditions. Another way of determining the voltage at no load is to open the switch between the generator and the motor and to drive the generator at no load by the auxiliary motor. Per cent rise in voltage is, by definition, the regulation of the generator.

If the machine under test is intended to be used as a motor, and per cent speed regulation between no load and full load is required, keep the voltage across the motor terminals constant by means of the booster field, and reduce the speed of the auxiliary motor until the circulating current is reduced to zero, or, more correctly, to that which the motor takes at no load. Another way of accomplishing the same result is to throw off the belt between the two machines and to run the motor at no load.

(c) *Temperature Run.* The adjustment is the same as before; the machine under test must have the same current which its armature carries at full load, same field and the rated speed. The other machine may be of any kind, provided it can take up the load delivered by the first machine. If the two machines are identical, the temperature run is made on both simultaneously.

Before beginning the run, cold resistances of the armature and the field must be measured. The machines are then run the required number of hours under load, and hot resistances are measured in order to calculate the temperature rise (§ 7). The temperature of the armature iron and of the commutator is measured by thermometers. Sometimes the temperature of the windings is also measured by thermometers, as a check on resistance measurements.

377. EXPERIMENT 18-A. — Efficiency, Regulation, and Temperature Rise by the Blondel Opposition Method. — The machines are connected as in Fig. 310; instructions for starting the set are given in the preceding article. The runs for efficiency, regulation and temperature rise may all be performed during the same laboratory period. Perform these tests from the standpoint of the machine under test running as generator, and at the end of the test make a few runs assuming the machine under test to be a motor.

(*a*) Begin the experiment by measuring cold resistances of the generator and the auxiliary motor. Bring the set up to speed and adjust the current at about 25 per cent overload. Take the necessary readings, and then gradually reduce the armature current to zero, so as to obtain complete curves of efficiency and regulation. Read generator volts, booster volts, field current, circulating amperes, speed, and the input into the driving motor. The latter reading should comprise armature amperes, volts, field amperes and speed. At the end of the test, take off the belt, and run the driving motor light, in order to determine its own losses.

(*b*) Belt the motor again, bring the set up to speed, and adjust the conditions for the temperature run, so as to obtain an appreciable temperature rise in an hour or less. An average machine ought to be able to stand at least 50 per cent overload in current and 25 per cent over-potential for this period of time. At the end of the run, measure hot resistances and temperature rise by thermometers.

Report. Plot curves of efficiency and voltage regulation to amperes output as abscissæ. Give data observed on temperature run.

378. Losses Supplied Mechanically — Hopkinson Method. — The above-described Blondel method of running two machines in opposition requires an auxiliary motor and a booster set. If it is desired to do away with the booster, the current between the two machines may be circulated by weakening the field of one, or by strengthening the field of the other. In this case the auxiliary motor (Fig. 310) supplies the copper loss as well as the iron loss and friction of the set. This method is simpler than Blondel's, and is quite commonly used in testing departments of electric manufacturing companies. The only objection to the method is that the iron loss in both machines is not the same, the fields being differently excited. This, however, is of no consequence for regulation and for temperature tests, as in this case the two machines need not be entirely identical. The method gives only approximate results for efficiency, but efficiency is usually determined from losses and not from an opposition test.

When a temperature run is required on both machines, each is run as motor half of the time to make the conditions the same. The difference between the true iron loss in the two machines is not so large as may appear from the difference in the exciting currents. With the proper setting of brushes, the field of the generator is weakened by the armature reaction, while that of the motor is strengthened, bringing them nearer to each other.

If P is the output of the auxiliary motor, total losses in each machine may be assumed to be $\frac{1}{2}$ P and

$$\text{efficiency} = \frac{(I - i)\,E}{IE + \frac{1}{2}P}$$

(see § 376). A similar formula may be deduced for the case when the machine under test is intended to be used as a motor.

379. EXPERIMENT 18-B. — Efficiency, Regulation, and Temperature Rise by the Hopkinson Opposition Method. — The method is explained in § 378; the directions for performing the experiment are similar to those in § 377.

380. Losses Supplied Electrically — Hutchison Method. — The losses may be supplied by the three methods enumerated under (3) in the table above. The theoretically correct method for determining efficiency is that of Hutchison, where the losses are supplied partly in series, partly in parallel. The connections are shown in Fig. 311. The method is similar to that of Blondel (Fig. 310), except that the auxiliary motor is dispensed with, and iron loss + friction are supplied directly from the line *pq* through the wires *ap* and *dq*.

The set is started from the line with the rheostat *R*, using the left-hand machine as a motor, the switch *s* being open. When the machines are up to speed, the generator field is excited, and when the voltages of the two machines are approximately equal, the switch *s* is closed. After this the required current is established between the two machines by suitably exciting the booster.

The field current must be the same in the two machines, at least for the efficiency test, in order to have the same iron loss. Copper loss is supplied by the booster, as in the Blondel method. It is shown below that the line supplies not only iron loss and friction, but also a small part of the copper loss; the correction is easily applied, and does not impair the accuracy of the method.

Kapp's method is obtained from that of Hutchison by omitting the booster; all the losses are supplied directly from the line. A current is made to circulate between the two machines by weakening the motor field. This method has an advantage of a greater simplicity, and is just as satisfactory for regulation and temperature runs. It does not give quite correct values for efficiency, both the iron loss and the copper loss in the two machines being different.

Potier's method is obtained from that of Hutchison by omitting the connections to the line and supplying all the losses from the booster. The field of the motor must in this case be stronger than that of the generator in order to give it an additional torque for overcoming the iron loss + friction of the set. With this method the copper loss in both machines is the same, but the iron loss is somewhat different, and the same remark applies to it as to Kapp's method.

381. Efficiency by Hutchison Method. — If Ei_o is the power delivered to the set from the line (Fig. 311), and ei_g that delivered by the booster, the expression $\frac{1}{2}(Ei_o + ei_g)$ represents total losses in each machine (assuming the losses equal), and the efficiency may thus be easily determined. This is, however, not quite correct, because the currents i_g and i_m in the two machines are somewhat different. If a greater accuracy is required, the copper loss should be figured out separately, as shown below.

With the notation shown in the diagram, we have

$$Ei_o + ei_g = i_g^2 r + i_m^2 r + P \quad . \quad . \quad . \quad . \quad . \quad (1)$$

This equation expresses that the power Ei_o supplied from the line, plus the power ei_g delivered by the booster, is equal to the copper loss in the armatures of the two machines, plus their iron loss and friction P. The fields being equally excited, the voltages induced in the two armatures are equal and opposite. Therefore, the booster voltage must be equal to the voltage drop in the armatures of both machines, or,

$$e = i_g r + i_m r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The current i_o supplied from the line is the difference between the motor current i_m and the generator current i_g, or

$$i_o = i_m - i_g \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Substituting e and i_g from (2) and (3) into (1), we get

$$P = Ei_o - i_m i_o r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

In other words, iron and friction loss, P, of the set is equal to the power Ei_o supplied from the line, less a correction $i_m i_o r$. The iron and friction loss in each machine is ½ P. The copper loss in the armature of either machine may easily be calculated from the ammeter readings, and the efficiency thus determined (taking account of the excitation loss).

382. EXPERIMENT 18-C. — Efficiency, Regulation, and Temperature Rise by the Hutchison Opposition Method. — The theory and the directions are given in §§ 380 and 381. The order in which the experiment is performed, and the requirements for the report, are the same as in § 377. At the end of the test, disconnect the line at p and q and run the set by supplying all the losses from the booster; this is Potier's method, mentioned in § 380. Then disconnect the booster and supply all the losses from the line; this is Kapp's method, mentioned in the same article.

383. Opposition Runs on Series Motors.— The methods given in the table (§ 374) with corresponding changes may be used for testing series motors, more particularly railway motors. Such motors are tested in large quantities, and it pays to provide special arrangements in order to facilitate handling the machines.

One of the methods used by the General Electric Company is similar to that shown in Fig. 310, except that the motors, instead of being belted together, are geared to a counter-shaft. This shaft is driven by an auxiliary motor, which supplies mechanical losses. The fields are connected in series with the armatures, and a booster supplies the copper loss.

A simpler method, which is a modification of Kapp's method, is shown in Fig. 312. The motors are coupled together and are connected electrically in opposition. The motor to the left is started from the line and drives the other machine, which acts as a generator and supplies current to the water rheostat; the switch S_1 is open. When the proper speed is reached, and the voltmeter V_1 shows nearly zero, the switch S_1 is closed. By increasing the resistance in water rheostat, the generator is made to send part of the current back into the motor, reducing the current drawn from the line. Finally the circuit-breaker S_2 is opened, and the machines run in opposition.

Shunting rheostats R_1 and R_2 are provided around the field windings F_1 and F_2, in order to regulate the speed of the set and the current circulating between the machines. The machine with a weaker field acts as a motor, the other as generator. To make the conditions during the temperature run equal, each machine is made to act half of the time as generator, and half of the time as motor.

384. EXPERIMENT 18-D. — Opposition Run on Two Series Motors. — Two identical railway motors are connected according to either scheme described in § 383 and run throughout the possible range of speeds, the purpose of the test being to obtain ampere-speed and efficiency curves. After this, a heavy overload is put on the motors for about an hour (temperature run). Measure cold and hot resistances, and also determine temperature rise by thermometers.

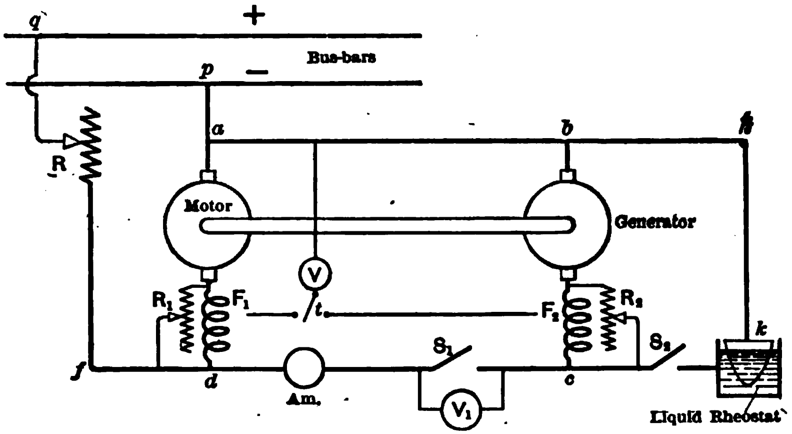

FIG. 312. An opposition test on two series-wound motors.

Report. Plot ampere-speed and efficiency curves of the motor; give temperature rise by resistances and by thermometers.

CHAPTER XIX.

TRANSFORMERS.

385. TRANSFORMERS are devices used for raising or lowering voltage in alternating-current circuits (Fig. 313). Their principal application is in power-transmission plants and in distribution of power by means of alternating currents. Aside from this, transformers are used in some cases for regulating voltage (Fig. 387), and for measuring purposes, (Fig. 45).

A transformer consists of two windings placed on the same iron core (Fig. 313); one winding — the primary — is connected to the power supply, the other — the secondary — to the load. The primary alternating current produces in the core an alternating magnetic flux, which in turn produces induced voltage and, consequently, currents in the secondary winding. By properly selecting the number of turns in the two windings, the voltage may be raised or lowered.

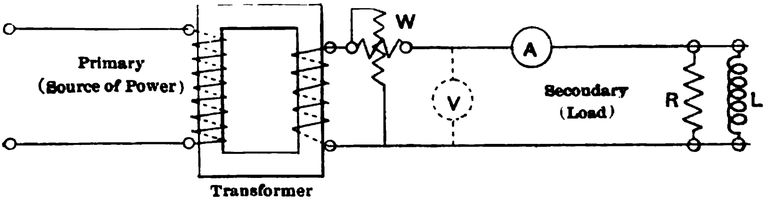

FIG. 313. Diagram of connections for loading a transformer.

386. Core-Type and Shell-Type Transformers. — There are two ways of arranging the windings and the iron core relative to each other: the core may be either inside the coils, or it may surround them (inside and outside). The first construction, the so-called *core-type transformer*, is shown in Fig. 314. The primary and the secondary coils surround the core, and are placed as close as possible to each other, in order to obtain a more perfect inductive action, and to minimize magnetic leakage. Transformers are usually immersed in oil, which keeps the transformer cooler in operation and also protects its insulation from moisture and deterioration.

A *shell-type transformer*, or one in which the iron core surrounds the coils, is shown in Fig. 315; the details are the same as in the core-type transformer.

Electrically the two types are equivalent, the difference being merely in the mechanical construction. Both types are used by different manufacturers, and some companies build transformers of either type.

387. Ratio of Voltages and Currents. — The fundamental property of the transformer is that the ratio of currents primary and secondary is practically equal to the inverse ratio of the corresponding voltages. This can be explained in two ways:

(*a*) As the losses in a transformer amount to only a few per cent of its rated capacity, it follows that practically all of the power supplied to the primary is delivered to the secondary. This power is measured by the product " volts times amperes," and so we have, to a very close approximation:

$$E_1 I_1 = E_2 I_2,$$

whence

$$\frac{I_1}{I_2} = \frac{E_2}{E_1},$$

where E_1 and E_2 are the primary and the secondary voltages, and I_1 and I_2 the corresponding currents.

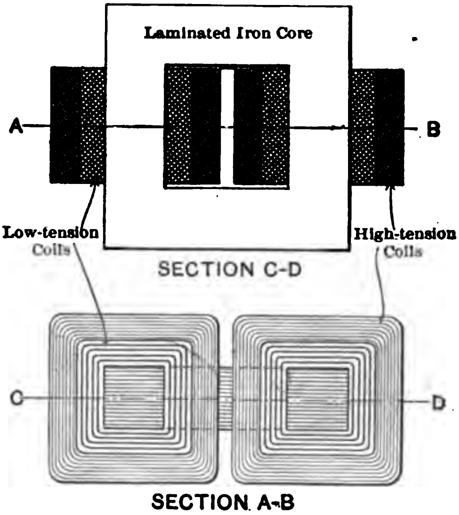

Fig. 314. A core-type transformer.

(*b*) Another explanation is based on the fact, that the current necessary to magnetize the transformer iron is very small (closed magnetic circuit, low flux density); therefore the number of primary ampere-turns n_1 is practically equal to the opposing number of secondary ampere-turns n_2, or

$$I_1 n_1 = I_2 n_2.$$

But in a transformer, as in any other apparatus based on induction, voltages are proportional to the number of turns, and we have again that

$$\frac{I_1}{I_2} = \frac{n_2}{n_1} = \frac{E_2}{E_1}.$$

388. EXPERIMENT 19-A. — Ratio of Voltages and Currents in a Transformer. — The purpose of the experiment is to make clear the numerical relations outlined in the previous article. It is well to have, for the experiment, a transformer with several taps, at least on one of the windings (Fig. 316), so as to be able to vary the number of active turns, and consequently the ratio of voltages and currents.

(*a*) Connect the primary winding to the supply, the secondary winding to a load; have an ammeter and a voltmeter inserted on both sides, or transfer the same instruments from one circuit to the other by means of suitable switches. Vary the load from zero to a maximum, and read volts and amperes on both sides.

(*b*) If possible, change the ratio of turns, and repeat the same experiment. Make one of the runs with an inductive load, say a choke coil, in order to see if the ratio of currents remains the same. Before leaving the laboratory, inform yourself as to the number of turns in the two windings.

(*c*) If two transformers are available, a complete power transmission may be realized in the laboratory. The voltage is stepped up, say from 110 volts to 2200 volts, and then stepped back at the other end of the line to 220, or any other desired voltage. For reading line volts, the voltmeter must be provided with a shunt transformer of a known ratio (Fig. 45).

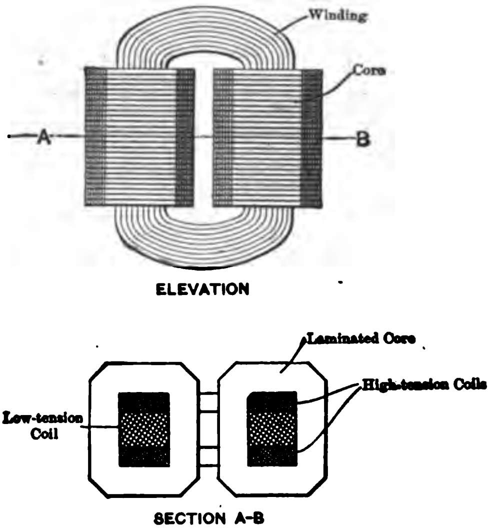

Fig. 315. A shell-type transformer.

Report. (1) Plot to amperes load as abscissæ: primary amperes, primary volts and secondary volts; also a curve of ratios of volts and of amperes. Show that the ratio of volts is very nearly the same as that of the number of turns, and as the inverse ratio of currents. The ratio of currents may be somewhat different at light loads, because the transformer consumes a small primary current, even with no current in the secondary. This so-called no-load current serves largely for magnetizing the transformer iron, and has a small component for supplying the core-loss. (2) Describe the experiment on power transmission with step-up and step-down transformers, and give the numerical data observed.

389. Voltage Drop in Transformers. — In the above discussion, the transformer windings are supposed to be ideal, possessing no resistance or reactance. In reality, when currents flow through the transformer, a certain drop takes place in its windings, so that if the primary voltage is maintained constant, the secondary voltage drops, as the load increases.

Suppose, for instance, that a transformer has such a ratio of the number of turns as 20 to 1. With the primary connected to a 2200-volt supply, the secondary voltage at no load is 110 volts. Suppose the voltage drop in the primary winding to be 2 per cent, or 44 volts at full load. This leaves 2156 volts to be transmitted into the secondary; with the ratio of transformation of 20 to 1, the secondary induced voltage is 107.8 volts. If the drop in the secondary winding is also 2 per cent, the voltage at the secondary terminals is but 105.6 volts, instead of 110 volts available at no load. Thus, as the load changes, the voltage fluctuates, producing an undesirable flickering of the lamps. Incandescent lamps are sensitive to a very few per cent variation of the rated voltage; therefore, good inherent regulation of transformers is of great practical importance.

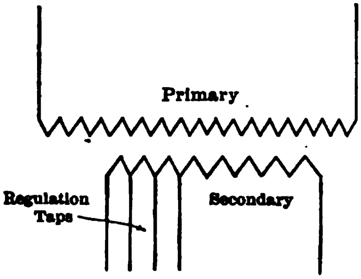

FIG. 316. Taps on a transformer winding, for voltage regulation.

Voltage drop in a transformer depends not only on amperes load but also on the power factor of the load. The explanation is the same as in Fig. 236 for transmission lines, and in Figs. 359 and 360 for alternators.

Voltage variation on the secondary side, with a constant primary voltage, may be determined either by actually loading the transformer, or it may be calculated from the measured resistances and reactances of the transformer windings. The second, or the indirect, method is always preferred in practice, because it gives more accurate results, and involves but a small expenditure of power. This method is described in §§ 408 to 411. In order, however, that the student may see a transformer in actual operation and be able to judge concerning the magnitude of voltage fluctuations, he should determine the regulation by actually loading a transformer on ohmic and inductive resistances, as described in the next experiment. It is advisable to determine the regulation of the same transformer, by the direct and the indirect methods, in order to be able to compare the results.

390. EXPERIMENT 19-B.—Loading a Transformer.—The purpose of the experiment is to determine the fluctuations of secondary voltage with inductive and non-inductive load. There should be provided for this test an ordinary lighting transformer, such as is used in cities for stepping the 2200-volt supply down to 110 volts or 220 volts for light and power in houses.*

The test consists in actually loading the transformer and varying the load and its power factor. Voltages must be measured very accurately, because the drop between no load and full load is but a few per cent. The best way to measure the primary voltage is to have another transformer connected to the same primary circuit and kept unloaded. The voltmeter should be provided with a double-throw switch to enable one to read quickly in succession the voltages of both loaded and the unloaded transformers. This is better than actually throwing off the load for measuring the no-load voltage, because this may somewhat change the primary voltage.

A shunt or potential transformer (Fig. 45) may be used with the voltmeter, for reading the voltage on the high-tension side. An ammeter and a wattmeter must be connected on the low-tension side, as shown in Fig. 313; it is well to have an ammeter on the high-tension side, that it may be possible to check the ratio of currents. First determine the regulation at non-inductive load. The primary volts must be kept constant; the secondary voltage decreases, as the load increases. If it should be impossible to keep the primary volts absolutely constant, the secondary volts must be corrected before plotting the curves; within the limits of fluctuations it can be safely assumed that the secondary voltage is directly proportional to the primary, so that if, for instance, the primary voltage was 2 per cent low when a reading was taken, the observed secondary voltage must be also increased 2 per cent before plotting the regulation curve.

After this test the regulation of the transformer must be investigated on inductive loads. With non-inductive load there is but one quantity to be varied, viz., amperes (or, which is the same, watts); on inductive loads there are three variables: amperes, watts and power factor. Any two of these can be taken as independent, and curves plotted giving the regulation of the transformer, say for constant watts, constant current or a constant value of the power factor. It is much easier to regulate the inductive and non-inductive resistances so as to keep either amperes or watts constant and to get the influence of the power factor on regulation.

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* If the distributing network is laid underground (cables), such transformers are usually placed in the basements of the houses; if the distribution is done by means of overhead wires, the transformers are mounted on the outside of one of the walls, or on a pole near the house.

NOTE. A student working with a 2200-volt transformer must remember that *it is dangerous* to come in contact with the high-pressure side of it; great care should be exercised in regard to touching any switches, wire, or instruments connected to the high-tension side. Rubber matting should be put on the floor for protection, so as to insulate the students from the ground, and the students should not touch either the walls, pipes or any objects that might be connected to the ground. In addition, rubber gloves should be provided for the student who reads the high-tension instruments, so that he could not touch them by mistake. *Directions of the instructor should be followed exactly,* and no changes attempted before consulting him.

An electrical engineer has to handle high-tension currents in his practical work, and it seems, therefore, natural that he should have some experience with it before leaving college; he should know, however, that a mistake or negligence may prove fatal, and should act accordingly.

(*a*) Apply a certain load, say 25 per cent overload, entirely non-inductive, and gradually decrease it, at the same time introducing more and more of an inductive load, — say a choke coil in parallel with the rheostat. Regulate the load so as to keep *total amperes constant,* and observe the variation of the secondary voltage as the power factor decreases. The same test can be repeated for 100 per cent load, 75 per cent load, etc.

(*b*) Now again take a certain non-inductive load and gradually add to it some inductive load, keeping *total watts constant.* Observe the regulation under these conditions. In this case it may be better to begin with the lowest power factor available (largest current), as otherwise the ammeter and wattmeter may be overloaded and damaged. Several curves should be taken, for various values of watts. A data-sheet, similar to the one shown below, is convenient for recording the readings.

	NON-INDUCTIVE LOAD.					INDUCTIVE LOAD....AMPS. CONST.					
	Prim. Amps.	Prim. Volts.	Sec. Amps.	Sec. Volts.	Sec. Watts.	Prim. Amps.	Prim. Volts.	Sec. Amps.	Sec. Volts.	Sec. Watts.	Power Factor.
Inst. No.											
Const.											

Report. Give the actual connections used during the experiment. Plot to power factor as abscissæ, curves of volts for constant amperes, as indicated in Fig. 361 in application to alternators. Plot on the same sheet curves of corresponding watts. As voltage drop in transformers is quite small, use a suppressed volt scale, or plot volts drop instead of the voltage itself. On another curve sheet plot similar curves for the tests in which watts were kept constant, showing also the corresponding amperes. Figure out per cent regulation at full non-inductive load, as per definition given in § 407.

391. Auto-Transformer. — It is not absolutely necessary to have, in a transformer, two separate windings; the line voltage may be changed by having one winding connected across the line, *and the load taken from a part of the same winding*, as in Fig. 317. The voltage across AB is to the voltage across BC as the ratio of the numbers of turns between these points. If, in addition to C, other taps are provided, as shown in the sketch, the secondary voltage may be varied within certain limits.

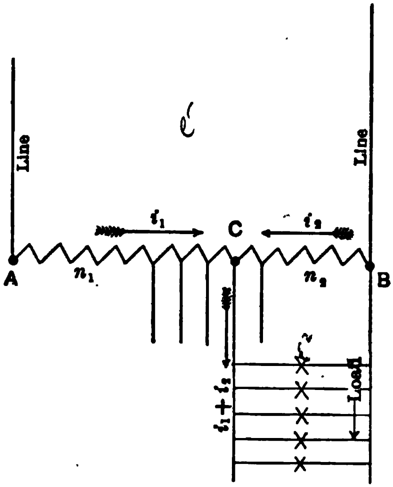

FIG. 317. A loaded auto-transformer.

Such a transformer with only one winding is called an *auto-transformer*. Theoretically it requires less copper than an ordinary transformer of same capacity. Its disadvantage is, however, that the high-tension and the low-tension windings are not insulated from each other. This excludes it from being used on ordinary lighting circuits where voltage is stepped down from 2200 volts. Auto-transformers are widely used for starting induction motors (§ 398) and for various purposes of regulation.

In an ordinary transformer a primary and a secondary current are distinguished. With an auto-transformer there are three different currents: current i_1 in the line and in the part AC in the transformer; current i_2 in BC, and the load current $(i_1 + i_2)$. The currents i_1 and i_2 are shown flowing in opposite directions, for the reason that the secondary current always opposes the primary in its magnetic action. We have then, as in § 387, that $n_1 i_1 = n_2 i_2$, where n_1 and n_2 are the numbers of turns in AC and BC.

Suppose, for illustration, that an auto-transformer has 120 turns between A and B, and the point C is so selected that there are 40 turns in the part BC. If the line voltage is 300 volts, the voltage across the load will be only 100 volts. The part AC has 80 turns as against 40 in BC; therefore the current i_1 needs to be only one-half of i_2 in order to produce the same magnetic action. If the load current $(i_1 + i_2)$ is equal to 60 amperes, two-thirds of it, or 40 amperes, are supplied by i_2, and one-third, or 20 amperes, supplied by i_1. With this distribution of currents, the same energy is delivered to the secondary as is supplied by the primary, no energy being created or lost in the transformer itself:

$$\text{Power} = 60 \times 100 = 20 \times 300 = 6000 \text{ watts.}$$

In general, if e_1 and e_2 are the line voltage and the load voltages respectively, four electrical conditions must be fulfilled in an auto-transformer:

$$\begin{aligned} n_1 i_1 &= n_2 i_2; \\ e_1 \div e_2 &= (n_1 + n_2) \div n_2; \\ \text{load current} &= i_1 + i_2; \\ e_1 i_1 &= e_2 (i_1 + i_2). \end{aligned}$$

The load current, flowing back to the point B, is divided again into i_1 and i_2; the former returns to the line, the latter circulates between the winding BC and the load.

An auto-transformer may be used for raising voltages as well as for lowering voltages. If, for instance, the voltage of the line is 100 volts, the line may be connected to the points B and C, and a voltage of 300 volts made available between A and B.

392. EXPERIMENT 19-C. — Loading an Auto-Transformer. — The purpose of the experiment is to illustrate the relations of the preceding article. The experiment is performed in a manner similar to experiment 19-B above. Use an auto-transformer with regulating taps, or one winding of an ordinary transformer, if it is provided with taps. In some cases an auto-transformer may be improvised by connecting in series the two windings of an ordinary transformer.

(*a*) First determine the ratio of voltages without load. Connect the two outside points of the winding to the source of supply, and measure the voltages between the various taps. Also apply a voltage across a part of the winding, and read the higher voltage resulting across the total winding. In doing so, be careful to have enough turns in the part of the winding connected across the line, so as not to draw an excessive current.

(*b*) Connecting to a suitable tap, load the transformer, as in Fig. 313, having ammeters connected so as to read the primary and the secondary currents and the current in the load. One voltmeter and one ammeter in connection with a polyphase board (§ 49) are sufficient for the purpose. Gradually increase the load from zero to a permissible maximum, reading all the currents and voltages.

(*c*) Repeat the same experiment with a few other taps, taken nearer to *A* or to *B*. In the latter case, be sure not to overload the part *BC* of the winding, as the current i_2 increases inversely to the number of turns. Before leaving the laboratory ascertain the number of turns between the various taps.

Report. Make a diagram of connections, and write on it per cent voltage at different taps. Plot i_1 and i_2 to load amperes as abscissæ. for a few taps. Show that the four conditions mentioned in the previous article are fulfilled.

TESTING INSULATION.

393. Samples of insulating materials, used in the construction of electrical machinery, are tested for dielectric strength, before the materials are accepted. This is done by applying to the sample a high voltage until it is pierced through by the electric spark. This is similar to testing mechanical strength of materials. In using insulating materials a factor of safety is assumed, as in the case of structural iron or machine parts. Thus, for instance, a sample of insulation, which can stand 10,000 volts as a limit, would not be used for a strain higher than 2000 volts in a lighting transformer, thus allowing for a factor of safety equal to five.

The most important materials used as electrical insulation are: Impregnated cloths and fabrics, treated paper and other fibrous materials, rubber, glass, porcelain and mica. Wire used for windings is usually double-cotton insulated (magnet wire); the coils, after being wound, are impregnated in special insulating varnishes which increase the insulating properties of cotton, and preserve it from atmospheric influences. A great majority of modern transformers are oil-immersed, so that oil must be added to the list of insulating materials.

High voltages necessary for testing are obtained by means of special transformers (Fig. 318), so-called testing transformers. No large output is required of such transformers, but they must give very high voltages, and these voltages must be easily adjustable within a wide range.

394. Description of a Testing Transformer. — A convenient arrangement for testing insulation, as well as for other high-tension experiments, is shown in Fig. 318. Two transformers are used: one is the testing transformer proper, the other serves for regulating the voltage. The testing transformer should preferably have no taps, to insure a better insulation. It is well to interpose a third transformer, the so-called insulating transformer, with a "one to one" ratio between the regulating and the testing transformer; this is an additional safeguard to the operator in case of defective insulation. The sample *I* to be tested is placed in the glass case *GG*. The switches *kk*, mounted on the doors of the case, are connected into the low-tension supply circuit. This is done for the safety of the operator: in order to reach the sample, he has to open the doors of the glass case, and to do this he has to open the switches *kk*, thus disconnecting the transformer from the source of supply.

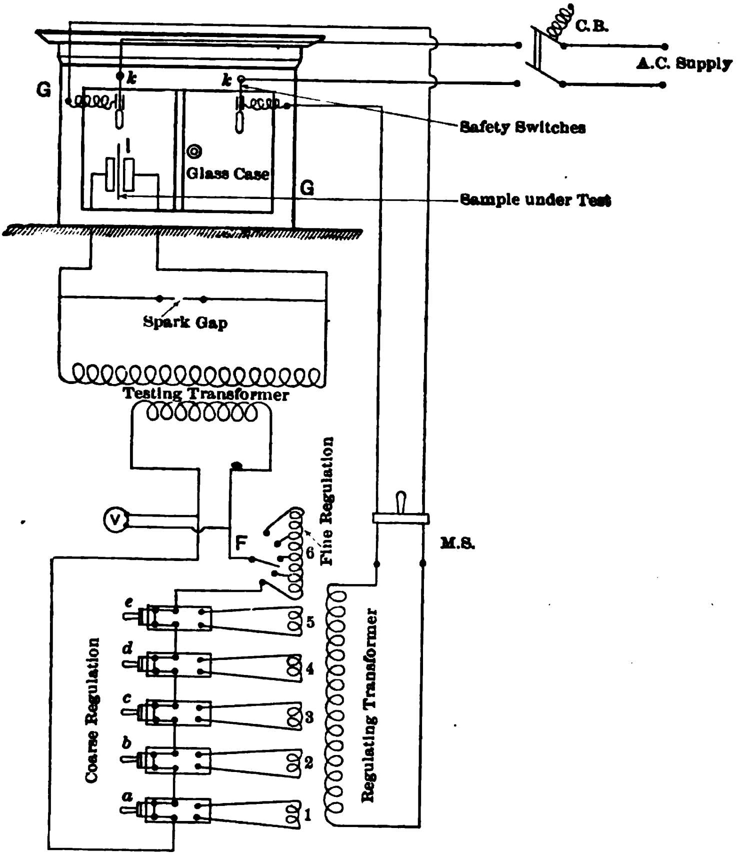

FIG. 318. Diagram of connections for disruptive tests on insulation.

The details of the circuit are as follows: The current from a low-pressure supply passes through the circuit-breaker C. B. and through the above-mentioned safety switches *kk* to the main switch M. S. and to the primary of the regulating transformer. The secondary of this transformer is wound in six sections, five of them being used for coarse regulation and the sixth for fine regulation. Coarse regulation is obtained by throwing the switches *a, b, c, d* and *e* to the right; fine regulation is done by the handle *F*. The secondary of the regulating transformer is connected directly to the primary of the testing transformer, the voltage being here raised many times. For instance, the regulating transformer may be made to raise the voltage from 110 to 2000 volts, and the testing transformer wound with the ratio 100:1; such a combination gives testing voltages up to 200,000 volts.

The electric pressure across the sample under test is measured by the voltmeter *V* connected to the low-tension side of the testing transformer, it being only necessary to multiply voltmeter indications by the ratio of transformation. Sometimes a spark gap is used in parallel with sample under test, as shown in the sketch. A table may be found in the *Standardization Rules of the American Institute of Electrical Engineers*, giving sparking distances for various voltages up to 300,000 volts. Some engineers do not trust the spark gap, and object to its being used. A spark gap is, however, a good precaution against an accidental rise of voltage beyond a desired limit; as soon as this limit is reached a spark jumps across the gap and the rush of current opens the circuit-breaker on the low-tension side.

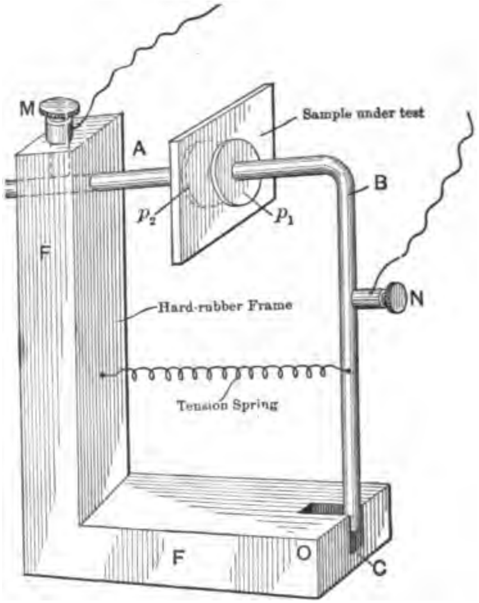

FIG. 319. A clamp for testing insulation.

395. Disruptive Strength of Solid Dielectrics, Compounds, and Oil. — Materials like Fuller board, cloth, mica, etc., may be conveniently tested in a clamp, shown in Fig. 319. The frame *FF* of the clamp is made of hard-rubber, the metal parts of brass. The tension spring insures the same tension being applied to the samples in all cases; the terminal plates p_1, p_2 have a loose fit in the supports and adapt themselves to the surface of the sample under test.

The disruptive strength of insulating materials is affected by many causes: In the first place it depends on the thickness of the sample; it could be naturally expected that a double thickness of insulation can stand a double voltage, but experience shows that this is not the case, and that the resistance to puncture increases more slowly than the thickness. If, for instance, a sample 0.01″ thick can stand 10,000 volts, a sample 0.02″ thick will be punctured at less than 20,000 volts. The cause of this is not definitely known; possibly, it is due to thicker samples being not as homogeneous inside as thinner ones. At any rate, several thin samples put together can stand more than one thick piece; therefore, the more important parts of insulation of high-tension transformers are always built of many thin layers, and not of a thick piece of insulation which may have faults inside. Mr. Baur, on the basis of his numerous tests, announced an empirical law (Baur's law) that dielectric strength increases as $d^{\frac{2}{3}}$ where d is the thickness of the insulating material. According to this law if a sample 1 mm. thick can stand 5,000 volts it takes a sample $2^{\frac{3}{2}}$ or about 2.8 mm. thick to stand 10,000 volts. Another investigator, Dr. Walter, assumes the law to be:

$$\text{break-down voltage} = a + bd,$$

where a and b are constants. This formula can be interpreted by saying that, in addition to the voltage bd for breaking the material proper, it takes an extra voltage a independent of the thickness of the sample to break its surfaces. It would be premature to decide between the two formulæ on the basis of the experimental data on hand. The only established fact is that the dielectric strength increases more slowly than the thickness; therefore, in all important cases tests should be performed on samples of exactly the same material that is intended to be used in the machine.

The other two factors which considerably influence the strength of insulation are: presence of moisture, and heat. Most of the insulating materials lose to a large extent their good insulating properties when subjected to moisture, or when they have not been sufficiently dried out before being used; also when heated up beyond a certain temperature.

Insulating varnishes, paints and impregnating compounds are tested by applying them in an even layer on some kind of cloth (preferably linen) and subjecting the cloth to a break-down test. By using always the same kind of cloth, testing it without varnish and with different kinds of varnishes, various brands can be compared, by the increase in the dielectric strength of the cloth.

Insulating oil is conveniently tested in the apparatus shown in Fig. 320, devised by Mr. C. E. Skinner. It consists of a glass tube A, filled with a definite quantity of oil under test, with two brass balls immersed in it as electrodes. These balls are set at a definite distance apart by means of a micrometer *D* placed on top of the apparatus, and an electric pressure is applied between them until a spark jumps through the oil. This break-down voltage is taken as a measure for the dielectric strength of the oil. By testing different oils in the same vessel, and with the same distance between the balls, a direct comparison of different kinds of oil is made possible. After each test the oil must be thoroughly shaken so as to mix with it the carbon formed by the electric arc. The brass balls must be kept well polished, as the sparking distance depends essentially on the state of their surfaces.

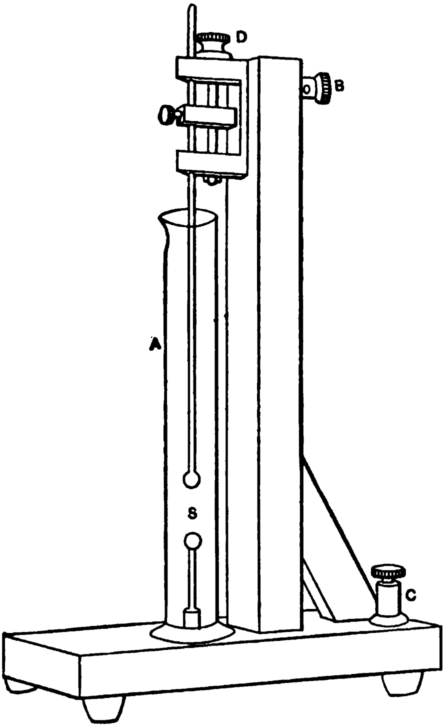

FIG. 320. A vessel for testing dielectric strength of oil.

For oil tests, other than disruptive strength, see an article in the *Electric Club Journal.* 1904, p. 227.

396. EXPERIMENT 19-D. — Insulation Tests with a High Tension Transformer.—

The purpose of the experiment is to afford experience in handling a high-tension transformer, and also to learn the principal properties of insulating materials used in electrical work. Before beginning the tests proper, the student should study most carefully the connections of the testing set, and be sure that all high-tension parts are properly insulated. The voltages used for test are dangerous to life, and utmost precautions and concentration of attention are required while performing the test.

Begin the experiment by puncturing a few samples of different materials, merely to observe the behavior of the transformer, of the circuit-breaker, spark gap, etc., and to get a general idea of the magnitudes of the pressures involved. After this, more systematic tests may be made on a specific material, such as fiber, paper or Empire cloth, of different thickness, and also using several layers of same or different materials as a compound insulation. Then the student may slightly moisten a few samples, or heat them up and see how this affects their dielectric strength.

It will be observed that these materials differ widely in the maximum voltage which they can stand, mica being the best insulator, and fiber and wood the poorest, they being punctured at many times lower pressure than mica, with the same thickness of the sample. Other insulating materials, Fuller board, treated paper, Empire cloth, etc., have disruptive strengths between these extremes. Different samples, taken even from the same piece of material, may give considerable discrepancies in results due to a lack of uniformity of structure and accidental impurities and faults. Reliable results can be obtained only as an average of hundreds of tests made on the same material, by a skilled observer. All that is expected from the student in this exercise is to get a few comparative data on the strength of different materials, and to learn how to handle a high-tension testing transformer.

Oil is tested as is described in the preceding article. In order that the student may see the importance of oil as insulation he should compare the voltage at which a spark jumps between the two brass balls in the oil-testing apparatus, with and without oil. It will be found that the voltage is much higher when the balls are immersed in oil. Having tested a sample of oil, add to it a few drops of water and see how markedly it affects the insulating properties of the oil; the same thing will be observed if a small amount of acid is added.

It is recommended that the student also puncture some other kinds of insulation important in electrical work, for instance, porcelain insulators, and insulated wires used on electric lines. These articles are tested under the most unfavorable conditions met with in practice, namely *wet*. Insulators are turned upside down and immersed in salt water which serves as one terminal; the other terminal is applied to the inside of the insulator, also filled with water. Insulated wires are tested immersed in water, the pressure being applied between the core and the water; it is required that the wire be immersed for twenty-four hours before the test.

Purely electrical tests as described above are not sufficient for finally approving an insulating material for use. It must possess, in addition, certain mechanical and chemical properties without which it could not be used in commercial apparatus. Thus, for instance, solid insulators should not be brittle, or affected by acid and fumes; they should be easily machined, not deteriorate with time, etc. All such tests are, for obvious reasons, outside the scope of the above laboratory exercise.

References. A very clear and complete book on *The Insulation of Electric Machines* is that by Turner and Hobart. This book contains most of the noteworthy results obtained by different investigators of insulating materials; moreover, it contains a complete bibliography on the subject (pp. 272–279). Chapter II of this book, treating on general properties of insulating materials, may be consulted in connection with this exercise, and also Chapter XXII, "Specifications for Insulation." The rest of the book is devoted to a detailed description of properties, tests and processes of manufacture of various insulating materials. Recommendations of the American Institute of Electrical Engineers, in regard to testing for dielectric strength, may be found in the report of their Committee on Standardization. For a series of good articles by Mr. C. E. Skinner, on testing insulation, see the *Electric Journal*, 1905, beginning on p. 615.

COMMERCIAL TESTS ON TRANSFORMERS.

397. Transformers are ordinarily tested for:

(*a*) *Efficiency;*

(*b*) *Temperature rise* during continuous operation;

(*c*) *Voltage regulation* determined by the voltage drop inside of the transformer itself.

The methods for testing transformers for these requirements are described below.

(*a*) *Efficiency.*

398. Efficiency from Losses. — The direct way for measuring the efficiency of a transformer would be to have it actually loaded and to read the output and the input by means of wattmeters in the primary and in the secondary circuits; their ratio would give directly the efficiency of the transformer. However, this direct method has serious drawbacks, and is hardly ever used in practice. It involves the use of a wattmeter on the high-tension side; it requires an utmost accuracy of calibration of the wattmeters because the difference in the values to be read is but a few per cent. Moreover, with large transformers the amount of power required for a full-load test is not always available. Therefore, it is customary to measure the losses in a transformer separately (at no load), and then to *calculate* the efficiency; this can be done with much simpler means and with a considerably greater accuracy, than is possible with the direct method.

The losses in a transformer consist of I^2R or copper losses in both windings, and of iron loss, also commonly called *core loss* (hysteresis and eddy currents). Copper loss depends on the load exclusively, and can be easily calculated for any load, if the resistances of both windings are known. Iron loss depends only on the magnetic flux, and is practically independent of the load. This follows from the fact that, the impressed voltage being constant, the flux is also nearly constant (neglecting the primary drop). Therefore the iron loss can be determined at no load and assumed to be the same at all loads. Resistances are usually measured by the drop-of-potential method; core loss is determined from the no-load reading of the wattmeter.

As an illustration, suppose that it is required to determine the efficiency of a 2200 to 110-volt, 5-kw. transformer at $\frac{3}{4}$ load. Let the resistances of the windings be 10.7 ohms and 0.026 ohm respectively, and the iron loss, as determined by a wattmeter at no load, be equal to 96 watts. At three-quarters load the output of the transformer is 3.75 kw., or 34.2 amperes at 110 volts. The primary current is $\frac{1}{20}$ of this, or 1.71 amperes; thus we have:

$$\begin{array}{lr}
\text{Primary copper loss} = \overline{1.71}^2 \times 10.7 & = 31.2 \text{ watts.} \\
\text{Secondary copper loss} = \overline{34.2}^2 \times 0.026 & = 30.4 \text{ watts.} \\
\text{Iron loss} \ldots\ldots\ldots\ldots\ldots & = 96 \text{ watts.} \\
\hline
\text{Total losses at } \frac{3}{4} \text{ load} & 157.6 \text{ watts.}
\end{array}$$

$$\text{Efficiency} = \frac{3750}{3750 + 157.6} = \text{about } 96 \text{ per cent.}$$

In this way, data for any load may be obtained, and an efficiency curve may be plotted from no load up to, say, 50 per cent overload.

399. EXPERIMENT 19-E. — Efficiency of a Transformer from its Losses. — The experiment comprises three distinct measurements:

(1) Ratio of transformation;
(2) Resistances of windings;
(3) Core loss.

The ratio of transformation is determined by voltmeters, as in § 388. The resistances of the windings are measured with direct current, by the drop-of-potential method. As the resistance of the low-tension winding is usually much smaller than that of the high-tension winding, it may be necessary to use different ammeters and voltmeters. Always removet he voltmeter leads when opening or closing the circuit especially on the high-tension side; the inductive kick may damage the instrument.

The core loss is measured by a wattmeter, as in Fig. 185. It is not necessary to measure it from the high-tension side; 110 volts applied on the low-tension side produce the same flux and give the same core loss as 2200 volts applied on the high-tension side. Take curves of core loss and magnetizing current from zero voltage to about 50 per cent overpotential. In performing this measurement, be sure to disconnect the high-tension side from the power supply, in order not to make a short circuit. Also remember that high-tension terminals become alive the moment a voltage is put on the low-tension terminals; in order to avoid the possibility of a fatal mistake, it is best to have the high-tension leads taped up.

Report. Give the ratio of transformation, the resistances of the windings, and plot curves of magnetizing current and iron loss, to volts as abscissæ. Plot an efficiency curve and separate losses, to kilowatts load as abscissæ, up to 50 per cent overload. Indicate by crosses on the same curve sheet the values of efficiency at full load and of the losses, when the supply voltage is 10 per cent above or below rated.

(*b*) *Temperdture Rise.*

400. — Safety of operation of a transformer depends essentially on the maximum temperature which it attains under load. An insulation which can stand a considerable over-potential at ordinary temperatures, easily breaks down when subjected to a sufficiently high temperature; transformer iron undergoes so-called "ageing" under the influence of heat, its properties being changed so that the hysteresis loss increases considerably. This in turn increases the temperature of the transformer. Therefore a test for temperature rise under actual load conditions is one of the most important practical transformer tests (Fig. 321).

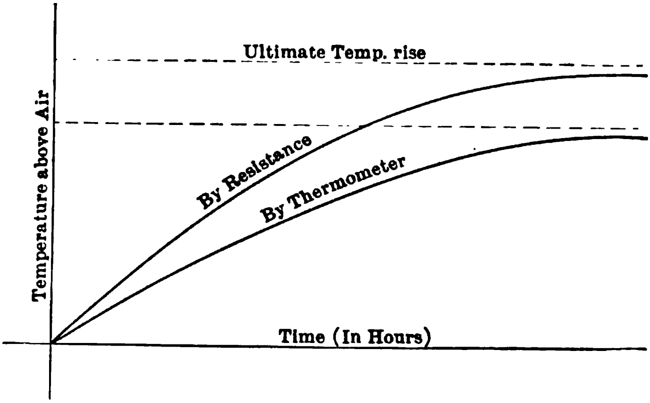

Fig. 321. Curves of temperature rise.

A transformer heats up in operation because of its core loss and of the I^2R loss in its windings. Core loss depends on the flux in the iron, or, which is the same, on the applied voltage, and is practically independent of the load; copper loss is directly proportional to the square of the current flowing through the windings. Therefore, in order to determine temperature rise in a transformer, it must be subjected to its full rated voltage and full-load current simultaneously.

Such a test offers no difficulties with small lighting transformers, which can be loaded on lamp rheostats, etc.; but with large high-tension transformers a load test is not only difficult and expensive, but in many cases impossible. When two similar transformers are available one transformer can be loaded on the other, and this latter connected back to the source of supply, so that no power is wasted except that necessary for supplying the losses in both transformers. This so-called " pumping back " method, also known as the "stray power," or "opposition " method, is often used not only in application to transformers, but to generators and motors as well (see Chapter XVIII).

401. Opposition Test on Transformers. — The connections for an opposition heat test on two transformers are shown in Fig. 322. *A* and *B* are the two transformers under test; their low-tension windings are connected in parallel to the power supply, and the high-tension windings are connected *in opposition to* each other. As the transformers are identical, no current flows through their secondaries, and but a small magnetizing current flows through the low-tension windings. At the same time full iron loss takes place in the cores of the transformers, they being subjected to the full rated voltage. If now a full-load current should be made to circulate in the windings, the transformers would be under the same conditions of heating, as under actual full load. As we are here concerned with I^2R loss only, it is not necessary that the heating current should be an alternating current of the right frequency; it may be a current of any frequency, or even a direct current. If no suitable alternating current is available, both the high- and the low-tension circuits can be opened at some place, and sources of direct current introduced, sufficient to produce the desired currents in the windings.

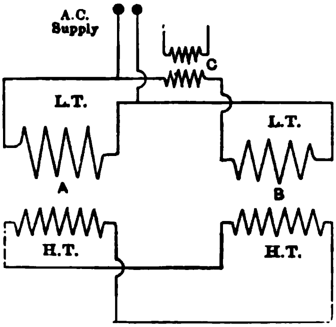

FIG. 322. Heat run on two transformers connected in opposition.

A more convenient method is to open but one of the circuits, say the primary, and to introduce there a comparatively low-voltage source of alternating current of any frequency. This auxiliary source is shown in figure in the form of a transformer *C*. By regulating the voltage of this source, a full-load current can be produced in the primaries of the transformers under test; this current induces in turn a full-load current in the secondaries, which thus need not be opened for any purpose. This is very convenient, since the secondaries are usually wound for comparatively high voltages.

In this test the transformers are under actual load conditions in so far as the losses and heating are concerned; at the same time, the energy supplied from outside is merely sufficient to cover the losses. For instance, with two 1000-kw. transformers having a full-load efficiency of about 98 per cent, the total expenditure of power with the above test would be but 40 kw.

When but one transformer is available, the same test may be performed by connecting in opposition the two halves of both windings.

402. Extrapolation of Heating Curves. — Commercial transformers usually attain their final temperature after a full-load run of 10 hours or thereabouts; therefore it is out of the question to attain this final state during a laboratory exercise. The following method makes it possible *to predict* the final temperature, and also to figure out approximately the maximum load which the transformer under test can carry continually. Let OMQ (Fig. 323) be the "heating" curve of a transformer, or the curve of temperature rise with time. Assume that only the part OM could be determined experimentally, and that it is required to find the position of the line PQ of the ultimate temperature rise, also to construct a portion of the curve beyond M. The method for doing this is given below without proof; for a mathematical proof of the formulæ used, the reader is referred to §§ 405 and 406.

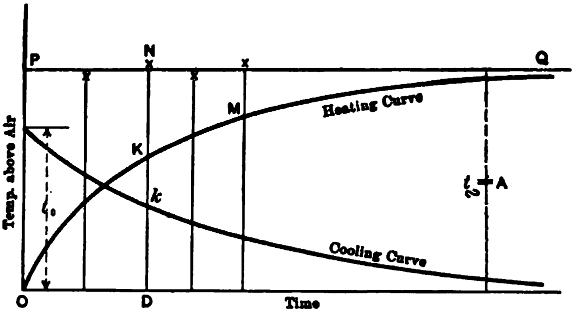

FIG. 323. Analysis of heating and cooling curves.

Let abscissæ of the heating curve, representing time, be denoted by x, and ordinates, or temperature rise above air, by T. Let the abscissa and the ordinate of the point M be x_4 and T_4. Divide x_4 into four equal parts corresponding to the points with abscissæ:

$$\left.\begin{aligned} x_1 &= x_1 \\ x_2 &= 2x_1 \\ x_3 &= 3x_1 \\ x_4 &= 4x_1 \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (1)$$

Measure the corresponding ordinates T_1, T_2, T_3 and T_4, and calculate the expression

$$z = \sqrt{\frac{T_4 - T_3}{T_2 - T_1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

The ultimate temperature rise, or the distance of PQ from the axis of abscissæ, may be calculated from any one of the four expressions below:

$$\left.\begin{aligned} A &= \frac{T_1}{1 - z} \\ A &= \frac{T_2}{1 - z^2} \\ A &= \frac{T_3}{1 - z^3} \\ A &= \frac{T_4}{1 - z^4} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

It is well to determine A from all the four expressions (3) and to take an average, in order to allow for irregularities of the curve.

If it is desired to continue the curve beyond M, take points $x_5 = 5\,x$, $x_6 = 6\,x$, etc., and calculate the corresponding ordinates T_5, T_6, etc., from the expressions

$$\begin{aligned} A &= \frac{T_5}{1 - z^5} \\ A &= \frac{T_6}{1 - z^6} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (4) \\ &. \quad . \quad . \quad . \quad . \\ &. \quad . \quad . \quad . \quad . \end{aligned}$$

which are analogous to (3), and in which A and z are known quantities.

Theory and experience show that heating curves such as OM are exponential curves (see equation (7) in § 405). Therefore, instead of extrapolating by means of the equations (4), OM may be plotted to PQ as abscissæ, using a logarithmic scale. This converts the curve into a straight line which is then easily produced to any desired points; the ordinates thus obtained are plotted again to an ordinary scale.

403. Use of the Cooling Curve. — In some cases the "cooling" curve of the transformer is available (Fig. 323), or the curve which gives temperatures of the transformer left to cool, after having been heated, the power being shut off. This curve may be used in addition to the curve OM in order to find the ordinate A and to extrapolate the heating curve beyond the point M. Namely, the theory shows (see § 405) that

$$T = A\left(1 - \frac{t}{t_0}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (5)$$

where T, as before, is an ordinate of the heating curve, such as DK, t is the corresponding ordinate kD of the cooling curve, and t_0 is the initial temperature on the cooling curve.

The cooling curve is supposed to be given throughout its whole working range; at any rate, being an exponential curve, it may be easily *extrapolated* by being plotted to a logarithmic scale, which converts it into a straight line.

Take a point K for which both $T = DK$ and $t = Dk$ are known; substituting these values in equation (5), determine $A = DN$. Do this for a sufficient number of points on the part OM of the heating curve, so as to be able to draw the line PQ of the ultimate temperature rise with sufficient accuracy. To find points on the heating curve beyond M the same equation (5) is used; the coefficient A is now known, and T can be calculated for any value of t.

It is interesting to note, that any point on the cooling curve may be selected for t_0, because only the ratio t to t_0 enters into calculations, and this ratio is constant for an exponential curve.

The above formulæ and deductions have a practical meaning only when the temperatures of different parts within the transformer are fairly uniform, which is the case in a transformer designed along rational lines. Otherwise, the temperature T in the heating curve has no definite meaning; T for some parts of the transformer may be far beyond the safe limit, though the average T may not be excessive. In oil-filled transformers differences of temperature in different parts are never very great, and it is sufficient for the above purpose, to take the temperature of oil as an indication of the average temperature of the transformer.

404. EXPERIMENT 19-F. — Heat Run on Transformers. — The experiment should be performed on two identical transformers, as in Fig. 322. An auto-transformer with regulating taps may be used for the "booster" transformer C. Insert thermometers into the oil, and upon the iron and windings; adjust the currents and the voltages to correspond to the desired load.

When performing this test the student should bear in mind that the secondary winding is subjected to a high voltage the instant a low voltage is applied to the primary winding. *Therefore the student must be careful not to put on any voltage whatsoever, until explicitly told to do so by the instructor, and should not touch any windings while the transformer is under test.*

The temperature rise of the windings must be calculated from the increase of resistance, the measurements being performed at the beginning, at the end of the run, and a few times during the test. This is necessary because the thermometers show the temperature of the external part of the insulation only, while the temperature inside of the coil may be appreciably higher (Fig. 321). The resistance is measured by the drop-of-potential method, using direct current (§ 7). In measuring the resistance during the heat test the readings must be made as rapidly as possible, in order to prevent the transformer from getting cooled off. The best scheme is to have a double-throw switch arranged so that the transformer can be instantly switched from alternating current over to direct current and back. Be sure to remove the voltmeter leads when the current is on and off, because a high induced e.m.f. during the variable state of the current may burn out the voltmeter.

(*a*) Measure the cold resistances, and then begin the temperature run proper. Keep the load constant, read thermometers every five minutes, and from time to time measure resistances, as a check. The time usually allotted for a laboratory experiment is not sufficient for obtaining the final temperature of the transformer. Unless the circumstances allow of taking a continuous run of from 8 to 10 hours, the student will have to extrapolate the heating curve, as explained in § 402. It is well to run the test on an overload, so as to get a sufficient rise of temperature within two or three hours.

(*b*) After the heat test take a core-loss curve within a wide range of voltages. This curve may be of use, should the results of the temperature run show that the temperature rise is too large or too small, for then it is possible to tell by how much to change the voltage (or the current) in order to bring the temperature rise to the desired value. Suppose, for instance, the final temperature rise of a transformer to be 70 degrees C. above the surrounding air. This is more than the safe limit, 50 degrees C., allowed for ordinary transformers. Now the rating of the transformer for continuous load can be figured out as follows: Suppose that the iron loss during the test was 550 watts and the copper loss 850 watts. Thus, a total loss of 1400 watts causes a temperature rise of 70 degrees; the loss that corresponds to a temperature rise of 50 degrees is therefore equal to

$$1400 \times \frac{50}{70} = 1000 \text{ watts},$$

which allows $1000 - 550 = 450$ watts for copper loss. Consequently, the current during the test was

$$\sqrt{\frac{850}{450}} = 1.37$$

times higher than should be allowed in continuous operation.

It may occur that a transformer is run at such a high voltage that the temperature limit is exceeded with iron loss alone, at no load. This will show that the transformer has been designed for a lower voltage, or a higher frequency; for the iron loss depends on the flux, and the flux decreases directly with the voltage and inversely with the frequency.

Report. Plot the core-loss curve to volts as abscissæ, and the heating curve to time as abscissæ. Calculate the ultimate temperature rise, as explained in § 402, or according to the method stated in § 403. It may occur that the load or the voltage selected for the test was too high for the transformer, so that the ultimate temperature rise A is beyond that which the insulation of the transformer can safely stand. If this is the case, the student is expected to find out how many hours the transformer can stand this load until the highest allowable temperature rise of 50 degrees C. is reached. Also, what is the maximum load and voltage which the transformer can carry indefinitely; in other words, what should be the true rating of the transformer.

405. Equations of Cooling and Heating Curves.—The mathematical expressions for heating and cooling curves (Fig. 323) will now be deduced. We shall make the assumption that the heat exchange between a warm body and the surrounding air is proportional to the excess of temperature of this body above that of the air. This assumption is confirmed by experience, at least within the limits of temperatures at which electrical machinery is operating.

(*a*) *Cooling Curve.* Let the excess of temperature of a cooling body over that of the air be t degrees Centigrade, and let K be the number of watts radiated in one second, when the difference in temperature between the body and the surrounding air is one degree Centigrade. Then the number of watts radiated during an infinitesimal element of time dx, is

$$Kt.\ dx.$$

On the other hand, if C is the number of watts it is necessary to communicate to the body in order to raise its temperature by one degree Centigrade, and dt is the decrease in temperature during the time dx, the body must have lost Cdt watts, radiated through its surface. Thus, we have

$$Kt\,.\,dx + C\,.\,dt = 0.$$

The integral of this equation is

$$\frac{K}{C}\,x = log\,\frac{t_0}{t},$$

where t_0 is the initial difference of temperature, i.e. when $x = 0$. The integration can be easily checked by differentiating the result. Denoting the ratio K to C by B, and eliminating the logarithms, we get the following equation of the cooling curve:

$$t = t_0 \,.\, e^{-Bx} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (6)$$

where e is the basis of Naperian logarithms; B is a coefficient which characterizes the thermal properties of the transformer.

(*b*) *Heating Curve.* In deducing the equation of the heating curve, we assume that W watts are being continually communicated to the body from an external source, and are converted into heat. In the specific case under consideration, W represents the iron loss and the copper losses in the transformer. The body loses by radiation, as before, $K\,T.\,dx$ watts and its increase in temperature dT is due to the difference $Wdx - KT.\,dx$ watts, or

$$W.\,dx - KT.\,dx = C.\,dT.$$

The integral of this equation is

$$T = A\ (1 - e^{-Bx}) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (7)$$

which can be easily checked by differentiating it. In this formula B has the same value as in (6), and $A = W \div C$ is the ultimate temperature rise after an infinite time (Fig. 323). Substituting the value of e^{-Bx} from (6) into (7) the equation of the heating curve is reduced to the following simple form:

$$T = A\left(1 - \frac{t}{t_0}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (5)$$

This is the formula used in § 403 above for extrapolating the heating curve.

406. Theory of Extrapolation of Heating Curves.—The method of extrapolation, given in § 402 without proof, may now be deduced from equations (6) and (7).

Writing (7) for the points corresponding to (1), we have:

$$\left.\begin{aligned} T_1 &= A\ (1 - e^{-Bx_1}\ ) \\ T_2 &= A\ (1 - e^{-2Bx_1}) \\ T_3 &= A\ (1 - e^{-3Bx_1}) \\ T_4 &= A\ (1 - e^{-4Bx_1}) \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (8)$$

To eliminate A, form the expression

$$\frac{T_4 - T_3}{T_2 - T_1} = \frac{e^{-3Bx_1} - e^{-4Bx_1}}{e^{-Bx_1} - e^{-2Bx_1}} = \frac{e^{-3Bx_1}\,(1 - e^{-Bx_1})}{e^{-Bx_1}\,(1 - e^{-Bx_1})} = e^{-2Bx_1},$$

from which

$$e^{-Bx_1} = \sqrt{\frac{T_4 - T_3}{T_2 - T_1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (9)$$

this expression is denoted for brevity by z in equation (2).

Substituting (9) in (8) the equations (3) are obtained, and the method is thus proved. Equation (5) is obtained by substituting e^{-Bx} from (6) in (7).

(c) *Voltage Regulation.*

407. The term "regulation" generally refers in electrical engineering to variations in voltage at the terminals of an apparatus, under different load conditions. More specifically, regulation is defined, in this country, as per cent rise in voltage, when the full rated load is suddenly thrown off the machine, all other conditions remaining the same. Thus, if a transformer gives its rated secondary pressure of 110 volts at full non-inductive load, and the voltage rises to 113 volts when the load is thrown off (the primary voltage being kept constant), the regulation of the transformer by definition is:

$$\frac{113 - 110}{110} = 2.73 \text{ per cent.}$$

The question of regulation is of great practical importance, particularly in case of lighting transformers, where a difference of even two per cent between the no-load and the full-load voltage is quite appreciable in the quality and quantity of light given by incandescent lamps. In transformers used on power circuits this is not so important, except where incandescent lamps are connected to the same circuit. In this case the requirements for regulation must be very strict, because the inherent regulation of a transformer on inductive loads is worse than on non-inductive load.

In many cases it is difficult to measure voltage drop and regulation under actual load conditions. Take, for instance, the case of a 2000-kw., 60,000 to 10,000-volt transformer: It is difficult to find a rheostat for loading such a transformer, and also a voltmeter that would measure a few per cent drop at such voltages with sufficient accuracy.

Therefore a method has been devised which permits the *pre-determination* or calculation of the regulation of a transformer from a simple *short-circuit test.* Such a test requires power to but a few per cent of the rated capacity of the apparatus, and may be performed with a comparatively low voltage. The method is analogous to that described in § 276 for the predetermination of regulation of transmission lines.

408. Predetermination of Regulation of a Transformer. — Voltage drop in a transformer is caused chiefly by the inductance of the windings; also to some extent by their ohmic resistance. The inductance is caused by leakage fluxes, which surround one of the windings, without being interlinked with the other winding.

The voltage OA_2 at the secondary terminals is smaller than the induced secondary e.m.f. OE_2 by the amount E_2A_2 representing the secondary drop. This drop again consists of two components, one parallel, the other perpendicular to the vector of the current i_2.

It will be noted, that the vector of the primary drop, E_1A_1, is added to the primary e.m.f., while the secondary drop E_2A_2 is subtracted from the secondary e.m.f. This is because the voltage at the primary terminals must overcome not only the counter-e.m.f. of the transformer, but the drop as well. On the contrary, there is no counter-e.m.f. in the secondary circuit, and the voltage available for the load is equal to that induced in the transformer, less the amount lost in its secondary impedance.

For practical purposes it is important to know the relation between the terminal voltages OA_1 and OA_2, and not between the induced voltages OE_1 and OE_2, which cannot be measured. To eliminate the latter, multiply all the sides of the polygon $OE_2B_2A_2$ by the ratio $n_1 \div n_2$ and substitute $(i_1n_1 \div n_2)$ for i_2; then turn the polygon over into such a position, that OE_2 coincides with OE_1. The new position of the polygon is $OE_1B'_2A'_2$. Completing the figure OA'_2KA_1 the relation may be obtained between the primary terminal voltage OA_1 and the secondary terminal voltage OA'_2 (reduced to the primary circuit). The difference between the two, or the total *equivalent* drop in the transformer, is represented by the vector A'_2A_1. This drop may again be imagined to consist of an ohmic component A'_2K and a reactive component KA_1. It will be easily seen from the figure that

$$\text{equivalent ohmic resistance } R = r_1 + r_2\left(\frac{n_1}{n_2}\right)^2;$$

$$\text{equivalent reactance } X = x_1 + x_2\left(\frac{n_1}{n_2}\right)^2.$$

The same expressions are developed in § 409, except that the equivalent resistance is there reduced to the secondary circuit. The above may also be considered as another proof of the proposition that resistances and inductances may be considered as transferred from one circuit into the other, when multiplied by the square of the ratio of the number of turns.

413. Kapp's Diagram. — It would be a tedious process to construct the diagram Fig. 325 separately for different values of load and power factor. With a certain value of current, the triangle A'_2KA_1 is independent of the power factor, the only variable being the angle ϕ_2 between the terminal voltage and the current. Mr. G. Kapp suggested a modification of the above diagram (Fig. 326), convenient for predetermining regulation at a given current with varying power factor of the load. The triangle *OKC* in Fig. 326 is the same as the triangle A'_2KA_1 in Fig. 325. In either case it is constructed from the result of a short-circuit test (Fig. 246). OE_r is the direction of the vector of the current, primary and secondary. The primary line voltage e_1 being constant, a semicircle is described from *C* as a center, with a radius equal to e_1. To find the secondary voltage e_2 (reduced to the primary circuit) draw the vector *OE* at an angle ϕ_2 corresponding to the power factor of the load. The length *OE* gives the value of the secondary voltage corresponding to this power factor.

With purely inductive load, the secondary voltage is a minimum, and is represented by the vector OE_x perpendicular to *i*. With a non-inductive load, the secondary terminal voltage is equal to *OE* in phase with the current *OI*. In case of a pure capacity load, the secondary voltage is equal to OE_c; it is larger than the applied voltage e_1 (reduced to the same number of turns). This is due to a partial resonance between the capacity of the load and the inductance of the transformer itself (see § 134).

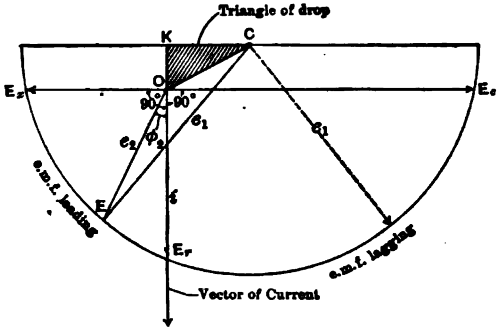

FIG. 326. Kapp's diagram of voltage drop and regulation.

If the secondary, instead of the primary voltage, is constant, a semi-circle must be drawn from *O* as a center; the vectors from *C* to the intersection with this semicircle then give the values of the primary voltage necessary for maintaining the required secondary voltage, with a varying power factor.

For other values of current, the triangle *OKC* must be changed correspondingly; the rest of the diagram remains the same.

The difficulty in applying Kapp's diagram to actual transformers is that the triangle *OKC* is comparatively small, and therefore the difference between *OE* and *CE* cannot be measured with a sufficient accuracy. For this reason some prefer to express analytically the relations shown in Fig. 325 and to figure out the required voltage from a formula, similar to that given in § 268. Another solution is to use an ingenious calculating device invented by Mr. F. G. Baum on which per cent voltages are read off directly for given percentages of ohmic and inductive drop (§ 277) and at any current and power factor.

CHAPTER XX.

ELECTRICAL RELATIONS IN POLYPHASE SYSTEMS.

414. A SIMPLE alternating current is entirely satisfactory for lighting, but up to this time the inventors have been unable to construct a satisfactory single-phase motor for ordinary commercial work. The single-phase commutator-motor is well adapted for railway work only, and the single-phase induction motor, although used to some extent, is not a satisfactory machine, except in small sizes. But, if the actions of *two or more alternating currents differing in phase are combined in one motor*, the motor is much improved, because *at no time is the power equal to zero* in all component circuits, or phases, at once.

Such combinations of alternating currents, differing in phase and mutually interlinked into one system, are called *polyphase systems*; their origin was a demand for a good alternating-current motor. Two types of motors are used on polyphase circuits: induction motors, based on the familiar principle of the revolving magnetic field; and synchronous motors, which are inverted polyphase alternators.

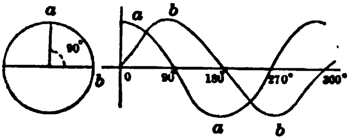

FIG. 327. Graphical representation of a two-phase system.

415. The Two-Phase System and the Three-Phase System. — Of all the possible combinations of alternating currents only the two simplest ones came into general use: the *two-phase system*, consisting of two currents with a phase displacement of 90 degrees, or quarter of a period (Fig. 327); and the *three-phase* system, consisting of three currents symmetrically displaced in phase by 120 degrees, or one-third of a period (Fig. 328). In these figures the sine-waves represent the instantaneous values of currents in separate phases, plotted to time as abscissæ. The time is expressed in degrees, one complete period of alternating current being equivalent to 360 electrical degrees. Instead of actually plotting the sine-waves the same relations are represented

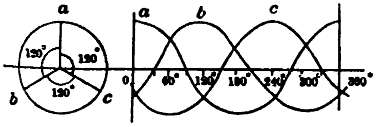

FIG. 328. Graphical representation of a three-phase system.

more simply by the radii or vectors drawn 120 or 90 degrees apart (see Figs. 333 and 338).

The understanding of electrical relations is easier in the two-phase system than in the three-phase system, because the former may be considered as a combination of two simple alternating-current circuits; volts, amperes and watts can be measured in each phase separately. For this reason the two-phase system is considered first.*

TWO-PHASE SYSTEMS.

416. The principal combinations of two single-phase circuits into a two-phase system are:

Two independent phases, Fig. 329;
Two-phase, three-wire system, Fig. 330;
Quarter-phase, star connection, Fig. 334;
Quarter-phase, mesh connection, Fig. 335.

In so far as the use of the line itself is concerned, all the above systems are electrically equivalent; the only difference is in the inside connections of generators and receiving devices.

The two-phase four-wire system is the one most commonly used, especially when, in addition to motors, a considerable amount of lighting is to be supplied; for the latter, each phase is used separately, the lamps being divided into two groups. The three-wire system gives a greater economy in copper, and reduces the number of insulators on the poles, but it has the disadvantage of being unsymmetrical. The quarter-phase star connection is used when it is desired to have a neutral point, for instance, in case of a three-wire, direct-current line fed from a two-phase rotary converter (Fig. 278). The quarter-phase mesh-connected system is obtained when a direct-current armature is tapped off at four equidistant points for producing two-phase currents. Such is the case in two-phase rotary converters and some revolving-armature alternators.

The electrical relations in all these varieties of the two-phase system will now be taken up more in detail.

417. Two Independent Phases.—This system is shown in Fig. 329; one-half of the total power is transmitted through each phase. The phases are entirely independent of each other, their only relation being that a phase difference of 90 degrees is maintained between the two.

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* A small experimental alternator, convenient for studying relations in polyphase systems, is made by the General Electric Company, especially for colleges. It has six independent armature windings with twelve leads brought out. By suitably connecting these leads, single-phase, two-phase, three-phase and six-phase currents may be produced, and voltages varied within wide limits.

by a suitable disposition of the armature windings in the alternator (Fig. 421). This phase relation comes into play in the windings of the motors supplied from the two-phase line, where the two currents produce a revolving magnetic field.

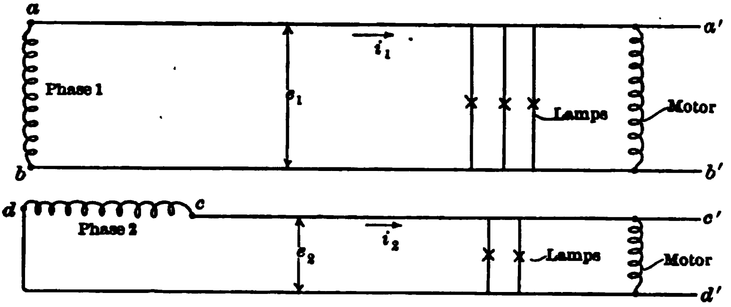

FIG. 329. Two-phase, four-wire system.

The relations between the current, the voltage and the power in each phase are exactly the same as in an ordinary single-phase line.

418. EXPERIMENT 20-A. — Electrical Relations in a Two-Phase Four-Wire System. — Wire up and load a two-phase alternator, as per Fig. 329. Measure power in each phase independently, and determine the total power delivered by the machine, with balanced and unbalanced load. Connect the generator to a two-phase induction motor in order to see the production of a revolving field. Observe the effect produced on the motor by reversing one of the phases, or both phases. Report the data and the phenomena observed.

419. Two-Phase Three-Wire System. — The connections are shown in Fig. 330; this system is obtained by connecting two wires *bb'*

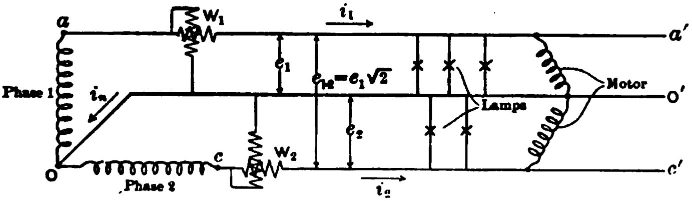

FIG. 330. Two-phase three-wire system.

and *dd'* (Fig. 329), belonging to different phases, into one, *OO'*, which is called the common return wire.

(*a*) The electrical relations with *balanced non-inductive load* are shown in Fig. 331. The currents i_1 and i_2 are represented by two vectors displaced by 90 degrees. The current in the return wire is equal to the sum of the two, and is therefore represented by the diagonal i_n. The load being non-inductive, the line voltages e_1 and e_2 are in phase with the corresponding currents. The voltage e_{1-2} between the two outside wires, being a geometrical difference between the two-phase voltages, is represented by a vector connecting the extremities of the vectors e_1 and e_2. It will be seen from the diagram that $i_n = i_1 . \sqrt{2} = i_2 . \sqrt{2}$, and $e_{1-2} = e_1\sqrt{2} = e_2\sqrt{2}$.

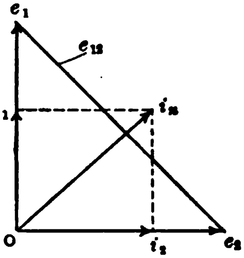

FIG. 331. Current and voltage relations in a two-phase system at a balanced load.

The student must clearly understand that the current i_n is a geometrical *sum* of the two-phase currents, while the voltage e_{1-2} is a geometrical *difference* between the two-phase voltages. The reason for this may be seen by considering instantaneous values. If 5 amperes are flowing at a certain instant through the wire aa' (Fig. 330), and 3 amperes in the wire cc', in the same direction, 8 amperes must flow back into the generator through the common return wire OO'. It is different in the case of voltages; suppose that the common return wire is grounded and the potential of the wire aa' at a certain moment is 100 volts above the ground, while the potential of the wire cc' is only 40 volts above the ground. It is evident, then, that the difference of potential between the two wires is 100−40=60 volts. Thus we must add currents and subtract voltages. The relations which are true algebraically for *instantaneous* values are true geometrically for the vectors of *effective values;* this explains the construction in Fig. 331.

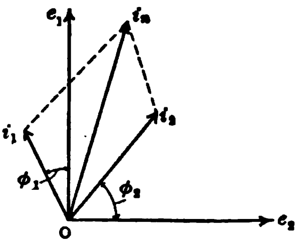

FIG. 332. Current and voltage relations in a two-phase system at an unbalanced load.

(*b*) When the *load is inductive,* the currents i_1 and i_2 (Fig. 332) are no longer in phase with the corresponding voltages, but are lagging behind by certain angles ϕ_1 and ϕ_2. The current i_n in the common return wire is a geometrical sum of the two, as before. The load may be different in the two phases; this is shown in the diagram by the vectors i_1 and i_2, being of different lengths and lagging by different angles (unbalanced load).

(*c*) *The resistance of the common return wire* is not always negligible, as is presupposed in the above diagrams. The diagram, shown in

Fig. 333, explains the way in which this resistance is taken into account. Assuming the load to be non-inductive and balanced, the vector of the voltage drop $e_n = i_n r_n$ in the return wire is in phase with the current i_n and at equal angles with both phase voltages e_1 and e_2. Subtracting this drop from the phase voltages, as is shown by the two vectors $- e_n$, we obtain the pressures available at the receiving end of the line. With non-inductive load, the currents i_1 and i_2 are in phase with these voltages, and the sum of the two currents is the current i_n.

It will thus be seen, that the effect of the resistance of the common return wire is to reduce the phase voltages and to increase the phase angle between them. If the voltage drop in the phase wires *aa′* and *cc′* need also to be taken into account, the vectors of this drop are again subtracted geometrically from the generator voltages e_1 and e_2.

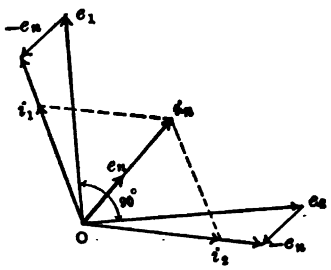

FIG. 333. Current and voltage relations in a two-phase system, taking into account the resistance of the return wire.

(*d*) *Electric power* is measured in the three-wire two-phase system by two wattmeters, as shown in Fig. 330. The sum of two readings gives the total power delivered by the generator. This is true for balanced as well as for unbalanced or inductive loads, for, even here, each phase may be considered independently.

420. EXPERIMENT 20-B. — Electrical Relations in Two-Phase, Three-Wire System.

— The purpose of the experiment is to make clear the electrical relations explained in the preceding article. Connect up the alternator and the load as per Fig. 330 and provide an ammeter, a voltmeter and a wattmeter which, by means of a suitable polyphase board (§ 49), may be connected into any of the wires. Perform the measurements in the following order:

(*a*) Balanced non-inductive load, relations as per Fig. 331.

(*b*) Unbalanced non-inductive load; same figure, but the vectors i_1 and i_2 are of different lengths.

(*c*) Inductive load, balanced and unbalanced, Fig. 332.

(*d*) Line wires and return wire of considerable resistance; relations as per Fig. 333.

Measure carefully volts, amperes and watts, so as to be able to construct the corresponding diagrams.

Report. Show in how far the theoretical relations, indicated in Figs. 331 to 333, check with the results of actual measurements.

421. Quarter-Phase System, Star Connection. — This system (Fig. 334) is obtained from the four-wire system shown in Fig. 329 by connecting together the middle points of the two phase-windings. Three kinds of voltages must be distinguished in the quarter-phase system:

(1) Phase voltages e_1 and e_2; these are the same as in the four-wire system, or in a three-wire system.

(2) Star voltages e_0, between the neutral point *O* and the points *a*, *b*, *c*, and *d*. It is easily seen that the star voltages are equal to one half the phase voltages.

(3) Mesh, or interlinked voltages between the wires belonging to different phases, such, for instance, as between *a* and *c*, or *a* and *d*. The voltage between *a* and *d* is the geometrical difference between the voltages *Oa* and *Od* (see § 419). Constructing a diagram similar to that shown in Fig. 331 it may be easily proven that the mesh voltage $e_{1-2} = e_0\sqrt{2} = e_1/\sqrt{2}$. The current and power relations are the same as with two independent phases.

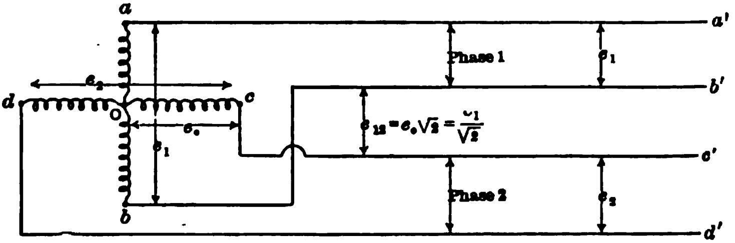

FIG. 334. Quarter-phase system, star connected.

422. EXPERIMENT 20-C. — Electrical Relations in Quarter-Phase Star-Connected System. — Either the generator, or the load, or both, should be connected in star, as per Fig. 334. If the middle points of the generator windings are not accessible, connect only the load in star. Balance the load and measure the three kinds of voltages mentioned in the preceding article. Then unbalance the load so as to make all the four star voltages different, and again measure all the currents, voltages and watts in order to be able to determine the phase angles. Perform similar measurements with an inductive load.

Report. Show how close the actual ratios between the voltages at balanced load, are to the theoretical ratio $\sqrt{2}$. Construct the diagrams showing the relations with unbalanced and inductive load. The vectors of the four mesh-voltages form a square (Fig. 336) of which the phase voltages are the diagonals. The star voltages are represented by the vectors drawn from a point inside of the square to its vertices. At balanced load this point coincides with the intersection of the diagonals; at unbalanced load the two points are different.

423. Quarter-Phase System, Mesh Connection. — The connections are shown in Fig. 335. Two kinds of voltages are distinguished:

(1) Phase voltages, e_1 and e_2.

(2) Mesh or interlinked voltages between the wires belonging to different phases, such as between *a* and *c*, or *a* and *d*. The phase voltages are $\sqrt{2}$ times larger than the mesh voltages, and are displaced from them in phase by 45 degrees. The proof of this is similar to that given for the three-wire two-phase system (Fig. 331).

The currents i_{1-2} flowing through the mesh are different from the line currents i_1 and i_2. As the power delivered to the line must be equal to that generated in the mesh, we have the relation

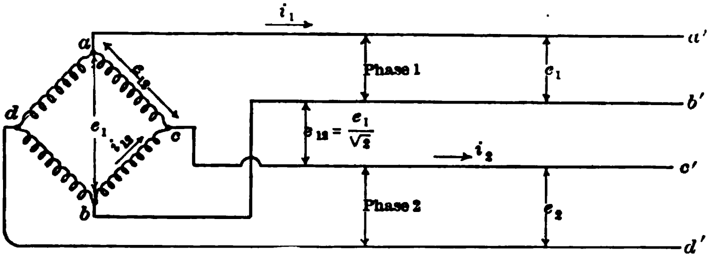

FIG. 335. Quarter-phase system, mesh connected.

$$2i_1e_1 = 4i_{1-2}\,e_{1-2}.$$

Substituting in this equation $e_1 = e_{1-2}\sqrt{2}$, we obtain that $i_1 = i_{1-2}\sqrt{2}$, or the currents in the mesh are $\sqrt{2}$ smaller than the line currents. The phase angle between i_1 and i_{1-2} is 45 degrees, as only under these conditions does the difference of two equal vectors give a $\sqrt{2}$ times larger vector (See Fig. 331).

These relations are shown graphically in Fig. 336. The currents in the mesh are represented by the sides of the square *acbd*, the line currents by the diagonals *ab* and *cd*. With unbalanced load these diagonals are no longer equal or perpendicular to each other, and the square is distorted into a parallelogram $a_1c_1b_1d_1$. The voltages are represented in their magnitude and relative phase position by the square *ACBD* and its diagonals.

424. EXPERIMENT 20-D. — Electrical Relations in Quarter-Phase, Mesh-Connected System. — Connect an alternator in mesh, or, if this is not possible, have only the load mesh-connected. Balance up the currents in the four branches and read all the currents, voltages, and watts. . Unbalance the load and take again all the necessary readings for constructing the diagram shown in Fig. 336. Watts should be read as a check on the phase relations. With unbalanced load, line currents are no longer in phase with corresponding voltages, even though the load be non-inductive. Repeat the same experiment with inductive load, and compare watts measured on the line with those actually consumed in the four branches of the mesh.

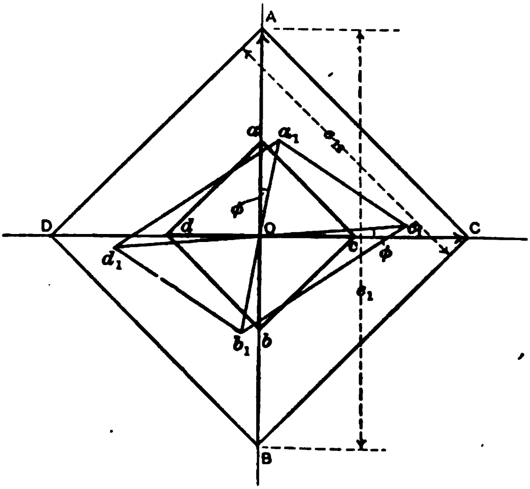

FIG. 336. Current and voltage relations in a quarter-phase system at a balanced and at an unbalanced load.

Report. Give the actual ratios of currents and voltages as found from the experiment. Select a few sets of readings at balanced and unbalanced load, and construct diagrams, as per Fig. 336, indicating both the currents and the voltages. Check the angles found by graphical construction with those calculated from the wattmeter readings.

THREE-PHASE SYSTEMS.

425. A representation of the three-phase system is shown in Fig. 328; three alternating currents or voltages *a*, *b*, and *c* relatively displaced by one third of a period, are there shown by sine waves and vectors. As in the case of the two-phase system (§ 416), various connections of generator windings are possible, all giving the same three-phase system on the line. The load may also be connected in two or three different ways.

Two schemes of three-phase connections are most commonly used: the star or *Y*-connection, shown in Fig. 337, and the Δ (delta), or triangle connection, Fig. 345. Both schemes are used in practice, and there are cases in which each kind of connections is preferable. The pros and cons of this question would hardly interest the student at this stage of his knowledge, and are therefore omitted. He should, however, become thoroughly familiar with current, voltage and power relations in both plans of connections.

In some special cases the so-called V-connection (Fig. 348) is used, also the T-connection (Fig. 349). The electrical relations in all these connections are explained below.

426. Y-Connection — Current Relations. — Let OA, OB, and OC (Fig. 337) be three resistances connected in Y and constituting the load of a three-phase system. Let $O'A'$, $O'B'$, and $O'C'$ be the corresponding armature windings, in which the three-phase currents are generated. The currents are relatively displaced by 120 degrees, as shown in Fig. 328. It may seem at first that a fourth wire, such as ONO', is necessary as a common-return conductor for the phase currents. Experience and theory show, however, that this conductor may in most cases be dispensed with, since the sum of the three phase-currents is identically zero at all moments, at least when the load is balanced, and the currents vary exactly according to the sine law.

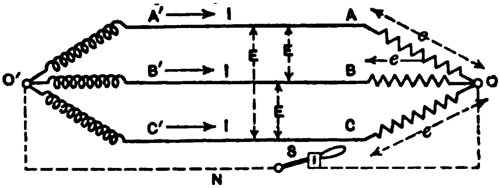

FIG. 337. Three-phase system, Y-connection.

The easiest way to see this is to add point by point the ordinates of the three curves shown in Fig. 328. It will be found that their sum is equal to zero for any instant of time. This is a direct result of a mathematical proposition, that the sum of sines of any three angles differing by 120 degrees is equal to zero. That is to say,

$$\mathrm{Sin}\ x + \mathrm{Sin}\ (x + \tfrac{1}{3}\,.\,2\pi) + \mathrm{Sin}\ (x - \tfrac{1}{3}\,.\,2\pi) = 0.$$

To prove this formula, expand the terms; the result is:

$$\begin{aligned}&\mathrm{Sin}\ x + \mathrm{Sin}\ x\,.\,\mathrm{Cos}\ \tfrac{1}{3}\,.\,2\pi + \mathrm{Cos}\ x\,.\,\mathrm{Sin}\ \tfrac{1}{3}\,.\,2\pi + \\ &\qquad + \mathrm{Sin}\ x\,.\,\mathrm{Cos}\ \tfrac{1}{3}\,.\,2\pi - \mathrm{Cos}\ x\,.\,\mathrm{Sin}\ \tfrac{1}{3}\,.\,2\pi;\end{aligned}$$

or, after reduction

$$\mathrm{Sin}\ x + 2\ \mathrm{Sin}\ x\,.\,\mathrm{Cos}\ \tfrac{1}{3}\,.\,2\pi.$$

But

$$\mathrm{Cos}\ \tfrac{1}{3}\,.\,2\pi = \mathrm{Cos}\ 120° = -\frac{1}{2},$$

so that we finally get

$$\mathrm{Sin}\ x - \mathrm{Sin}\ x = 0,$$

which proves the above formula.

The vectors of the three currents I_1, I_2, I_3, are shown in Fig. 338, the current in the return wire is a geometrical sum of the three. This vector sum is evidently = 0, which is a graphical proof of the above proposition.

427. Y-Connection — Voltage Relations. — Two kinds of voltages are distinguished with the Y-connection; phase voltages e, such as between O and A, or O' and A', etc.; and line voltages E, such as AB, AC, and BC. Theory and experiment show that

$$E = e\sqrt{3}.$$

This is because the voltage between, say A and B, is a difference of the voltages OA and OB, or

$$= e\ \text{Sin}\ x - e\ \text{Sin}\ (x + \tfrac{1}{3}.\ 2\pi) = e\sqrt{3}.\ \ \text{Sin}\ (x - 30\ \text{degrees}),$$

so that, apart from the phase displacement of 30 degrees, the amplitude of the voltage E between A and B is $\sqrt{3}$ times greater than that of e between A and the neutral point O.

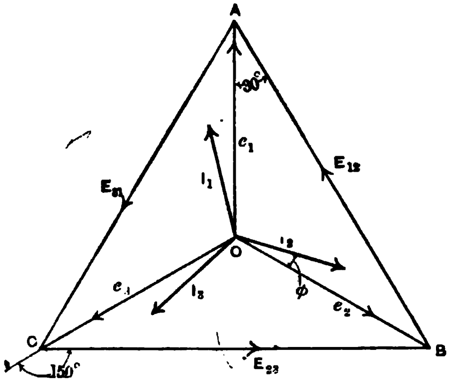

Fig. 338. Current and voltage relations in a Y-connected, three-phase system at a balanced inductive load.

The same voltage relations are shown graphically in Fig. 338. The line voltages E are represented by the sides of the triangle ABC, the phase voltages e by the rays OA, OB, OC. The line voltages are relatively displaced by 120 degrees (note the arrow heads), as are the phase voltages. It will be seen directly from the diagram that the line voltages are $\sqrt{3}$ times larger than the phase voltages, and that the phase angle between the two is 30 degrees.

With balanced non-inductive load the vectors I_1, I_2, and I_3 of the line currents are in phase with the voltages OA, OB, and OC. With inductive load they are lagging behind by an angle ϕ, depending on the amount of inductance in the load.

The diagram shows that even with a non-inductive load the currents are not in phase with the voltages E across the line, but in phase with the voltages e between the corresponding lines and the neutral point.

428. Y-Connection — Unbalanced Load. — When the load becomes unbalanced, or currents do not vary exactly according to the sine wave, equalizing currents flow through the return wire ONO', if such is provided. Otherwise, voltages and currents in the three line wires are considerably distorted. In practice, these currents never attain any considerable magnitude, so as to necessitate an extra conductor: Al that is required is to have the points O and O' connected to the ground · the equalizing currents find their way through the ground.

The current in the return wire, or through the ground, is equal to the geometrical sum of the three line currents, as shown in Fig. 339 The load being unbalanced, the three vectors I_1, I_2, I_3 are shown o: different length and at different angles to the corresponding voltages $e_1, e_2 e_3$. To find the geometrical sum of the three currents, draw from the extremity of the vector I_1 the vector AB equal and parallel to I_2; from B draw the vector BC equal and parallel to I_3. The resultant vector OC, closing the polygon, represents the current I_n in the return wire, in its magnitude and phase position.

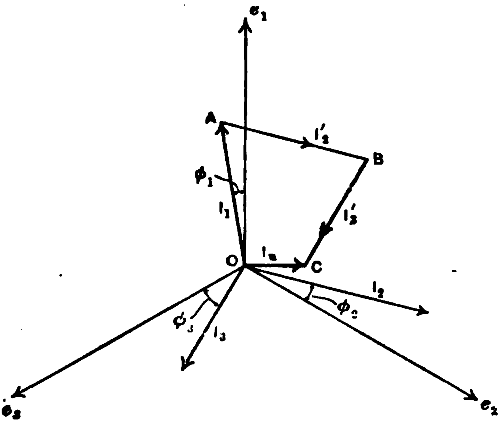

FIG. 339. Current and voltage relations in a Y-connected three-phase system with a low-resistance neutral connection.

429. Influence of the Resistance of the Neutral Wire. — In some exceptional cases the resistance of the neutral wire may be quite considerable, or a poor ground connection may exist in the system. So long as the load is balanced, this may not be noticeable but should the load become unbalanced, an appreciable difference o: potential arises between the two neutral points, O and O' (Fig. 337). This usually causes a further unbalancing of the voltages.

The electrical relations are indicated in Fig. 340. Let e_n be the voltage between the two neutral points O and O' (Fig. 337); the actual voltages at the terminals of the load are reduced (geometrically) by this amount. Subtracting e_n from the three generator voltages e_1, e_2 e_3, the three load voltages e_1', e_2', e_3' are obtained. If the load is non-inductive the three currents I_1, I_2, I_3 are in phase with these voltages. Adding the three currents together, as in Fig. 339, the current in the common return wire is obtained. meters he iary wattmeters wit so that the indication in Fig. 340 correspond to the case of an in the return circuit, for instance, when one of the neutrals is insulated from the ground. In this case the sum of the three currents must be $= 0$, because i_n is of necessity $= 0$.

430. Measuring Power in Y-Connection. — Total power W delivered to three resistances connected in Y (Fig. 337) is the sum of the watts developed in each phase, or $W = 3Ie$, where I and e designate the *effective* values. The neutral point is not always accessible, so that the voltages e cannot always be measured. It is customary therefore to figure out the power in terms of the line voltage E. With the relation $E = e\sqrt{3}$ (Fig. 338) we get:

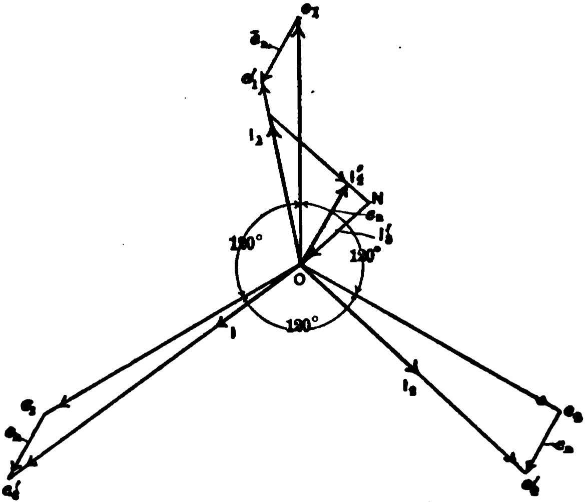

Fig. 340. Current and voltage relations in a Y-connected three-phase system, with insulated neutral points.

$$W = EI\sqrt{3}.$$

If there is a phase displacement between the currents and the voltages e, due to some inductance in the load, the above expression must be multiplied, as in single-phase circuits, by Cos ϕ, and we have

$$W = EI\sqrt{3}.\ \mathrm{Cos}\ \phi.$$

There are two ways of connecting wattmeters for measuring power in three-phase circuits: the three-wattmeter method (Fig. 341) and the two-wattmeter method (Fig. 342). The first method is self-evident: the power is measured in each phase separately and the results added together. This method gives correct results for total power under all circumstances, whether the load is balanced or not.

The two-wattmeter method is the one generally used in practice, since the neutral point O is not always accessible, also because with this method only two wattmeter readings are necessary, instead of three. This method is based on the fact that any of the three wires may be considered as a return wire for the currents flowing in two other wires. With the wattmeter connections, shown in Fig. 342, the wire B is taken as the return wire, so that the three-phase system is reduced to two single-phase systems A-B and C-B. The power in the first system is proportional to the current in A times the voltage between A and B. The power in the second is proportional to the current in C times the voltage between C and B. The wattmeters are connected so as to measure the power according to these expressions. The total power is equal to the sum of two simultaneous wattmeter readings, and the method is correct with balanced as well as with unbalanced loads.

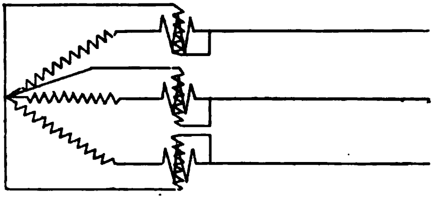

FIG. 341. The three-wattmeter method of measuring power.

431. Some Remarks on the Two-Wattmeter Method. — The following remarks may make the practical application of the method easier:

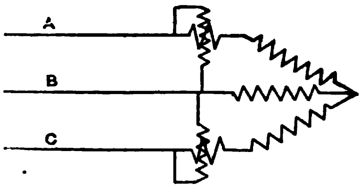

FIG. 342. The two-wattmeter method of measuring power.

(*a*) The two component readings are equal to each other only with balanced non-inductive loads; under all other conditions they are different (see § 432). When the power factor is below 50 per cent, one of the wattmeters begins to give negative deflections. If such is the case, either its current or voltage terminals must be reversed and a difference of the two readings taken, instead of their sum.

(*b*) It makes no difference which of the three wires is selected as the return wire: this affects only the component readings, but not the sum, which represents the total power in the system, and is always the same at a certain load.

(*c*) It is not always convenient to have two separate wattmeters and to read them simultaneously. Polyphase wattmeters are on the market for this purpose; they consist of two ordinary wattmeters with moving elements mounted on a common shaft, so that the indications are automatically added together; the pointer indicates on the scale the total power. The connections are made as in Fig. 342.

(*d*) Where a polyphase wattmeter is not available, it is still possible to use one wattmeter instead of two, by having double-throw switches arranged so that this wattmeter can be switched over from one phase to the other (§ 49). Of course, this is permissible only when the load is steady, while the wattmeter connections are being changed.

(*e*) If the load is perfectly balanced, it is sufficient to measure the power in one phase only, as shown in Fig. 343, and to multiply the results by three. It is assumed in Fig. 343 that the neutral point of the load is not accessible; therefore an *artificial* neutral point O is created by connecting three high resistances or inductances x in "Y" and connecting the potential winding of the wattmeter between one of the line wires and this neutral. If the neutral point of the generator or load is accessible, this complication is of course unnecessary.

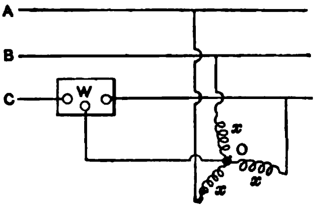

FIG. 343. Measuring power with an artificial neutral point (at balanced load).

(*f*) The two-wattmeter method should not be used when the neutrals are thoroughly grounded and the load is badly out of balance.

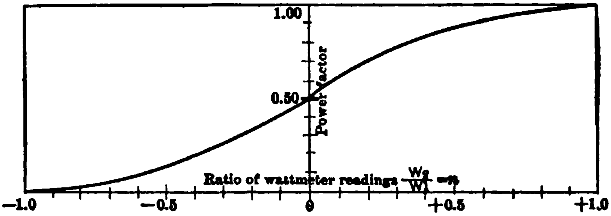

FIG. 344. The watt-ratio curve, for use with the two-wattmeter method.

In this case we have actually a four-wire system, so that three wattmeters are necessary for measuring total power, the fourth wire being a common return wire for the three others.

432. Watt-Ratio Curve. — With the two-wattmeter method, two component readings are not equal to each other, when the load is inductive, though it may be balanced. The ratio of the two readings depends on the power factor of the load only, and not on its magnitude. A curve may be plotted (Fig. 344) showing the power factor of the load as a function of the ratio of wattmeter readings. This curve is sometimes useful in estimating the power factor, or as a check on the power factor figured from the ammeter, voltmeter, and wattmeter readings.

The equation of this curve may be derived as follows: Let the two wattmeters be connected as in Fig. 342; the first wattmeter measures the product (see Fig. 338)

$$W_1 = E_{1-2} I_1 \operatorname{Cos} (30^\circ - \phi);$$

the second wattmeter registers

$$W_2 = - E_{2-3} I_3 \operatorname{Cos} (150^\circ - \phi).$$

The minus sign is used in order to make the last expression positive; Cos $(150^\circ - \phi)$ is in itself negative for small values of ϕ, while the wattmeter connections are such that with small ϕ (high power-factor) both readings are positive.

The second reading may also be represented identically by

$$W_2 = E_{2-3} I_3 \operatorname{Cos} (30^\circ + \phi).$$

The ratio of the smaller to the larger reading, with balanced load, is

$$n = \frac{W_2}{W_1} = \frac{\operatorname{Cos} (30^\circ + \phi)}{\operatorname{Cos} (30^\circ - \phi)},$$

or, denoting the power factor Cos ϕ by f, we get, after some transformations,

$$n = \frac{W_2}{W_1} = \frac{f\sqrt{3} - \sqrt{1 - f^2}}{f\sqrt{3} + \sqrt{1 - f^2}}.$$

This is the equation of the curve shown in Fig. 344.

If it is desired to calculate the power factor from a given ratio of wattmeter readings, the above equation becomes;

$$f = \operatorname{Cos} \phi = \frac{1}{\sqrt{1 + 3\left(\frac{1 - n}{1 + n}\right)^2}}$$

The reading W_1 is always positive, when ϕ is positive (current lagging); W_2 is positive as long as ϕ is below 60 degrees. When $\phi = 60$ degrees, $W_2 = 0$, and the ratio $n = 0$; the power factor of the load $f = \operatorname{Cos} \phi = 50$ per cent. This is the point at which the ratio curve crosses the axis of ordinates.

With non-inductive load $W_1 = W_2$; $f = 1.00$, and $n = 1$. With purely inductive load $\phi = 90$ degrees, $W_1 = - W_2$; $n = - 1$. Other points on the ratio curve are calculated in a similar way from the above given formula. It should be clearly understood that the ratio curve refers to *balanced* load only.

433. EXPERIMENT 20-E. — Electrical Relations in a Y-Connected System, with Non-Inductive Load. — The purpose of the exercise is to make clear the relations derived in §§ 426 to 431. The measurements may be performed by connecting three resistances in *Y* to a three-phase supply. It is better, however, to have them connected to a small experimental alternator, the neutral point of which is accessible, so as to be able to investigate the effect of the common return wire.

It would be rather cumbersome to have an ammeter, a voltmeter, and a wattmeter for each phase; more than that, the results would not be directly comparable unless the instruments were in exact calibration. It is preferable, therefore, to use a "polyphase board" (§ 49) so arranged that the instruments can be connected in all three phases in succession, without opening the main circuit. With this device, one ammeter, one voltmeter and one wattmeter are sufficient for all polyphase measurements.

The experiment should be performed in the following order:

(1) *Balanced load.* Have the alternator *Y*-connected and load it on three resistances also *Y*-connected, as in Fig. 337. Adjust the resistances so as to have the same current in the three phases, and verify that:

(*a*) The sum of the three currents is identically zero. To prove this, insert an ammeter into the neutral wire; if the phases are well balanced the ammeter shows practically no deflection. Moreover, opening and closing the switch *S* in the neutral wire does not affect the ammeter readings in the three phases.

(*b*) There is no difference of potential between *O* and *O′*, so that these points can be permanently grounded.

(*c*) Voltage *E* across the line is equal to $e\sqrt{3}$, where *e* is the phase voltage (Fig. 338).

(*d*) The power measured by the two-wattmeter method is the same as that determined by the three-wattmeter method; moreover, the result does not depend on which phase is selected as the return wire.

(2) *Unbalanced load.* Now unbalance the load; immediately a current will flow in the neutral, when the switch *S* is closed. Take readings with the switch *S* open and closed, and with the same load resistances. It will be seen that the neutral wire tends to keep the three phase voltages *OA*, *OB*, and *OC* equal. Without the neutral they are different, and there is also a difference of potential between *O* and *O′*. Introduce an appreciable resistance into the neutral wire, and observe its influence on the currents and voltages in the three phases.

In performing the experiment have before you the three diagrams (Figs. 338 to 340) and read all necessary data so as to be able to construct these diagrams. Have all the readings recorded systematically on a data sheet, such as shown below.

	Amperes.				Line Volts.			Phase Volts.			Watts (3-wattm. Meth).			Watts (2-wattm. Meth).		Total Watts.
	A	B	C	O	AB	BC	CA	AO	BO	CO	AO	BO	CO	A—AB	C—CB	
Inst. No.																
Const.																

Report. Give the actual ratios between the line voltages and the phase voltages, and compare them to the theoretical ratio $\sqrt{3}$. Plot curves showing the influence of unbalanced load in one phase on currents and voltages in the three phases, with and without the neutral wire. Select a set of readings at unbalanced load and construct a diagram as per Fig. 339. Construct a diagram for unbalanced load, without the neutral wire, as indicated in Fig. 340. Check the angles obtained by graphical construction with those calculated from wattmeter readings.

434. EXPERIMENT 20-F. — Electrical Relations in a Y-Connected System with Inductive Load. — This experiment supplements the preceding experiment. It is not necessary to repeat all the measurements made at non-inductive load, but merely those which offer new features with inductive load. Verify that two component readings in the two-wattmeter method are different even with a balanced load, when it is inductive (§ 432). Measure, as a check, the same power according to the three-wattmeter method (Fig. 341). Take readings at various values of power factor in order to verify the theoretical equation of the watt-ratio curve (Fig. 344). Observe the fact that when the power factor is equal to 50 per cent, one of the wattmeters reads zero; when the power factor is reduced still further it becomes necessary to reverse the terminals and to take the difference of the two readings, in order to have the same power as indicated by the three-wattmeter method.

Connect an unbalanced load, with and without the neutral wire, and take all the readings necessary for constructing the diagrams shown in Figs. 339 and 340.

Report. Give, in the form of curves, or of a table, the values of power measured by the two-wattmeter and the three-wattmeter methods. Plot the theoretical watt-ratio curve and the curve actually observed. Construct typical vector diagrams for balanced and unbalanced load.

435. Delta-Connection — Current and Voltage Relations. — The connections are shown diagrammatically in Fig. 345; Fig. 346 shows the same connections when each phase is loaded separately, for instance; when the load consists of incandescent lamps. Electrically the schemes shown in Figs. 345 and 346 are equivalent. Delta-connection differs from the Y-connection in that it has no neutral point, and the phase load is subjected to the full line pressure E. On the other hand, with Y-connection each phase consumes the full line current I, while in a delta-connected load the current i in each phase is only $I \div \sqrt{3}$ of the line current I. This can be proved in exactly the same way as it was proven before that $E = e\sqrt{3}$: namely, with the positive directions of the currents shown in Fig. 345 we have, that the current I in the line wire A is a difference of the currents i in AB and AC. This difference is

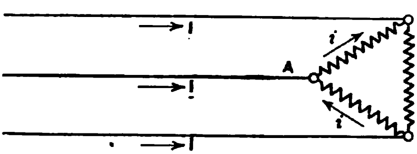

FIG. 345. Three-phase system, "delta"-connection.

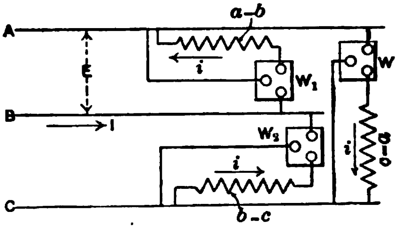

FIG. 346. A three-phase system loaded in "delta."

$$i \operatorname{Sin} x - i \operatorname{Sin} \left(x + \tfrac{1}{3} \,.\, 2\pi\right) = \sqrt{3} \,.\, i \operatorname{Sin} \left(x - 30^\circ\right)$$

so that, apart from the phase difference of 30 degrees, the current $I = i\sqrt{3}$.

The same relations are shown graphically in Fig. 347. The three line voltages E are represented by the sides of the triangle ABC. The currents in the load Bk, Bl, Bm are in phase with these voltages, provided the load is non-inductive. The line currents are represented by the sides of the triangle klm. They are in phase with the imaginary Y-voltages OA, OB and OC (compare Fig. 338). It will be seen from the figure that the line currents are $\sqrt{3}$ times larger than the load currents, and that the phase difference between the two is 30 degrees. The same result is obtained above graphically.

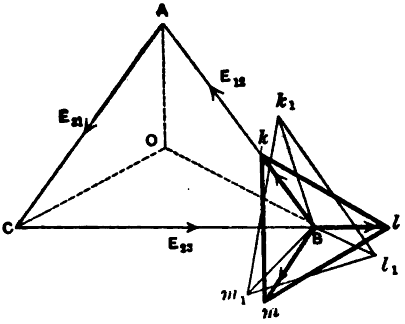

FIG. 347. Current and voltage relations in a "delta"-connected three-phase system at a balanced and at an unbalanced load.

When the load is unbalanced, or inductive, the triangle of currents klm is distorted into a triangle $k_1l_1m_1$. The load currents are no longer equal to each other, nor in phase with the corresponding voltages; the relation still holds true, that each I is a geometrical difference of the two adjacent i's.

436. Delta-Connection — Power Relations. — The three-wattmeter method in application to the delta-connection is shown in Fig. 346, the power being measured separately in each phase of the load. The *two-wattmeter* method (Fig. 342) is the one commonly used in practice. In fact, *when the two-wattmeter method is used it is not necessary to know in which way the load is connected, since one of the line wires can always be assumed to be a return wire for the other two line wires.* The single-wattmeter method, as shown in Fig. 343, can also be used equally well, provided, of course, that the load is balanced in all three phases, as in the case of three-phase motors.

The power delivered by an alternator is the same whether the load is connected in Y or in delta, provided that the line voltage and the line currents are the same in both cases. With the Y-connection, each phase consumes a current I at a pressure $e = E \div \sqrt{3}$, or a power equal to $IE \div \sqrt{3}$. With the delta-connection each phase consumes a current $I \div \sqrt{3}$ at a voltage E, or again the same power $IE \div \sqrt{3}$. The total power W in the three phases is also the same in both cases and is equal to $3 \times EI \div \sqrt{3}$, or $EI \,.\, \sqrt{3}$, as is proven above.

437. EXPERIMENT. 20-G — Electrical Relations in a Delta-Connected System, at Non-Inductive Load. — The purpose of the experiment is to make clear the numerical relations deduced in §§ 435 and 436. Connect three non-inductive resistances as per Fig. 346, and adjust them so as to have the same current in the three phases. The generator, used as the source of power, may be either Y- or delta-connected, since the relations are studied on the load only. As a matter of fact, no special alternator is required, and the load may be connected to the laboratory power supply. Use a data sheet, as in § 433. Make the following measurements:

(*a*) Measure the ratio between the currents in the line and in the delta, and see how close they check with the theoretical ratio $\sqrt{3}$.

(*b*) See if the power measured by the three-wattmeter method, as per Fig. 346, checks with that measured by the two-wattmeter method, as per Fig. 342. Make this comparison with balanced and with unbalanced load.

(*c*) Gradually unbalance the load in one of the phases, first increasing it and then reducing it below that in the other two phases. Note the effect on the currents, voltages, and phase relations in all three phases and in the line.

(*d*) Open the circuit in one of the phases of the load and compare the resulting voltages and currents to those with balanced load.

(*e*) Select an unbalanced load and take all the readings necessary for constructing the diagram shown in Fig. 347. Measure watts in order to be able to check the phase angles.

Report. Figure out the ratio between the line currents and the load currents; compare it to the theoretical ratio $\sqrt{3}$. Give the results of comparison of the two-wattmeter and the three-wattmeter methods. Plot curves showing the effect of underloading or overloading a phase; use amperes in the unbalanced phase as abscissæ. State your findings when one of the phases was opened. Construct a diagram, as in Fig. 347, for a balanced and for an unbalanced load.

438. EXPERIMENT 20-H. — Electrical Relations in Delta-Connection, at Inductive Load. — The experiment supplements the one preceding. It is not necessary to repeat all the measurements specified there, but only those which offer some difference with inductive load, in particular the runs (*b*), (*c*), and (*e*).

439. V-Connection. — The three-phase combination, shown in Fig. 348, is known as the V-connection. BA and BC represent the secondary windings of two transformers, or two generator windings. The load is connected as usual, either in *Y* or in delta. The *V*-connection is obtained from the delta-connection by omitting the generator or transformer winding between *A* and *C*, thus compelling the two other phases to take up the whole of the load. With three windings, each carries one third of the load; with *V*-connection each winding carries one half of the load.

If, for instance, the total load is 300 kw., three 100-kw. transformers are required with *Y*- or delta-connection, and two 173-kw. transformers with *V*-connection.* The total cost of the latter is less, and this is the advantage of the *V*-connection over the delta scheme. In large power houses transformers are sometimes connected in delta, and, should one of them be disabled, the two others temporarily carry the load in *V*, possibly being loaded above their rated capacity.

It should be clearly understood that only generator, transformer, and motor windings may be connected in *V*, and not ordinary ohmic or inductive resistances. This is because the load in the three phases remains constant only with the proper phase relation between the e.m.f.'s induced in *AB* and *BC*. This condition is fulfilled in generators and motors, the windings in the two phases being suitably spaced relative to each other. It is also fulfilled in transformers, because they transmit phase relations, practically without distorting them.

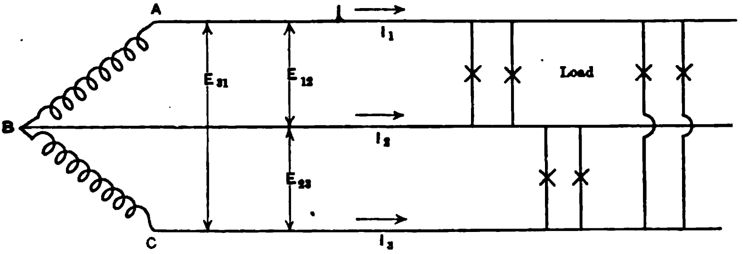

FIG. 348. Three-phase system, V-connection.

It may seem at first that the increase in current in the windings *AB* and *BC*, after the winding *AC* has been removed, should be $50 - 33\frac{1}{3} = 16\frac{2}{3}$ per cent. A little consideration will show, however, that the current increases $\sqrt{3}$ times, because the additional current, which the windings have to carry, is out of phase with their former currents,

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* Using two transformers instead of three brings in a phase displacement of $\cos 30° = \sqrt{3}/2$, so that the apparent output of each transformer becomes 173 kw., instead of 150 kw. See explanation above.

For assuming the line currents to be I in both cases, the currents in the transformer windings are also necessarily equal to I with V-connection, as may be clearly seen from Fig. 348. With delta-connection the currents in the transformer windings are equal to $I \div \sqrt{3}$ (see § 435).

The increase in the currents being greater than the increase in the power to be delivered by each transformer, the currents in V-connected transformers must be out of phase with the voltages, even at non-inductive load. One way to prove this is as follows: total power with balanced load is $EI\sqrt{3}$; hence, each of the windings has to supply $EI\sqrt{3} \div 2$. The voltage at the terminals of each winding is E, and the current flowing through it is I; therefore the phase displacement ϕ between the two must be such that $\text{Cos}\ \phi = \sqrt{3} \div 2$, in order that the power may be represented by the above expression. The angle ϕ corresponding to the above cosine is 30 degrees.

The same relations may be deduced from Fig. 347. When the transformer winding AC is taken out, its current Bm must be carried in series by the two other transformers. The transformer AB has to carry its own current Bk and the current Bm, the sum of the two being km. In the same way the current in the transformer BC is changed from Bl to ml. It will be seen, that the currents are thus increased $\sqrt{3}$ times, and a phase displacement of 30 degrees is produced.

It follows from the above that the power in a V-connected system cannot be determined by merely taking volts times amperes in the two transformers, even with a non-inductive load. The power should in all cases be measured by the two-wattmeter method, as in Fig. 342.

440. EXPERIMENT 20-1. — Electrical Relations in a V-Connected System. — The purpose of the experiment is to afford practice in connecting transformers in V, and to make clear the numerical relations explained in the preceding article. Unless a special V-connected generator is available (see footnote to § 415), the connections should be established through two transformers, whose primaries are connected in V to the source of supply, and the secondaries are connected in V to the load. It is well to have a third transformer in order to observe the difference between the delta and the V-connection.

(*a*) Apply a balanced non-inductive load and measure currents and voltages with the transformers in delta and in V. Use a wattmeter to prove the phase difference of 30 degrees spoken of in the preceding article. Unbalance the load, and take all the readings necessary for constructing a vector diagram of current and voltage relations.

(*b*) Unbalance the load in the phase *AB* or in the phase *BC* by a certain per cent and measure the resulting unbalancing in currents and in watts supplied by each transformer. Now unbalance the phase *AC* *by the same amount,* and again measure amperes and watts in both transformers. The practical significance of this experiment is this: In some cases two *V*-connected transformers supply a balanced load of three-phase induction motors, and it is desired to connect to the same supply some single-phase lighting. The question to decide is, whether the lighting should be connected to the phase *AC*, where the unbalancing would be taken up by both transformers, or to one of the other phases where it would affect one of the transformers more than the other.

(*c*) Apply a balanced inductive load and investigate the relation between the power factor of the load and the phase angle between the currents and the voltages in the transformers themselves. It will be found, that this angle, which was equal 30 degrees at non-inductive load, increases for one transformer and decreases for the other, as the load becomes more inductive. This shows that at non-inductive load the current is lagging in one transformer, and is leading in the other.

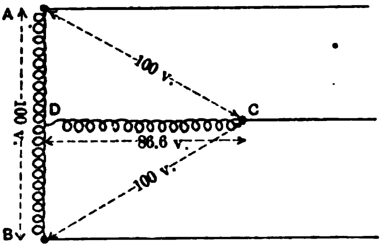

Fig. 349. Three-phase system, T-connection.

Report. Give the diagram of connections used; figure per cent increase in currents with the change from delta- to *V*-connection. Determine the actual phase displacement in the transformers, and show how it varies when the load becomes inductive. Give an explanation of this on the basis of the diagram, Fig. 347. Give your findings in regard to unbalanced load, and corroborate them by a vector diagram.

441. T-Connection.— Besides *V*-connection there is another way of producing three-phase currents with two windings only; this is the *T*-connection shown in Fig. 349. The two windings there shown represent either the secondary windings of two transformers or two phases of a special generator. The three-phase line is connected at *A*, *B*, *C*, the voltages being the same (100 volts) between any two wires. The voltage induced in the winding *DC* is de-phased 90 degrees against the voltage induced in *AB*, as shown in Fig. 350. But the voltage across

AC, being a geometrical sum of *DC* and of one half *AB*, has a phase displacement of 120 degrees from *AB*. The number of turns in the winding *DC* is

$$\sqrt{3} \div 2 = 86.6 \text{ per cent}$$

of that in *AB*. The reason for this may be seen from Fig. 350, as only under such conditions is *ABC* an equilateral triangle.

The line currents with balanced load are represented by the vectors I_1, I_2, and I_3; the former two are displaced by 30 degrees from the voltage *AB*. Thus, with *T*-connection the currents in the transformer *AB* are 30 degrees out of phase with the voltage induced in the transformer, with non-inductive balanced load. One of the currents is leading the e.m.f., the other is lagging behind it. Therefore, when the load becomes inductive, the phase angle increases in one half of the transformer and decreases in the other half. The current I_3 is in phase with the voltage *DC* at non-inductive load.

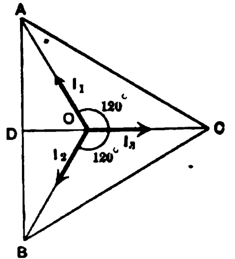

Fig. 350. Current and voltage relations with T-connection.

The power delivered to the line is measured, as usual, by the two-wattmeter method. The following is a check calculation showing that the total power *W* in two *T*-connected transformers is equal to $IE\sqrt{3}$, as with the other three-phase connections. Taking the three currents (Fig. 350), and multiplying them by the corresponding voltages and values of the power factor, we have:

$$W = DA \,.\, I_1 \,.\, \text{Cos } 30° + DB \,.\, I_2 \,.\, \text{Cos } 30° + DC \,.\, I_3 \; :$$

$$W = \frac{E}{2} \,.\, I \,.\, \frac{\sqrt{3}}{2} + \frac{E}{2} \,.\, I \,.\, \frac{\sqrt{3}}{2} + \frac{E\sqrt{3}}{2} \,.\, I$$

or

$$W = EI\sqrt{3},$$

which was to be proved. Each transformer carries one half of the total power.

442. EXPERIMENT 20-J. — Electrical Relations in a T-Connected System. — The purpose of the experiment is to illustrate the electrical relations explained in the preceding article. If a *T*-connected generator is not available, take two transformers, connect their primary and secondary windings in *T*, and then connect them to the supply and to the load respectively. One of the transformers must have a tap *D*

(Fig. 349) in the center of each winding, the other transformer must have a lower number of turns (86.6 per cent). In practice, both transformers are made identical, and both are provided with taps at 50 per cent and 86.6 per cent of their windings, so as to be interchangeable.

(*a*) Measure, at no load, all the voltages shown in Fig. 350. If the transformer *DC* has additional taps, investigate the effect of the magnitude of voltage *DC* on the line voltages *AB*, *BC* and *CA*.

(*b*) Provide a balanced non-inductive load, measure currents, voltages and the power in both halves of the transformer *AB*, in order to check the theoretical phase difference of 30 degrees. Gradually introduce some inductance into the load and repeat the same measurements; it will be found, that the phase difference decreases in one transformer and increases in the other. It will also be found, that the line voltages become somewhat unsymmetrical, because the drop of voltage in the two halves of the transformer is different with leading and lagging currents. In order to reduce this unbalancing, the coils on the halves *AD* and *BD* are interposed to bring them into a closer inductive relation.

(*c*) Unbalance the load, first in the phase *AB*, and then in the phase *AC* by the same per cent; measure currents and power in the two transformers in order to determine in which case the unbalancing affects the system less.

Report the results and show in how far they corroborate the theory. Construct vector diagrams of current and voltage relations (Fig. 350).

443. Determination of Size of Conductors in Polyphase Lines.— The relations in polyphase circuits being somewhat more complicated than in ordinary direct-current and single-phase circuits, the beginner sometimes encounters difficulties in understanding these relations, particularly in application to the most important practical question with which an operating engineer is most likely to deal,viz., figuring the size of conductors for a two-phase or a three-phase transmission line. The necessary explanation as to how a three phase or two-phase line can be reduced to an equivalent single-phase line and calculated as such is given in § 264. A numerical example may be of use here, in showing that polyphase lines can also be figured out directly from the relations derived above.

The problem usually presents itself in some such form as this: 500 kw. are to be transmitted by means of a three-phase system, over a distance of 4 miles, with a loss of 8 per cent of the power delivered; the pressure at the receiving end of the line is to be 6600 volts. What is the size of conductors to be used?

Let us first assume the load to be non-inductive; the current in each wire is then, according to the formula $W = EI \cdot \sqrt{3}$,

$$I = \frac{500}{6.6 \times \sqrt{3}} = 43.8 \text{ amp.*}$$

The loss of power allowed in the line is $0.08 \times 500 = 40$ kilowatts, or $40{,}000 \div 3 = 13{,}333$ watts in each wire. The resistance R of each wire is determined from the equation $I^2R = 13{,}333$, from which $R = 6.95$ ohms. Resistances are usually given in wire tables per 1000 feet of conductor; the resistance of our wire per 1000 feet must be

$$\frac{6.95}{4 \times 5.28} = 0.329 \text{ ohm},$$

which corresponds approximately to No. 5 Brown & Sharp gauge.

If the load is inductive, the power factor being, say, 80 per cent, a heavier wire is required, because it takes a larger current to transmit the same power at a lower power factor. As the loss in the line is proportional to the square of the current, the resistance per 1000 feet now has to be $0.329 \times (0.80)^2 = 0.210$ ohm; this corresponds to about No. 3 B. & S. wire.

If the transmission is to be effected by means of the two-phase four-wire system the size of the wire is figured out as follows: With 500 kw. transmitted by two phases, each phase transmits 250 kw., so that the line can be calculated as a single-phase line of the same voltage, transmitting 250 kw. The current at non-inductive load is $250 \div 6.6 =$ about 38 amperes, and the resistance of the line

$$R = \frac{0.08 \times 250{,}000}{38^2} = 13.9 \text{ ohms.}$$

This gives

$$\frac{13.9}{5.28 \times 4 \times 2} = 0.329 \text{ ohm per 1000 feet},$$

or the same size of wire, as figured above for the three-phase system. This shows that there is a saving of 25 per cent in copper with the three-wire system as compared to the two-phase system, because, the size of the wire being the same in both cases, there is one wire less with the three-phase system.

PHASE TRANSFORMATION.

444. There are cases in which it is desirable to transform two-phase power into three-phase power, or vice versa. Thus, for example, power is sometimes generated by means of two-phase alternators, then the voltage is raised, the system being simultaneously transformed into a three-phase system for transmission and distribution. The advantage of this arrangement is that only three line wires are required instead of four. On the contrary, in some cases two-phase local distribution is preferred, especially for lighting, where each phase is used separately; the three-phase supply is then converted into a two-phase four-wire combination. These transformations are usually accomplished by means of the so-called Scott system described below.

---

* The power being given in *kilowatts*, it is convenient to measure the pressure in *kilovolts:* 6600 volts = 6.6 kilovolts.

445. The Scott Two-Phase to Three-Phase Transformation. — A *T*-connection is used with this system on the three-phase side (Fig. 351); the other windings of the transformer are connected directly to the two-phase supply. In *T*-connection (Fig. 349), as well as in the ordinary two-phase connection (Fig. 329), the two voltages generated in the windings are displaced in phase by 90 degrees. The only difference is that in the two-phase system the two windings *ab* and *cd* are used independently, while in the *T*-connection the corresponding windings are interconnected at *B*, and the three-phase line is connected to the three remaining terminals *A*, *C*, and *D*. Moreover, in the two-phase system, both windings have the same number of turns, while on the three-phase side the winding *AB* must have only 86.6 per cent of the turns of *CD* in order to produce an equilateral triangle of voltages (compare Fig. 350).

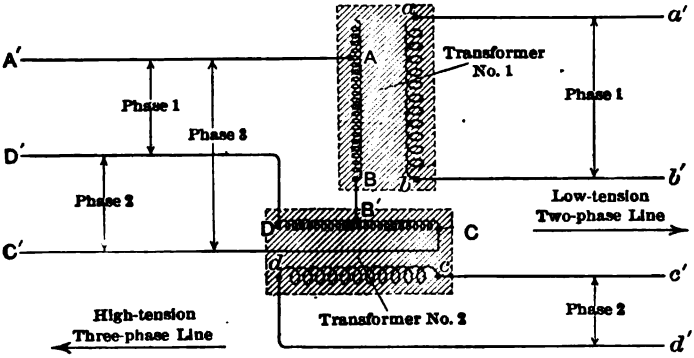

FIG. 351. The Scott two-phase to three-phase transformation.

446. EXPERIMENT 20-K. — The Scott System of Polyphase Transformation. — Connect two transformers, as per Fig. 351, one side to the source of supply, the other side to the load. Use a polyphase board (§ 49) so as to be able to measure currents, voltages, and watts on the three-phase side and on the two-phas[illegible] Connect first a balanced non-inductive load, and take all the readings necessary for determining the actual phase relations of voltages and currents. Unbalance the load on one side, and see how it affects the supply; do this with all the phases, because the system is not symmetrical. Perform a similar test with an inductive load; note in all cases the unbalancing of voltages resulting from an internal drop in the transformers.

Report. Give the connections actually used, the ratios found with inductive and non-inductive load; explain by means of vector diagrams the results obtained.

447. Six-Phase System. — It is mentioned in § 505, that the copper loss in the armature of a rotary converter decreases with the increase in the number of slip-rings, because the alternating currents are more evenly distributed in the armature. As the heating in the armature determines the output and the rating of the rotary converter, it becomes advantageous with large rotaries to increase the number of slip-rings to six. It would be out of the question to have six-phase currents generated and transmitted to rotary substations. This is, however, not necessary, as six-phase currents can be easily produced in the substation itself, from the two-phase or three-phase currents delivered from the power house, as is explained below.

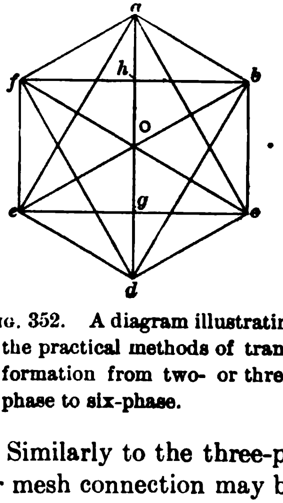

FIG. 352. A diagram illustrating the practical methods of transformation from two- or three-phase to six-phase.

Similarly to the three-phase and quarter-phase systems, either star or mesh connection may be used with the six-phase system (Fig. 352). The star voltages are represented by the rays *Oa*, *Ob*, *Oc*, etc. The mesh voltages are *ab*, *bc*, *cd*, etc. It will be seen that in this case star voltages are numerically equal to mesh voltages, because *Oab*, *Obc*, etc., are all equilateral triangles.

448. Three-Phase to Six-Phase Transformation. — The transformation is based on the fact that in a six-phase system three of the phases may be considered as merely three other phases reversed. Thus, with a star connection, the voltage *Of* is the reversed *Oc*, etc. With the mesh connection the voltage *ab* is the reversed voltage *de*, etc.

Fig. 353 shows three transformers whose secondaries are wound in two sections each; the primaries of the transformers, not shown in the figure, are connected in *Y* or delta to the three-phase supply. The two terminals of each section are marked by *s* (start) and *f* (finish). To get a star-connecte[illegible]x-phase combination, connect together all the terminals marked *s*; this gives the neutral point *O*; the terminals *f* are connected to the six slip-rings of the rotary, in the order 1, 2, 3, etc., as indicated.

If the mesh connection is desired, the finish of coil 1 is connected to the start of coil 2, finish of coil 2 to start of coil 3, etc.

The above-described combination of transformer windings is not the only one possible for converting three-phase into six-phase currents. As a general proposition, there are as many separate methods of transformation as there are possible combinations for connecting the six vertices, *a*, *b*, *c*, *d*, *e*, *f* (Fig. 352), with lines of three directions. This last limitation is necessary because there are but three voltages to begin with, and the phases may be only reversed, but not shifted by a certain angle.

These combinations are as follows:

(1) Connecting *a* to *b*, *b* to *c*, etc.

(2) Connecting the vertices to the point *O*.

(3) Connecting them by three diagonals, *ad*, *be*, *cf*.

(4) Connecting by two deltas, *aec* and *fbd*.

The first two methods are explained in Fig. 353; the method (3) does not require double secondaries; the method (4) may be obtained by properly connecting the double secondaries, shown in Fig. 353.

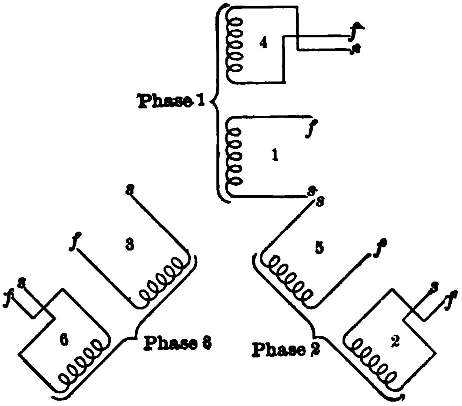

FIG. 353. Transformation from three-phase to six-phase by means of three transformers, each with two secondary windings.

By using the *V*- or the *T*-connection, it is possible to convert a three-phase system into a six-phase system using two transformers only; thus,

(5) By using two *V*'s, *eac* and *fbd*.

(6) By using two *T*'s, *ecga* and *fbhd*.

The primaries of the two transformers are connected in *V* or *T* respectively; the secondaries are wound in two sections, the four windings being connected in double *T* or double *V*.

449. Two-Phase to Six-Phase Transformation. — This transformation may be understood directly from the method (6) mentioned in the previous article, only the primaries of the two transformers, instead of being connected in *T*, are connected each across one phase of the supply, as in the Scott system (Fig. 351).

450. EXPERIMENT 20-L. — Six-Phase Transformation. — The purpose of the experiment is to verify experimentally the relations derived in §§ 448 and 449. At least two transformers must be available, or preferably three, in order to be able to try all the six combinations mentioned above. The transformers should be provided, if possible, with double secondary windings and suitable taps on the primary windings, for obtaining identical secondary voltages with different connections.

Combinations (1) and (2) can be tried on open circuit; the other combinations may be investigated on a closed circuit only, because otherwise not all of the six vertices of the hexagon are interconnected. For instance, with method (4) there is no electrical connection between the points *a*, *e*, *c* on one hand, and *f*, *b*, *d* on the other hand; this connection is established through the load only.

A regular six-phase rotary converter is the most natural load for this experiment; if such is not available, an experimental rotary, such as is shown in Fig. 388, will answer the purpose, since it will interconnect the phases in at least one point. If no rotary is available, the system may be interlinked by producing an artificial neutral point *0* connected to the six vertices by equal resistances or inductances, connected in star.

Report. Give the actual electrical connections used during the test and the voltages observed. Plot the observed voltages graphically, as a vector diagram, and compare the theoretical ratios, shown in Fig. 352, with those actually observed.

451. Transformation from a Polyphase to Single-Phase System. — It is sometimes desirable to connect a single-phase load to a three-phase supply, *without unbalancing the phases.* Such a transformation is physically impossible, without bringing into play a revolving device which can store electrical energy at a certain part of an alternation, and deliver it again at another time. The reason for this is, that the energy delivered by a three-phase alternator is constant at all times, while that taken by a single-phase load fluctuates between a certain maximum and zero, or even becomes negative, if the load is inductive With a revolving motor-generator set, interposed between the alternator and the load, the difference between the energy delivered by the three-phase system and that taken in by the load is stored as a part of the momentum of the revolving part of the set, to be delivered at the next moments. The motor-generator is dispensed with if an unbalancing of the polyphase system is not objectionable. Thus, on single-phase railways, two tracks, or various sections of the same track, are supplied from different phases of the same polyphase alternator, the load being at times out of balance.

The inverse transformation from a single-phase to a perfect polyphase system is impossible for the same reason, without the medium of a revolving motor-generator set. An imperfect two-phase system (split-phase arrangement) can be produced by suitable combinations of resistance, inductance and capacity. See §§ 81 and 536.

CHAPTER XXI.

ALTERNATORS AND SYNCHRONOUS MOTORS—OPERATING FEATURES.

452. An alternator is an electrical machine for producing alternating currents. The essential parts of a modern alternator are shown in Fig. 354. It consists of a stationary armature in which currents are induced, and of the revolving magnetic poles whose function it is to induce these currents. The exciting current, which energizes these magnet poles, is conducted into the revolving part through the two slip rings shown.*

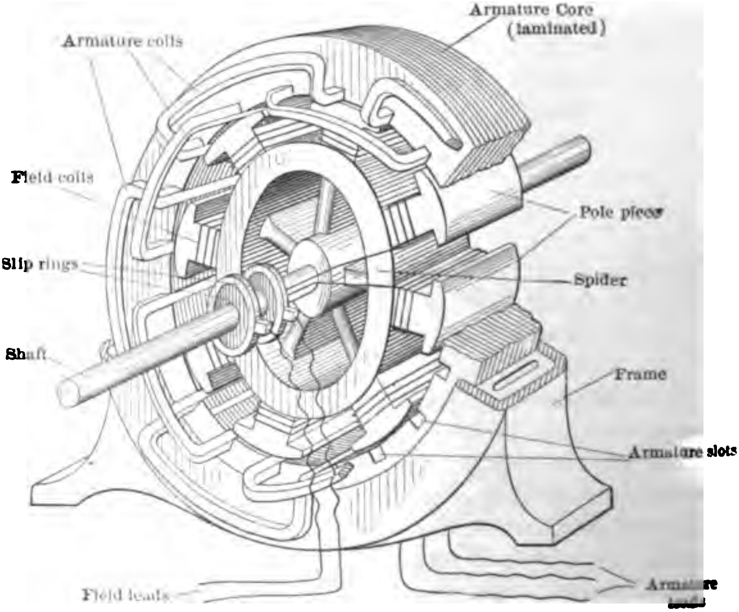

FIG. 354. The principal parts of an alternator, or of a synchronous motor.

The particular alternator shown in the sketch is an eight-pole, three-phase machine; it has four armature coils for each phase, or one side of a coil under each pole. Part of the coils have their free ends bent upward to leave room for the other coils. The armature core is laminated and is held together by a cast-iron frame.

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* There is a type of alternator, so called *inductor* type, in which both the exciting and the armature windings are stationary; the pole projections alone are rotated, and the e.m.f. is induced, due to periodical changes in the reluctance of the magnetic circuit.

By varying the speed of the machine, the frequency of the generated current is varied: for instance, the above machine, when driven at a speed of 900 r.p.m., gives 7200 alternations per minute, or 60 cycles per second. Whenever a pole passes under a conductor it produces one impulse, or one alternation. With 8 poles, 8 alternations are produced during one revolution of the machine, or 8 × 900 = 7200 alternations in one minute. One positive and one negative alternation give one complete period of alternating current; thus 7200 alternations make 3600 periods (or cycles) per minute, or 60 cycles per second. For a description of direct-reading frequency indicators see § 474 below.

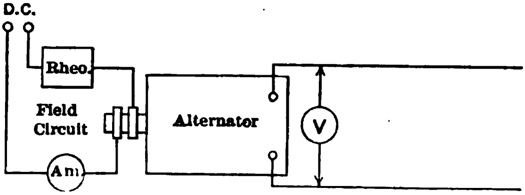

Fig. 355. Diagram of connections of a single-phase alternator at no load.

453. EXPERIMENT 21-A. — No-Load Characteristics of an Alternator. — The purpose of the experiment is to investigate the dependence of the voltage of an alternator on exciting current and speed. If the machine is a single-phase alternator, it is wired up as in Fig. 355; for three-phase connections see § 425. It is preferable to perform this experiment on a single-phase machine because the relations to be investigated are the same as in a polyphase machine, while the wiring and the measurements are much simpler.

Run the machine at its rated speed and increase the field excitation in steps; read the corresponding alternating voltages and field amperes. Take readings also with decreasing field current, so as to eliminate the influence of residual magnetism. Repeat the same test with a higher and a lower speed. A frequency meter, if available (see § 474), should be used to give direct readings of frequency, and will make it easier to keep the speed constant.

Report. Give a short description of the machine, and plot the voltage curves to field amperes as abscissæ (Fig. 381). Figure out the corresponding frequencies in alternations per minute and in cycles per second. Show that with the same exciting current the induced voltages are proportional to speed.

454. Load Characteristics of Alternators.—When a load is put on an alternator, as in Fig. 356, the terminal voltage decreases, because of a voltage drop in the machine itself. This drop depends upon the magnitude of the load and on its power factor. Therefore when the load varies, the terminal voltage fluctuates.

One of the most important problems in the operation and design of alternators is that of reducing the fluctuations of the voltage with varying load. These fluctuations are particularly objectionable on lighting circuits, because of the accompanying flickering of the lamps. Therefore, certain requirements and guarantees in regard to the maximum fluctuations in voltage are usually introduced into contracts regarding the performance of alternators. These requirements are now usually stated in accordance with the recommendations of the American Institute of Electrical Engineers, in the form of a guarantee of a certain *inherent regulation* of the machine (see *Report of Committee on Standardization, Section on Regulation*).

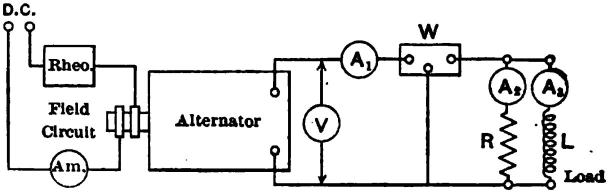

FIG. 356. Diagram of connections of a loaded single-phase alternator.

The definition of the term "regulation," as given there, may be best explained by means of a numerical example. Suppose an alternator to be connected to the line, driven at the rated speed, and loaded to its full capacity *on a non-inductive load*, at the rated voltage, say 2200 volts. Then, let the load be thrown off and the voltage measured again at the same field current and the same speed. Suppose it to be 2330 volts; *by definition*, the regulation of the machine is

$$\frac{2330 - 2200}{2200} = 5.9 \text{ per cent.}$$

The practical meaning of the regulation is as follows: Suppose the above alternator to be running in the evening, at practically full load, on incandescent lighting, current being supplied to residences through 2200-volt to 110-volt house transformers. The lamps are burning at their rated voltage (110 volts). As time advances, lamps are gradually turned off, the load on the machine decreases and the voltage rises, unless the field current is at all times properly regulated. When only a few lamps remain burning, the machine is running at practically no load, and the voltage would be about 2330 volts, or

$$2330 \times \frac{110}{2200} = 116.5 \text{ volts}$$

at the lamps; this is too high for 110-volt lamps, and would considerably shorten their life.

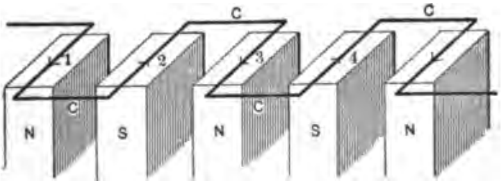

Fig. 357. Relative position of poles and armature winding when the induced e.m.f. is a maximum.

Of course, the switchboard attendant in the power house watches the voltage and regulates the field rheostat accordingly. But, since the load fluctuates constantly, it is very difficult to keep the voltage fairly constant, *if the machine has a poor inherent regulation.*

The better the desired regulation of the machine, the more expensive it is to build. For this reason, a knowledge of the factors affecting regulation becomes of great commercial importance, as well as of purely engineering interest.

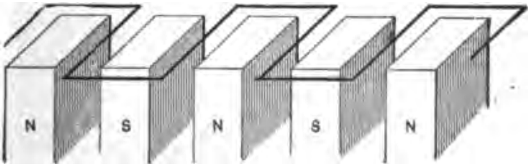

Fig. 358. Relative position of poles and armature winding when the induced e.m.f. is zero.

455. Influence of Power Factor of the Load on the Internal Voltage Drop.—Voltage [illegible] is considerabl[illegible]ased when the load is *inductive* (arc lamps, induction motors, etc.); the lower the power factor the worse being the regulation of the machine. This may be explained in two different ways:

(1) The more the current lags behind the induced e.m.f., the greater is the demagnetizing action of the armature upon the field. At a high power factor the maximum of the current wave in the armature occurs

when the armature conductors occupy the position shown in Fig. 357. The maximum of the voltage wave occurs at about the same time (at high power factors, the maximum of the current occurs when the voltage is near its maximum, and this occurs when the conductors are in the position shown). With a low power factor the maximum of the current occurs nearer the position indicated in Fig. 358, or one quarter of a period later than for unity power factor. It will be easily seen, that in the second case the demagnetizing action of the armature currents is much more effective, the field of the machine is weakened more, and the terminal voltage reduced to a larger extent, than at a high power factor.

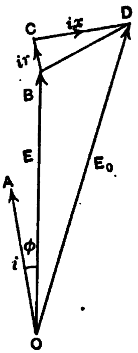

FIG. 359. Voltage drop in an alternator, at a high power factor.

(2) Another way of explaining the influence of power factor on regulation is by considering the ohmic and the inductive drop in the armature. The relations at a high power factor are shown vectorially in Fig. 359. *OB* is the terminal voltage of the machine; *OA* is the current, lagging behind it by an angle ϕ, corresponding to the power factor of the load. The ohmic drop *ir* in the armature is represented by e vector *BC* parallel to the vector of the current; e inductive drop *ix* by *CD* perpendicular to the same. le induced e.m.f. of the machine, *OD*, is the geoetrical sum of the terminal voltage and the voltage op in the armature. With regard to the components f the induced e.m.f. it can be said that the part *DC* is lost in the armature reactance, the part *CB* in its resistance; the rest, *OB*, is available at the terminals of the machine.

Fig. 360 represents a similar diagram for the same values of induced e.m.f. and armature current, but *for a lower power factor* (angle ϕ is larger). It will be readily seen that, for obvious geometrical reasons, the terminal voltage *OB* is lower in this case, than in Fig. 359.

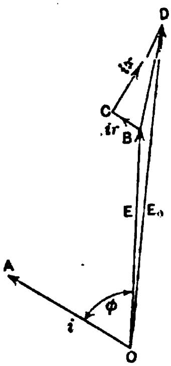

FIG. 360. Voltage drop in an alternator, at a low power factor.

In reality both factors, the demagnetizing action of the armature and the armature inductance, are present simultaneously, so that the above two explanations should be taken in their entirety to cover the actual phenomenon.

456. Performance Curves. — As was stated above, the regulation, or the voltage drop in an alternator, depends not only upon amperes output, but also upon the power factor of the load. For this reason the regulation of an alternator cannot be represented by one curve, as in case of a direct-current generator (Fig. 267), but requires a set of curves, either as shown in Fig. 361 or in Fig. 362. The first set gives the terminal voltage as a function of variable power factor, for constant values of the armature current. The second set of curves gives the values of terminal voltage as a function of variable armature current, for constant values of the power factor. The two sets are equivalent, and one can be plotted from the other. In taking these curves on an actual machine it is easier to keep the current constant, than to maintain a constant power factor; therefore the results are usually plotted as shown in Fig. 361. Then, if desired, the ordinates of these curves can be recombined so as to obtain the curves shown in Fig. 362.

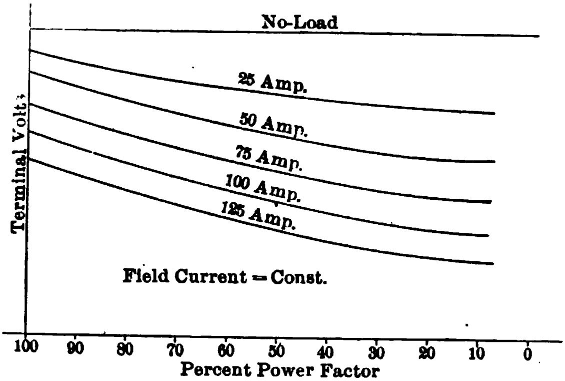

FIG. 361. Voltage characteristics of an alternator, as depending on the power-factor of the load.

457. EXPERIMENT 21-B. — Voltage Characteristics of a Loaded Alternator. — The purpose of the experiment is to obtain regulation curves (Fig. 361) showing the influence of the current, and of the power factor of the load, on the terminal voltage of an alternator. In order to facilitate the experiment, a single-phase machine should be selected. Qualitatively the same phenomena are observed in polyphase alternators, while the measurements are more complicated.

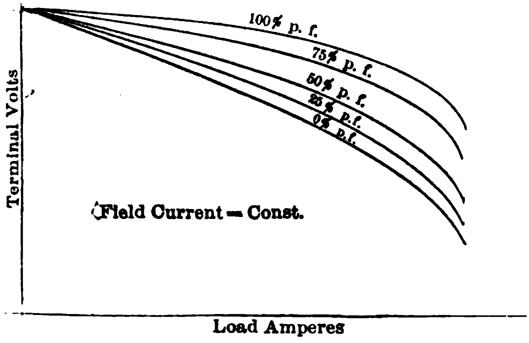

FIG. 362. Voltage characteristics of an alternator, as depending on the load current.

The connections are shown in Fig. 356; they are the same as in Fig. 355, save that a load is added, consisting of a resistance R, and an impedance coil L in parallel with it. The current may be regulated separately in either branch, and read on the corresponding ammeters A_2 and A_3; the ammeter A_1 measures the total current of the machine. In reality, all three currents are usually read on the same ammeter which is connected in succession into the three parts of the circuit, by means of a suitable multi-throw switch, or polyphase board (§ 49). In some cases, especially with large machines, it is convenient to use a synchronous motor as a load, with or without ohmic resistances in parallel. By regulating the field current of the motor, it may be made to take a large leading or lagging wattless component and thus vary the power factor of the load (see § 470).

Begin the experiment by determining the value of the field current, at which the machine gives its full rated (non-inductive) current at normal voltage. This field current and the speed of the machine must be kept constant, while the curves, shown in Fig. 361, are taken. Begin the readings with the most unfavorable conditions — lowest power factor and heaviest current (about 25 per cent overload). Keep the current constant and gradually raise the power factor to 100 per cent by reducing the inductive component of the current and increasing the current in the non-inductive resistance. This will give the lowest regulation curve. Take all the other curves in a similar manner, and finally throw off the load altogether and measure the no-load voltage (the upper horizontal line).

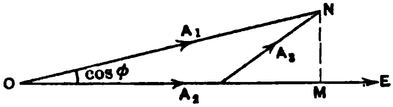

Fig. 363. Diagram of currents shown by the three ammeters in Fig. 356.

For each point on the curves read volts, amperes and watts; also read separately the amperes in the inductive and the non-inductive branches of the circuit (ammeters A_2 and A_3). It will be seen that the arithmetical sum of these currents is always larger than the total current shown on the ammeter A_1, since the two currents A_2 and A_3 *are not in phase with each other;* A_1 represents their geometrical sum (Fig. 363).

The readings may be conveniently recorded on a data sheet similar to the one shown below.

Report. (1) Plot the curves shown in Fig. 361, and supplement them by a curve showing *per cent* regulation at full-load current, with different values of power factor (§ 454).

(2) Explain by means of an example how to convert these curves into the curves shown in Fig. 362.

(3) Take a few sets of readings of total and component amperes, and construct triangles, as shown in Fig. 363. From such triangles the power factor can be determined without the use of a wattmeter. See if the values of the power factor determined as the ratio of *OM* to *ON* check with those figured from the wattmeter readings.

	Load Amps. Constant at ——— Per Cent Full Load Rating.						
	Amperes.			Volts.	Watts.	Field Amps.	Speed.
	Non-Ind.	Ind.	Total.				
Inst. No.							
Const.							

458. Maintaining Constant Voltage — In the above explanations and while performing experiment 21-B, it is assumed, that the field current is not varied, and that no attempt is made to keep the voltage constant. In actual operation, however, the terminal voltage of the machine, rather than the field current, is kept constant, and the field excitation is varied according to the load. It is of importance to know the limits within which the field current must be varied in order to keep the voltage constant (Fig. 364). The designer is interested in this in order to give proper dimensions to the field coils and to the field rheostat. The operating engineer wants to know this curve in order to be able to judge how sensitive the alternator is to changes of load, and how difficult it will be to maintain the proper voltage regulation. The curve shown in Fig. 364 is called the *excitation characteristic;* it gives the values of field amperes necessary for maintaining a consant terminal voltage with different loads. The horizontal line shows the value of field current which gives the same terminal voltage at no load.

459. EXPERIMENT 21-C. — Excitation Characteristics of an Alternator. — The connections are the same as in Experiment 21-B; similar load tests must be performed, and the same data sheet may be used. The only difference is that now the terminal voltage is kept constant, and the field current is varied accordingly.

The regulation of an alternator depends essentially on the degree of saturation of the magnetic circuit; with highly saturated pole-pieces even a considerable number of demagnetizing ampere-turns on the armature has less effect than a smaller number with unsaturated poles. Therefore a machine with highly saturated poles gives, under equal circumstances, a better regulation, though, of course, it requires more copper on the exciting coils.

In order to see the effect of saturation, take two sets of curves, with different terminal voltages. Select one of the voltages as high as it is possible to obtain with highly inductive load, by cutting out all the resistance of the field rheostat. The other voltage must be selected considerably lower — if possible, below the knee of the saturation curve (Fig. 151). In each case begin the run with the lowest power factor obtainable. Gradually increase the power factor up to 100 per cent, keeping the current constant, and, at the same time, varying the exciting current so as to keep the terminal voltage constant. Then throw off the load and reduce the field current so as to get the same voltage again. This will give the ordinate of the horizontal line shown in Fig. 364. For each setting of the load read field amperes and watts; keep volts, armature amperes and the speed constant.

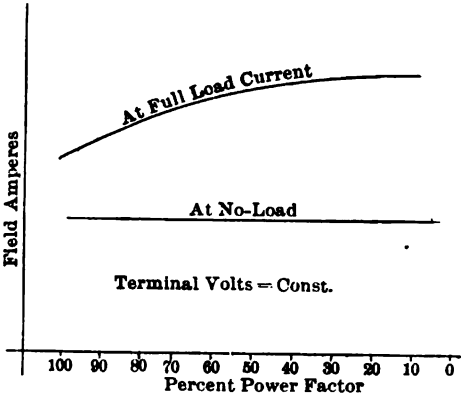

FIG. 364. Excitation characteristics of an alternator.

460. Tirrell Regulator. — Large variations of voltage common with alternators make an automatic pressure regulation highly desirable. Numerous attempts at compounding alternators have proven to be impracticable as they are too complicated.* Another solution, namely, placing an automatic regulator *outside* of the machine, proved to be more of a success. The Tirrell regulator, described in § 309 in application to direct-current generators, may be used with some modifications for maintaining constant voltage on alternators.

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* An original type of self-exciting and compounded alternator, designed by Mr. E. F. Alexanderson for the General Electric Co., apparently has good chances for commercial success. See *Transactions of the A. I. E. E.*, January, 1906.

The regulator is represented schematically in Fig. 365. As is explained in the description of the direct-current regulator, it periodically short-circuits the field rheostat of the machine, or, in this case, the field rheostat of the exciter. The relative durations of short-circuit and open circuit are made to depend on the terminal voltage of the alternator. The exciter rheostat is short-circuited between the two relay contacts, when the differential relay under the contact arm is deënergized. The deënergizing is done by the main contacts, which are closed and opened by the direct-current control magnet shown to the left. The duration of the contact is determined by the position of the right-hand lever actuated by the alternating-current control magnet.

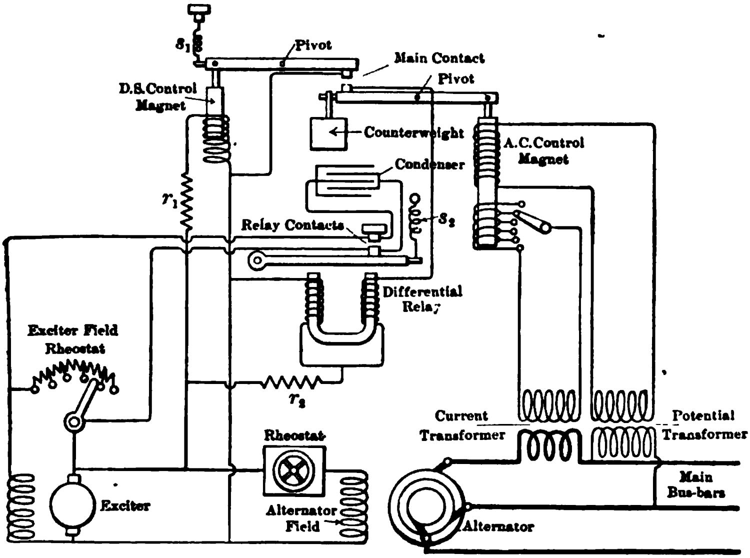


FIG. 365. Diagram of connections of the Tirrell voltage-regulator as used with an alternator.

Assuming first the main contacts to be open, only the left-hand leg of the differential relay is energized and the relay contacts are opened. This brings down the exciter voltage, and the spring s_1 closes the main contacts. A current flows through the right-hand leg of the differential relay, destroying its magnetism. Thereupon the relay contact is closed, and the exciter voltage suddenly increased. The D. C. control magnet overcomes the action of the spring s_1 and again opens the main contacts, etc. Thus, both contacts are all the time vibrating, and maintain the right exciter voltage. If the voltage on the alternating-current side should increase above normal, the A. C. control magnet is pulled upward. This brings the main contacts farther apart, and reduces the time of short-circuit of the exciter rheostat; the voltage is thus brought back to its normal value. If the A. C. voltage should be below normal, the same solenoid brings the main contacts closer together.

If it is desired to have the alternator over-compounded — to give a higher voltage with increasing load — a series winding is provided on the A. C. control magnet connected to the main line through a series transformer. Its action is such, that when the load increases, the main contacts are brought closer together. The number of turns on this winding is adjustable for different degrees of over-compounding. The condenser shown across the relay contacts serves for suppressing sparking at these contacts. The other details of the apparatus are clearly seen in the diagram. In its general appearance the regulator is similar to the direct-current regulator, shown in Fig. 274.

461. EXPERIMENT 21-D. — Study of the Tirrell Regulator. — The regulator, described in preceding article, is connected as shown in Fig. 365, and its performance observed under various conditions met with in practice. Vary the load of the machine, in turn, gradually and suddenly; vary its speed, setting of the rheostats, adjustments of springs, etc. Connect the compound winding and observe its action with various values of power factor of the load. Investigate the action of all factors separately, and report results systematically and with necessary explanations.

ALTERNATORS IN PARALLEL.

462. Two direct-current machines can be put in parallel as soon as their voltages are approximately equal (§ 312.) But before switching two alternators in parallel three conditions must be fulfilled; namely, the machines must

(1) Give the same voltage;
(2) Have the same frequency;
(3) Be *in phase* with one another.

Unless all these requirements were fulfilled, one of the machines would send part of the current into the other machine, instead of sending it to the line. Only when the e.m.f.'s of both alternators are equal *at all moments*—which means the fulfillment of the above three conditions—are their actions properly combined, and each can send its current to the line.

The process of bringing an alternator to the voltage, frequency and phase of another alternator is called *synchronizing* (the word literally means "bringing in the same time"). Let us investigate what would occur if two alternators should be connected to the same bus-bars without all of the above-stated conditions being fulfilled.

(1) If the two machines are excited so as to give *different voltages*, the other two conditions being fulfilled, a wattless current circulates between the two machines, leading in the machine excited lower, and lagging in the machine excited higher. This has the effect of strengthening the field of the first machine and weakening the field of the second machine; the resultant voltage at the bus-bars is, therefore, somewhere between the voltages of the two machines. The set may still work satisfactorily, provided this wattless current is not too heavy, but it does not help the operation and gives an unnecessary I^2R loss in the armatures of both machines. Should this wattless current become sufficiently large, it may heat up the machines or open the main circuit-breakers, though the external load may be quite small.

(2) If the machines have *a different frequency*, the current which they supply is an *interference* current, without any definite frequency; it could not be used either for lamps or motors. To see this clearly, draw two sine waves of frequencies differing by, say, 5 per cent. one from the other, and add them together. Moreover, under such circumstances each machine would be partially short-circuited on the other; ı fact, if coupling without synchronizing is attempted, the circuit-rotecting devices immediately open the circuit.

(3) If the machines are *not in phase*, in other words, if the maxima f their e.m.f. waves do not occur at the same moments, a current rculates between the two machines, under a pressure equal to the fference of their e.m.f.'s, and we have again, in an exaggerated manner, e conditions described above under (1). It may also be pointed out re, that if the two machines have *different wave forms*, equalizing rrents are always present to some extent, but unless the wave forms e very much different, these currents do little harm aside from causg additional heating of the machines, and lowering somewhat their ficiency.

Knowing the frequency of the supply and the number of poles of the achine to be synchronized, it is easy to determine the speed necessary r the desired frequency. If, for instance, the machine has 30 poles, gives 30 alternations during one complete revolution; so that if the equency of the supply is 7200 alternations per minute, the machine must volve 7200 ÷ 30 = 240 times per minute in order to give this number f alternations. The simple rule is: the speed of an alternator in r.p.m. found by dividing the frequency of the supply (in alternations per inute), by the number of poles of the machine. For a description of irect-reading frequency indicators see §§ 475 and 476 below.

After the right speed has been approximately obtained, the field of the alternator is so adjusted as to give about the same voltage as that of the bus-bars to which the machine is to be connected. It only remains then to bring the machine *into phase* with the voltage at the bus-bars. This is done either by means of properly connected incandescent lamps (synchronizing lamps), or special instruments, so-called synchroscopes, or synchronism indicators.

463. Synchronizing Lamps. — Synchronizing lamps are connected as shown in Fig. 366, around the main switch which connects the machine to the bus-bars. As long as the alternator is not in phase with the line, equalizing currents circulate through it, and the lamps *a* serve both to reduce and to indicate these currents. By varying the speed of the alternator, it is possible to extinguish the lamps; this will show that the machine is in perfect synchronism, and the switch can be closed. The sketch illustrates the case of a three-phase machine; with a single-phase machine one of the lines, say CC_1, is omitted.

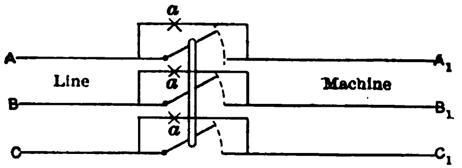


FIG. 366. Synchronizing lamps.

Some prefer to have synchronizing lamps crossed, as shown in Fig. 367. The machine is in synchronism when the lamps glow the brightest. As an advantage of this arrangement, it is claimed that should a lamp burn out during the process of synchronizing, the operator would immediately notice it, while with the first arrangement he may judge, by the lamps being extinguished, that the machines are in perfect synchronism. Thus he may close the switch while the machine is altogether out of phase, and, unless the protective devices (fuses or circuit-breakers) operate promptly, the machine may be damaged by the rush of current. However, the possibility of a synchronizing lamp burning out is rather remote, and, with two- or three-phase machines, the burning out of one set of lamps would not affect the others. On the other hand, it is generally considered that it is easier to observe moments of total extinguishing than moments of maximum brilliancy.

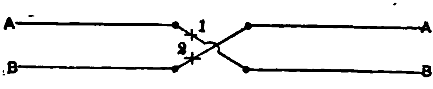


FIG. 367. Crossed synchronizing lamps.

With three-phase machines, crossing synchronizing lamps in two phases is very convenient, especially when the lamps are arranged in a circle, as in Fig. 368. In this case, maximum brightness occurs in the three sets of the lamps in rotation, so that the light appears traveling along the circle. The direction in which the light rotates depends on whether the speed of the machine is low, or high. When the machine is in synchronism, the lamps marked "3" are dark, while the lamps " 2 " and " 1 " glow brightly.

If two 220-volt single-phase machines are to be put in parallel, at least four ordinary 110-volt synchronizing lamps should be used in series, because at certain moments during synchronizing the e.m.f.'s of the machines may be acting in the same direction instead of in opposition, thus giving 440 volts. It is even better to have, under these conditions, 5 lamps in series, so as not to let them glow too brightly; then it is easier to observe the periods of the extinguishing of the light. With two three-phase machines of the same voltage, the pressure across the lamps in each phase can never exceed 220 volts, so that two 110-volt lamps in series are sufficient, though three lamps may give a better service.

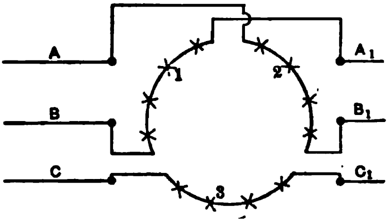


FIG. 368. Synchronizing lamps arranged in a circle.

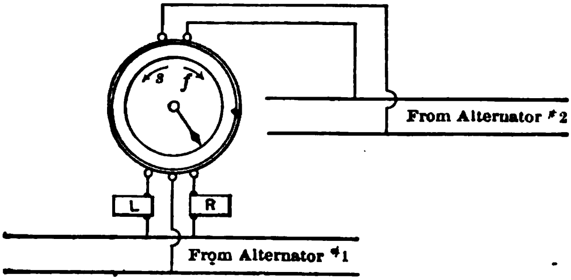


FIG. 369. Connections to a single-phase synchronism indicator.

464. Synchronism Indicators. — During the last few years, synchronizing lamps have gradually given place to special synchronizing instruments, so-called synchroscopes, or synchronism indicators. These devices have the appearance of ordinary switchboard instruments (Fig. 369), except that the pointer has no retaining spring or weight, and is free to revolve through 360 degrees. When the speed of the alternator to be synchronized is low, the pointer revolves in one direction; if it is high, it rotates in the opposite direction. When the speed is right, the pointer stands still; and when the machine is "in phase," the pointer shows zero, indicating that the main switch may be closed.

In their construction synchronism indicators resemble small alternating-current motors. The field is connected to one of the machines, or to the line, and the armature to the machine to be synchronized (Fig. 370). When the frequencies of the two machines are different, the resultant field in the synchronism indicator constantly changes its position, making the armature revolve in one or the other direction. When the frequency is the same, the field is stationary in space; when finally the machines are in phase with each other, the field occupies such a position that the pointer, connected to the armature, shows zero.

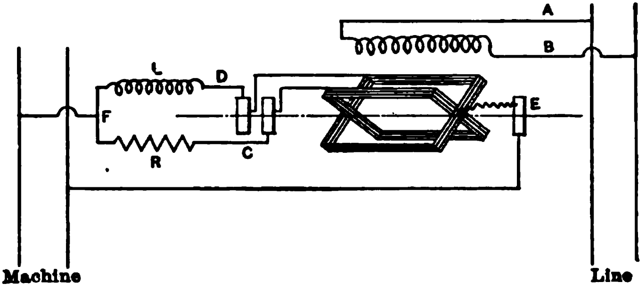


FIG. 370. The revolving armature of a General Electric synchronism indicator.

The synchronism indicator, illustrated in Figs. 369, 370 and 371, is adapted for use with single-phase alternators, though it may be used with polyphase alternators as well. The stationary field is connected to the bus-bars (line); the revolving armature is connected to the machine to be "thrown in," through some inductance L and resistance R, usually an incandescent lamp. The armature is of the drum type; it has two coils rigidly fastened at right angles to each other and connected in series. Their junction is connected through the collector ring E to the binding post marked E. The other two terminals are brought out, also through collector rings, to binding posts C and D (Fig. 371).

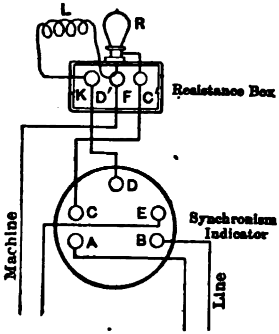


FIG. 371. The actual connections between the split-phase box and the synchronism indicator.

R and *L* are used for "splitting" the phase of the armature current, so as to produce a revolving field, as in single-phase induction motors (see § 536). The two currents in the armature coils are displaced in phase, and the coils themselves are also displaced geometrically; under these conditions the two currents produce a revolving magnetic field. *A* and *B* are the terminals of the stationary field winding of the synchroscope; it is excited from the main bus-bars.

Thus, we have in this device a stationary pulsating field produced by one machine and a revolving field (split-phase arrangement) produced by the other machine. If the frequencies of both machines are equal, there is a certain stable position of the armature of the synchronism indicator, in which position the interaction between the two fields is practically zero. If, however, the frequencies are different, one sine wave continually changes its phase relation with respect to the other, and the position of stability of the armature also continually changes. This causes the armature to revolve at a speed equal to the difference of the frequencies.

No split-phase arrangement is used in synchroscopes which are specially built for two- or three-phase service, and all the windings are stationary; the revolving element consists merely of a light iron vane. The construction is similar to that of the power-factor meter shown in Fig. 85, except that shunt transformers are used instead of the series transformers *T*. One of the machines is connected to the three-phase winding *W* which produces a revolving magnetic flux. The coil *P* inside of this winding is connected to one of the phases of the other machine. The revolving flux produced by *W* may be considered as consisting of two pulsating components; when the machines have the same frequency, one of these components is destroyed by the action of the coil *P*, and the magnetic vane is held in the direction of the other component. When the machines are not in synchronism no perfect compensation is possible, and the magnetic vane revolves, following the rotating magnetic field.

465. Remarks on Synchronizing. — Some additional remarks may be useful for performing the experiment on synchronizing alternators, described below.

(1) For synchronizing high-tension alternators, the synchronizing lamps and synchronism indicators are connected through potential transformers.

(2) Ordinary voltmeters can be used for synchronizing, instead of lamps; a voltmeter can be safely connected between the machines because of its high resistance. When the machines are in synchronism, the voltmeter pointer comes to zero; otherwise it swings to and fro.

(3) In synchronizing three-phase machines, lamps must be provided in at least two phases. It is not sufficient to have the lamps in one phase only, because when A is connected to A_1 (Fig. 366), B may be connected to C_1 and C to B_1, thus causing a partial short-circuit.

(4) Automatic synchronizing devices have recently been put on the market; they operate on the relay principle and automatically close the main switch when the machine is in synchronism. It is not possible at this writing to pass judgment regarding the practicability of these automatic synchronizers, but should they prove to be a success, they will materially reduce the amount of time necessary for synchronizing large alternators. An automatic synchronizer is described in the *Electric Journal*, 1907, p. 490.

(5) With two *direct-current* generators working in parallel, the amount of power delivered by one of the machines can be increased by increasing its field excitation (§ 312). Such is not the case with *alternators*: Increasing the excitation merely produces *wattless* currents, which tend to demagnetize the field and thus reduce its strength to the former value. The only way to increase the *output of an alternator*, working in parallel with other machines, is to increase the *input into the driving motor*, or the prime mover. Then the *working* component of the current is also increased.

466. EXPERIMENT 21-E. — Exercises in Synchronizing Alternators. — The experiment should be conducted as follows:

(1) Practice synchronizing a single-phase alternator, or one phase of a polyphase alternator, using it as a single-phase machine. After this has been successfully accomplished, the synchronizing of three phases may be tried. The student is warned against closing the main switch too soon; the lamps should remain extinguished several seconds at a time, in order to be sure that the machine is in synchronism and at the right speed.

(2) Synchronize with lamps crossed, as per Figs. 367 and 368.

(3) Try synchronizing with a voltmeter.

(4) Connect up and observe the operation of a synchronism indicator.

Repeat each experiment several times, and form your opinion as to how long it should take to synchronize the machine under ordinary commercial conditions, from the time it is started from rest until it is connected to the bus-bars and takes its share of load.

(5) See what effect it has on the machine if it is switched in without having the three necessary conditions fulfilled (§ 462): the voltage of the machine not brought up to the proper value, the machine not in

phase with the supply voltage, etc. This produces a partial, or even a complete, short-circuit; hence, be sure that the machine is properly protected by a reliable circuit-breaker, which will instantly open the circuit.

(6) Vary the excitation of the alternator without increasing the input into the driving motor: It will be found that the current may be increased several times, while the wattmeter reading will be comparatively little affected; this will show that the increase in current is due to a wattless component. Some increase in the wattmeter reading will be, however, noticeable, due to a larger copper loss with heavier currents (see under (5) in the preceding article).

(7) After the machine has been synchronized and connected t
supply, open the main switch of the driving motor; the set w
tinue to rotate, the generator having evidently become a motor
the other machine. To make this still more evident, instead o
the motor switch, leave the direct-current motor connected to it
and gradually increase its field current. The counter-e.m.f. of t or a
will increase, and the current drawn from the line will diminish.
this counter-e.m.f. becomes higher than the e.m.f. of the supply ose
motor begins to send current back into the circuit, thus becoming
direct-current generator. At the same time, the power generated by the alternator will diminish, and finally the wattmeter will begin to read in the opposite direction, showing that the machine now takes in some power, instead of generating it; in other words, it is acting as a *synchronous* motor.

Thus, by properly regulating the energy relations in the driving motor, the alternator can be made to operate either as a generator, or as a motor.

Report. (1) State how long it should take an experienced man to synchronize the alternator with lamps; with a voltmeter; with a synchroscope.

(2) Describe the phenomena observed when the main switch was closed without having the machine properly synchronized.

(3) Plot, to alternator field current as abscissæ, the results of the run (*b*), specified above. Plot the armature current, watts, working component of the current, its wattless component and the current input into the driving motor. Give an explanation of the curves.

(4) Plot, to the field current of the driving motor as abscissæ: currents in the armatures of both machines, volts, watts, and the efficiency of the set. In plotting watts consider the output of the alternator as positive, the input into the synchronous motor as negative; use the opposite signs for the driving motor.

OPERATING FEATURES OF SYNCHRONOUS MOTORS.

467a. It has been pointed out in § 466 [under (7)] that an alternator may act under certain circumstances as a motor, and is called in this case a *synchronous* motor. The action of such a motor may be explained as follows: Suppose the machine to be single-phase and to be brought up to the required speed by some external means. Assume that at a certain moment, the relative position of the pole-pieces and of the armature winding is such that the winding attracts the pole-pieces (Fig. 357). As the machine is supposed to revolve synchronously, the pole-pieces change their position during one alternation of the supply current
ne pole pitch, so that the north poles come in place of the south
Elect. and vice versa. At the same time the direction of the armature
(5) is reversed, so that the mutual force between the two is again
amount, and not repulsion.
increasi other speed but synchronous, there would be a repulsion at
nators: ments and an attraction for the rest of the time; the resultant
which aken over a considerable length of time is zero. This shows
forme is necessary to bring the machine up to its synchronous speed,
wo re it is capable of developing mechanical power. The same expla-
tion holds true for each phase of a two-phase or a three-phase machine,
the actions of the phases being added together.

Another explanation limited to the case of polyphase synchronous motors is, that the polyphase armature winding produces a revolving field (see § 520) which rotates synchronously in the air-gap. The field poles of the machine must revolve at the same speed in order that there be a constant attraction between the two magnetic fields; otherwise south poles and north poles are brought together in succession and the resultant attractions and repulsions neutralize each other.

467b. Starting Synchronous Motors. — The above explanation of the action of synchronous motors shows that they must be started and brought up to full speed, before being capable of carrying a load. The following means are used for starting synchronous motors and bringing them up to speed:

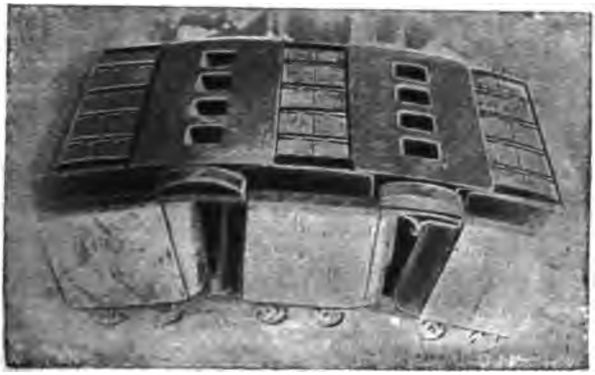
(1) A small induction motor, usually mounted on the same shaft with the synchronous motor, and disconnected from the supply after the synchronism has been reached.

(2) If a source of direct current is available, the exciter machine belted or direct-connected to the motor, may be used for starting.

(3) Synchronous motor itself (if polyphase) may be converted i an induction motor and started as such.

For further details see § 506 on starting rotary converters.

practically the same as are described there. A type of dampers, used with revolving field machines for preventing hunting, is shown in Fig. 374. The dampers consist of heavy copper wedges driven in between the tips of the pole-pieces, and are identical in their action to those shown in Fig. 392.



374. Copper bridges (dampers) between the poles of an alternator or a synchronous motor, to prevent hunting.

FREQUENCY METERS.

474. Instead of measuring speed of alternators by speed indicators, or tachometers, it is convenient to have an instrument which reads directly in cycles or alternations, and may be placed on the switchboard at any distance from the machine. Such instruments are called *frequency meters*, and are often found in large generating stations and sub-stations. The type largely used in this country is based on a purely electrical split-phase arrangement. Another type, popular in Europe, utilizes vibrating reeds attuned to different frequencies. These two types of frequency meters are described in the following articles.

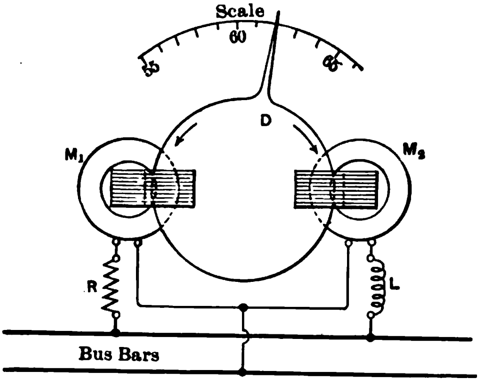


FIG. 375. An induction-type frequency meter (Westinghouse).

475. Split-Phase Frequency Meter. — The frequency meter shown in Fig. 375 is essentially a combination of two induction voltmeters described in § 42. Two split-phase electromagnets M_1 and M_2 are each of the same construction as the one shown in Fig. 43. They act in opposite directions on the aluminum disk D, thus constituting a differential voltmeter. To make the instrument respond to changes of frequency, the winding of one of the electromagnets is connected in series with some inductance L, and the other winding with some resistance R. The current in the branch M_1 containing the resistance is practically independent of frequency; the current in the branch M_2 decreases as the frequency increases, thus giving preponderance to the electromagnet M_1. To each frequency there corresponds a definite position of the aluminum disk, and the instrument is calibrated in cycles per second. The calibration is made by connecting the frequency meter to an alternator, the speed of which is measured directly by a speed counter.

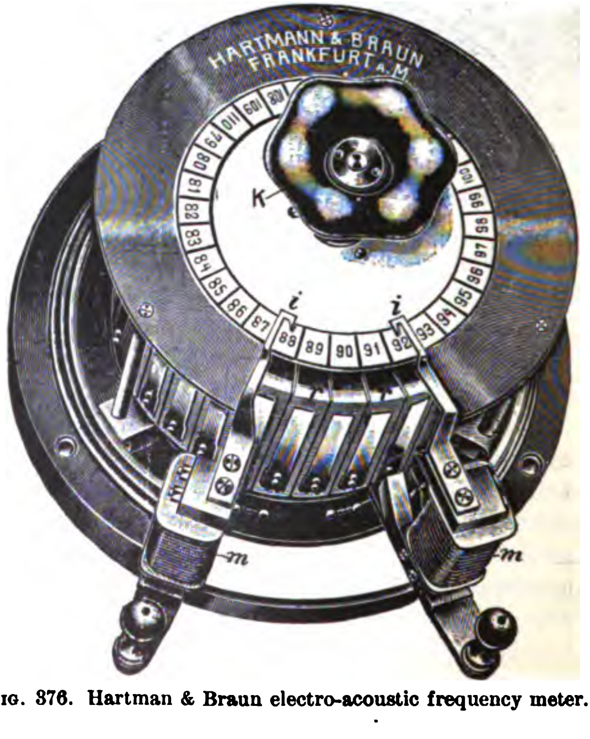


FIG. 376. Hartman & Braun electro-acoustic frequency meter.

476. Resonance or Vibrating Reed Frequency Meter. — The frequency meter shown in Fig. 376 is based on an entirely different principle, namely, on that of vibrating steel reeds. A steel strip, Fig. 377, fastened at one end and free at the other end has a natural vibration period, and is easily set into vibrations by outside impulses of the right frequency. The reeds are made of different length and different inertia, so as to vibrate at different frequencies, and are mounted in a circle. By means of the handle the reeds may be brought in succession before the poles of two laminated electromagnets shown in front. These electromagnets are connected to the alternating-current supply, the frequency of which it is desired to measure. When one of the electromagnets is opposite the correct reed it vibrates violently producing a distinct tone; the corresponding frequency is read on the scale.

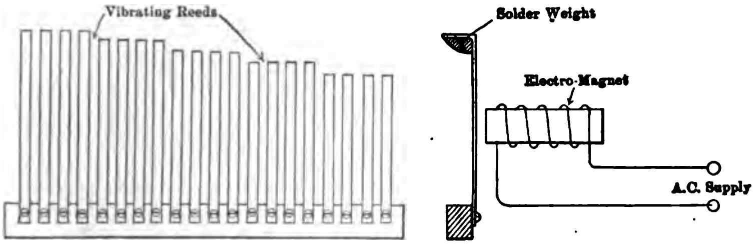


FIG. 377. Measuring frequency of alternating current by means of vibrating steel reeds.

The two electromagnets are at first set close to each other, so as to act on the same reed; then, when the proper reed has been located, the magnets are separated so as to inclose three or five reeds with the right reed in the center. This practically stops its vibration, and the sound which would be annoying, while deviations from the standard frequency may be noted on the adjacent reeds.

It has been found that the reeds may be made to vibrate by currents of double their normal frequency if direct current be allowed to flow at the same time through separate windings on the cores of the electromagnets, thus polarizing the reeds in one direction. This doubles the useful range of the instrument. The explanation is, that a non-polarized reed is attracted by the current of either direction, while a polarized one is attracted by impulses of one direction only, being repelled by the opposite impulses. Consequently, a double of the original frequency of current is required to make the reed vibrate at its natural frequency.

Frequency meters, built on the same principle for switchboard service, have electromagnets extending over all the reeds. The vibrating reed is observed on the dial, instead of being noted acoustically. This makes the instrument direct-reading without the necessity of turning the knob. The same principle is applied to tachometers intended to be read at a distance. An interrupter is belted or direct-connected to the shaft, the speed of which is to be measured, and a battery is connected to the frequency indicator, through the interrupter. The number of interruptions or vibrations depends on the speed of the shaft, and the instrument may be calibrated in revolutions per minute, or in any other desired units.

CHAPTER XXII.

ALTERNATORS AND SYNCHRONOUS MOTORS—COMMERCIAL TESTS.

477. THREE important points in the performance of alternators with respect to which certain guarantees are usually made in contracts of sale, are—efficiency, temperature rise, and per cent regulation.

These guarantees are usually expressed about as follows:

(1) The efficiency of the machine shall not be less than . . . . per cent at full load, not less than . . . . per cent at three-quarter load, etc.

(2) Temperature rise, under definitely specified conditions of operation, shall not be more than . . . . degrees Centigrade.

(3) Regulation, or voltage drop between no load and full load, shall be within . . . . per cent.

In many cases it is out of the question to test large alternators in actual operation, especially before they leave the factory. Various test methods have been devised for checking the above three points, without expenditure of much power. The most important of these tests are described below.

EFFICIENCY.

478. The efficiency of alternators and synchronous motors is determined in practice by the same two methods, as the efficiency of direct current machines, viz., from the losses (Chapter XVII), and from an opposition run (Chapter XVIII). For general information concerning efficiency and losses see §§ 350 to 353; most of the explanations given there for direct-current machines are applicable, with self-evident changes, to alternating-current machines. Here, also, iron loss and friction are determined by either of the three methods used with direct-current machines, namely: (*a*) the machine is driven mechanically by a small auxiliary motor; (*b*) the machine is driven electrically as synchronous motor; (*c*) by the retardation method. The first of these methods is the one most used in testing alternators.

479. EXPERIMENT 22-A.—Efficiency of an Alternator from Losses.—For this test, drive the machine by a small motor at the rated speed, with various values of exciting current, and note the input into the driving motor. Finally take off the belt and determine the losses in the driving motor itself. For details, see the conclusion of § 353.

Measure the resistances of the field and of the armature by the drop-of-potential method. Before leaving the laboratory, ascertain the rated field current at full load, or assume it with sufficient margin above that at no load, unless an actual load test can be performed.

In machines wound with heavy conductors, not sufficiently subdivided, copper loss in the armature is considerably increased by eddy currents in these conductors. In such cases it is well to determine the actual copper loss in the armature from the so-called short-circuit test. The armature winding is short-circuited through suitable ammeters, and the machine is driven at the rated speed, with such an excitation as to give the full rated current in the armature. The iron loss is negligible, the field being very weak, so that the input into the driving motor corresponds to the copper loss and friction in the alternator. The friction is determined separately by driving the machine without excitation, and the copper loss in the armature calculated as the difference of the two.

Report. Show actual connections used, and describe the performance of the test. Plot an efficiency curve for non-inductive load, and also plot separate losses to kilowatts output as abscissæ. The excitation loss is usually calculated from the formula i^2r, where i is the exciting current and r the resistance of the field winding. This is not quite correct; the loss in the field rheostat must also be charged to the machine, because it constitutes an unavoidable loss of energy, and is present in every alternator. It is more correct, therefore, to figure out this loss by the formula ei, where e is the standard voltage of the direct-current supply, or of the exciter. Another way is to figure out the efficiency of the alternator without the excitation loss, and to state kilowatt loss in the field separately.

480. EXPERIMENT 22-B. — Efficiency of an Alternator from the Air-Box Test. — All the losses taking place in electric machines are finally converted into heat. This gives a method for determining the sum total of the losses by measuring the quantity of heat (in thermal units) developed in a machine during a certain period of time. The machine under test is inclosed in an air-tight box with walls made of poor conductors of heat, and is run under such conditions as to have all the losses present to their full value. A definite quantity of air per minute is blown through the box, and its temperature measured at the intake and at the exhaust. After a certain number of hours of run, the temperature of the machine becomes constant, all the heat developed being conveyed away by the air. Knowing the rate of air discharge, and the difference of temperature, the thermal units developed may be calculated from the specific heat of air. Converting them into watts (see § 714) gives the total losses of the machine.

The difference of temperatures of the air is usually quite small and must be measured very accurately. Sensitive electrical thermometers with an accuracy of at least 1/10 of a degree are well adapted for the purpose. The air-box method has been applied in a few cases for acceptance tests of very large alternators, in power houses where it was not possible to check the guaranteed efficiency by any other method.

The experiment can be arranged so that it will not be necessary to know the absolute quantity of air supplied by the blower. For this purpose the machine is stopped, and a *known* amount of power is supplied within the box, for instance, by passing a current through the armature windings, from the outside. The blower must continue to run at the same rate as during the regular test. From the difference of temperatures after steady conditions have been established, the total loss in the machine can be calculated. Let, for instance, the difference in temperature during the load test be 10 degrees C., while it was only 5 degrees C. when 2500 watts were delivered to the armature from outside. Evidently the total loss in the machine was

$$2500 \times \frac{10}{5} = 5000 \text{ watts.}$$

481. Efficiency from an Opposition Run.—Two alternators may be tested for efficiency under actual load conditions without expenditure of much power by loading them in opposition (Chapter XVIII). Thus, two identical machines may be driven at full load by a small auxiliary motor, as is explained in § 378 in application to direct-current machines. The machines must be rigidly coupled together so as to preserve the same frequency and the same phase relation throughout the test. An ammeter, a voltmeter and a wattmeter are connected between the machines. One machine acts as a generator, the other as a motor. Any desired load, at practically unity power factor, may be obtained by varying the relative angular position of the armatures, by means of the coupling. Leading or lagging components are added by varying the field excitation of one of the machines.

Let the input into the auxiliary driving motor (corrected for the losses in the motor itself) be w, and the wattmeter reading be W. The efficiency of both machines may be assumed to be the same; let it be = η. The electrical power produced by the machine acting a generator is converted into mechanical power in the machine acting a motor, and is delivered back to the generator through the shaft an the coupling. The driving motor merely supplies the losses of the se

Expressing these relations mathematically, we have: the input int the synchronous motor being W, its mechanical output, delivered t the shaft is ηW; in addition to this the auxiliary motor supplies to the shaft an amount of power = w. Thus the power available for drivin the generator is $\eta W + w$. The efficiency of the generator being η, it electrical output is $\eta\ (\eta W + w)$; on the other hand, we know from the wattmeter reading that the output of the generator is W. Thus we have the condition

$$\eta\ (\eta\ W + w) = W.$$

This is a quadratic equation with respect to η; the positive solution is

$$\eta = -\frac{\frac{1}{2}w}{W} + \sqrt{\left(\frac{\frac{1}{2}w}{W}\right)^2 + 1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

With the aid of this formula the efficiency of either machine may be calculated from the observed values of W and w. The copper loss in the field must be taken into account separately.

482. Behrend's Split-Field Test. — As two identical machines are not always available, Mr. B. A. Behrend proposed (after a suggestion by Mr. W. M. Mordey) to connect two halves of the same alternator in opposition to each other, using one half as generator, and the other as synchronous motor. The connections, in application to a three-phase machine, are shown in Fig. 378. The armature winding is short-circuited through three ammeters; the field winding is divided into two halves, connected in opposition, and each provided with a regulating rheostat. The same scheme is shown more in detail in Fig. 379. In order to conduct the current to the middle point of the field winding, an extra slip ring c is provided, in addition to two regular slip rings a and b. Or, else, the middle point of the winding is simply connected to the spider, and a temporary brush is mounted on one of the bearings, so as to make a contact with the shaft of the machine. The current in each half of the field is regulated independently by the rheostats R_1 and R_2, and ammeters A_1 and A_2. Only two ammeters are used in the armature circuit, because, if the currents in the two phases are equal, the third current is necessarily the same, the three vectors forming an equilateral triangle.

When the currents in the two halves of the field are equal, no currents flow in the armature winding, the e.m.f.'s induced in two halves of the armature being equal and opposite. This corresponds to the case of two identical machines coupled in opposition and excited to the same degree. By weakening the current in one half of the field winding, the two e.m.f.'s induced in the armature are unbalanced, and a current flows through the ammeters *AA*. The machine does not deliver any useful output, so that the input into the driving motor is merely sufficient to cover the losses in the machine itself. By having the same armature current and the same field current (on the strongest side) as at full load, the input into the driving motor represents the sum total of the losses in the alternator under test. The copper loss in the field must be calculated separately, since the exciting current is supplied from an independent source.

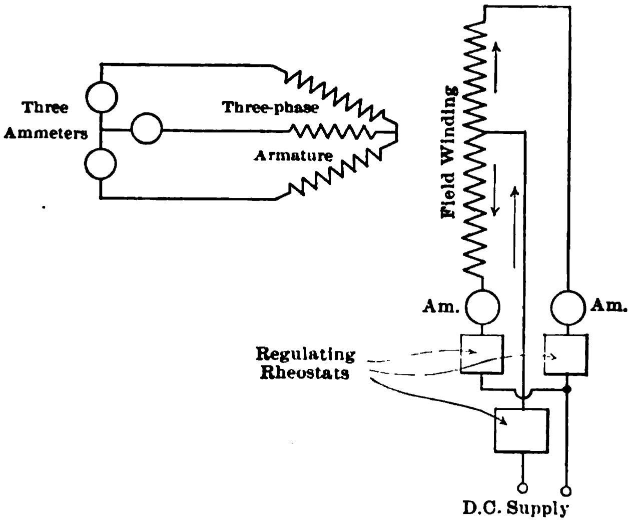


FIG. 378. Diagram of Behrend's split-field method for testing alternators.

In order to determine the proper value of the exciting current on the "generator" side of the machine, the voltage is measured across one set of the armature coils, by puncturing the insulation with sharp points on the voltmeter leads. Suppose, for instance, that the machine under test is a 2200-volt, 150-ampere, 24-pole, 3-phase, *Y*-connected alternator. The voltage between each terminal and the neutral point is $= 2200 \div \sqrt{3} = 1271$ volts. A 24-pole machine has 12 groups of armature coils per phase (see Fig. 354); thus, the voltage across one set of coils is $1271 \div 12 = 105.9$ volts. The two rheostats, R_1 and R_2, must be adjusted so as to produce 150 amperes in the ammeters *AA*, and 105.9 volts across a set of armature coils. Then the losses in the machine are the same as under actual load conditions.

An objection may be raised, that iron loss is smaller than under actual load conditions, only one half of the poles having the required field strength. Mr. Behrend proved, however, by direct tests that the core loss determined from a no-load run is practically the same as with the split-field test.

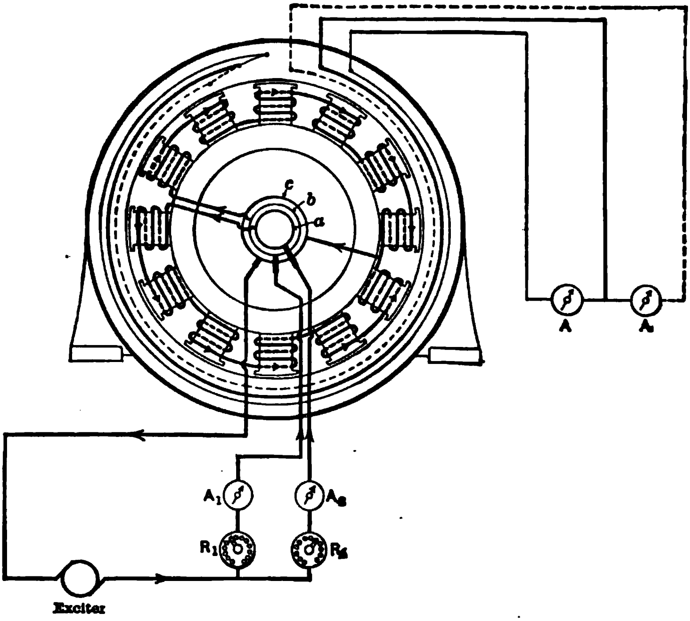


Fig. 379. Connections for Behrend's split-field test on a three-phase alternator.

If the corrected input into the driving motor is w, and the rated output of the machine is W,

$$\text{Efficiency} = \frac{W}{W + w + \text{field } i^2r}$$

Behrend's method is limited to alternators with a considerable number of poles, not less than eight or ten. Evidently with the field connections, shown in Fig. 379, there are two adjacent north poles, and two adjacent south poles, which cause an irregularity in the induced e.m.f. With a considerable number of poles this irregularity does not seriously affect the results.

483. EXPERIMENT 22-C. — Efficiency of an Alternator from an Opposition Test. — The experiment is performed either on two identical machines, as explained in § 481, or by using the split-field arrangement shown in Fig. 379. Take readings on partial loads, at full load, and also on an overload, so as to be able to plot a complete efficiency curve. It is well to determine the losses separately, as in § 479, so as to check the efficiency from the losses.

TEMPERATURE RISE.

484. The rise of temperature, due to the losses, is of importance in four parts of the machine:

(1) Armature windings.
(2) Armature core.
(3) Field winding.
(4) Collector rings.

The temperature rise in the two latter parts depends on field current only, and may be determined by running the machine a certain number of hours with the field fully excited, and the armature circuit closed or open, as the case may be. The temperature of the collector rings is measured by thermometers; that of the field coils by thermometers, as well as by the increase in resistance. For copper, resistance increases by 0.42 per cent for each degree Centigrade.

It is more difficult to obtain the correct conditions for temperature rise in the armature, because of an exchange of heat between the core and the winding. For this reason, both copper and iron must be brought up to the temperatures during the test, which they are supposed to have during the actual operation. This means, that a full-load current must be flowing through the armature windings, and at the same time a normal flux maintained in the core. Several methods have been proposed for obtaining these conditions without the expenditure of power, corresponding to full-load output. Some of them are described below, and their limitations indicated.

485. Separate Heat Runs for Copper Loss and Core Loss.—When machines of a standard type, on which heat runs have been previously made, are tested, the purpose of the temperature test is chiefly to ascertain if the new machine may be expected to give the same temperature rise in actual operation, as the machines previously tested. In this case it is permissible to run the machine first on open circuit for iron loss only, and then with the armature short-circuited for copper loss alone. If the temperature rise under these conditions is not more than with the machines previously built, which proved satisfactory in operation, the test is considered as conclusive.

Of course, such a run is satisfactory for the manufacturer, but not for the customer; a modification described in the next article makes it possible to approach much more closely the actual temperature rise.

486. Hobart and Punga's Temperature Test. — With this method the machine under test is run *alternately* on iron loss and on copper loss so as to get the same *average* amount of losses converted into heat in the armature, as under actual load conditions. An example may serve to illustrate this method: Suppose the armature copper loss in an alternator to be equal to 20 kw. and iron loss 36 kw. The machine is run, say, 10 minutes with the armature short-circuited and with such a current flowing through it as to give 60 kw. loss. Then the machine is run 20 minutes on open circuit with a field excitation such that it gives an iron loss equal to 54 kw.; after this it is run again on copper loss, etc. Under such conditions the same *total* amount of heat is supplied to the machine as if 20 + 36 kw. were supplied continually. Indeed, 60 kw. supplied ⅓ of the time are equivalent to 20 kw. supplied all the time, and 54 kw. converted into heat ⅔ of the time are the same as 36 kw. supplied continuously. At the end of several hours the machine reaches the same temperature as if these losses were supplied simultaneously all the time, with the advantage that only a small amount of energy is necessary when either iron or copper loss is supplied alone.

This method implies a knowledge of the magnitude of iron loss; the same may be determined by driving the machine at no load, first excited, and then with the field circuit open. The difference of input into the driving motor gives the iron loss of the alternator.

In order to save time and power, it is advisable to heat up the machine by an overload, viz., by running it for a time with higher armature currents and higher field excitation than are calculated for the heat run. When the machine is warmed up, the copper loss and the iron loss are reduced to their correct values. With this method the machine assumes its final temperature in a much shorter time than it would otherwise.

Temperature rise in the field winding with this test may not be the same as in regular operation. When the machine is running on iron loss, the field is above normal, while it is considerably below normal during the periods of run for copper loss. It is possible, however, to so adjust the relative duration of the two parts of the run and the values of the losses, that the *total* amount of heat developed in the field winding will be the same, as under actual load conditions. For details of the necessary calculations, see the original paper in the *Electrical World*, 1905, Vol. 45, p. 759; the method is also explained in Hay's *Alternating Currents*, p. 175.

487. Heating the Armature with Direct Current. — It may seem at first that the most natural way to conduct the heat run on an alternator would be to run it with full field on iron loss, and at the same time send a direct current through the armature windings, to produce the required copper loss. There are, however, two great objections to this method:

(1) The armature must be so connected, that the total induced A. C. voltage will be equal to zero; otherwise a short-circuit would be produced through the source of direct current.

(2) The stationary field produced by the direct current in the armature windings should not induce high e.m.f.'s in the field winding.

With a single-phase machine, direct current may be sent through two halves of the armature winding connected in opposition; a three-phase machine may be temporarily connected in delta, and direct current introduced by opening one of the vertices of the delta. There are also some other combinations possible, with the use of transformers, etc., but, as a matter of fact, this method is very little used in practice.

488. Heat Run by Opposition. — If a second alternator of suitable size is available, the machine under test may be run in connection with the other machine used as a synchronous motor. By under-exciting a synchronous motor, it may be made to take a large apparent input with a comparatively small expenditure of power. The arrangement is similar to that described in § 481, except that in this case it is not necessary to have the two machines rigidly coupled together.

Instead of using two separate machines, the two halves of the machine under test may be connected in opposition, as in Fig. 379, and an adjustment of currents and voltages made, as explained in § 482. The machine runs with the same copper loss and the same iron loss, as under actual load, and gives therefore the correct temperature rise. This method gives satisfactory results only when the number of poles is not less than eight; hence, unfortunately, it cannot be used with modern high-speed turbo-generators.

489. Critical Comparison of the above Methods. — When only approximate results are required, or the test is merely a check to compare the machine with machines previously built, separate heat runs for copper loss and iron loss (§ 485) are sufficient.

For machines with a sufficient number of poles, Behrend's opposition test (§ 488) seems to be the most practical.

Turbo-generators may be loaded either on synchronous motors (§ 488), or heated by the Hobart and Punga's method (§ 486).

With the advent of the steam-turbine, large manufacturing companies begin to provide facilities on their testing floors, such as to test sets up to 2000 kw. and more under actual load conditions. In some cases, an artificial load for temperature run is provided even in power houses, where the machines are installed. Whenever possible, temperature run should be performed with the machine actually loaded, because this permits the determination simultaneously of the regulation of the machine, and the testing of its mechanical features, such as heating of bearings, vibration, etc.

490. EXPERIMENT 22-D. — Heat Run on Alternators. — The purpose of the experiment is to make practical application of the methods described in §§ 485 to 488, rather than to obtain actual numerical data on temperature rise. A regular temperature run takes at least six hours, and with large machines even ten or twelve, while the methods themselves may be illustrated in a considerably shorter time. It is therefore recommended that the student connect up the machine under test, and run it for a few minutes according to the various methods described above, merely to make clear to himself the connections and the operation.

After this, select one of the methods, say Hobart and Punga's, or Behrend's, and run the machine for an hour or two. Take regular temperature readings by thermometers every five or ten minutes, and check them from time to time by the increase in resistance. Select such an overload as to obtain a considerable temperature rise in the comparatively short time allotted for the experiment.

Report. Give the actual connections used, the readings obtained with various methods, and plot heat curves as far as the run was carried. If desired, extrapolate the heat curves by the method described in § 402.

VOLTAGE REGULATION.

491. The meaning of the term "regulation" is explained in § 454; the physical significance of the three factors affecting regulation is given in § 455. These factors, in the order of their importance, are:

(1) Armature reaction.
(2) Armature inductance.
(3) Ohmic drop.

It has also been shown that regulation and voltage drop depend on power factor of the load, as well as on the value of the current. The methods by which regulation of alternators can be determined from actual tests, or predetermined by calculation, are described below.

492. Regulation at Power-Factor Zero. — According to the accepted definition of regulation, it refers to the variation of the terminal voltage, at a power factor of 100 per cent. In testing machines, however, and for purposes of investigation, it is much more convenient to consider voltage drop and regulation at the power factor zero. The reasons are as follows:

(1) In many cases it is easier to obtain a large load at a power factor practically zero, than a full non-inductive load. Moreover, the regulation varies considerably at high values of power factor, but is practically constant at any value of power factor, say below 20 per cent. This may be seen by consulting Figs. 361 and 364.

(2) The drop at a power factor zero is the largest possible, and is therefore more acceptable as a measure of the quality of the machine. Besides, in calculating regulation at other values of power factor, on the basis of an observed regulation at zero power factor, less error is committed than by beginning with the regulation at the power factor of 100 per cent, and figuring out from it the regulation at other values of power factor.

(3) The theory of armature interference is much simpler at a power factor zero, because the armature winding produces only a demagnetizing, but no distorting action on the field (Fig. 358).

In view of the above considerations, there is a tendency on the part of some progressive engineers to introduce into contracts, and into general engineering practice, *regulation at a power-factor zero as a standard*, instead of regulation at a power factor of 100 per cent, used at present.

Regulation at power factor zero may be conveniently represented by two curves, NV and OV_0 (Fig. 380). The former gives the values of the terminal voltage to field current as abscissæ, with a full-load current, lagging 90 degrees, flowing through the armature. On the other hand, the curve OV_0 gives terminal voltage at no load. If gA is the rated voltage of the machine, the regulation at the power factor zero is represented by the per cent ratio A_0A to gA. The curve NV may be obtained experimentally by loading the machine on an under-excited synchronous motor, on induction motors running light, on choke coils, etc.

The method for *predetermining* the regulation at other values of power factor, from the regulation observed experimentally at a power factor zero, is explained in §§ 501 and 502 below.

493. EXPERIMENT 22-E. — Regulation of an Alternator from a Synchronous Motor Load. — The experiment is performed as in § 457, but the power factor of the load (synchronous motor) is kept always below 20 per cent. It is important to have a synchronous motor of sufficient size, so as to obtain enough points on the curve NV (Fig. 380). Begin with the heaviest current which the generator can carry and with the highest voltage possible at this load with a low power factor. The load is adjusted by regulating the field current of the synchronous motor. Gradually reduce the excitation of both machines, so as to get a curve similar to NV. Take another curve for a smaller value of the armature current, etc. Finally take the no-load saturation curve OV_0.

No exact wattmeter readings are required, except that the power factor must be sufficiently low; actual experience shows, that when the power factor is below 20 per cent, terminal voltage is practically independent of its actual value, and depends on the field current and the armature current only.

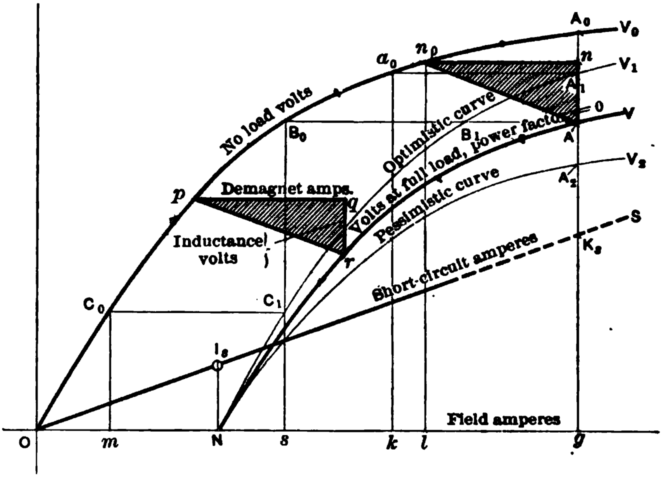


Fig. 380. Curves of voltage regulation of an alternator at zero power-factor.

Report. Plot the observed curves to field amperes as abscissæ (Fig. 380); determine per cent regulation at power factor zero. Figure out per cent regulation at power factor 100 per cent and 80 per cent by the method described in § 501, or by the more accurate method shown in § 502.

494. Regulation by Split-Field Test. — Instead of using a synchronous motor, one-half of the alternator under test may be converted into motor (Fig. 379): this is the same (Behrend's) method as is described in § 482 for determining efficiency, and in § 488 in application to temperature run. By suitably adjusting the field current on the "generator" and the "motor" side of the machine, a full-load current may be made to circulate through the armature. This current is practically wattless, because of the high inductance and comparatively low resistance of the armature windings; thus the conditions are practically the same as with the machine loaded at a power factor zero.

The terminal voltage is determined either by a voltmeter, as explained in § 482, or is calculated from the so-called short-circuit curve (Figs. 380 and 381). The short-circuit curve is obtained by short-circuiting the armature windings upon themselves, through suitable ammeters, and gradually increasing the excitation, with the fields connected as in actual operation. The curve gives the values of the field current necessary for driving different currents through the armature windings. It is usually a straight line, the saturation under the conditions of the test being quite low.

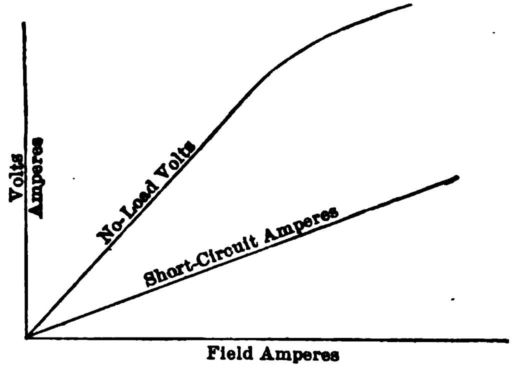


FIG. 381. No-load and short-circuit characteristics of an alternator.

The short-circuit curve is used for calculating the fictitious terminal voltage of a machine with fields connected as in operation, in the following way: Assume for a moment that the " generator " half and the " motor " half are two separate machines. The voltage gA at the generator terminals is equal to the sum of the motor counter-e.m.f. plus an excess voltage, necessary for driving the current through the impedance of the motor armature. It will be seen from Fig. 373 that when the power factor is zero, the two latter e.m.f.'s are simply added arithmetically, and not vectorially. Referring again to Fig. 380, let Og be the excitation of the generator, and Om that of the motor. Also let ON be the field-current necessary for driving the full-load current NI_s through the armature windings (here we make use of the short-circuit curve).

Thus, the generator must supply ampere-turns Om and ON to balance the action of the motor. Making $ms = ON$, we obtain the total excitation Os necessary for balancing the action of the motor. To this excitation there corresponds the voltage sB_0, which the generator has to supply at its terminals. On the other hand, the actual excitation of the generator is Og; having the two points B_0 and g, we find the point A on the required regulation curve NV. Other points on the curve NV are constructed by using different pairs of values of Om and Og, obtained from the test.

The method is strictly correct only when the saturation curve between O and B_0 is practically a straight line, so that the exciting current is proportional to induced voltages; this is very nearly the case under the conditions met with in practice.

We assumed in the above, that the generator and the motor are two separate machines, and showed how to figure out the terminal voltage gA without actually measuring it. The method can be applied directly to a machine with two halves connected in opposition, as in Fig. 379 where the separating points between the "generator" and the "motor" side of the armature are not accessible, since they travel synchronously with the field. The reasoning is exactly the same as above; it is not necessary to divide by two the induced voltages or the short-circuit current, as it may seem at first on the ground that only one-half of the armature coils are active. A little consideration will show, that the numerical relations are exactly the same as if the whole machine were used as generator, or as motor.

495. EXPERIMENT 22-F. —Regulation of an Alternator from Behrend's Split-Field Test. — The purpose of the experiment is to determine the regulation of an alternator at power factor zero. The theory of the method is explained in the preceding article; the connections are shown in Fig. 379.

(*a*) Bring up the field current on both sides as high as possible, and reduce it on the "motor" side, until a full-load current flows through the armature. Read the voltage across one set of armature coils, as explained in § 482, and exciting amperes in both halves of the field. Gradually decrease the field current on both sides, so as to keep the armature current constant, read again: field amperes, the voltage across the armature coil, etc. Take similar curves with other values of armature current.

(*b*) After this test take the no-load saturation and the short-circuit curves, as in Fig. 381, and measure the resistances of the armature windings.

Report. Plot the curves OV_0, OS, and NV, as in Fig. 380. See if the terminal voltages measured by the voltmeter check with those predetermined from the short-circuit curve. Figure out per cent regulation at power factor zero, 80 per cent, and 100 per cent, as explained in §§ 501 and 502.

496. Optimistic and Pessimistic Limits of Regulation. — The question of predetermination of regulation of alternators, without an actual load test, must be considered as only partly solved at present: The methods in use are but approximations to the actual complicated phenomena of armature interference. At the same time, it is a comparatively easy matter to predetermine *the limits* NV_1 and NV_2 (Fig. 380) between which lies the true regulation curve NV. These limiting curves were strikingly called by Mr. Behrend the "optimistic" and "pessimistic" curves. They are useful in cases, in which it is impossible to get the true regulation curve, or where it is deemed sufficient to know the limits.

The optimistic and the pessimistic curves may be easily plotted from the no-load saturation curve OV_0 and the short-circuit curve OS. It is explained in § 455, that the armature interference (neglecting ohmic drop) is caused partly by armature reaction, partly by armature inductance. The optimistic curve is obtained on the assumption that *all* of the armature interference is caused by the armature reaction, the pessimistic curve — on the assumption that *all* of it is caused by the inductance.

(*a*) The reasoning in plotting *the optimistic curve* is as follows: When the armature is short-circuited, and a rated current NI_s flows through it, the armature ampere-turns are equal to the field ampere-turns, because no field is left to induce a voltage for the external circuit. Therefore ON represents the demagnetizing armature ampere-turns with full-load current. These ampere-turns are the same at any terminal voltage, provided the current is the same and the power factor is near zero, as on short-circuit test. Thus, if an excitation Os is necessary to produce a certain voltage sB_0 *at no load*, an excitation larger than this by the amount equal to ON is necessary to produce the same voltage *with full-load wattless current* flowing through the armature. Plotting $B_0B_1 = ON$ we obtain a point B_1 on the full-load regulation curve NV_1.

The reasoning is the same for all points on the curve NV_1; they are obtained by merely plotting segments C_0C_1, B_0B_1, a_0A_1, etc., equal to ON. This would be correct, if the no-load saturation curve OV_0 were a straight line, so that the same number of ampere-turns ON would equally affect the voltage with all values of the exciting current. As this is not the case, the curve NV_1 is not the true regulation curve. A little consideration will show, that it gives values of terminal voltage which are too high; for this reason it is called the optimistic curve.

(*b*) In plotting *the pessimistic curve*, it is assumed that all of the drop in the armature is caused by the inductance of the armature windings, as in Fig. 360. Take a point K_s on the short-circuit curve, such that the current gK_s equals 3 times the normal current NI_s. It takes the voltage gA_0 to drive this current through the armature, hence, to drive the current NI_s through the armature, with the same excitation Og, would take 3 times less voltage. Let A_0A_2 be $= \frac{1}{3}gA_0$; then the rest of the voltage, gA_2, is available in the external circuit. Repeating a similar operation for determining other points on the curve OS and connecting the points, such as A_2, the pessimistic curve NV_2 is obtained.

A simple method for locating points on the pessimistic curve is as follows: To find the point corresponding to an ordinate, such as gA_0, connect O to A_0 and from N draw a parallel to OA_0; the point of intersection A_2 of this parallel with gA_0 lies on the pessimistic curve. To prove this, denote gA_0 by y and gA_2 by y_2, and suppose the current gK to be n times larger than the rated current NI_s. It takes a voltage y to drive the current gK_s through the armature; consequently, it will take $y \div n$ to drive NI_s through the armature. The rest, y_2, is available at the terminals of the machine. We have, thus:

$$y_2 = y - (y \div n),$$

or

$$\frac{y_2}{y} = \frac{n-1}{n} = \frac{gK_s - NI_s}{gK_s}.$$

But

$$\frac{gK_s}{NI_s} = \frac{Og}{ON};$$

substituting, we find

$$\frac{y_2}{y} = \frac{Og - ON}{Og} = \frac{Ng}{Og}.$$

Consequently, the line connecting the points O and A_0 is parallel to that connecting N and A_2; whence the above construction.

The reasoning in constructing the pessimistic curve would be correct if it were not for the effect of saturation in iron; a little consideration will show that the points, obtained by the method described above, lie below the actual regulation curve, whence the name pessimistic curve.

It is interesting to note, that if the no-load saturation curve OV_0 were a straight line, both the pessimistic and the optimistic curves would coincide, and would be represented also by a straight line parallel to the no-load saturation curve and passing through the point N.

497. EXPERIMENT 22-G. — Determination of Limits of Regulation of an Alternator. — The purpose of the experiment is to obtain the optimistic and the pessimistic curves of an alternator, as explained above (Fig. 380). The necessary tests comprise the determination of the no-load saturation curve and the short-circuit curve, Fig. 381. It is generally known, that polyphase machines give better regulation than single-phase machines. To observe this, take two short-circuit curves; with but one phase short-circuited, and with three phases short-circuited. When a phase is short-circuited between the terminals, with the machine Y-connected, two phases of the winding are connected in series. It is interesting to take another short-circuit curve, with one phase short-circuited between the terminal and the neutral point. In performing this test, measure the open-circuit voltages in the two other phases.

Report. Plot, for the three-phase machine, all the curves shown in Fig. 380, except the actual regulation NV, unless it has been determined from a previous test, for instance, as in § 493. Plot similar curves for the machine running single-phase, and compare the results, corresponding to the same value of field current. Plot also the voltages induced in the phases which were not short-circuited, and explain the physical meaning of these voltages. If actual regulation tests on the machine are available, show that with the three-phase machine, the actual regulation is nearer to the optimistic limit than with the single-phase machine. In comparing the single-phase and the three-phase data do not forget to reduce the voltages to the same number of turns per phase.

498. Predetermination of Regulation by Torda-Heymann Method. — The optimistic and the pessimistic curves (§ 496), which give two limits of actual voltage regulation, would coincide if the no-load characteristic were a straight line (Fig. 380). In good modern two-phase and three-phase machines the actual regulation is considerably closer to the optimistic than to the pessimistic limit, the influence of the armature reaction being much more prominent than that of the inductance of the armature winding.

On the basis of these two facts, Dr. Torda-Heymann* developed a method of predetermination of regulation, in which the "optimistic" method of construction is used, but the curve obtained is corrected for the effect of saturation in iron, and thus is made to represent the actual regulation of the machine. The effect of saturation Dr. Torda-Heymann expresses in what he calls the "apparent reluctance" of the machine, or the ratio of the exciting ampere-turns to the correspondin magnetic flux at no load. But the exciting ampere-turns are proportional to the field current, and the flux is proportional to the induced voltage. Hence, referring to Fig. 382, the apparent reluctance at the field current Og is proportional to the ratio $Og \div gA_0$. The coefficien

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* *Electrician* (London), 1904, Vol. 53, p. 6.

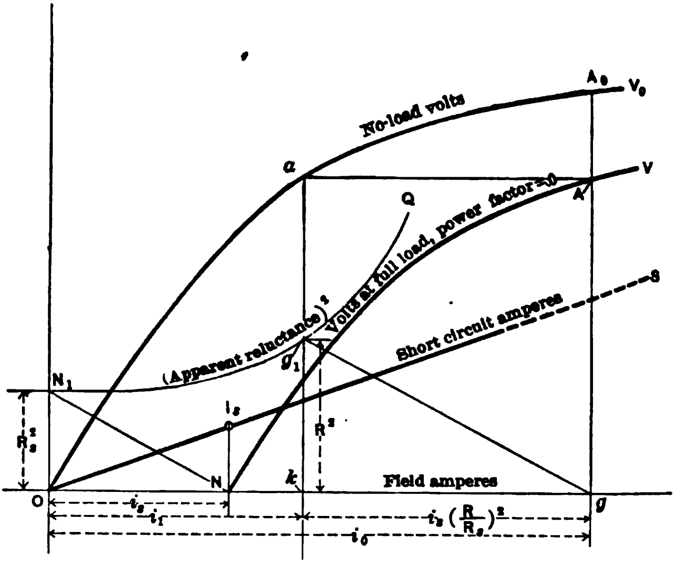


Fig. 382. The Torda-Heymann method for predetermination of regulation at zero power-factor.

of proportionality has no bearing on the method, the relative values only being used.

According to the optimistic method (§ 496a), the difference kg between the abscissæ at no load and at full load corresponding to the same terminal voltage is equal to ON (Fig. 380), or with the notations in Fig. 382,

$$i_1 = i_0 - i_s.$$

As has been pointed out, this is incorrect, because the same number of demagnetizing ampere-turns produce less effect on the flux with a higher saturation in the magnetic circuit than at a lower saturation. The correct relation is

$$i_1 = i_0 - i_s \cdot f(R),$$

where R is the *apparent reluctance* of the machine, as defined above.

Dr. Torda-Heymann has found theoretically and confirmed by the results of various tests that the function $f(R)$ may be represented, with a sufficient accuracy, by the ratio $(R \div R_s)^2$, where R is the reluctance corresponding to the actual saturation at the point k (Fig. 382), and R_s is the lowest limit of reluctance, with no saturation whatever. Thus, we have

$$i_1 = i_0 - i_s \cdot \left(\frac{R}{R_s}\right)^2.$$

In order to construct the curve NV of actual regulation at power factor zero, from the no-load and short-circuit curves, according to Torda-Heymann's method, plot first the curve N_1Q giving, to any arbitrary scale, apparent reluctances *squared*, preferably to such a scale that $ON_1 = 1$, 10, 100, etc. Take an exciting current, such as Ok, calculate the value $kg = i_s\ (R \div R_s)^2$ and thus locate the point g. Complete the rectangle $gkaA$; the point A lies on the required voltage regulation curve. The point g may also be located graphically by drawing g_1g parallel to N_1N; this may be easily proved from the similarity of the triangles g_1kg and N_1ON.

Torda-Heymann's method should be considered as a semi-empirical one, because of the correction $(R \div R_s)^2$; but it gives good results in practice. It is also quite flexible, because the correction may be given any other form to suit a particular type of alternator.

499. EXPERIMENT 22-H. — Predetermination of Regulation of an Alternator by the Torda-Heymann Method. — The curves required are the no-load saturation and the short-circuit curve (Fig. 381). The experiment is performed exactly as in § 497. In fact, the same data may be used for working out the results of both experiments. It is well to have an actual regulation curve, determined experimentally, so as to check it with the predetermined regulation, according to the Torda-Heymann method.

500. Regulation at Power Factors other than Zero. — With non-inductive load the current in the armature reaches its maximum in the relative positions of the poles and the conductors shown in Fig. 357. The reaction of the armature consists in this case in a strengthening of the original field on one side and a weakening of it on the other side of each pole. By following the armature interference from point to point in various positions of the armature winding, it may be shown that the armature reaction only distorts the field instead of weakening it, as when the power factor is zero (Fig. 358). Such a reaction is sometimes called the transversal reaction, while the demagnetizing action is known as the direct reaction. With power factors between zero and

100 per cent both kinds of armature reaction are present simultaneously. This circumstance makes the theoretical evaluation of the armature reaction, and the predetermination of voltage drop, rather intricate; the problem cannot be considered at present as solved, at least for practical uses. The methods described below are but a rough approximation to the actual conditions, but are used for the lack of a more accurate method.

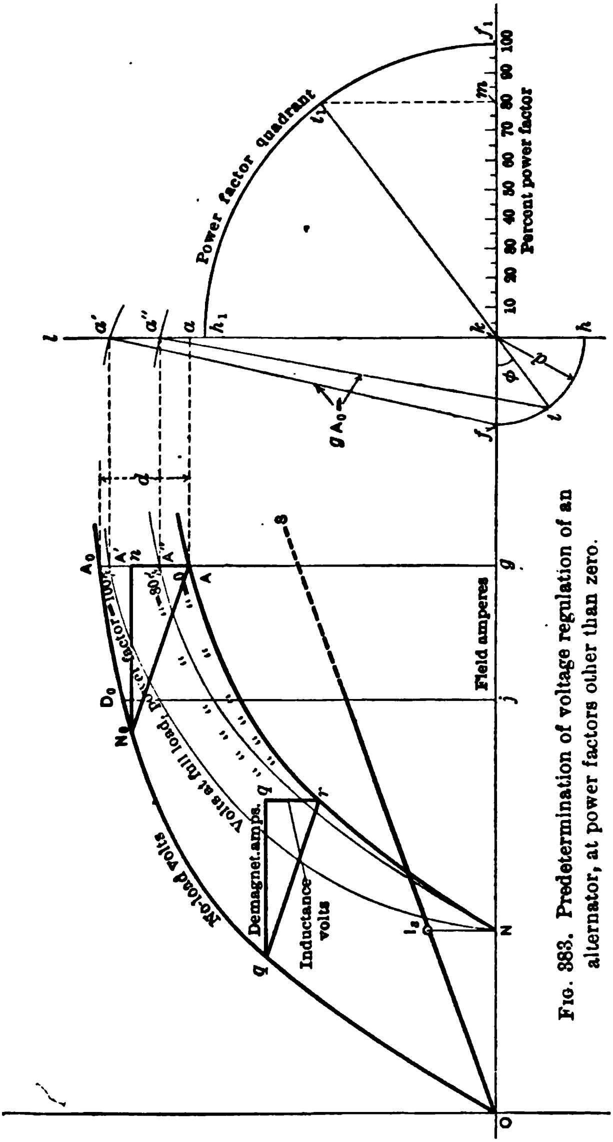


FIG. 383. Predetermination of voltage regulation of an alternator, at power factors other than zero.

501. Kapp's Diagram. — Kapp's method for predetermining regulation of alternators, at power factors other than zero, is shown in Fig. 383. The curves OA_0 and NA are the same as in Fig. 380, and are supposed to be determined from experimental data. Referring to the vector diagram, Fig. 360, the inductive drop ix is in phase with the terminal voltage E, when the power factor of the load is zero. Moreover, neglecting the ohmic drop ir, the induced e.m.f. E_0, and the terminal voltage E, are in this case in phase with each other, and the inductive drop is an *algebraical* difference of the two. At other values of power factor it is a *geometrical* difference of the two. In Fig. 383, gA_0 is the induced voltage, gA is the terminal voltage, and AA_0 is the total drop at power factor zero, due to both armature inductance and armature reaction.

At any other value of power factor, these three vectors form a triangle, such as *kta''*, the angles depending upon the power factor of the load. The side $ta'' = gA_0$ represents the no-load voltage; the side $tk = AA_0$ is the drop in the armature. Their difference *ka''* represents the terminal voltage of the machine at this particular power factor, and at the same current, to which the curve NA refers. Projecting a'' on the ordinate gA_0, the point A'' is obtained on the regulation curve for this power factor. At a power factor of 100 per cent, the triangle assumes the aspect *fka'*; at a power factor zero it is reduced to a straight line *hka*, similar to gAA_0. If complete regulation curves are desired, the same construction must be repeated for other ordinates of the saturation curves OA_0 and NA.

In order to read the power factor directly, it is convenient to draw the power factor quadrant $h_1t_1f_1$ with a radius such that it may be conveniently divided into 100 parts. The power factor is then read in per cent on the radius *kf*. This is evident when we remember that *km* is proportional to the cosine of the angle ϕ, and this cosine, by definition, is the power factor of the load.

An objection to this method of predetermining regulation of alternators is that it combines into one vector $A_0 A$ all the factors of the armature interference, which factors in reality follow different laws. A more accurate method is that by Potier described in the following article.

502. Potier's Diagram. — The method is similar to that shown in Fig. 383, except that the armature reaction is separated from the armature inductance before the diagram is constructed. Referring to Fig. 380, let n_0n represent the demagnetizing ampere-turns on the armature; when the exciting current is $= Og$, the actual excitation corresponds to Ol, the rest, lg, being destroyed by the armature reaction. To the excitation Ol corresponds the induced voltage ln_0, or ng. From this voltage the inductive drop nA must be subtracted, giving the terminal voltage gA. Thus, the total action of the armature interference is represented by the triangle n_0nA, n_0n representing the action of the armature reaction, nA that of the armature inductance. This triangle shows also the errors inherent to the pessimistic and the optimistic methods (§ 496). In the optimistic method, the horizontal line

a_0A_1 is taken instead of the hypothenuse n_0A; in the pessimistic method A_0A_2 is taken instead of the same hypothenuse n_0A.

If n_0A were known in its magnitude and direction, the regulation curve NV could be easily plotted from the no-load curve OV_0 by drawing parallel lines, such as n_0A, pr, etc. On the other hand, when OV_0 and NV are given, the vector n_0A may be determined by trials. Potier makes a tracing of the curve NV and moves this parallel to itself until it coincides as closely as possible with the curve OV_0. The direction of the movement determines the position and the magnitude of n_0A. Having found n_0A, the demagnetizing ampere-turns n_0n and the inductive drop nA are easily found.

Now Potier assumes, that the demagnetizing ampere-turns n_0n are the same with all values of power factor, but are subtracted geometrically, instead of arithmetically, from the field excitation Og. This gives again a diagram similar to that shown on the right side of Fig. 383, except that exciting amperes are used instead of corresponding voltages. The radius $kt = n_0n$; $ta'' = Og$. The result, ka'', gives the effective ampere-turns at the power factor corresponding to the angle ϕ. Plotting $Oj = ka''$, we find the voltage jD_0 actually induced in the armature. From this voltage, the inductive drop nA must be subtracted geometrically, as in Fig. 360, in order to obtain the terminal voltage at this load. If greater accuracy is required, the ohmic drop ir shown there may also be taken into account.

The angle ϕ, used in the construction of the triangle tka'', is not quite correct, since it must be the phase displacement between the *induced* voltage and the current, and not that between the *terminal* voltage and the current. Having constructed the diagram shown in Fig. 360, the angle AOD may be used for constructing the triangle tka'' in a second approximation, new value of ka'' found, etc.

The Potier method is not quite correct, because the armature reaction varies in reality, according to a much more complex law than is assumed there.

503. Literature References. — Those especially interested in the difficult question of regulation of alternators will find further information in the following publications:

Guilbert, A series of articles on the subject in the *Electrical World*. 1902–1903.

Torda-Heymann, *Electrician* (London), 1904, Vol. 53, p. 6.

Hobart and Punga, *Trans. of the A.I.E.E.*, 1904, p. 291.

Blondel, *Trans. Internat'l Elec. Cong.*, 1904, Vol. 1, pp. 620 and 635.

Rushmore, ibid., p. 744.

Henderson and Nicholson, *Journal of the Inst. of Elec. Engs.* (British), 1905, p. 465.

Rushmore's article has quite a complete bibliography on regulation of alternators, up to 1904. A compromise between an experimental determination and a predetermination of the regulation curve at power factor zero, is made possible by the methods of Blondel and Fischer-Hinnen, in which but one point of the curve NA needs to be determined experimentally (Hay, *Alternating Currents*, p. 160).

CHAPTER XXIII.

ROTARY CONVERTERS.

504. In certain cases of engineering practice it is necessary to transform direct current into alternating current, or vice versa. The most important case of this kind is in connection with electric railways, where high-tension alternating currents delivered from the power house must be converted into 500 volts direct current, in so-called substations, supplying the trolley lines with current.

One of the possible solutions for converting alternating into direct current is to use a motor-generator set. An induction or synchronous motor is direct-connected to and drives a direct-current generator; the energy supplied to the motor in the form of two- or three-phase currents is in this way transformed into direct current.

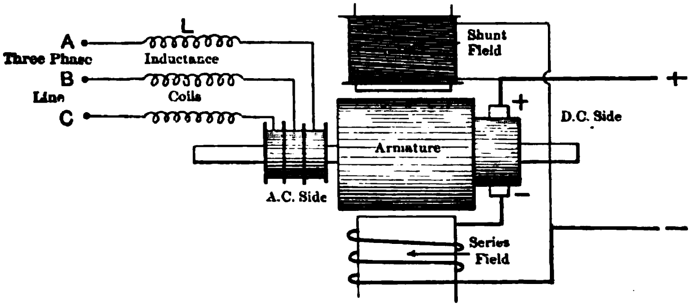


Fig. 384. A compound-wound three-phase rotary converter.

Another solution is to use the so-called *rotary converter*, which is a combination *in one machine* of a synchronous alternating-current motor and a direct-current generator.

505. General Description of the Rotary Converter. — A rotary converter is shown schematically in Fig. 384. In its construction it resembles a direct-current machine (Fig. 264), only, in addition to a commutator and brushes for direct current, it has two or more slip rings, connected to the same armature winding as the commutator. Fig. 385 shows diagrammatically the armature winding, the commutator, and the slip rings of a two-pole, single-phase rotary converter.

Fig. 386 shows a similar machine, provided with three slip-rings for converting three-phase currents into direct currents, or vice versa.

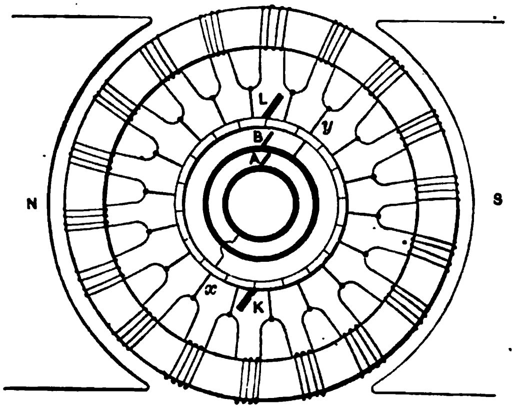


FIG. 385. Armature connections in a single-phase rotary converter.

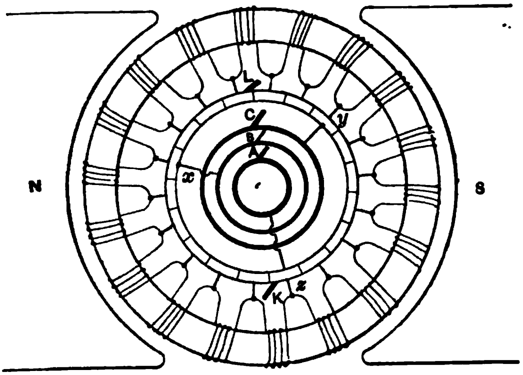


FIG. 386. Armature connections in a three-phase rotary converter.

The action of a rotary converter can be understood from these two figures. When a machine such as is there shown, is driven mechanically, as a generator, it delivers direct current through the commutator and the brushes *K*, *L*, at the same time delivering an alternating current through the collector rings *A*, *B*, or *A*, *B*, *C*. When so doing, it is called a *double-current* machine. The direct and the alternating currents are superimposed in the armature, but are used in separate external circuits. Now, both direct- and alternating-current generators are invertible in their action. A direct-current generator may be converted into a direct-current motor, and an alternator into a synchronous motor. So, when direct current is sent from outside through the terminals *L* and *K* of the machine, it runs as a direct-current *motor:* at the same time it can continue to supply alternating current from its collector rings. In this case it acts as a rotary converter, *receiving* direct current and *delivering* alternating current. The more power is taken from the alternating-current side, the larger becomes the direct-current input; the latter being always equal to the output plus the losses in the machine itself. Instead of driving the rotary as a direct-current motor, it can be driven from the alternating-current side, as a synchronous motor; then direct-current power is available, as an output, at the commutator side of the machine. This is the way in which rotary converters are used in railway work. The above-described rotary converters are two-pole, ring-wound machines. Commercial rotaries are multipolar, polyphase, drum-wound machines, and represent a development of the above simplest type.

Rotary converters are provided with two, three, four or six slip-rings, according to the number of phases of the alternating-current supply. Single-phase converters are seldom used; three-phase machines are provided with three slip-rings, two-phase rotaries have four slip-rings. Very large rotary converters are sometimes supplied with six slip-rings, and the three-phase supply is converted into a six-phase system by means of special transformers (see § 447).

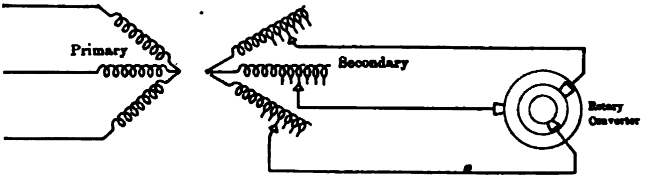


FIG. 387. Starting a rotary converter at a reduced voltage.

It can be proved theoretically that, with the same output, the I^2R loss in the armature decreases with an increasing number of slip rings; the reason being that the alternating-currents are more uniformly distributed in the armature winding. Thus, a six-phase rotary converter has a higher efficiency than a four slip-ring (two-phase) rotary converter; the three-phase rotary converter has a slightly lower efficiency than the two-phase rotary converter. All polyphase rotaries are considerably superior to the single-phase rotary, in that they give a larger output with the same size machine, and can stand a higher overload without falling out of step. This is due to the revolving field produced in the armature by polyphase currents.

506. Starting Rotary Converters. — Rotary converters, used for converting direct current into alternating, are started as ordinary shunt motors, and are brought up to the speed corresponding to the desired frequency of alternating current. If the rotary is intended to be run from the alternating-current side, as is the case in railway substations, it must first be brought up to the required speed and *synchronized* (§ 462), as any alternator or synchronous motor. Three methods are used for starting and synchronizing rotary converters:

(1) The rotary is started from the direct-current side, as a shunt motor, and synchronized by regulating the field and the starting resistance.

(2) A small induction (or direct-current) motor is provided on the shaft of the rotary converter; this motor being used for bringing the machine to the required speed.

(3) The rotary itself is converted into an induction motor during the period of starting, by opening its field circuit and switching the alternating current on the armature. A revolving field is thereby produced in the armature; this field induces eddy currents in the pole-pieces, which act as the squirrel-cage secondary of the induction motor. The machine starts, and when synchronism is reached, the direct-current excitation is switched on; the rotary continues to run as a synchronous motor. It is not advisable to start rotaries in this way on full voltage because of the inrush of current which may damage the machine itself, or be objectionable for the line. Taps are usually provided on the transformers for starting rotaries at about one half of the rated voltage (Fig. 387).

507. EXPERIMENT 23-A. —Exercises in Starting Rotary Converters.—The purpose of the exercise is to make clear the three methods of starting explained in the preceding article. Wire up the machine on the direct-current side as a shunt-wound motor (Fig. 284); on the alternating-current side provide synchronizing lamps, or a synchroscope (§§ 463 and 464).

(1) Start the machine from the direct-current side and bring it up to the right speed by regulating the field current and the starting rheostat, if necessary. Synchronize the machine as is done in the case of alternators. When the machine is in synchronism, close the main switch on the alternating-current side, and open the main switch on the direct-current side, taking care not to open the field circuit of the machine. The rotary will continue to run as a synchronous motor, supplying its own excitation, and may be loaded electrically on the direct-current side.

(2) Now provide a small induction motor, either belted or direct-connected to the rotary, and start the rotary by means of this motor. When synchronism is reached, switch the rotary on to the alternating-current line and open the auxiliary-motor circuit. In using this method of starting, it is convenient to have the secondary resistance of the induction motor adjusted so that the motor brings the rotary exactly to the required speed. If this is not feasible, some resistance is connected between the slip rings of the rotary converter. This resistance

constitutes a load which can be adjusted so that the induction motor will bring the rotary exactly to the required speed, and thus save time in synchronizing.

(3) After having practiced sufficiently with the above two methods of starting, the student may try to start the rotary converter from the alternating-current side by utilizing the eddy currents induced in the pole-pieces and dampers (Fig. 392). In performing this experiment, it must be borne in mind that the rotary, in starting, takes several times its normal current, so that the line should be suitably protected by fuses, or circuit-breakers. Another important precaution is to protect the field winding against high voltages induced in it by the revolving flux produced by the armature. This is done by subdividing the field winding into sections by suitable switches. *Never touch the field circuit* when the machine is being started from the alternating-current side, as the induced voltages may be dangerous to life. They gradually decrease to zero, as the machine approaches synchronism. Then the direct-current excitation is switched in, and the machine continues to run as a synchronous motor. Try starting at various voltages (Fig. 387), and measure current inrush and time which it takes to bring the rotary up to synchronous speed; also measure voltages induced in the field winding.

Report. Give diagrams of the exact connections used in the three methods of starting; give starting currents and voltages; time which it took in each case to bring the rotary to synchronism, and total time required to have the rotary running in regular operation. Give your opinion as to which method of starting is preferable in certain cases.

508. Ratio of Voltages on Direct- and Alternating-Current Sides.—There is a definite ratio between the direct- and the alternating-current voltages in a rotary converter, since both are induced in the same winding. In the single-phase rotary converter (Fig. 385) the maximum instantaneous value of alternating voltage is equal to the direct voltage: This is because the induced e.m.f. is the same between the direct-current brushes and the alternating-current brushes, when the points x and y of the winding come under the brushes K and L. At the same time this is the largest alternating voltage that can be induced in the armature. Therefore, the effective value of the alternating voltage is

$$1 \div \sqrt{2} = 0.707$$

times direct-current voltage. For instance, if it is desired to have 550 volts on the direct-current side, $550 \times 0.707 = 388$ volts must be applied between the collector rings A and B.

The same ratio 0.707 holds true for the two-phase rotary converter, provided the voltage is measured between the slip-rings *a* and *b*, or *c* and *d*, belonging to the same phase (Fig. 388). Otherwise the ratio is again reduced in proportion to $1 \div \sqrt{2}$ (Figs. 335 and 336) so that, the effective voltage, for instance between *a* and *c*, or *a* and *d*, is only one half of the direct-current voltage.

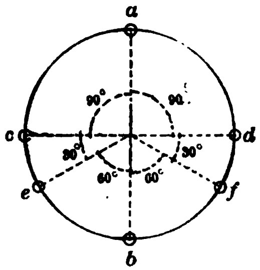


FIG. 388. Disposition of taps to six slip-rings, for producing either single-phase, two-phase, or three-phase currents.

If taps for the slip rings are taken at a distance less than 180 degrees, such as, for instance, *a* and *e* in Fig. 388, the alternating-current voltage between these slip-rings is reduced in the ratio *AC* to *AB* (Fig. 389), where *AB* represents the voltage induced between two slip-rings connected to the armature at points 180 electrical degrees apart. It may at first seem, that the voltage between the points *A* and *C* is to the voltage between the points *A* and *B* as the lengths of the corresponding arcs. However, a glance at Fig. 385 will show that the voltages induced in separate coils of the armature reach their maximum at different times, and must therefore be added geometrically. The sum of the infinitesimal arcs between *A* and *C* is the chord *AC*, and the same is true for the chord *AB*.

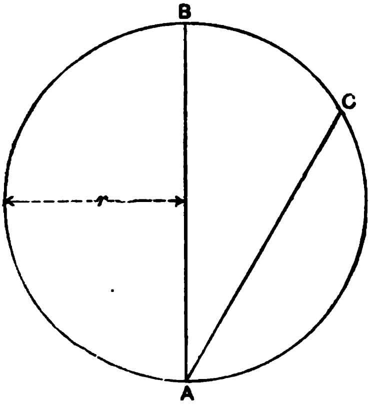


FIG. 389. Ratio of voltages in a rotary converter.

In a three-phase rotary (Fig. 386) the chord *AC* corresponds to 120 degrees, so that

$$AC = \tfrac{1}{2}\,AB\,.\,\sqrt{3} = 0.866AB.$$

This means, that assuming a certain voltage on the direct-current side, the voltage between the slip-rings of a three-phase rotary converter is only 86.6 per cent of that of a single-phase or two-phase rotary converter. It has been shown that the alternating voltage of a single-phase rotary converter is equal to 0.707 of the direct-current voltage. Thus we finally get that the ratio between the A.C. and the D.C. voltages in a three-phase rotary converter is

$$0.866 \times .707 = 0.612.$$

It is desired that the student deduce, in a similar manner, the ratios between the direct-current voltage and the alternating-current voltages in four-, six-, and twelve-phase rotary converters. These voltages correspond in Fig. 388 to the chords *b-c*, *b-e* and *e-c*.

509. EXPERIMENT 23-B. — Ratio of Voltages in a Rotary Converter. — A rotary converter convenient for such an experiment should have six slip-rings (Fig. 388), tapped at the points of the armature marked *a*, *b*, *c*, *d*, *e*, *f*. The taps are spaced so that voltages can be obtained corresponding to either 1, 2, 3, 4, 6, or 12 phases. Run the rotary from the direct-current side and measure all the possible combinations of voltages between the different slip-rings. Make several runs with different speeds, different values of field current and with various voltages at the direct-current brushes. For very accurate work the machine must be driven mechanically by some outside source of power in order to have two voltages from open circuit. For all practical purposes the converter may be run from either the direct or alternating-current side, without materially affecting the ratios of the voltages.

Report. Give the ratios of voltages, as calculated from the experiment, and compare them to the theoretical ratios derived above. Show whether these ratios vary with the voltage, speed and excitation of the machine, and give possible causes for this.

510. Ratios of Direct and Alternating Currents. — The ratio between the current taken in on the direct-current side of a rotary converter and that delivered on the alternating-current side may be easily deduced from the fact, that the power output is equal to the power input, less the losses in the armature itself. This, of course, refers only to the power component of the alternating current; the wattless component may have any value according to the properties of the load or the excitation of the machine (§ 470). Suppose, as an example, that a three-phase rotary connected to a 110 volt direct-current supply, is delivering 12 kilowatts of non-inductive power on the alternating-current side. The currents on both sides of the rotary can be figured out as follows: The three-phase voltage (disregarding voltage drop) is equal to a $110 \times 0.612 = 67.3$ volts. To deliver 12 kilowatts at this voltage a current of

$$\frac{12000}{67.3\sqrt{3}} = 103 \text{ amperes per phase}$$

is required. If there were no losses in the armature, the direct current input would also be 12 kilowatts, or

$$\frac{12000}{110} = 109 \text{ amperes.}$$

To this current must be added a small current necessary for supplying the armature losses. This latter current is determined by running the machine at no load; it is used for overcoming iron loss and friction of the machine.

If, in the above example, the load were inductive, the power supplied from the direct-current side would still be the same, as long as the *true* power remains equal to 12 kilowatts. The necessary wattless component of the current is generated in the armature itself, without being supplied from the direct-current line.

When a rotary converter is running from the alternating-current side, supplying a load on the direct-current side, the *watts* input is equal, as before, to the watts output on the direct-current side, plus the losses in the machine itself, but the alternating *amperes* may vary within comparatively wide limits, with the same direct-current output. This is due to the rotary converter taking in a leading or lagging wattless component according to the value of field excitation. In this respect it is similar to the synchronous motor (§ 470). Knowing the output of the machine and the ratio of the voltages, the power component of the alternating current may be calculated; it represents the lowest limit of alternating current possible with a given direct-current load. The rotary converter may be made to actually take in this minimum current, at a certain value of field current. By strengthening the field, the converter is made to take in a leading component, in addition to the power component; by weakening the field a lagging component is taken from the line (Fig. 372).

511. EXPERIMENT 23-C. — Operating a Single-Phase Rotary Converter from the Direct-Current Side.

— Start the rotary from the direct-current side, as a shunt-wound motor; provide inductive and non-inductive load for the alternating-current side, as in the case of a single-phase alternator (Fig. 359). Keep the voltage on the direct-current side and the field current constant, adjusting the latter so as to have the required speed at no load. This speed must be such as to give the rated frequency for which the rotary converter is designed. First apply a non-inductive load, read volts and amperes on both sides, and the speed of the machine. Take a series of such readings from no load up to about 50 per cent overload. Make similar runs at lower values of power factor, keeping it constant for each run. Or else, keep amperes constant during each run, and vary the power factor, as in Fig. 361.

The student must be careful not to let the machine run away while loading it on reactances. Wattless currents, flowing in the armature, weaken the original field; this has the familiar effect of increasing the speed of the direct-current motor. Some rotaries are provided with a centrifugal speed-limiting device which opens the direct-current circuit, as soon as the speed exceeds a certain predetermined limit. It is well to repeat the same experiment with several values of field current and of the voltage at the direct-current terminals.

In the above test, both the frequency and the alternating voltage are variable. In practice, either one of them or both are required to be kept constant. This can be done within certain limits by regulating the field current and the direct-current voltage. Take a few readings of this kind, in order to determine the limits within which field current and direct-current voltage must be varied in order to keep either the alternating voltage, or the frequency, or both, constant.

Report. Plot to amperes input into the armature: direct and alternating voltages, amperes output, speed, and efficiency. If possible, plot on the same curve sheet the values of the power factor, in order to show its influence on the speed of the machine.

512. EXPERIMENT 23-D. — Operating Two-Phase or Three-Phase Rotary Converters from the Direct-Current Side. — The experiment is performed in exactly the same way as the preceding experiment, except that the load is two-phase or three-phase, instead of single-phase. If performed on the same machine (Fig. 388), it will show to what extent the rotary is more efficient when the load is taken from three or four slip-rings, than when loaded single-phase.

513. EXPERIMENT 23-E. — Operating a Single-Phase Rotary Converter from the Alternating-Current Side. — Connect up and synchronize the rotary, as explained in § 507; provide a load on the direct-current side, and have an ammeter in the field circuit. Adjust the field current so as to have a power factor of 100 per cent, with full-load current on the direct-current side. Vary the load, if possible, from zero to 50 per cent overload, without adjusting the field rheostat, thus allowing the field current to follow the variations of the direct voltage. This corresponds to the practical case of a fluctuating load, when the attendant does not adjust the voltage. Read amperes, volts and watts; also note the field current.

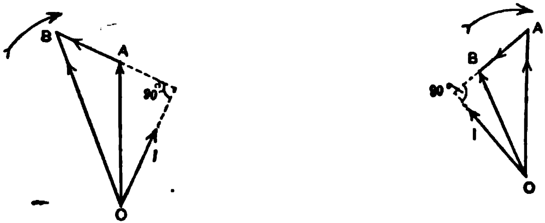
Repeat the same run with such an initial value of the field current as to give about ninety per cent lagging power factor at full load. Make a third run with 90 per cent leading power factor, under the same conditions of load.

After this, make one or more runs with the field current constant: this is done by adjusting the field rheostat, before each reading. If feasible, test the overload capacity of the rotary, by raising the load until the machine falls out of step. Be sure that the machine is not damaged by the overload, and that it is protected by a suitable circuit-breaker on the alternating-current side. Requirements for the report are similar to those in § 511.

514. EXPERIMENT 23-F. — Operating a Two-Phase or a Three-Phase Rotary Converter from the Alternating-Current Side. — The experiment is similar to the preceding one, and is intended to bring out the difference in the operation of a polyphase converter as compared to that of a single-phase machine. Note higher efficiency, better regulation, and larger overload capacity of the machine when operated two-phase or three-phase.

Report. See § 511.

515. Compounding Rotary Converters. — In many cases it is desirable to have rotary converters compounded, or provided with an additional series winding (Fig. 384), for the same reason that direct-current generators are compounded, viz., in order to have a constant voltage at variable load: It is sometimes advantageous to have the rotary even *over-compounded*, so as to compensate for voltage drop in long feeders (§ 306). It may at first seem, that a series winding alone, shown in Fig. 384, should be sufficient to produce the desired effect. But it must be remembered that, in a rotary converter, the ratio of alternating- to direct-current voltage is nearly constant, since they are induced in the same winding by the same magnetic field: Therefore, the only effect of a series winding would be to produce leading wattless currents which would weaken the field to nearly its original value. *In order to increase the voltage on the direct-current side, the alternating voltage must be raised accordingly.*



FIGS. 390 and 391. Diagrams showing that presence of inductance raises voltage with a leading current, and lowers voltage with a lagging current.

This is accomplished in practice by producing electric resonance (§ 134). An over-excited rotary converter takes in leading currents similarly to a synchronous motor (§§ 470 - 471). An inductance L is connected in series with the line (Fig. 384), and these leading currents flowing through it raise the voltage at the slip rings above that of the line. This is a case of the voltage resonance shown in Fig. 143.

The same relations are shown in Fig. 390: OA is the vector of the line voltage; OI is the line current which is supposed to be leading, the machine being over-excited. The vector of the voltage drop, AB, in the inductance coil, is perpendicular to OI. It will be seen from the diagram that the resultant voltage OB at the collector rings is higher than the line voltage OA. When the current is lagging, as in Fig. 391, OB is less than OA, so that in this case the inductance L makes the conditions still more unfavorable. Thus, it must be remembered, that *an inductance in series with the line helps to raise the voltage, provided the rotary is compounded, or over-excited, so that a leading current is produced.*

The arrangement operates automatically, and by properly selecting the inductance coils and the number of turns of the compounding winding, it is possible not only to keep the voltage on the direct-current side constant, within wide variations of the load, but even to have the machine over-compounded to compensate for the drop in long feeders.

516. EXPERIMENT 23-G. — Exercises in Compounding Rotary Converters. — Connect the rotary for running from the alternating-current side, and provide suitable inductance coils, as shown in Fig. 384. Apply a moderate load to the direct-current side, and vary the shunt-field current throughout wide limits so as to observe the combined action of leading currents and of the inductance in the line. The series field is not connected in during this test. Now, from the increase in the shunt current, necessary for keeping the voltage constant, figure out the required number of turns in the series winding, in order to produce the same effect automatically. If the number of turns of the shunt winding is not known, it can be determined as is explained at the end of § 307. Put the calculated number of series turns on the machine and verify the predetermined performance. Then increase the number of series turns so as to get a certain specified amount of over-compounding, and if the machine is over-compounded too much, put a shunt across the series winding, as is done in case of compound-wound generators (Figs. 271 and 272).

Change the inductance in the line, and repeat the same experiment in order to see the influence of the inductance on the number of series turns required for compounding. Take a complete curve of voltage regulation for at least one combination of inductance and series windings.

Report. Show by a diagram the connections used during the experiment; plot curves showing the variations of the current and watts input, power factor, and voltages with varying shunt-field current, with a constant direct-current load. Give the results of calculating the necessary number of series turns, and show in how far the actual performance checks with the calculations. Give results showing the influence of the change in inductance on the voltage characteristics of the machine.

517. Alternating-Current Voltage Regulators. — The above-described method of keeping the voltage on the direct-current side of a rotary converter constant, by a combination of inductance in the line and series winding on the machine, while automatic in its action, has certain limitations. Therefore, in some cases, voltage regulators, independent of the rotary, are used on the alternating-current side.

(*a*) The simplest method is shown in Fig. 387: the secondary winding of the transformers supplying the rotary are provided with taps taken to a dial switch on the switchboard. By operating this switch, the voltage between the slip rings can be varied within certain limits, so as to keep the direct-current voltage constant. In some cases the dial switch is operated automatically by a small motor, which is started and stopped by a relay, sensitive to the variations of the voltage. Such regulators have two disadvantages: The regulation is not continuous, but in steps, and the main circuit is broken for an instant while changing from one step to another. These disadvantages are eliminated in so-called induction regulators.

(*b*) Induction regulators are transformers having a fixed secondary winding, and a primary winding which can be moved within certain limits so as to vary the inductive action between the two windings. The secondary winding is connected in series with the line, the primary is connected across the line. In this way the line voltage can be "boosted" within certain limits in accordance with the variation of the load. Induction regulators may also be operated either manually or by a small auxiliary motor.

(*c*) Still another method to keep the direct-current voltage of the rotary converter constant, is to provide a separate booster set, the armature of the booster being in series with the direct-current line. The voltage is regulated at will by varying the field excitation of the booster.

518. Hunting and its Prevention. — A trouble, called "hunting," is sometimes experienced in operating rotary converters: The pointers of the ammeters on the alternating-current side begin to swing without any corresponding changes in the load, and the machine begins to give a humming noise, indicating that its speed varies periodically. This "pumping" grows worse and worse until the converter falls out of step, and the circuit-breakers on the alternating-current side open the circuit.

This trouble is supposed to be due to the natural period of swing of the rotary converter armature: anything unsteady in the circuit creates favorable conditions and intensifies these swings. Non-uniformity of rotation of the steam-engines, or a faulty governor may act as such factors, causing periodical inrushes of current into the armature of the rotary converter. If the impulses happen to occur at intervals sufficiently close to the period or to a multiple of the period of natural swing of the armature, the inrushes are intensified, causing still greater non-uniformity, etc.

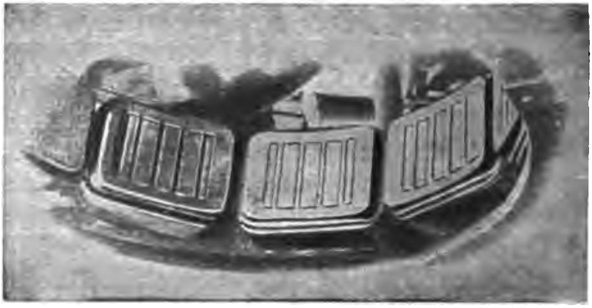


FIG. 392. Copper dampers on the poles of a rotary converter, to prevent hunting.

This phenomenon is much more noticeable with 60-cycle rotary converters, than with 25-cycle machines, because the former have more poles for the same speed; the angular distance between the consecutive poles is smaller, hence, the same angular deviation of the armature from its true position means a greater electrical deviation. For this reason some engineers object to using 60-cycle rotary converters, and prefer to use motor-generator sets, in which the synchronous motor and the direct-current generator may be given different numbers of poles. Sometimes induction motors are used in place of synchronous motors, and in this way all "hunting" troubles are eliminated.

The best remedy known against hunting is to provide the pole-pieces with so-called "dampers" (Figs. 392 and 374). These are solid pieces of copper mounted on the pole-pieces, as close as possible to the armature. Polyphase currents in the armature produce a revolving flux which travels synchronously in the direction opposite to the rotation of the armature, and thus is stationary in space, merely strengthening or weakening the field produced by the poles. When, however, the armature is "pumping," the armature flux is no longer stationary in space, but follows the oscillations of the armature. This swinging of the flux induces currents in the dampers: a heavy electromagnetic torque is produced, tending to keep the armature at the right speed. In some respects, the action of the dampers is analogous to that of the squirrel-cage secondary of induction motor.

Another remedy which may be used in the case of an emergency is connecting inductance coils in the alternating-current circuit, whether there is a series winding or not (Fig. 384). Such inductance coils limit the inrush of current during oscillations and prevent them from attaining dangerous proportions. This remedy should not ordinarily be applied, since it means a considerable voltage drop and a poor regulation.

Still another remedy is to provide the rotary with a heavy fly-wheel, its inertia limiting possible oscillations of the revolving part. This is also a makeshift, but may be useful in the case of an emergency, especially with small rotaries.

519. EXPERIMENT 23-H. — Comparative Study of Remedies against Hunting. — This experiment can be performed only when conditions can be created in the laboratory, such as to produce appreciable hunting; for instance, by supplying the rotary with current from an alternator driven by a steam-engine having a high degree of non-uniformity of rotation. Synchronize the rotary and try to run it from the alternating-current side, first without any auxiliary appliances mentioned above. Then gradually introduce more and more self-induction into the line and observe the improvement in operation of the rotary. See what happens when more inductance is put in than is necessary. Take readings at no load, at a partial load, and at full load. There are two ways of expressing the "degree" of hunting or pumping. One is by per cent variation of angular velocity of the armature; the other, by per cent current fluctuations as shown on the ammeters. The non-uniformity of rotation may be observed on a sensitive tachometer, such as a magneto-generator connected to a milli-voltmeter. A chronograph may also be used, in the same way as for determining the degree of non-uniformity of rotation of steam- and gas-engines.

Disconnect the inductance coils, and have a fly-wheel direct-connected or belted to the rotary. See if this remedy is as efficient as the inductance coils, and note the performance of the rotary with and without the fly-wheel.

Finally provide the rotary with suitable dampers and investigate their influence at various loads.

Report the observed performance of the rotary, and your conclusions in regard to hunting and its prevention.

CHAPTER XXIV.

INDUCTION MOTORS — OPERATING FEATURES.

520. THE induction motor is the most popular type of alternating-current motors, and finds a wide field of application. Its action is based on the principle of the *revolving magnetic field*, produced by alternating currents displaced in phase relative to each other (polyphase currents, Chapter XX). At least two currents of different phase are necessary in order to produce a revolving field; but for the economy and convenience of transmission of electric power (see §§ 443 and 444), it is preferable to have three alternating currents displaced in phase by

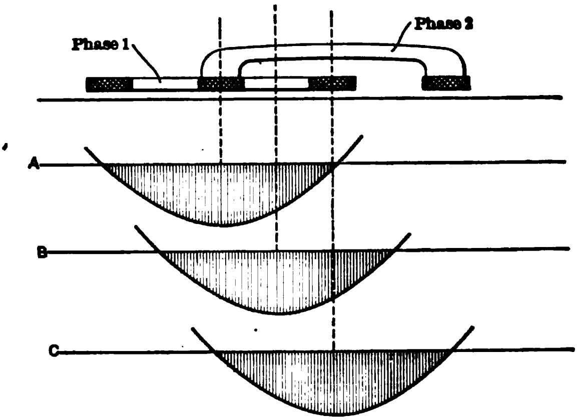


FIG. 393. The progression of a magnetic field produced by two-phase currents.

120 degrees (Figs. 327 and 328). Accordingly, induction motors are wound for two- or three-phase circuits.

Fig. 393 shows a moving magnetic field, produced by two alternating currents, displaced in phase. The currents in the two coils are displaced in phase by 90 degrees (Fig. 327), so that the current in one phase reaches its maximum when the current in the other phase passes through its zero value, and vice versa. The coils themselves are also displaced geometrically with respect to each other. When the current is at its maximum in phase 1, phase 2 is inoperative, and the magnetic field has the position denoted by *A*. One quarter period later, the current in phase 2 reaches its maximum, and the field occupies the position denoted by C. At an intermediate moment, when currents are flowing in both windings, the field occupies an intermediate position, denoted by B. It will thus be seen that during one half of an alternation of the supply currents, the magnetic field is shifted from the position A to the position C. During the next half alternation the field moves farther to the right by the same amount, etc.

This simple scheme explains how it is possible to produce a moving magnetic field with stationary windings, provided that the currents flowing through these windings are displaced in phase, and the windings themselves are displaced geometrically with respect to one another. For a detailed study of revolving magnetic field, see § 558 and succeeding sections.

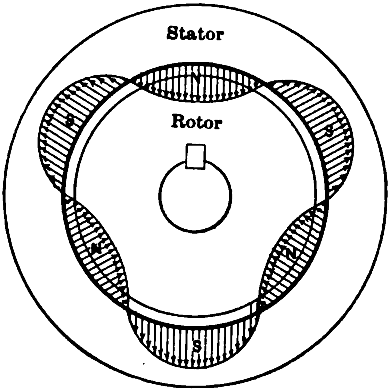


FIG. 394. The field in the air-gap of a six-pole induction motor.

521. Stator and Rotor.— Fig. 394 represents the magnetic field of a six-pole induction motor, at a certain moment of time. This field, produced by the windings placed on the stationary part of the motor, or the *stator*, travels along the air-gap at a speed which depends upon the frequency of the supply currents.

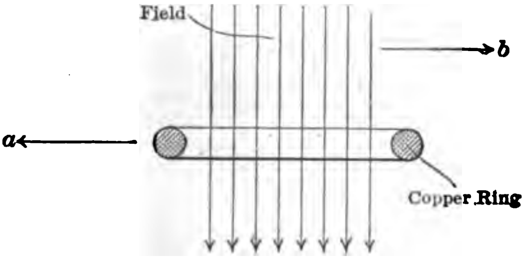


FIG. 395. Reaction between a magnetic field and a closed conductor.

The revolving part, or the *rotor*, consists of an iron core, and a winding short-circuited upon itself. The revolving magnetic flux induces secondary currents in this winding, and, by its magnetic action upon the flux produced by these currents, exerts a torque which makes the rotor follow the revolving flux. This is but a modification of the famous Arago experiment in which a copper disk was made to follow a revolving permanent magnet.

The action of the revolving magnetic flux on the rotor may also be explained with reference to Fig. 395. Assuming the copper ring to be stationary and the magnetic field to be moving in the direction *b*, currents will be induced in the ring, in such a direction as to exert an attraction upon the field. The copper ring will have a tendency to follow the field, overcoming a mechanical force that may be applied to it in the direction *a*. This follows directly from the law of conservation of energy: by following the field the currents in the ring are decreased, while otherwise they would increase indefinitely.

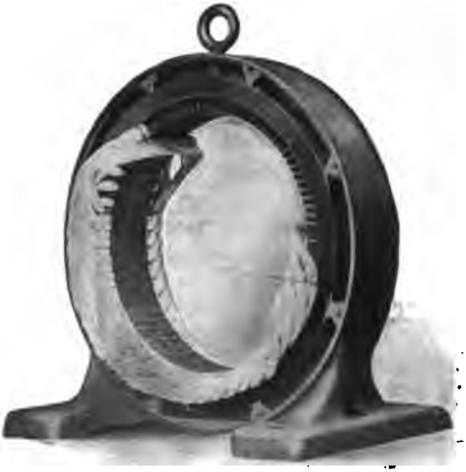


Fig. 396. The stator of an induction motor.

The stationary part, or the stator of an induction motor, is shown in Fig. 396. It consists of a laminated iron frame, held by a cast-iron housing and provided with a regular three-phase or two-phase winding, similar to that of an alternator; the winding is connected to a line, which supplies the motor with power.

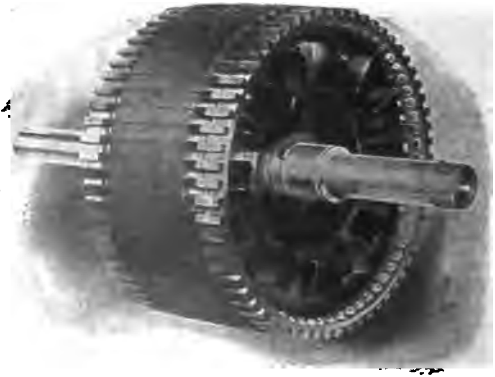


Fig. 397. The squirrel-cage rotor of an induction motor.

The rotor, or the secondary part of the motor, is shown in Fig. 397; it consists of a laminated iron core mounted on a cast-iron spider and provided with a winding short-circuited upon itself. This particular rotor has a winding of the so-called "squirrel-cage" type, which consists of copper bars slightly insulated from the iron and connected on both ends to metallic rings which complete the circuit.

522. Slip and Torque. — The speed of the revolving magnetic field depends on the frequency of the line currents and the number of poles of the stator. During one alternation of the supply current the field travels from one pole to the next pole, therefore the number of revolutions of the rotating field per minute is equal to the number of alternations (frequency) of the system divided by the number of poles of the stator winding. For instance, on ordinary 7200 alternation lighting circuits the revolving field of a 6-pole induction motor makes $7200 \div 6 = 1200$ revolutions per minute.

The speed of the rotor is lower than that of the revolving field, in order that the lines of force may cut the secondary conductors. In practice, the speed of the armature at full load is a few per cent less than that of the revolving field; for instance, the actual speed of the above-mentioned motor would be somewhere near 1140 r.p.m. This difference in speed is called *the slip* of the induction motor, and is usually measured in per cent of the speed of the revolving field, which latter speed is also called the *synchronous speed.* Thus the synchronous speed of the above motor is 1200 r.p.m. and the slip at full load is

$$\frac{1200-1140}{1200} = 5 \text{ per cent.}$$

The slip depends on the load of the motor and increases with it. At no load the motor runs almost synchronously, because a very small difference of speed between the revolving field and the rotor is sufficient to induce currents in the latter, furnishing the torque necessary for overcoming the friction and windage. As the load increases, the slip also increases, the rotor runs more slowly, and the induced currents become larger; in consequence, the torque on the motor is increased, as it ought to be. When the load exceeds a certain limit, the motor "pulls out" and comes to a stop. The overload capacity of an induction motor is usually measured by the ratio

$$\frac{\textit{pull-out torque}}{\textit{full-load torque}}.$$

This ratio varies from 1.3 to 2.5 and higher, according to the size and type of the motor and the purpose for which it is designed.

523. Starting Devices. — Induction motors cannot be started by simply switching them on to the line (except in very small sizes), because the rush of current during the first moments would be too large. The starting is done at a voltage considerably lower than that of the line. and then, as the motor gains in speed, the voltage is increased in a few steps to that of the line. The gradations of the voltage are obtained by means of two *V*-connected auto-transformers with several taps (Fig. 398); a controller or switch is provided, usually immersed in oil, for safety and durability. The transformers and the switch are inclosed in a common case, and the apparatus is called an "auto-starter" or "compensator" (Fig. 399).

The auto-starter connections are such that at starting, the transformers are connected to the line, while the motor windings are connected across a certain part of the transformer windings, giving a lower voltage. After the motor has reached a considerable speed, the switch is turned to the "running" position, in which the auto-transformers are entirely disconnected from the line and the motor is directly connected to the line.

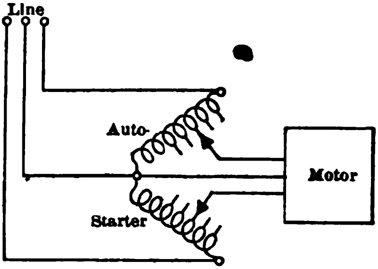


FIG. 398. Starting an induction motor by means of two V-connected auto-transformers.

An induction motor with a squirrel-cage secondary is essentially a constant-speed motor, as the difference in speed between no load and full load is but a few per cent. The starting torque of this type of motors is not particularly high, the motor being started on a low voltage.* For driving machine shops and similar purposes, where no high starting torque is required, squirrel-cage induction motors are well adapted because of their simplicity. There are cases, however, in which a variable-speed duty with high starting torque is required, as, for instance, in crane, hoisting and similar work, for which direct-current series-wound motors are otherwise used. For such service the rotor of the induction motor is made phase-wound, with the same number of poles as the stator. The ends of the winding are connected to three slip-rings, whence connections are made to a three-phase variable rheostat. At starting, the rheostat is all connected into the circuit and the motor started at the full line voltage, with a powerful torque equal to several times full-load torque. Then the rheostat is gradually cut out and if necessary used for speed regulation: the more resistance is inserted into the circuit, the lower is the speed of the motor.

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* According to the general theory of induction motors, the torque decreases as the square of the voltage. For torque is proportional to the product — flux times secondary current. But secondary current, being induced by the flux, is proportional to it. Therefore, torque is proportional to the square of the flux. Now, the flux is proportional to the counter-e.m.f., or, roughly, to the applied voltage. Therefore, torque is proportional to the square of the applied voltage.

524. EXPERIMENT 24-A. — Exercises in Starting and Speed Control of Induction Motors. — The experiment is intended to familiarize the student with the two above-described methods of starting induction motors. If a motor is available which has a phase-wound secondary with slip rings, it is well to perform this experiment on such a motor in order to be able to compare both methods of starting. Wire up the motor so as to be able to use at will either an auto-starter or a resistance in the secondary.

(1) Compare the two methods of starting with regard to the magnitude of the current which is necessary for starting the motor at a certain load: Put on a load, say by means of a Prony brake, and start the motor, *first*, by switching the motor on the full line voltage, but having a high resistance in the secondary, the resistance being gradually cut out as the motor gains speed; *secondly*, by the usual method, with the secondary winding short-circuited, and the primary voltage reduced by means of an auto-starter.*

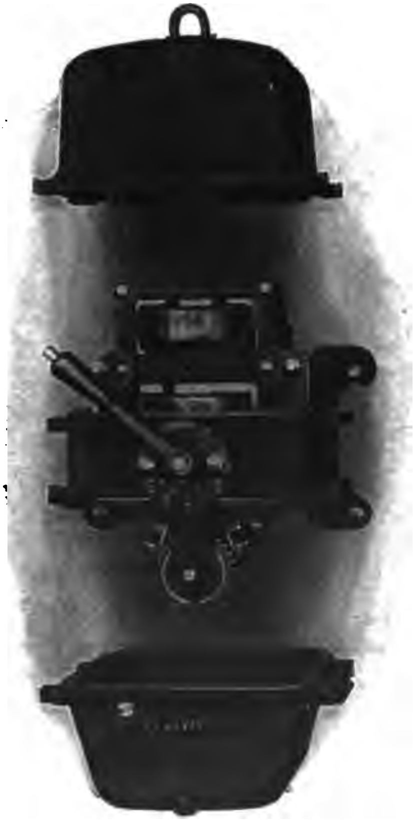

FIG. 399. An induction-motor starter.

The starting current depends on the amount of resistance inserted into the secondary, and on the starting voltage, if an auto-starter is used. Therefore, for a fair comparison, the best starting secondary resistance and the lowest possible starting voltage must first be found. This can be done, for instance, for the full-load torque. Knowing the rated horse-power of the motor and its normal speed, the full-load torque can be easily calculated and the brake adjusted for this torque. Then a high resistance is introduced into the secondary, and is gradually cut out, until the motor starts. This is assumed to be the right value of the starting resistance for the first step of the starting rheostat. In a similar way, different taps on the transformers of the auto-starter are tried, until a voltage is found at which the motor starts with the same brake load. This is taken as the right tap for the first notch of the auto-starter.

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* The starting torque is different in different positions of the secondary relative to the primary winding. In order to have comparable results the motor must in all cases be started from the same position.

Having thus found the proper setting of the starting apparatus for both methods of starting, start the motor at a few different values of starting torque, using both methods of starting in succession Try to read the first rush of the current and its subsequent variation as closely as possible; also note the number of seconds that it takes for the motor to attain full speed. Do this with the purpose of forming a judgment concerning the relative advantages of both methods of starting. It will be noted that leaving some resistance in the secondary circuit may considerably improve the starting torque, when the auto-starter is used, though this will somewhat lower the efficiency and increase the slip at full speed.

In making the above comparison, it must, of course, be kept in mind that a squirrel-cage motor is a less expensive and simpler machine than one with a phase-wound rotor; also that starting by means of auto-transformers is simpler and more reliable than by means of a three-phase rheostat in the secondary. Therefore the former method of starting should be invariably used where a high starting torque is not required and a large starting current is not objectionable.

(2) After this, the motor with the phase-wound secondary should be tested with regard to its speed control. Bring the motor up to its full speed by gradually short-circuiting the secondary rheostat, and put on approximately full load. Now keep the torque (not horse-power) constant, and gradually reduce the speed by again introducing resistance into the secondary. This corresponds to a practical case when a certain weight is lifted by a crane at varying speeds. Read amperes primary and secondary and the corresponding speeds with different values of secondary resistance.

Speed control by varying the primary voltage is never used, because decreasing the applied voltage rapidly reduces the magnetic flux and consequently the torque of the motor. If time permits, try this method of control, by connecting the starting resistance into the primary, instead of the secondary circuit, and observe the behavior of the motor. The same can be done by using the several starting positions of the compensator, if it is large enough to carry the current for a considerable time.

Report. Arrange the data in the form of curves, or a table, showing the comparative behavior of the motor with the two methods of starting; show, in particular, the amount of starting current and the time that it takes to bring the motor up to speed. In regard to speed control, give curves showing the speed of the motor and the current input, as a function of the amount of resistance in the secondary circuit.

PERFORMANCE TESTS.

525. — The operating features of an induction motor may be best judged from a set of curves, such as shown in Fig. 400. All the data are plotted to horse-power output as abscissæ. Some prefer to plot them to amperes, or watts input, others to pounds torque. There is not much difference between these three methods; one justification for plotting it to horse-power is that such curves are better understood by non-electrical engineers. The three most important curves are those of efficiency, power factor, and speed. The other curves may also be useful for various purposes, and are usually included on the curve-sheet. The curve marked "true H.P. Input" is obtained by dividing watts input into the motor by 746. Similarly, the ordinates of the "Apparent H.P. Input" curve represent the volt-ampere input divided by 746. The ordinates of the efficiency curve are equal to the ratios of the ordinates of the output and the true input curves. The ordinates of the power factor curve represent the ratio of true input to apparent input. In addition to these curves an "Apparent Efficiency" curve is sometimes plotted, giving the ratio of the output to apparent input.

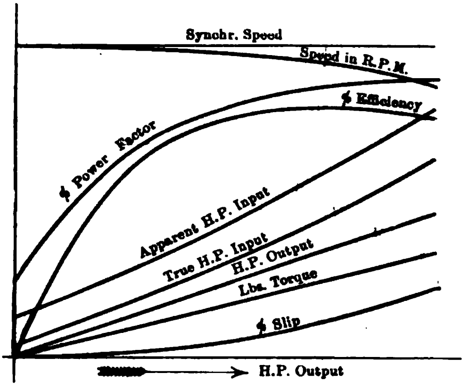


Fig. 400. Performance curves of an induction motor.

The above performance curves may be obtained from a direct brake test, or they can be calculated from the losses in the motor; sometimes they are predetermined from a vector diagram. Each of these three methods will be described in detail in the course of this and the following chapters. The general arrangement of the brake test is the same as is described in §§ 331 to 333, in application to direct-current motors.

526. Measuring Input. — The electrical power-input to a three-phase motor is measured by the so-called two-wattmeter method, shown in Fig. 401. One of the line wires, say *B*, is assumed to be a common return wire for two other wires: The power is measured between the wires *A* and *B*, and then between *C* and *B*, and the wattmeter readings are added together in order to get the total input into the motor. Accordingly, the series winding of one wattmeter is connected into the line *A*, and its shunt winding across *A* – *B*. The series winding of the other wattmeter is connected into the line *C*, and the potential winding across *C* – *B*. With the use of a polyphase board (§ 49) only one wattmeter is needed; it is connected in succession in two positions shown in Fig. 401, and the readings added together.

Theory and experience show that this method gives correct results for the total power input on balanced, as well as on unbalanced loads. However, the two component readings are equal only when the load is balanced and non-inductive (power factor of 100 per cent). With an induction motor the load is practically balanced in the three phases, but the power factor is always less than 100 per cent. Accordingly, one of the wattmeter readings is always smaller than the other. On light loads, when the power factor of an induction motor becomes less than 50 per cent, one of the wattmeters begins to give negative deflections. Then it is necessary to reverse the potential leads of this meter, and to take the *difference* of the two readings, instead of their sum.

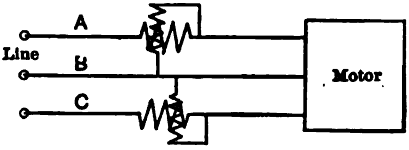


FIG. 401. Measuring power input into a three-phase motor by the two-wattmeter method.

For this reason it is advisable to begin the test at the maximum load (say 25 per cent overload) where the power factor is surely higher than 50 per cent, and then reduce the load by steps to zero. Then one cannot miss the point at which it becomes necessary to reverse the leads of one of the wattmeters. For further details in regard to the two-wattmeter method, see §§ 430 to 432. If a polyphase wattmeter is available, total power is obtained from one reading.

It would hardly be practicable to provide separate ammeters and voltmeters for each phase; a special "polyphase board" is used by means of which the same instruments may be connected in succession in the three phases. Any of the polyphase boards described in § 49 can be used for the purpose.

527. Measuring Frequency and Speed. — Measuring the speed of an induction motor deserves special attention. The speed depends on the load and on the frequency of the supply currents, as the latter determine the speed of the revolving field. If the power for testing is taken from a commercial supply, the frequency may at times be several per cent above or below the normal, and unless the exact frequency is known at the time when the speed is taken, the speed determination is of little value.

When the generator is accessible, its speed may be measured simultaneously with that of the motor, so that the two speeds refer to the same frequency. If the generator is not accessible, a small synchronous motor may be run from the same line to which the induction motor is connected. As the speed of the synchronous motor is always equal to that of the generator and automatically follows the variations in speed of the latter, the synchronous motor will give at any moment the actual speed of the generator. Special instruments, so-called frequency meters (see § 474), may also be used for measuring the frequency of the supply.

Instead of measuring the speed of the motor and the frequency of the supply, it is much preferable to measure simultaneously the speed and the slip (§ 522) of the induction motor; their sum gives the synchronous speed, and consequently the frequency of the supplied currents. If, for instance, the motor speed is 702 r.p.m., and the slip 22 r.p.m., the synchronous speed is 724 r.p.m. If the motor is a 10-pole machine, the frequency of the supply is at that particular moment 7240 alternations per minute, instead of the standard 7200. In plotting the speed curve, the corresponding correction must be made.

Slip usually constitutes but a few per cent of speed; at the same time an accurate knowledge of its value is of considerable importance to the designer as well as to the user of the motor. When slip is determined as a difference between the synchronous speed and the actual speed of the motor, a small error in the determination of either may lead to considerable error in the value of the slip figured out as the difference of the two. For this reason it is preferable to measure slip and speed directly and independently of each other, and to determine the frequency of the supply as the sum of the two. This leads us to the description of slipmeters.

528. Stroboscopic Slipmeters. — The name "stroboscopic" is usually applied to devices based on the peculiarity of our eye to preserve a continuous impression when the frequency of flickering of the light exceeds a certain limit.

(a) *Sectored-Disk Slipmeter.* A simple slipmeter based on this principle is shown in Fig. 402: A pasteboard or sheet-metal disk with white sectors painted on it is mounted on the shaft of the motor. The number of sectors should be equal to the number of poles of the motor. An alternating-current arc lamp, fed from the same supply as the motor, is placed before the disk. If the motor could revolve synchronously the white sectors would appear stationary; as, however, the speed of the rotor is a few per cent below synchronism, the sectors appear to the eye slowly rotating in the direction opposite to that of the shaft.

The reason for this phenomenon is that the light of the alternating-current arc lamp is actually extinguished once during each alternation of the current. With synchronous rotation of the disk, each sector has just enough time during one alternation to occupy the position of the preceding sector. Thus, when the lamp relights, the eye of the observer finds the sectors in seemingly the same position in space, as before. The flickerings of the light occurring several thousand times per minute, the phenomenon appears as continuous. When, however, the rotor lags behind synchronism, the sectors have not enough time to move through one polar division during one alternation of the current, so that when the lamp lights up the next time, the eye finds the sector in a slightly different position. This lagging behind is going on continually, and the sectors appear to the eye slowly moving backward, while in reality they are revolving at a high speed in the direction of rotation of the shaft.

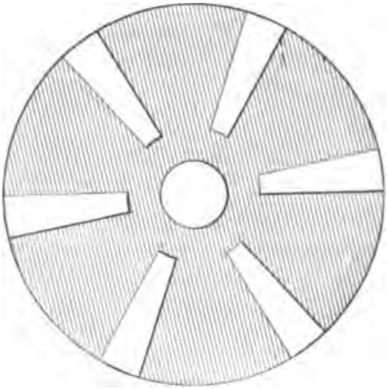


FIG. 402. Disk with painted white sectors, for measuring slip of induction motors.

The higher the slip of the motor, the faster are the sectors moving through the field of vision; the number of the *apparent* revolutions of the disk is a direct measure for the slip. In order to count the sectors more conveniently, it is advisable to limit the field of vision so as to see only one sector at a time. Suppose, for instance, that 120 sectors have passed through the field of vision in one minute, while the speed of the motor, as measured by an ordinary speed-counter, was 1765 r.p.m.; let the motor be a four-pole, 60-cycle machine. With these data the slip of the motor and the actual frequency of the supply is figured out as follows:

With 120 sectors passing through the field of vision during one minute, the motor skipped 120 alternations of the supply. As the motor has four poles it took $120 \div 4 = 30$ slip revolutions to skip 120 alternations. Hence, the synchronous speed of the motor was $1765 + 30 = 1795$ r.p.m., and the slip amounts to

$$\frac{30}{1795} = 1.67 \text{ per cent.}$$

The frequency of the supply when the speed was measured was equal to $1795 \times 4 = 7180$ alternations per minute instead of the standard frequency 7200.

It will be seen from the above example that the stroboscopic slipmeter is more suitable for 25-cycle motors and for light loads than for high-frequency motors and for heavy loads, because in the latter case the sectors pass too rapidly through the field of vision, and it is difficult to count them.

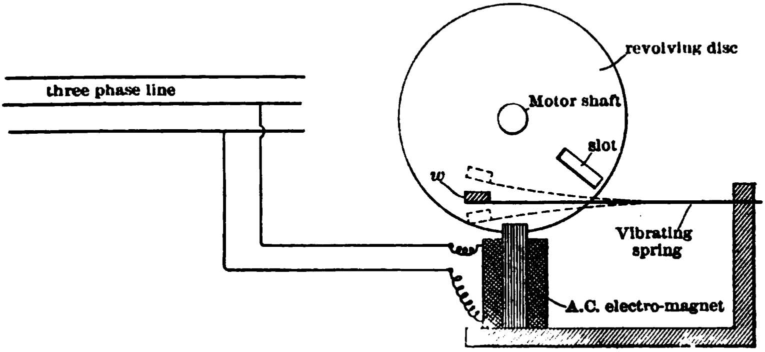
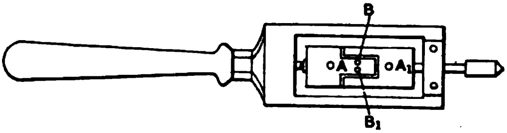


FIG. 403. A vibrating-reed slipmeter.

(*b*) *Vibrating-Reed Slipmeter.* Another stroboscopic slipmeter, which does not require an arc lamp, was suggested by Professor Perkins of the University of Tennessee. It consists of an alternating-current electromagnet (Fig. 403) connected to the source of supply and provided with a steel reed near one of its ends. Alternating current flowing through the electromagnet sets the reed into synchronous vibrations. The reed is loaded at its extremity with the weight *w*, so as to make its natural period of vibration correspond to that of the supply. A disk with a slot in it is mounted on the shaft of the motor, and the vibrating reed is viewed through this slot. If the rotor were revolving synchronously, the reed would appear to the eye stationary, because it would be viewed by the observer always during the same part of its vibration. As, however, the rotor lags behind synchronism the reed appears slowly moving up and down. The number of strokes per minute is proportional to the slip, as in the case of the arc-lamp slipmeter.

The reed may vibrate at a frequency some multiple of that of the supply, and it is safer to calibrate this instrument experimentally, by measuring the slip of a motor by some other method. This will give a constant by which to multiply the number of strokes in order to find the number of alternations skipped per minute.

529. Commutator Slipmeters. — In places where many induction motors are tested regularly, it is desirable to have more convenient devices for measuring slip than the above-described stroboscopic slipmeters. A device of this kind is shown in Figs. 404 and 405. In its principle it is a commutator with as many segments, as the motor has poles. This commutator is pressed against the end of the motor shaft, as an ordinary speed-counter, and at the same time is connected through a resistance to the power supply and to a sensitive ammeter. If the speed of the motor were exactly synchronous, the impulses of the current, sent through the commutator into the ammeter, would be always at the same point in the alternating e.m.f. wave, and its indication would be steady. As, however, the motor *lags* behind the revolving field, these impulses occur now at the maximum of the wave, now at the intermediate values, and the needle of the ammeter swings at a speed equal to the *difference* of the speeds of the revolving field and the rotor, and hence proportional to the slip of the motor. If the slip is not too high, the number of swings per minute can be easily counted and the slip calculated. An alternating-current bell (polarized bell) can be used instead of an ammeter, and the number of strokes per minute counted.



404. A commutator-type slipmeter.

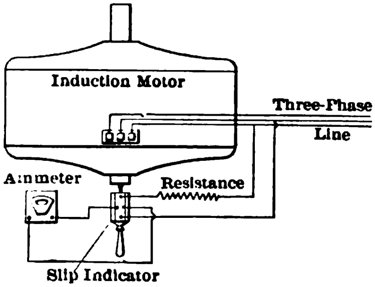


FIG. 405. Electrical connections to the slipmeter shown in Fig. 404.

The difficulty in counting the number of impulses, when the frequency is high and slip is considerable, is the same with this device as with the stroboscopic slipmeters. The Bianchi slipmeter (Fig. 406), while similar in its principle to that shown in Fig. 404, has an attachment for automatically registering the number of revolutions of slip. The impulses produced by the revolving commutator *D*, instead of being counted by the observer, are sent through the electromagnet *M*, which actuates a ratchet-and-pawl recording mechanism *q*, through the permanent magnet *c*. The number of revolutions of slip is thus recorded on the dial Z_2; at the same time the number of revolutions of the rotor is shown on the dial Z_1, connected to an ordinary speed-counter. The operation of this device is entirely automatic; it is pressed against the shaft of the motor at *p*, as an ordinary speed-counter, and is held for, say, one minute. The reading on one scale Z_1 gives the actual speed of the motor; the reading of the other scale gives the number of revolutions of slip; adding the two gives the synchronous speed of the motor, and consequently the frequency of the supply.

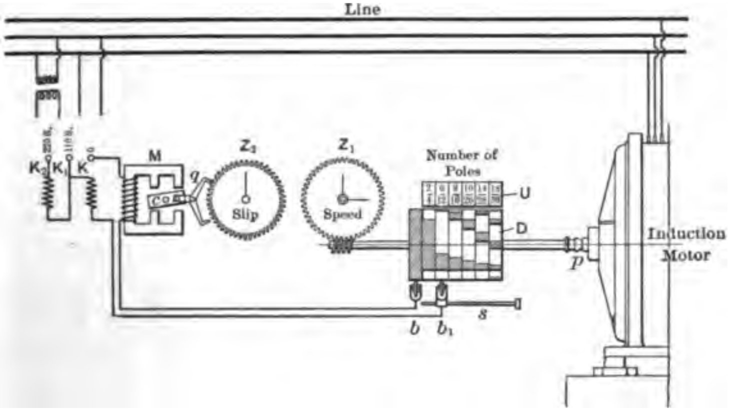


FIG. 406. The Bianchi automatic slipmeter.

The slipmeter is connected to the line either between the terminals *K* and K_1, or *K* and K_2, according to the voltage of the motor. For higher voltages a shunt-transformer is used. The drum *D* is provided with different combinations of contacts, to make possible the use of the device with motors of various numbers of poles. The setting is accomplished by the screw *s*, which moves the roller brush b_1. The number of poles is indicated on the scale *U*.

Date..................................Section.....

Observers:

....

....

....

....

Length of Brake Arm...............................

Tare or Zero Brake Reading........................

Make of Motor.....................................

Type — Open, semi-enclosed, enclosed, ventilated frame { Yes. / No.

Volts..................................Amps. per terminal..........

H.P......................Mfrs' No..............No. of Poles........

Speed { At full load.................... / At no load.................... Alts..............Cycles.....

Phase...............................Connections Y or Δ.

Type of Rotor { Phase wound. / Squirrel Cage. Fans { Yes. / No.

Method of Starting...

	Amperes.			Volts.			Watts.		Speed.		Slip indic.	Torque.
	A	B	C	AB	BC	CA	A — AB	C — CB	Motor.	Synchr.		Lbs.
Inst. No.												
Const.												

530. EXPERIMENT 24-B. — Brake Test on an Induction Motor. — The purpose of the experiment is to obtain performance curves of an induction motor, as shown in Fig. 400. Wire up the motor as per §§ 523 and 526, and let it run light for about 30 minutes to warm up the bearings. Then load the motor to about 25 per cent above its rated capacity, and, keeping the brake load as steady as possible, read volts, amperes, watts, pounds brake load, speed and slip. Reduce the load by approximately equal steps to zero, taking at each point the same readings as above. The form of data sheet shown on the opposite page will be found convenient for recording the readings.

Report. Give a diagram of connections used, and plot curves, as per Fig. 400. For one point on the curves give the methods and numerical calculations by which the ordinates were obtained. In figuring out results it is well to first multiply torque (at 1 foot radius) by speed and divide by 5252; this will give horse power for abscissæ. Watts and volt-amperes, divided by 746 give true and apparent horse power input. True efficiency, apparent efficiency and power-factor curves represent the ratios of the ordinates of the curves of output and input. Speed, slip and torque are plotted directly from the test data.

531. Objectionable Features of the Brake Test. — Those who have performed brake tests on induction motors know the difficulties involved in getting accurate data. The two principal complications of the test are: The necessity for using two or three wattmeters (or changing the wattmeter from one phase to another), and the difficulty in keeping the brake load constant while the speed or slip is measured. Because of these difficulties, and also because a comparatively large number of men are required for this test, a method has been developed which does away with the Prony brake, the output being calculated from the input and the losses in the motor.

The motor is belted to a generator, a blower or any other machine giving a steady load, which can be varied at will. The experiment is performed in exactly the same manner, as in the above-described brake test, except that no brake readings are taken. At the end of the test the belt is taken off and readings are taken of amperes and watts at no load. After this the motor is stopped and the resistances of the primary windings are determined, usually by the drop-of-potential method. From the last two readings the losses in the motor can be calculated; subtracting them from the input, the output is obtained without the necessity of having a brake. Experience shows that more reliable results can be obtained by this method, than by a direct brake test.

532. Losses in the Induction Motor. — The performance test described in the preceding article involves figuring out losses in the induction motor. These losses consist of copper loss in the primary and in the secondary windings, and of no-load losses (iron loss and friction). The latter can be assumed practically constant at all loads and having the same value as at no load (hence the name "no-load losses"). The primary copper loss can be easily calculated if the primary current and the resistance of the primary winding are known. The secondary copper loss is proportional to the per cent slip, as is shown below.

Suppose first the resistance of the secondary winding to be zero; the slip would be practically zero at any load because the secondary currents necessary for producing a torque could be induced with an infiintesimal difference in speed between the revolving flux and the rotor. If, however, the secondary winding has an appreciable resistance, a certain finite difference in speed is required in order to induce the same torque-producing currents. Thus, the necessary torque is obtained at a sacrifice of a certain per cent of speed. The corresponding loss of output is converted into the I^2R heat in the secondary winding.

The details of calculation of the output of the motor from its input and the losses may be best shown in a numerical example. A 10 horse-power, 440-volt, 3-phase induction motor was found to take 800 watts at no load; the no-load current was 5.5 amperes per phase, primary resistance 0.75 ohm per phase. From this data, iron loss and friction should first be computed; these are equal to 800 watts less a correction for the copper losses in the primary and the secondary windings. But the secondary copper loss is negligible at no load, because the slip is very small. Thus the correction amounts to $3 \times 5.5^2 \times .75 = 68$ watts, and the losses in question are equal to

$$800 - 68 = 732 \text{ watts.}$$

Take now a point on the load curve, for instance, corresponding to an input of 15 amperes per phase. Suppose that the power reading at this input was 9920 watts, and the slip 5.4 per cent. The primary copper loss is

$$3 \times \overline{15}^2 \times 0.75 = 506 \text{ watts,}$$

therefore

$$\text{Output} + \text{Sec. copper loss} = 9920 - (506 + 732) = 8682 \text{ watts,}$$

the last number representing the input into the secondary.

The secondary copper loss constitutes a part of the input into the secondary, proportional to per cent slip. Thus, in our case,

$$\text{Sec. copper loss} = 8682 \frac{5.4}{100} = 469 \text{ watts.}$$

Therefore

$$\text{Output} = 8682 - 469 = 8213 \text{ watts} = 11 \text{ horse-power.}$$

Having thus determined the output corresponding to a given input, the efficiency, the torque, etc., may be figured out as in the brake test before.

NOTE. It is not quite correct to subtract both iron loss and friction from the primary input. Most of the iron loss takes place in the primary, while the friction reduces the secondary output. The correct way would be to subtract the primary copper loss and the iron loss from the input into the primary; the result will represent the input into the secondary. The same being reduced by per cent slip represents the power developed on the shaft of the motor; subtracting the friction from it the power is obtained actually available for useful work. This accurate procedure is never used in practical testing, because there is no very simple method for separating iron loss from friction in an induction motor. The error committed is, however, so small, as to be negligible for practical purposes.

533. EXPERIMENT 24-C. — Performance of an Induction Motor, from Losses. — The motor is belted to a generator, or a blower, serving as a load, and a series of readings are taken as explained in § 530, except that the brake readings are omitted. At the end of the load test, careful readings should be taken of amperes and watts at no load, and the resistances of the primary windings determined. Measure (by thermometers) the temperature of the windings while measuring their resistance.

Report. The same curves should be plotted as in the report in § 530, the results to be derived by the method described in article 532. In figuring out the primary copper loss, use the value of the primary resistance, which it has at a temperature 50 degrees C. above the temperature of the room.

It should be borne in mind that when a resistance is measured between two terminals of a Y-connected induction motor, the result thus obtained represents the double resistance per phase. Therefore, in order to obtain the *average* resistance per phase, take the resistances between the terminals $A-B$, $A-C$, and $B-C$, add them together, and divide the result by six.

Plot on a separate sheet, curves of total losses, primary and secondary I^2R losses, and iron loss + friction, against horse-power output as abscissæ, so as to have a clear expression of the relative importance of these losses at various loads.

A modification of the above test is possible when a carefully calibrated generator or an accurate transmission dynamometer, such as the one described in § 332, is available as a load. No wattmeter readings are necessary in this case, and the input can be figured out from the output and the losses. If wattmeter readings are taken, they may be used as a check on the results. It is evident that such a check is always possible with the ordinary brake test, since the input can be figured out from the brake readings and the losses, as well as read directly on the wattmeter. Such a check is always desirable when the test is of particular importance.

SINGLE-PHASE INDUCTION MOTORS.

534. — It is a well-known fact that when a two-phase or a three-phase induction motor is running, and the connection between one of the phases and the line is broken, the motor continues to run, provided the load is not too heavy. In this case it is said to operate as a single-phase motor. The motor will not start, however (without special devices), when connected to but one phase of the supply. Induction motors wound single-phase are used to some extent in places where most of the load consists of lighting, and where, therefore, power distribution is effected by single-phase lines. Such motors require special devices for starting; some of these are described in §§ 536 to 538.

535. Action of Single-Phase Induction Motors. — The theory of single-phase induction motors is much more complicated than that of polyphase motors, because the currents and the fluxes in the former have rather an irregular and complicated form. The following simple explanation of the action of a single-phase induction motor is given without any claim to its absolute accuracy.

A single-phase stator winding can produce but a pulsating magnetic flux, and not a revolving one. Thus, when the motor is at rest, the rotor acts as the secondary of a transformer, and there is no reason why the motor should start in either direction. From a well-known mechanical law, the pulsating magnetic flux may be resolved into two fluxes revolving in the opposite directions. These two revolving fluxes produce equal actions on the secondary, and the actions are mutually destroyed.

But when the motor has been brought up to a certain speed by external means, the *relative* speed between one of the imaginary revolving fluxes and the rotor is decreased, while the other is increased. The dissymmetry produced is of such a nature as to give preponderance to the magnetic flux revolving in the same direction with the armature. The flux revolving in the opposite direction produces high-frequency secondary currents which are considerably out of phase with the flux, and therefore give only a small torque, which slightly reduces the useful torque of the motor.

For the theory of single-phase induction motors see Dr. McAllister's *Alternating Current Motors*, and B. A. Behrend's *Induction Motor.* An elegant and accurate theory may be also found in an article by Goerges in *Elektrotechnische Zeitschrift*, 1903, No. 15. See also § 562.

536. Starting with an Auxiliary Phase. — A revolving field is necessary in order to make an induction motor self-starting. This field may be produced by adding a second phase-winding to the main stator winding of the single-phase motor (Fig. 407). It is shown in § 520 that two windings, displaced relatively to each other, produce a revolving field, only when the currents flowing in these windings are out of phase with each other. In the case of the single-phase motor both windings are of necessity connected to the same phase of the supply, so that, if the circuits of the two windings were identical, the currents flowing through them would be in phase with one another, and no revolving field would be produced.

In order to shift or "split" the phase of the current in one of the windings, some resistance, inductance or capacity is connected in its circuit: This makes the phase displacement either larger or smaller than in the other phase, and a revolving field is produced. When the motor is started and a considerable speed is reached, the circuit of the auxiliary phase is opened, either automatically, or by hand, and the motor continues to run single-phase. To reverse the motor, the connections to either the main or the auxiliary phase are reversed.

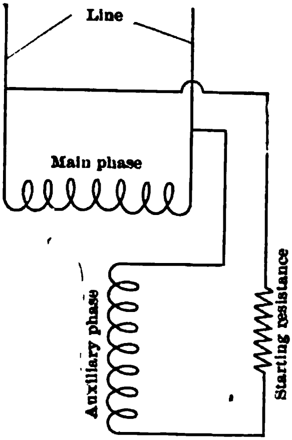


FIG. 407. Connections for starting a single-phase induction motor by means of an auxiliary phase.

A special case of single-phase motors is that of fan motors in which the auxiliary phase is produced by copper rings permanently short-circuited upon themselves, — so-called "shading coils" (Fig. 43). The principle shown there in application to a measuring instrument is also used in single-phase induction motors. Very small motors may also be started without any auxiliary phase, by simply giving an energetic pull on the belt or pulley.

The revolving field produced by means of an auxiliary phase is a rather imperfect "elliptical" field at the best, and no appreciable starting torque can be expected from single-phase induction motors. The motors are often supplied with friction clutches which are engaged only after the motor has reached a considerable speed. This is usually done automatically, operating on the principle of centrifugal force. A small starting torque must be had, if desired, at the expense of an excessive starting current.

537. Three-Phase Motors on Single-Phase Circuits. — There are cases in which it becomes desirable or necessary to operate three-phase induction motors on single-phase circuits, for instance, when one of the line wires or a transformer is put out of commission. A good starting arrangement is shown in Fig. 408. Two of the phase windings.

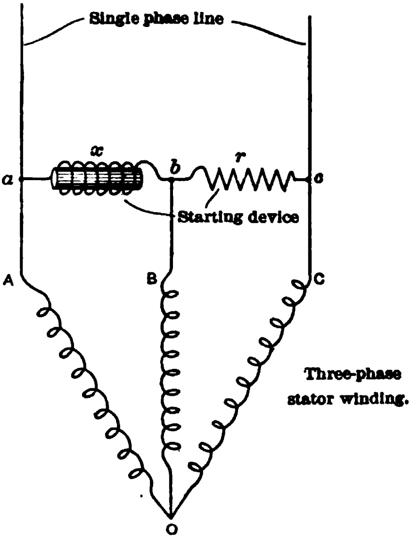


FIG. 408. Starting a three-phase induction motor on a single-phase circuit.

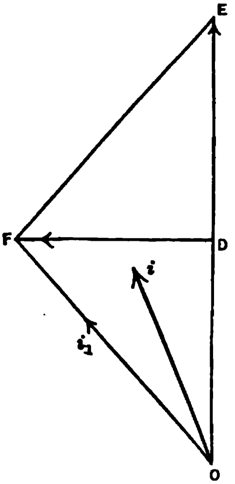


FIG. 409. Current and voltage relations with the connections according to Fig. 408.

OA and OC, are connected in series across the line and constitute the "running," or the main phase. The third winding OB is used for starting only, in connection with an inductance x and a resistance r. After the motor has reached a considerable speed, the circuits of the starting device and that of the auxiliary phase are opened.

The vectorial relations are represented in Fig. 409: OE is the line voltage, Oi the current in the main phase AC, lagging behind OE because of the inductance of the winding. Voltages across r and x are represented by OF and FE respectively. The current i_1 through r and x is in phase with OF and lags considerably behind FE. The voltage across the starting phase is represented by the vector

DF, since this phase is connected between the middle point of AC and the junction of r and x. It will be seen from the sketch that the voltage DF may be made to lag considerably in phase behind OE, so as to satisfactorily imitate a two-phase combination. The diagram represents the conditions taking place when the auxiliary phase is open; as soon as a current begins to flow through OB, the phase relations become less favorable and the phase angle between DF and OE decreases. Still, by suitably selecting r and x, a torque can be produced sufficient for starting the motor.

538. Starting as Repulsion Motor.—There is a class of single-phase motors, Fig. 410, which possess quite a considerable starting torque; this is obtained, however, at the expense of a commutator and brushes. The particular motor, shown in the sketch, has a stationary field winding FF, connected to the line, and producing a pulsating field, as shown by the arrows. The armature A is similar to that of direct-current machines, and is provided with a commutator. The brushes b b are shifted against the direction of the field, and are short-circuited upon themselves, thus forming a closed armature circuit. The pulsating field induces secondary currents in the armature, and the latter tends to move into such a position as to be subjected to minimum induction. This is impossible because the direction of the magnetic axis of the armature is determined by the position of the brushes; so the armature revolves continuously, trying to come into a stable position.

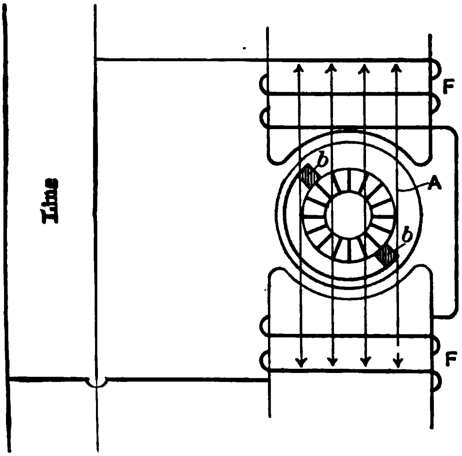


FIG. 410. Connections of a repulsion motor.

Such motors are called "repulsion" motors; they are seldom used at present on account of difficulties in commutation, and their so-called *series* characteristics. The speed of the motor greatly increases as the load decreases, the repulsion motor being in this respect similar to a direct-current series-wound motor (Fig. 287).

But there are good single-phase induction motors on the market, which *start as repulsion motors*. When a certain speed is reached, the commutator is automatically short-circuited, and the brushes are removed from the commutator by a centrifugal device to avoid unnecessary friction. The objections against regular repulsion motors are not valid in this case, and the combination motors give good satisfaction, especially if starting does not have to be done too frequently.

539. EXPERIMENT 24-D. — Exercises in Starting Single-Phase Motors. — A regular single-phase induction motor, with an auxiliary phase, should be provided for this experiment. Connect up the motor as shown in Fig. 407, or according to the diagram supplied with the motor. Start the motor at no load, and determine the first current inrush, current taken by the running phase, current taken by the starting phase, and time which it takes for the motor to reach full speed. If any adjustment in the starting phase is possible, vary the conditions so as to obtain the best results, viz., minimum starting current, or minimum time necessary to reach full speed. If the motor can be started with a small load, put on a brake and determine maximum starting torque possible with various values of starting resistance in the auxiliary phase. Read line amperes, amperes in both phases, ohms starting resistance, and pounds torque.

If a regular single-phase motor is not available, use an ordinary three-phase induction motor, connected as per Fig. 409. The resistance r and the inductance x should at first be adjusted so as to get any starting torque whatever; then by varying their values the starting conditions should be improved, as far as possible. After this either r or x should be kept constant, and curves taken showing the influence of the other factor. All currents, voltages and watts should be read, as far as possible, in the several branches of the circuit, so as to be able to construct diagrams, like that shown in Fig. 408. Voltages and currents should also be read with the phase OB open, in order to see the influence of the current in this phase on the current and voltage relations in the circuit $a—b—c$. If a starting torque can be obtained, the values of this torque should also be investigated.

Should the motor, available for test, be intended to be started as a repulsion motor, investigate the first current inrush, the power factor at start, time which it takes to reach the final speed, and maximum starting torque at which the motor can be started. If the brushes are adjustable, make a study of the influence of their position upon the starting characteristics of the motor.

Report the results of the tests, and construct vector diagrams showing current and voltage relations, such, for instance, as in Fig. 409. Plot curves giving starting amperes, torque, etc., as a function of the starting resistance or inductance, in case an auxiliary starting phase was used. For the repulsion motor plot the same values to the position of the brushes as abscissæ.

540. EXPERIMENT 24-E. — Brake Test on a Single-Phase Induction Motor. — The purpose of this test and the curves to be taken are similar to those indicated in experiment 24-B (§ 530). The wiring is much simpler, there being but one phase. Particular attention should be paid to an accurate determination of the pull-out torque; low overload capacity is one of the weaknesses of the single-phase induction motor, so that the ratio of the pull-out torque to the full-load torque should be considered when judging of the quality of the motor.

541. EXPERIMENT 24-F. — Performance of a Single-Phase Induction Motor from Losses. — The reasons for determining the performance of a single-phase motor from the losses, instead of by direct brake test, are the same as are given in § 531 for polyphase induction motors. The experiment is conducted in exactly the same way, as explained in § 533. The only difference in figuring out the results is in the expression for the secondary copper loss. It is shown in § 532 that in per cent of secondary input, the secondary copper loss in a polyphase induction motor is equal to per cent slip. With the single-phase motor the secondary copper loss is *twice* as large as per cent slip. For instance, if the slip of the motor at a certain is load 4 per cent, the secondary copper loss amounts to 8 per cent; in other words, 8 per cent of the input into the secondary are converted into heat in rotor windings. For a theoretical proof of this property of single-phase induction motors, see the references given in § 535.

CHAPTER XXV.

INDUCTION MOTORS — SPECIAL STUDY.

1. PREDETERMINATION OF PERFORMANCE.

542. ACTUAL load tests on induction motors, as well as on any other type of electrical machinery, are avoided, whenever possible, since they involve a considerable expenditure of energy, and special appliances, if accurate results are required. In machines beyond a certain size the waste of power and the arrangements for load make load tests practically impossible. Methods have been developed, therefore, for *predetermining* the performance of a machine, under actual load conditions, from a few simple readings taken at no load. Such a method in application to polyphase induction motors is given in the following articles.

The first step towards predetermination of performance is indicated in §§ 531 and 532, where it is shown that the output of an induction motor may be calculated from the input and the losses. But, a load test is necessary even there, in order to obtain the values of power factor and speed, corresponding to a given current input. It will be shown that power factor and speed may also be predetermined from no-load readings, thus making a load test with all its complications unnecessary.

543. Circle Diagram. — Let OE in Fig. 411 represent the impressed voltage in one of the phases, and OI the current in the same phase at a certain load; the angle ϕ between the two corresponds to the power factor of the motor at this load. *Theory and experiment show that, when load varies, the current changes in such a way that the locus of its vector is represented by a semicircle* $I_0 I K$.

The current at no load is represented by the vector OI_0; the current is small and the phase angle EOI_0 large, the power factor being low. The no-load current may be considered as consisting of a wattless component Om, which produces the magnetic flux, and of a power component mI_0, which overcomes iron loss and friction in the motor. As the load increases, the point I_0 moves along the circle, the value of the current increasing, as well as the power factor. When the motor becomes overloaded the power factor decreases again, due to a more pronounced influence of the magnetic leakage. When the load is such that the motor stops, the current which it takes at standstill is represented by a certain vector OI_s. The current OI_s is called the short-circuit current, or, more correctly, the current with the armature locked.

Thus, if the circle has been determined for a given motor, the power factor Cos ϕ may be measured from the diagram for any given value of the input current OI.

The proof of the proposition, that the locus of the points, such as I, is a semicircle, is given separately in §§ 554 and 555. We will here confine ourselves to the exposition of the application of the circle diagram. Apart from any theoretical proof, this may be accepted as a result of numerous direct experiments. It may be verified from any accurate load test by plotting the vectors of current to the observed values of the phase angle ϕ; it will be found that the load points lie on a semicircle.

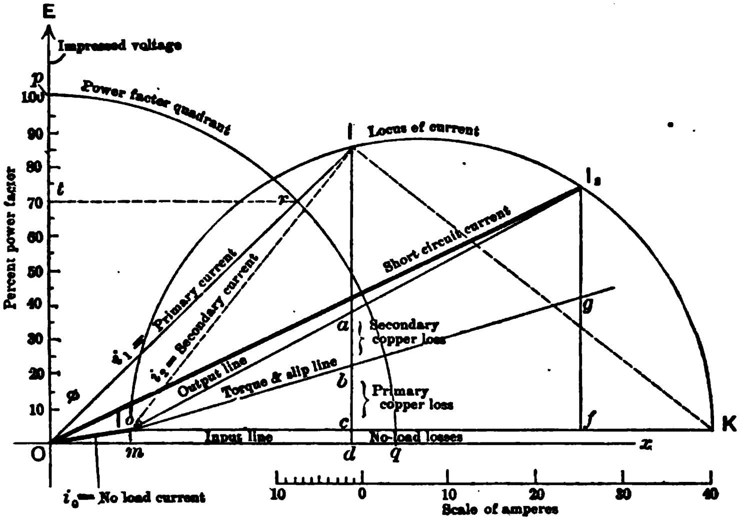


FIG. 411. The circle diagram for determining the performance of an induction motor (Heyland).

The circle may be constructed by knowing only one point on it, if, in addition, the position of the point I_0 is known. As such a second point, the point I_s is usually selected, the vector OI_s representing the current with the armature locked. The vector OI_0 is known from the no-load readings. Thus, the construction of the circle is made possible from no-load readings, and readings with the armature locked. In both cases no brake is required, and the necessary measurements are both simple and accurate.

OI being the primary current, its component Id in phase with the voltage OE represents the working component of the current: hence, Id multiplied by the voltage gives true watts input per phase. Thus the circle gives watts input and the value of the power factor, for any given value of the primary current.

Power factor Cos ϕ, equal to the ratio of Id to OI, can also be read directly in per cent on the quadrant prq. Namely, Cos ϕ is equal to the ratio of Ot to Or; but Or, the radius of the circle, being constant, Cos ϕ is directly proportional to Ot. By selecting Op equal to Or such as to be conveniently divided into 100 parts, power factor is read off directly in per cent, on the line OE.

It will now be shown, that output, torque, slip, efficiency, — in fact all the data for plotting the performance curves (Fig. 400) — may be predetermined from the same circle diagram.

544. Graphical Representation of the Losses. — The current cd, equal to I_0m, represents the no-load losses in the motor, being the working component of the no-load current OI_0. Thus, cd accounts for the iron loss, friction, and for the copper loss corresponding to the no-load current.

We shall now prove that ba may be made to represent the secondary copper loss, at the input corresponding to the ordinate Id. The secondary current at this load (reduced to the primary circuit) is represented by the vector I_0I. This is permissible since the primary current OI may be considered as consisting of two parts; magnetizing component OI_0 and the other component I_0I, which produces ampere-turns equal and opposite to the secondary ampere-turns. Thus, we have to prove that ab is proportional to $\overline{I_0I}^2 \times r_2$, where r_2 is the secondary resistance of the motor, also reduced to primary terms (§ 409).

The triangles II_0c and II_0K being similar, we have that I_0c is equal to $\overline{II_0}^2 \div I_0K$. Thus, the loss in question is proportional to the segment I_0c. Now, by drawing any two lines such as I_0I_s and I_0g, radiating from the point I_0, it will be seen that the segment ab which these lines cut from the ordinate Id is proportional to I_0c. Therefore, the secondary copper loss may be represented by ab, provided the rays I_0I_s and I_0g are properly selected.

The primary copper loss $i_1^2r_1$ cannot be represented with exactness in a similar simple way; it may be shown, however, that it can be represented with sufficient accuracy by the segment bc. Namely, assuming the triangle OII_0 to be *approximately* a right-angle one, we have

$$i_1^2 = i_2^2 + i_0^2.$$

Multiplying this expression by r_1 we get that the primary copper loss $i_1{}^2r_1$ may be approximately represented by

$$i_2{}^2r_1 + i_0{}^2r_1.$$

We have already seen that $i_0{}^2r_1$, being the primary copper loss at no load, is already taken into account as a part of the ordinate *cd*. The other term, $i_2{}^2r_1$, has the same form as the secondary copper loss, except that r_1 is substituted for r_2. Thus it can also be represented as a segment, such as *bc*, cut off from the input *Id* by two lines radiating from the point I_0.

The proper position of the line I_0g is found from the fact, that the whole input, with the armature locked, is converted into losses, since no useful work is performed by the motor under these conditions. The distance between the lines I_0K and Ox, accounts for iron loss, friction, and the copper loss corresponding to the no-load component of the primary current; consequently, the ordinate I_sf must account for the secondary copper loss and the primary copper loss corresponding to the component I_0I of the primary current.

The expression $\overline{I_0I_s}^2 \times r_1$ is calculated first, and plotted to the suitable scale as the ordinate *fg*; the rest, gI_s, represents the secondary copper loss with the armature locked. In motors provided with a squirrel-cage secondary this loss cannot very well be determined experimentally, or calculated directly.

Thus, by drawing the lines I_0I_s and I_0g, all the losses in the motor may be separated, and the output calculated, corresponding to a given input.[*]

545. Output and Torque. — The ordinate *Id* representing the true input into the motor, and *da* being the sum total of the losses, the difference of the two, or *Ia*, represents *the output of the motor, in amperes*. Multiplying it by the voltage and dividing by 746 the output in horsepower is found.

The ordinate *Ib* is a sum of the output and the secondary copper loss, hence it represents working amperes input into the secondary: It is proportional to the energy transmitted from the primary into the secondary circuit, through the mechanical torque existing between these two parts of the motor. Therefore, *Ib* represents the torque of the motor. To obtain the torque in foot-pounds, *Ib* must be multiplied by the voltage, and divided by the *synchronous* speed of the motor. Synchronous speed should be taken in this case, and not the actual speed, because the diagram is constructed from the standpoint of the primary part of the motor. The torque is considered here as the input into the secondary, produced by a revolving magnetic flux which rotates synchronously.

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[*] The circle diagram, explained here, is accurate in all respects, except in the part where the primary ohmic loss is accounted for. In motors above 1 or 2 horsepower, the primary copper loss is usually so small in itself, that some inaccuracy in its value does not appreciably alter the results. With very small motors it is preferable to obtain performance curves directly from a brake test, the more that it can be performed much easier than with larger motors. Another alternative is to use more complicated diagrams, for instance such as described by Mr. Specht in the *Electrical World and Engineer*, 1905, p. 388.

Up to this time, the vectors, such as *OI*, *Id*, etc., have been supposed to represent currents in amperes, and to be converted into watts by being multiplied by the voltage of the supply. Instead of multiplying each separate ordinate, it is more convenient to change the scale of the circle so as to obtain vectors directly in watts, instead of amperes. Then the input *Id*, the output *Ia*, and all the losses may also be read in watts. If it is desired to express the input and the output in horse-power, instead of watts, the scale for the circle may be selected so that the ordinates will read directly in horse-power. This latter method for constructing the circle diagram is adopted in the instructions given in § 549 below.

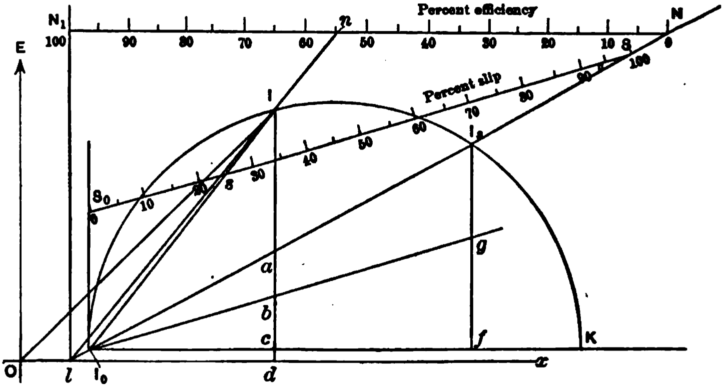

FIG. 412. Determination of efficiency and slip from the circle diagram.

546. Efficiency Scale.—Efficiency may be determined from the circle diagram (Fig. 411) as a ratio of the output *Ia* to the input *Id*. Some prefer to read the efficiency directly from the diagram, without taking the ratio. This is done by drawing an efficiency scale NN_1 (Fig. 412). Produce the line I_0I_s to the intersection with the axis Ox at the point l, and draw the vertical line lN_1. The efficiency scale NN_1 is a horizontal line drawn at any desired distance from the axis of abscissæ, and is only limited by the condition that its zero N should lie on the line $I_0 I_s$, and the point N_1, corresponding to 100 per cent efficiency, must be on the same vertical with the point l. To find the efficiency corresponding to a certain input Id, connect l to I and produce lI to its intersection with the efficiency scale at n. Nn gives the efficiency directly in per cent, while nN_1 represents the total losses, also in per cent of the input.

The proof of this construction is as follows: The triangles lNN_1 and lad are similar, therefore

$$ad \div ld = lN_1 \div N_1 N.$$

The triangles lnN_1 and lId are also similar, so that we have

$$Id \div ld = lN_1 \div nN_1.$$

Dividing the first expression by the second, we obtain

$$ad \div Id = nN_1 \div N_1 N,$$

or, in words,

$$\text{total losses} \div \text{input} = nN_1 \div 100.$$

This shows that nN_1 represents per cent losses; the rest, Nn, evidently represents the per cent efficiency.

547. Slip Scale. — It is pointed out in § 532 that in polyphase induction motors, per cent slip is equal to per cent secondary copper loss. This loss is represented in the diagram (Fig. 412) by ab; the input into the secondary is equal to Ib. Consequently, the slip of the motor at this particular load is equal to per cent ratio of ab to Ib.

Instead of figuring out the values of this ratio for various loads, it is convenient to have a scale on which per cent slip can be read off directly as is explained above for the efficiency.

Any line parallel to $I_0 g$, such as $S_0 S$, may be used as a slip scale, provided its zero point lies on the same vertical with the point I_0, and the division corresponding to 100 per cent slip is on the line $I_0 I_s$. For any load, such as Id, produce $I_0 I$ to its intersection with the slip scale at s. $S_0 s$ reads slip directly in per cent; the remainder, sS, gives the actual speed of the motor, in per cent of synchronous speed.

The proof of this construction is as follows: The triangles $I_0 S S_0$ and $I_0 ab$ are similar, so that

$$ab \div I_0 b = I_0 S_0 \div S_0 S.$$

The triangles $I_0 s S_0$ and $I_0 Ib$ are also similar; consequently,

$$Ib \div I_0 b = I_0 S_0 \div S_0 s.$$

Dividing the first expression by the second, we obtain

$$ab \div Ib = S_0 s \div S_0 S,$$

or, in words,

$$\text{secondary copper loss} \div \text{input into the secondary} = S_0 s \div 100,$$

which proves the construction.

548. Maximum Input, Maximum Output, etc. — The circle diagram (Fig. 412) gives directly the points at which various performance values obtain their maximum:

(1) Maximum input corresponds to the point at which the tangent to the circle is parallel to the axis Ox.

(2) Maximum torque corresponds to the point at which the tangent to the circle is parallel to the line I_0g.

(3) Maximum output corresponds to the point at which the tangent to the circle is parallel to the line I_0I_s.

(4) Maximum efficiency corresponds to the point at which the tangent from l touches the circle.

(5) Maximum power factor corresponds to the point at which the tangent from O touches the circle.

In plotting the performance curves from the diagram it is well to have all these points of maximum marked first, as they add considerably to the accuracy of the curves.

549. Instructions for Constructing a Circle Diagram. — The following instructions are given in order to enable the student to construct quickly and without hesitation a circle diagram (Figs. 411 and 412) from the results of a test, and to plot the corresponding performance curves, shown in Fig. 400. The instructions are given for three-phase motors, and the points of difference indicated for two-phase motors.

(1) *Experimental Data.* The following data must be obtained experimentally, as is explained in §§ 551 to 553: Total no-load watts W_0, no-load amperes i_0 (per phase); total watts W_s with the armature locked (short-circuit watts), short-circuit current i_s (per phase). All of the above at the rated terminal voltage E of the motor. Resistance R (average) of the stator between two terminals.

(2) *Power Factor Quadrant.* Select, on a sheet of cross-section paper, convenient reference axes Ox and OE (Fig. 411). Take Op equal to 100 per cent power factor to a convenient scale; the scale should be preferably the same as used for plotting the power-factor curve on the curve sheet (Fig. 400). With Op as a radius strike the quadrant prq. For any vector of primary current, such as OI, the power factor is represented by the ordinate Ot of the point r, at which this vector intersects the quadrant.

(3) *Input.* Figure out the apparent horse-power input with the armature locked, $i_sE\sqrt{3} \div 746$ h.p., and plot it to a convenient scale as the vector OI_s, at a power factor $= 100W_s \div i_sE\sqrt{3}$ per cent. In the same way, plot the apparent horse-power input at no load, $OI_0 = i_0E\sqrt{3} \div 746$ h.p., the power factor being $100W_0 \div i_0E\sqrt{3}$ per cent.

Through I_0 and I_s draw a semicircle KI_sI_0, having its center on the horizontal line IK_0, passing through the point I_0. For any point, such as I, the vector OI represents the apparent horse-power input, the ordinate Id measures to the same scale true horse-power input. Maximum input which the motor can take corresponds to the point at which the tangent to the circle is parallel to the axis Ox. It is advisable to select the same scale for horse-power, that will be used on the curve sheet: the ordinates can then be transferred directly, without having them scaled off.

For two-phase motors substitute in the above formulæ the coefficient 2 in place of $\sqrt{3}$.

(4) *Primary Current.* The vector of the primary current coincides with the vector OI of apparent input. If OI is measured in horse-power, the primary current per phase $i_1 = 746 OI \div E\sqrt{3}$ amperes. For two-phase motors substitute 2 in place of $\sqrt{3}$.

(5) *Output.* Connect I_0 with I_s. Now Ia measures the output in horse-power, to the same scale to which Id measures the input. Maximum output corresponds to the point on the circle, at which the tangent to the circle is parallel to I_0I_s. To find the point corresponding to the *rated* output of the motor, draw a line parallel to I_0I_s, at a vertical distance from it, equal to this horse-power. This line will intersect the circle in two points; the point to the left is the one required, since it corresponds to a higher power factor and a higher efficiency.

(6) *Torque.* Figure out the expression

$$3 \times \left(i_s \frac{I_0I_s}{OI_s}\right)^2 \times \tfrac{1}{2}R \div 746 \text{ h.p.}$$

(reduced primary copper loss with the armature locked), plot it to the horse-power scale as fg, and draw I_0g. For any input, such as Id, the ordinate Ib measures torque in *synchronous* horse-power.

$$\text{Torque in ft. lbs.} = \frac{\text{torque in synchron. h.p.} \times 5252.}{\text{synchron. rev. per min.}}$$

The maximum or the pull-out torque corresponds to the point, at which the tangent to the circle is parallel to I_0g. For two-phase motors use in the above formula the coefficient 2 instead of 3, and R instead of $\frac{1}{2}R$.

(7) *True Efficiency.* The true efficiency of the motor is equal to the ratio of Ia to Id. If it is desired to measure the efficiency directly, produce I_0I_s to l (Fig. 412) and draw the vertical line lN_1. The efficiency scale NN_1 is then drawn at such a distance from the axis Ox as to get the length NN_1 conveniently divisible by 100. It is best to

use for NN_1 the same scale which is intended to be used for efficiency on the curve sheet. To get the efficiency corresponding to the input Id lay a straight edge between the points l and I: the efficiency is read directly in per cent at the point n on the same line.

(8) *Apparent Efficiency* is calculated as the ratio of Ia to IO.

(9) *Slip.* Per cent slip may be calculated as the ratio of ab to Ib. If it is desired to read the slip directly, draw the vertical line I_0S_0 (Fig. 412) and a slip scale S_0S, parallel to I_0g. Select the distance at which the slip scale is drawn such as to have S_0S conveniently divisible into 100 parts. Or else, select S_0S equal to the synchronous revolutions per minute of the motor, to the scale used for the speed curve on the curve sheet. To find the slip for a point, such as I, lay a straight edge between the points I_0 and I: S_0s measures the slip, and sS the actual speed of the motor, either in per cent of the synchronous speed, or in actual revolutions per minute.

(10) *Plotting Curves.* Select on the circle several points such as I and mark the corresponding horse-power output as abscissæ on the curve sheet (Fig. 400). For each point, I, measure on the diagram: power factor, true and apparent input, torque, efficiency, slip and speed as explained above, transferring the lengths directly to the curve sheet. Determine also from the diagram the points of maximum input, maximum output, maximum power factor and maximum efficiency. Mark all these points on the curve sheet and then draw the curves.

550. Test Data for Predetermination of Performance. — In explaining the construction of the circle diagram, it was stated that the data, necessary for predetermining the performance curves of an induction motor, are:

(1) Ampere and watt input with the motor running idle.

(2) The same with the armature locked.

(3) Resistance of the stator winding.

It may seem at first that these tests, together with the necessity of constructing a diagram, and other calculations, should take more time than an ordinary brake test. Experience shows, however, that better results are obtained by the indirect method, even with less experienced observers, than is possible with a brake test. It is easier to perform a few simple runs in succession, than one complicated test.

The tests for obtaining the above data will now be described in detail; explicit instructions for constructing the diagram are given in the preceding article.

551. EXPERIMENT 25-A. — Measuring Resistances of Induction Motor Windings. — This experiment, with the two following, is intended to supply data, necessary for the predetermination of performance of the induction motor by means of the circle diagram (§ 549). Aside from this, the resistance may be measured for some other purpose, for instance, in connection with a heat run, or as a check on the construction of the motor. The resistance of the stator windings of an induction motor is usually measured by the drop-of-potential method. Direct current is applied between pairs of terminals of the motor in succession, and volts and amperes are read. Then the current is changed and the readings are taken again. It is advisable to have at least six sets of readings for each pair of terminals. Insert a thermometer into the winding so as to know the temperature to which the calculated resistance refers.

Report. If the motor is Y-connected, a resistance measured between two terminals corresponds to two phases in series, and should be divided by 2 in order to get the resistance of one phase. It is customary to first get the averages of all the readings between a certain pair of terminals, and then calculate the resistance per phase from these averages. If only the average resistance per phase is required, the three averages are added together and the result divided by 6.

If, however, a fault in the winding is suspected, or for any other reason it is desired to know the resistance of each particular phase, the problem is reduced to three equations with three unknown quantities:

$$\begin{aligned} x + y &= R_{1-2}, \\ x + z &= R_{1-3}, \\ y + z &= R_{2-3}; \end{aligned}$$

from these equations the individual phase resistances x, y, and z can be easily determined.

It is left to the student to prove that resistances can be determined in the same way when the motor is Δ-connected, so that *in performing the test it is not necessary to ascertain the character of connections in the motor.*

A good way to get an average resistance from several readings of volts and amperes, is to plot volts to amperes as abscissæ and through the points thus obtained to draw a straight line passing through the origin. The trigonometrical tangent gives directly the value of the average resistance.

552. EXPERIMENT 25-B. – No-Load Characteristics of an Induction Motor. — The purpose of this test is to determine watts and amperes taken by the induction motor at no load, the data to be used in the construction of the circle diagram, together with the data of the preceding and the following experiments.

Wire up the motor as for regular operation and have resistances or transformers with taps provided in the primary circuit, in order to vary the voltage at the motor terminals (Fig. 398). Have a voltmeter, an ammeter, and a wattmeter connected in the primary circuit through a polyphase board (§ 49), in order to be able to use these instruments in all the phases in succession.

In measuring the power input by the two-wattmeter method (§ 431) it must be remembered that the power factor at no load is usually below 50 per cent, so that the *difference* of two readings must be taken, instead of their *sum*.

Run the motor for some time to obtain steady conditions of friction and lubrication. Then raise the terminal voltage about 50 per cent above the rated: read volts, amperes, watts and speed, the latter merely as a check on the frequency. Reduce the terminal voltage in steps, taking similar readings. Go to the lowest limit of the voltage, at which the motor is capable of running.

Report. Plot no-load amperes per phase and total watts, to terminal volts as abscissæ. Mark the values at the rated voltage, these values to be used in constructing the circle diagram. Explain the peculiar shape of the curves at low voltages.

553. EXPERIMENT 25-C. — Short-Circuit Characteristics of an Induction Motor. — The purpose of this test is to determine watts and amperes taken by an induction motor with the armature locked, the data to be used in the construction of the circle diagram, together with the data of the two preceding experiments.

This test should be performed very carefully, because the current with the armature locked is several times larger than the rated current of the motor, especially in motors of considerable size. Here, as in the previous test, it is advisable to take complete curves of amperes and watts, by varying the applied voltage. Begin with the highest voltage that the armature can stand, while the motor is yet cold, and then rapidly take the readings, gradually reducing the voltage. In many cases it is impossible to get readings at the rated voltage, as the winding heats up too rapidly.* In such cases the readings are taken for lower voltages, and the curves *extrapolated* to the rated voltage, as is explained below. This is easily done because the curve of amperes is practically a straight line, and the curve of watts is a parabola.

In measuring watts be sure to mark on the data sheet whether the two readings are to be added or subtracted; in other words, whether the power factor is above or below 50 per cent (§ 432). An easy way to determine this is to let the motor run at no load and to connect the wattmeter so that the two readings are positive. If they remain positive with the armature locked, the power factor remained below 50 per cent. If it becomes necessary to reverse the connections in one of the phases, the power factor is above 50 per cent.

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* Should the windings get very hot, run the motor for a time at no load, to cool them off by the draft of air produced.

Report. Plot amperes per phase and total watts, to terminal volts as abscissæ. Mark the values corresponding to the rated voltage, these values to be used in constructing the circle diagram.

If it is impossible to get readings at the rated voltage, the curves must be extrapolated. The curve of the current is theoretically a straight line passing through the origin: in practice, it often has a small curvature near the origin, probably due to a peculiar condition in iron with very low saturation. These lower points should be left out of consideration and a straight line drawn through the other points; this line may not pass through the origin.

The curve of watts is theoretically a parabola:

$$\text{Watts} = CE^2,$$

where C is a constant. The easiest way to extrapolate this parabola is to figure out the working component of the current, since this component is proportional to the voltage:

$$\text{power component} = \frac{\text{Watts}}{E} = CE.$$

Plot this power component to volts as abscissæ; it will give an approximately straight line. Produce this line up to the rated voltage; the ordinates of this straight line, multiplied by the corresponding voltages, give the ordinates of the watt-curve.

554. First Proof of the Circle Diagram. — No proof was given in § 543 above for the statement that the locus of the primary current in an induction motor is a semicircle. Two proofs will be given now, one based on reducing the induction motor to an equivalent stationary transformer, the other on an analysis of magnetic fluxes in the motor.*

Instead of considering the rotor revolving and delivering mechanical power, it may be assumed as locked and loaded electrically on non-inductive external resistances. The same currents can be produced in the primary windings, by suitably varying the resistances, as if the rotor were revolving and were loaded mechanically.

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* The author's experience is that either the first or the second proof appeals more to the reader, depending upon differences of temperament, and of preparation.

By stopping the motor, the secondary induced e.m.f. is increased $n_1 \div (n_1 - n_2)$ times, where n_1 is the synchronous speed of the motor, and n_2 is its actual speed: The revolving flux cuts the secondary conductors at a rate proportional to $(n_1 - n_2)$ when the rotor is revolving; with the armature locked the speed at which the conductors are cut by the flux is equal to n_1. The frequency of the secondary currents being thus greatly increased, the reactance of the secondary is thereby also increased the same number of times. By adding such an external resistance that the rotor resistance is increased $n_1 \div (n_1 - n_2)$ times, the same current and the same electrical relations are obtained in the secondary, as with the rotor revolving; consequently, the primary current will have the same value and the same power factor. Varying the external resistance is equivalent to varying a brake load with the armature revolving. In this way the induction motor is reduced to a stationary transformer, with an abnormal leakage and with a secondary load of non-inductive resistances, varying in value from zero to infinity.

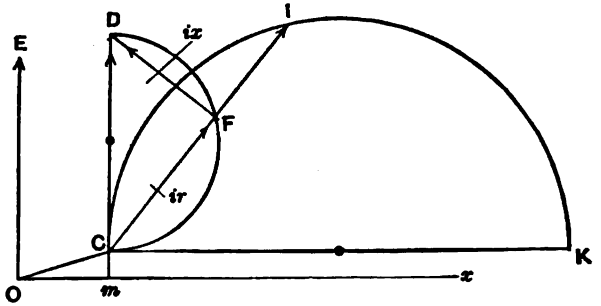

FIG. 413. First proof of the circle diagram: the induction motor is reduced to an equivalent transformer.

Such a transformer can be replaced by an equivalent resistance and an equivalent reactance (see § 408), and the problem is reduced to merely this: *A non-inductive load is connected to a constant-potential line, with some resistance and some inductance in series with it* (Fig. 240). *The locus of the current vector when the load changes from zero to infinity is to be determined.* This problem is solved in Fig. 413. *CD* represents the vector of the line voltage, which is constant; *CI* is the current at a certain load. *CF*, in phase with the current, represents the resistance drop; *FD*, perpendicular to the current vector, represents the reactive drop. *CD*, the geometrical sum of *CF* and *FD*, is constant; as the latter two vectors are perpendicular to each other, the point *F* moves on the semicircle having *CD* for its diameter. The equivalent reactance being constant under all conditions of load, the vector *FD* is proportional to the current, and represents it to a certain scale. Thus, it will be seen that when the load varies, the extremity of the vector of the current moves on a semicircle. This semicircle, *CIK*, has its diameter perpendicular to *CD*, because *FD* is perpendicular to the true position of the current vector.

The above proof applies to the induction motor, since it has been demonstrated before that the circuits in the two cases are electrically equivalent. The only difference is that, in addition to the load current *CI*, the induction motor takes a magnetizing current *Om* and a small power component *mC* for overcoming iron loss and friction. This brings the total primary current to the value *OI*, and the diagrams, shown in Figs. 411 and 413, become identical.

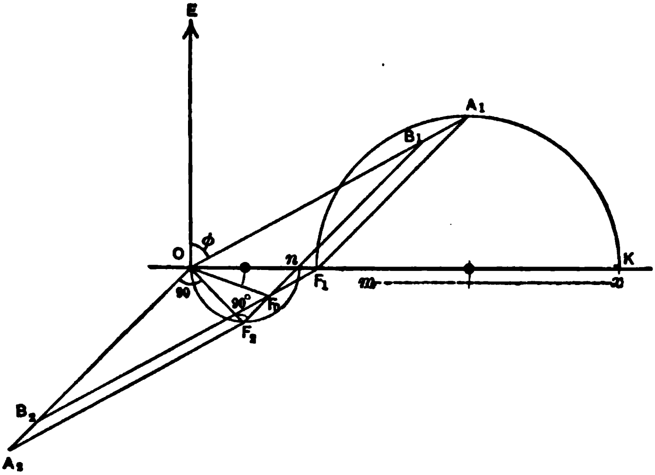

FIG. 414. Second proof of the circle diagram: relation between the primary and the secondary fluxes.

555. Second Proof of the Circle Diagram. — The above given proof, while correct in its reasoning, does not consider the phenomena in the induction motor in their particular physical aspect, but reduces the problem to a combination of fictitious resistances and reactances. The following proof is based directly on a consideration of the magnetic fluxes, currents and voltages in the motor.

Let *OE*, Fig. 414, represent the applied e.m.f. at the terminals of the motor, and OA_1 the vector of the primary current in one of the phases. This current may be considered to represent, to a certain scale, the primary ampere-turns, or the stator magnetomotive force. Let OA_2 represent the secondary magnetomotive force, or the secondary current reduced to the primary circuit. The actual magnetic fluxes in the primary iron, in the air-gap, and in the secondary iron are produced by these two magnetomotive forces. Only a part OB_1 of the primary magnetomotive force is effective in producing the flux in the secondary iron, because of the primary leakage. The resultant magnetomotive force in the secondary, OF_2, is the geometrical sum of OA_2 and OB_1. OF_2 may be also considered to a certain scale as the secondary magnetic flux.

Similarly the primary magnetic flux OF_1 is produced by the total primary magnetomotive force OA_1 and the part OB_2 of the secondary magnetomotive force; the rest B_2A_2 being lost as the secondary leakage.

The primary flux OF_1 is perpendicular to the vector of the voltage OE (neglecting small ohmic drop in the primary winding). The secondary current OA_2 is in phase with the secondary induced e.m.f., and is, therefore, perpendicular to the secondary flux OF_2, which produces this e.m.f. The applied voltage OE being constant, the primary flux OF_1 is also constant under all conditions of load. The segment On is also constant, being a definite part of OF_1. This is because the triangles OA_1F_1 and OB_1n are similar, and the leakage ratio, B_1A_1 to OA_1, remains constant under all conditions of load.

Thus we have two conditions: (1) OF_2 is perpendicular to OA_2, or, which is equivalent, perpendicular to F_2B_1. (2) On is constant under all conditions of load. From these two conditions follows directly that, when the load varies, the point F_2 moves on the semicircle having On for its diameter.

It can be shown that the point A_1 also moves on the semicircle.. The secondary current OA_2 is not only perpendicular to OF_2, but is also proportional to its value, because the flux OF_2 induces the secondary e.m.f. The segment F_1A_1 is parallel and proportional to OA_2 and consequently is perpendicular and proportional to OF_2. Therefore, when OF_2 describes a semicircle, F_1A_1 also describes a similar semicircle; in other words, the point A_1 moves on the semicircle F_1A_1K.

But A_1 is the extremity of the vector of the primary current OA_1, and we thus arrive at the proof, that the locus of the extremity of the current vector is a semicircle. To make the diagram Fig. 414 identical with that in Fig. 411, the origin O must be shifted down to the line mx, in order to account for iron loss and friction proportional to mI_0.

556. Predetermination of Performance of Single-Phase Induction Motors. — Theory and experiment show that the locus of the primary current for the single-phase induction motor is a semicircle, as for the polyphase induction motor, so that the predetermination of its performance by means of a circle diagram is possible. However, the relations in the diagram are somewhat different from those deduced above, and the scope of use of single-phase motors does not warrant a detailed description of the corresponding diagram here. Moreover, single-phase induction motors are built in small sizes only, and there is no difficulty in obtaining the performance characteristics, either from a direct brake test, or from the losses.

Those interested in performance diagrams of single-phase induction motors should consult Dr. McAllister's *Alternating-Current Motors* and B. A. Behrend's *Induction Motors.*

2. STUDY OF THE REVOLVING MAGNETIC FIELD.

557. It has been the experience of the writer, that the average electrical engineer has a rather indefinite idea concerning the rotating magnetic field, *as a physical phenomenon.* This lack of knowledge in turn makes difficult an understanding of the theory and design of alternating-current machinery: the mathematical relations found in books often seem arbitrary and not sufficiently real; nor does the engineer see his way clear for original research in this branch of electrical engineering. It is with the view of giving a clearer *experimental* foundation for the knowledge of the revolving magnetic field that the following experiments are arranged. They are not regular commercial tests, but rather experimental demonstrations of the validity of the assumptions, made in practice, in regard to revolving magnetic flux.

558. Component Fluxes and Resultant Flux. — An elementary explanation is given in § 520 of how a *revolving* magnetic field is produced by a combination of two or more *pulsating* fluxes displaced in phase both geometrically and electrically. We shall now give the numerical relations between the values of the component fluxes and the intensity of the resultant flux, produced by three-phase currents.

Suppose the *maximum* magnetic density produced in the air-gap by the current *in one phase* to be B (Fig. 415); the density at a certain point, having the abscissa x, and at a time t is then

$$B \text{ Sin } mt \text{ Cos } x,$$

assuming the flux to be distributed *in space* according to the sine wave, and also varying *with the time* according to the same law. The density produced at the same point and at the same time by the second phase is

$$B \text{ Sin}\left(mt - \frac{2\pi}{3}\right)\text{Cos}\left(\frac{2\pi}{3} - x\right),$$

and that produced by the third phase:

$$B \text{ Sin}\left(mt + \frac{2\pi}{3}\right)\text{Cos}\left(\frac{2\pi}{3} + x\right).$$

Adding these three expressions, in order to get the resultant flux density, when all three currents are acting simultaneously, we get, after suitable mathematical transformations:

$$1.5\, B \operatorname{Sin}\, (mt - x).$$

This result being interpreted means that the maximum density, produced by the three phases, is only 50 per cent higher than that produced by each phase separately. Moreover, it means that this resultant maximum density *travels* along the air-gap at the angular velocity *m*, being distributed in space at any moment according to the sine law. The practical importance of this result is that in figuring out the magnetic flux of an induction motor with a given excitation, or vice versa, the calculations may be reduced to one phase by the above factor 1.5.

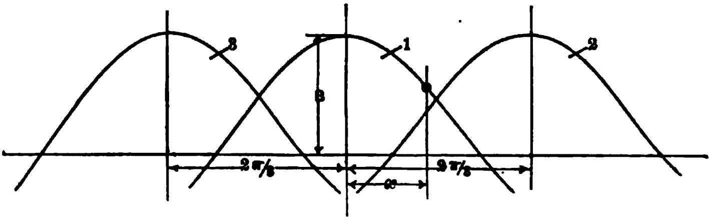

Fig. 415. Magnetic fluxes produced by three phases in the air-gap of an induction motor.

The factor 1.5 is correct only with sine-wave distribution of currents and fluxes. Factors differing from 1.5 are sometimes used in designing induction motors, and it is claimed that they represent more closely the actual form of the field, as influenced by the distribution of the winding. It has been shown experimentally, however, that the actual revolving flux follows quite closely the sine-wave law, with practically any distribution of windings. The reason probably is that higher harmonics are suppressed by the damping effect of the secondary winding.

559. EXPERIMENT 25-D. — Study of the Revolving Magnetic Field Produced by Alternating Currents. — The purpose of the experiment is to observe the relations explained in the preceding article. In order to simplify the measurements, the rotor should be taken out and a plain laminated iron core put in its place: the same offers a closed path to the lines of force, without influencing the field by induced secondary currents. If such a core is not available, the regular squirrel-cage rotor can be used, with one of the end-rings removed, so as to have the secondary circuit open. If the motor has a phase-wound secondary, the circuit is simply opened at the slip rings. In all cases the rotor is kept stationary during the experiment.

An exploring coil of a width corresponding to the pole pitch is placed on the secondary, and is connected to a suitable alternating-current voltmeter. Single-phase or polyphase currents are sent through the primary winding of the motor, and the magnetic flux in the air-gap is studied by measuring the voltage induced in the exploring coil.

(1) Excite one phase of the stator, between a terminal and the neutral point, by a single-phase alternating current, and investigate the magnetic field thus produced, by changing the position of the rotor, step by step, and measuring the induced secondary volts. It will be found, that the induced voltage is a maximum, when the secondary coil is held opposite one of the primary coils, and is zero in the position between two stator coils. It varies gradually in intermediate positions, as it should with the distribution of the flux, shown in Figs. 415 and 393–394.

(2) Repeat the same experiment with two phases in series (a double number of slots per pole).

(3) Change the connections between the coils so as to get half the number of poles, and again investigate the field.

(4) Now excite all three phases by three-phase currents, so as to produce a revolving magnetic field. It will be found, that the voltage induced in the exploring coil is practically constant in all positions. The explanation of this is, that the same field, traveling along the air-gap, will always cut the exploring coil, wherever it is placed. It will also be found that the induced voltage is not three times larger than that induced by one phase, but only about 50 per cent larger. This is due to the maxima of the magnetizing current in the three phases not occurring simultaneously, as is shown in the preceding article.

(5) In order to see more clearly that a *revolving* magnetic flux is actually produced, the central core is taken out, and a light pivoted piece of soft iron put in its place. It will be found, that this piece of iron will tend to revolve, attracted by the flux. Instead of soft iron, a copper or aluminum cylinder can be used; eddy currents induced in it by the revolving flux will be sufficient to set it in rotation. Reversing two primary leads reverses the direction of rotation of the magnetic field, and consequently that of the armature. In performing this last experiment the student must be careful not to overload the primary winding: the motor takes much more current when the central iron core is taken out.

Report. Plot curves showing the form of the single-phase field with one phase excited, and with two phases in series. Explain the difference in its shape and magnitude by constructing the theoretical form of the field. Give the ratio obtained between the values of single-phase field and three-phase field; if it should be much different from the theoretical ratio 1.5, explain the cause.

560. The Determination of the Instantaneous Values of a Revolving Flux. — The preceding experiment gives only general information concerning the revolving field. If a more detailed investigation is desired, it becomes necessary to "stop" the alternating currents at a certain instant, or fix them at certain instantaneous values, until the form of the field is investigated by means of an exploring coil. Then another set of instantaneous values may be taken, the corresponding field explored, etc. This is conveniently done by means of a direct current, instead of alternating: the values of the currents in the three stator windings are adjusted to correspond to some three instantaneous values of a three-phase combination. The flux will be the same, as is produced instantaneously by the alternating currents, with an advantage that it can be kept at this value indefinitely, until the distribution of the magnetic field has been properly investigated.

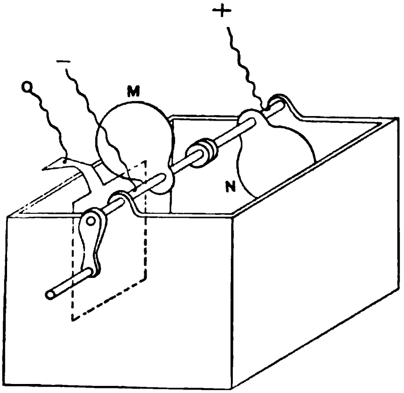

FIG. 416. A device for producing sinusoidal currents from a direct-current three-wire supply.

The exploring coil must be made very narrow so as to measure the field at a given point only; the coil is connected to a ballistic galvanometer, or a fluxmeter, instead of to a voltmeter (see §§ 139 and 140). The field is measured by placing the coil in a desired place in the air-gap, and then suddenly pulling it out. Instead of the coil being pulled out of the field, the main circuit can be opened, with the same effect on the galvanometer; the deflection is, in both cases, proportional to the flux. If the constant of the galvanometer is known, not only the relative distribution of the flux, but its absolute value can also be determined.

In order to more easily obtain any three values of direct current, corresponding to some instantaneous values of three-phase currents, a special liquid rheostat may be used, shown in Figs. 416 and 417. It consists of three glass jars *A*, *B*, *C*, such as are used for storage batteries, one for each phase. Each jar has two iron or nickel electrodes *M*, *N*, mounted on a shaft and insulated from each other. One electrode or blade is connected to the positive, the other to the negative pole of a three-wire direct-current supply. The liquid itself is connected to the neutral wire of the supply, through the winding in which it is desired to regulate the current. According to whether the positive or the negative blade dips into the liquid, the current flows through the winding in one or the other direction. The strength of the current depends on the amount of immersed surface of the blade; by giving the blades a suitable shape, a current is produced, varying nearly as a sine wave, when the handle is uniformly rotated. Three such rheostats are combined, as shown in Fig. 417, with the three pairs of blades set at 120 degrees apart: Regular low-frequency three-phase currents are thereby produced in the stator winding. The apparatus can be stopped in any desired position for studying an instantaneous value and distribution of the field.

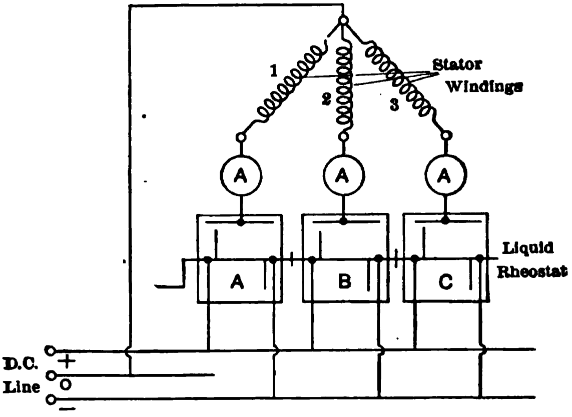

FIG. 417. Three-phase rheostat for a study of the revolving field in an induction motor.

If such a special rheostat is not available, three ordinary wire rheostats can be used, and the current in each phase adjusted separately. A table may be made, giving the instantaneous values of three sinusoidal currents, say, every 10 or 15 degrees.

561. EXPERIMENT 25-E. — Exploring Induction-Motor Field with Direct Current. — The purpose of the experiment is to obtain the actual form of the magnetic field in the air-gap of an induction motor. In order to observe the instantaneous values of the field, the stator is excited with direct current, and the intensity of the field is measured with a ballistic galvanometer or a fluxmeter, as explained in § 560. First explore the field with but one phase excited; then connect two phases in series, so as to have a different disposition of the single-phase winding. Use a narrow exploring coil connected to a ballistic galvanometer, and, keeping the current constant, move the coil through 360 *electrical* degrees, taking discharges through the galvanometer every 5 or 10 degrees, by opening the circuit. Take an extra curve with a large exciting current, in order to see if a high degree of saturation in iron appreciably changes the field curve. Galvanometer deflections, plotted to the angular positions of the exploring coil as abscissæ, give a curve of the relative distribution of the magnetic flux in the air-gap.

After this, investigate the field with all the three phases excited through the above-described liquid rheostat. Take the first curve with such a position of the blades that the current in one of the phases is a maximum. Then gradually change the position of the blades by steps of 10 or 20 degrees until a position is reached in which the current is a maximum in another phase. Take a galvanometer reading with each position of the blades and read the corresponding currents in the three phases. The results plotted in the form of curves give a clear view of the field *traveling* along the air-gap.

An oscillograph, or the point-to-point method (see Chapter XXVII) may be used in this study, in place of the tedious process of exploring the field by a ballistic galvanometer. The particular arrangement can be easily devised by a person familiar with the use of wave-tracing devices.

Report. Give a sketch of the arrangement of the winding in the slots; plot curves of the single-phase field with one phase, and with two phases excited in series. Check the form of the field from the number of slots per phase and the disposition of the winding. Give curves showing actual currents in the three phases, to the angular positions of the blades as abscissæ. Plot curves showing the progression of the three-phase field in the air-gap. See if the form of the three-phase field can be predetermined from the shape of the single-phase field.

562. Magnetic Field in the Single-Phase Induction Motor.—When a single-phase induction motor (§ 534) is stationary, but is connected to the line, its field is merely pulsating, as in a transformer. When the motor is revolving, two kinds of secondary currents are induced in the rotor: (1) by the "transformer" action of the pulsating primary field, and (2) because of the secondary conductors cutting the lines of force of the primary field (generator action). These latter currents produce a field displaced by 90 electrical degrees in space from the primary field, and also displaced by 90 degrees in phase relation. The "generator action" currents, and consequently the "perpendicular" field, are proportional to the speed of the motor. This field is of zero value at standstill, and gradually increases to a value nearly equal to that of the primary field, as the motor gains in speed.

The result is that the single-phase induction motor has a nearly uniform *revolving* field at synchronous speed, while the field is only a pulsating one at standstill. At intermediate speeds an irregular elliptical field is produced.

The existence of the perpendicular field, produced by the armature currents, and its variation with the speed, are important features in the theory of the single-phase induction motor; it is therefore interesting to explore this field experimentally. When the motor has an auxiliary phase, as in Fig. 407, or when a two-phase or a three-phase motor is used running single-phase (Fig. 408), the investigation is made by measuring the e.m.f., induced in the idle phase, as is explained below.

563. EXPERIMENT 25-F. — Exploring the Field of a Single-Phase Induction Motor. —The purpose of the experiment is explained in the preceding paragraph. The main phase of the motor is connected to the line — if necessary, through a suitable resistance, or an auto-transformer, to reduce the voltage. The rotor is driven by some outside power, in order to maintain a desired speed. An alternating-current voltmeter is connected across the auxiliary phase: its indications are proportional to the "perpendicular" field.

It will be found that the voltage induced in the auxiliary phase gradually increases from zero, practically to that across the main phase, as the speed increases from zero to synchronism. This, of course, presupposes, that both windings have the same number of turns; otherwise the voltages must be reduced to an equal number of turns.

Measure volts, amperes, speed and power factor in the main phase. In order to determine the phase angle between the primary and the secondary flux, connect the main phase at one point to the auxiliary phase: Read the volts across each phase separately and then across the two phases in series, so as to get a triangle of voltages. With the arrangement shown in Fig. 408 the auxiliary phase is already connected at *O* to the main phase, so that the phase relations may be determined without any additional connection.

Report. Plot all observed values to speed as abscissæ. Construct a triangle of voltages for some speed near synchronism; from the power factor in the main phase determine the actual phase angle between the primary and secondary field; this angle must be nearly 90 degrees. In drawing conclusions take into consideration the difference in the number of turns primary and secondary, and the distribution of the winding in the slots (breadth factor).

CHAPTER XXVI.

ARMATURE WINDINGS.

564. It has been customary heretofore to study armature windings by means of diagrams and sketches; it is much more profitable, however, to study them on models, by actually putting dummy coils in slots. This chapter is devoted to such a study, the exercises in winding being supposed to be performed in the laboratory.

The alternating-current windings, being simpler than the direct-current windings, are taken up first.

1. ALTERNATING-CURRENT WINDINGS.*

565. The simplest single-phase armature winding is shown in Fig. 357; it consists of conductors 1, 2, 3, etc., placed in the slots of a laminated-iron armature core (not shown) and connected in series by the end connectors *C*, *C*. The distance between the conductors is equal to that between the centers of the consecutive poles *N*, *S*, *N*, *S*, . . . of the machine, so that the conductors are subjected simultaneously to the action of the magnetic flux, and the e.m.f.'s induced in all of them are added together.

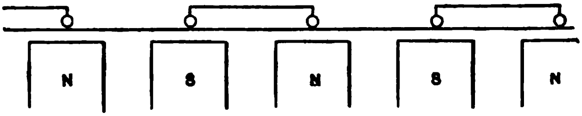

Fig. 418. A single-phase winding with one slot per pole.

In the relative position of the armature winding and the poles, shown in Fig. 357, the induced e.m.f. is a maximum, since the conductors are in the densest part of the magnetic field. In the position shown in Fig. 358 the total induced e.m.f. is zero, because the conductors do not cut any lines of force; this position differs by 90 *electrical* † degrees from that in Fig. 357. In a position, differing by 180 electrical degrees from that in Fig. 357, the conductor arrives at the center line of the pole *S*, and the induced e.m.f. becomes a maximum in the opposite direction (one alternation). When the next N-pole is reached, a complete cycle of alternating current has been accomplished.

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* For a more comprehensive treatment see Kinzbrunner's *Alternating-Current Windings.*

† In a two-pole machine this corresponds also to 90 *geometrical* degrees.

The same winding is shown in Fig. 418: the small circles represent the conductors in the armature slots, and the lines between them the end connections. In many machines more than one slot is used per pole; with two slots per pole, the winding has the aspect shown in Fig. 419. or in Fig. 420. Both schemes are electrically equivalent.

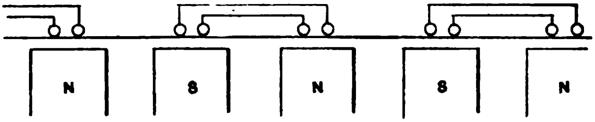

FIG. 419. A single-phase winding with two slots per pole.

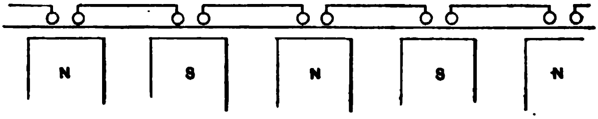

FIG. 420. Another arrangement of free ends of coils for the winding shown in Fig. 419.

Polyphase windings are formed by providing the same armature core with two or three independent single-phase windings in the right

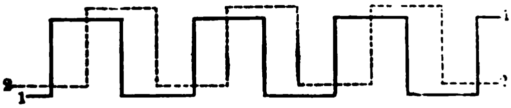

FIG. 421. A two-phase winding.

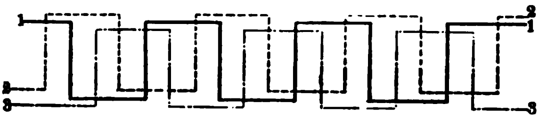

FIG. 422. A three-phase winding.

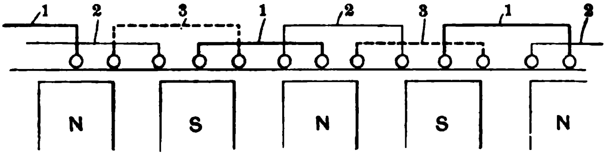

FIG. 423. Overlapping of free ends of coils in a three-phase winding.

positions relative to one another. A two-phase winding is shown in Fig. 421: It consists of two windings identical with those in Fig. 418, and displaced by 90 electrical degrees. When the e.m.f. in one of the windings reaches its maximum, the e.m.f. in the other winding is zero, and vice versa. Fig. 422 represents a three-phase winding; it consists of three single-phase windings displaced by 120 electrical degrees with respect to one another. The separate phases are marked by 1, 2, and 3. The same winding is shown in Fig. 423.

Alternating-current windings used in practice are a development of these simple schemes and can be easily deduced therefrom.

566. Bar Winding and Coil Winding.— Low-tension alternators require a comparatively small number of conductors of a large cross-section; high-tension machines must have a large number of conductors of a comparatively small cross-section. Therefore, windings in low-tension machines are often made of insulated copper bars with soldered cross-connections at the ends (Fig. 424). In high-tension machines the number of bars in series becomes so large that it is necessary to use regular coils, consisting of many turns of wire (Fig. 425). Coil windings and bar windings offer some distinctive features, and will therefore be considered separately. It must be remembered, however, that the distinction is merely of a mechanical and geometrical nature. From an electrical standpoint, the two types of winding are equivalent. A certain number of conductors must be placed on the armature and connected in series, in order to produce a given e.m.f.; it does not make any difference whether they are formed as separate bars or bunched in coils.

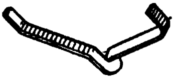

FIG. 424. The shape of end connectors.

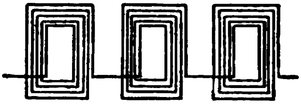

FIG. 425. Connections between the coils belonging to the same phase.

There are two distinct types of coil winding: Chain winding, Fig. 428, and double-layer winding, Fig. 429. Both types may be used with the machines having open slots; the coils are machine-wound and are placed directly into the slots. With half-closed slots, chain winding only is used; the methods of placing the winding into the slots are described in § 571 below.

567. Experimental Frame. — A stator frame (dummy) should be provided in the laboratory, for the convenient study of alternating-current windings. Such a frame may be made from boards glued together and turned true on the inside. The teeth are formed separately, as a molding, and nailed to the frame, leaving spaces for the slots. Grooves should be left in the teeth for driving in wooden wedges to hold the coils in place.

Wooden bars may be used instead of metal conductors; cross-connectors (Fig. 424) should be preferably made of metal and fitted into slots made at the ends of the bars. For exercises in chain winding, regular coils should be used, or colored cord, which the student may wind into the slots of the frame. Coils for double-layer winding, Fig. 436, should preferably be obtained from manufacturers.

It is convenient to have a frame with a number of slots such as 72 or 96, which is divisible by several numbers, so as to be able to use various numbers of poles.

568. Form of Induced E.M.F. — The wave form of an induced e.m.f. depends on the number of slots per pole per phase, and on the arc subtended by the pole-pieces. To find the theoretical wave form in a given machine, imagine the poles moving and note the instantaneous e.m.f.'s induced in a group of coils. Thus, in Fig. 418 the e.m.f. is zero so long as a pole is between two conductors. When the edge of the pole comes opposite a conductor, the e.m.f. suddenly reaches its maximum value, and remains constant until the other edge of the pole passes by the conductor. In reality, the e.m.f.-curve is not altogether flat, but has rounded corners, due to a "fringing" of the magnetic flux around the poles.

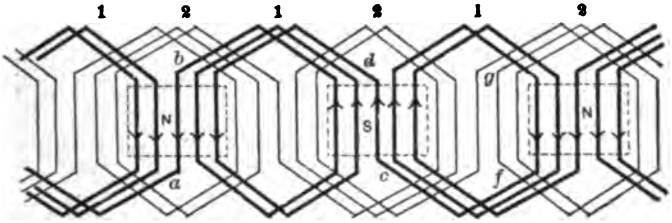

FIG. 426. A two-phase bar winding with five slots per pole per phase.

With two slots per pole per phase (Fig. 419), first one slot comes under the influence of the pole, and then two, so that the curve of the induced e.m.f. has an intermediate step. With three slots per pole per phase the induced e.m.f. is practically a sine wave.

Having an actual alternator, it is possible to compare the theoretical wave form with that actually observed, by the methods described in Chapter XXVII. By connecting two phases of a three-phase machine in series, and using it as a single-phase machine, a winding is obtained having a double number of slots per pole per phase; this is a simple method of investigating the influence of the number of slots on the wave form of the induced e.m.f.

569. Bar Winding. — A standard two-phase bar winding with five conductors per phase per pole is shown in Fig. 426; conductors belonging to one phase (No. 1) are indicated by heavier lines. The five conductors under one pole are placed in 5 slots; this gives a good e.m.f. wave-form, and considerably reduces the armature inductance. By following the connections, it will be seen that all the conductors of the same phase are connected in series, so that the e.m.f.'s induced in them are added, and do not counteract each other. It will also be seen that three sizes of bars are necessary with this kind of winding: Long bars, such as *ab*, for passing from one coil to the next one; short bars, such as *cd*, for the same purpose; and medium bars, such as *fg*, inside of each coil. The cross-connectors have the form shown in Fig. 435, so that by having long and short ends on each side of the core it is possible to make the connections shown in Fig. 426, without the cross-connectors touching one another. A similar type of winding can be used for three-phase machines.

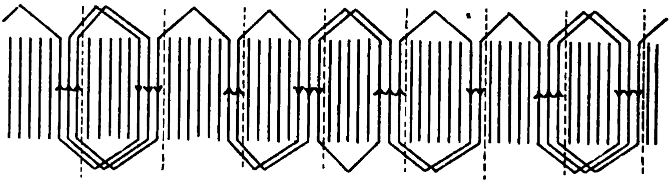

FIG. 427. An unsymmetrical three-phase winding, eight slots per pole.

In some cases it is convenient to use a standard frame for a special machine, and it may occur that the number of slots is indivisible by the number of poles, or by the number of phases required with this special machine. The frame can nevertheless be used by allowing a somewhat unsymmetrical winding. Suppose, for instance, that a manufacturing company is regularly using a 96-slot frame for 16-pole three-phase alternators of a certain size; this corresponds to 6 slots per pole, or 2 slots per pole per phase. Suppose that for some reason a customer desires to have a machine with a somewhat higher speed, say, such that it becomes necessary to use only 12 poles, instead of 16, in order to keep the same frequency. This means 8 slots per pole, a number indivisible by the number of phases. The problem is solved as shown in Fig. 427. Three bars are used for two consecutive poles, and only two bars for each third pole. No trouble is experienced in operation with such an apparently unsymmetrical winding.

Another alternative in such a case is to have the machine wound for *T*-connection (Fig. 349). With 8 slots per pole, the phase-winding

AB is placed in 4 slots and the winding *CD* in 4 slots. A part of the latter winding is left idle, a tap being taken at 86.6 per cent of it. This arrangement is particularly convenient when a machine must generate, according to circumstances, either two-phase or three-phase currents, for instance in supplying currents for testing purposes.

570. EXPERIMENT 26-A. — Exercises with Alternating-Current Bar Windings. — An experimental frame should be provided for this exercise, as described in § 567. Select a number of poles possible with the number of slots of the frame and place a single-phase winding, using three different lengths of bars, as in Fig. 426; have them properly cross-connected. Take a few other combinations, possible with two-phase and three-phase machines, and place a sufficient number of bars in the slots, to show clearly the disposition of cross-connectors. Finally select a number of poles such as to get an unsymmetrical winding, shown in Fig. 427.

Report. Give diagrams of windings investigated during the experiment; make a table of all the combinations and numbers of poles possible with a given frame. Show by an example how to determine the theoretical wave form of the induced e.m.f. from the disposition of the winding (§ 568). For the wave form thus obtained, determine the ratios of the effective value and of the average value to the maximum ordinate. Assume for the same example a certain magnetic flux per pole, and calculate the voltage of the machine, at a speed corresponding to a standard frequency. Take the flux such as to get in the teeth a density of about 90,000 magnetic lines per square inch. Assume that about 65 per cent of the periphery is covered by the poles, and allow a reasonable amount for fringing.

571. Chain Winding. — The chain-winding consists of separate coils, and is shown in Fig. 354 in application to a three-phase machine. Fig. 419 shows a similar winding with two slots per pole per phase; one phase only being indicated.

The coils belonging to the same phase are connected in series; sometimes the two groups of coils are connected in parallel in order to use the same winding with two different voltages.

Some of the coils with this type of winding have straight heads, others have their heads bent outward to avoid crossings; the idea is clearly seen in Fig. 428. Each coil has one head straight, the other bent. In some cases both heads of one coil are straight, those of the other bent, as is explained below.

Chain winding may be used with either open or half-closed slots. The advantages of half-closed slots are: (1) A lower flux density in the air-gap, and therefore less exciting ampere-turns; (2) A more uniform distribution of the magnetic flux in the air-gap; (3) A lower magnetic density in teeth, and therefore a smaller iron loss. On the other hand, the winding with half-closed slots is much more expensive, since it is more difficult to wind.

With open slots, coils are wound on special formers, by winding machines, some with both heads bent, others with straight heads. The coils with bent heads are placed in the slots first, and then the coils with straight heads.

With half-closed slots there are three ways of winding:

(1) The coils are wound by hand in the slots, using suitable molds for the heads, and threading the wire to and fro. First the coils with bent heads are wound, and then the coils with straight heads over and between them.

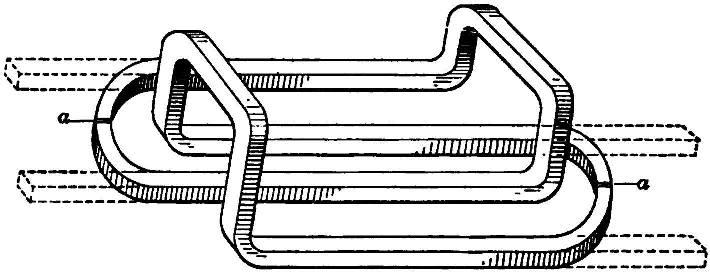

FIG. 428. Relative position of coils, which are opened at *a* before being put into slots.

(2) The coils are machine-wound and have the form shown in Fig. 428, one head being straight, the other bent. Before being put into the slots the coils are cut at the places marked *a*, *a*, and straightened out, as indicated by dotted lines. They are then shoved into the slots, the wires are bent back, soldered together, and carefully insulated turn by turn. This is such an expensive and tedious process that in many cases it is preferred to wind them in the slots by hand.

(3) The coils are machine-wound, but the wires are not bound or taped together. A coil is placed on the machine, and each side is threaded into the corresponding slots wire by wire. This winding is used in some common types of small induction motors (Fig. 396).

572. EXPERIMENT 26-B.—Exercises with Alternating-Current Chain Windings.—An experimental frame should be used in this exercise, similar to the one described in § 567; use form-wound coils, and cord of three colors to distinguish the phases. The properties of chain windings are explained in the preceding article. Select a suitable number of poles, place in the slots a single-phase winding, and properly interconnect the coils. Do the same with a two-phase and a three-phase winding, using in each case a different number of poles. Place a sufficient number of coils, in order to see clearly how the heads are interlinked.

The requirements for the report are similar to those in § 570.

573. Double-Layer Winding.—The double-layer winding (Fig. 429) is used with open slots only, the coils being form-wound. The ends of the coils have the same form as in direct-current armatures (see Fig. 436).

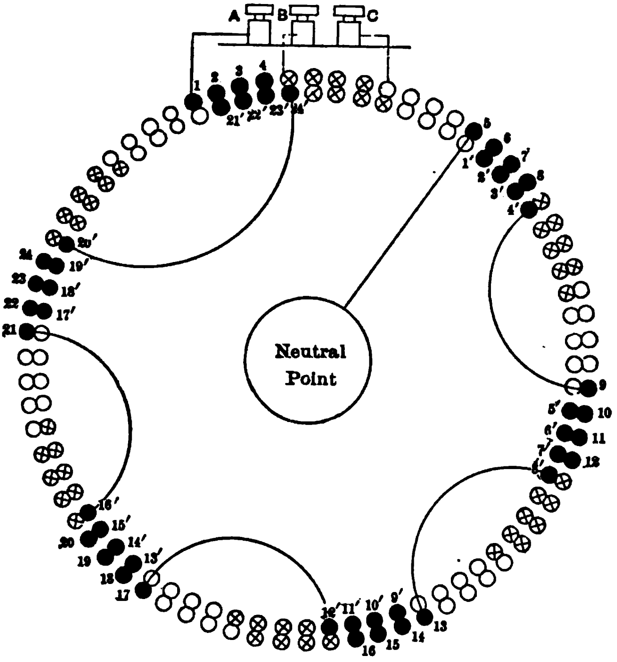

FIG. 429. Double-layer winding of a six-pole three-phase induction motor.

To make the winding symmetrical, one side of each coil is placed in the lower half of the slot, the other side in the upper half. Fig. 429 represents the disposition of coils in a six-pole, three-phase stator with 72 slots (four slots per pole per phase). Conductors belonging to different phases are marked by different circles; the conductors in one of the phases are numbered, the two sides of each coil being denoted by the same number. The three phases are wound exactly alike. Therefore the cross-connections between the groups of coils are shown in the sketch for one phase only, those in the two other phases being omitted for the sake of clearness. The four coils of each group, such

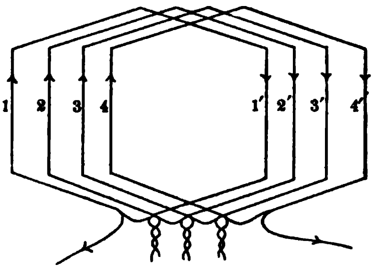

Fig. 430. Interconnection of coils belonging to the same pole and phase, in the winding shown in Fig. 429.

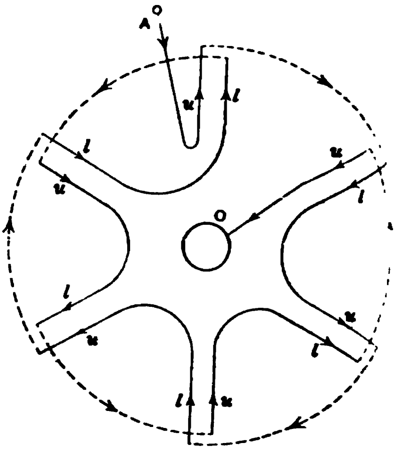

Fig. 431. Series connection of the groups of coils, shown in Fig. 429.

as coils 1, 2, 3, 4, are connected in series, as shown in Fig. 430; then the separate groups are again connected in series, as per Fig. 431. In this diagram the conductors in top of the slots are denoted by u (upper), those in bottom by l (lower). The dotted lines connect the conductors belonging to the same coil; the inner curved lines represent the cross-connections between the groups of coils, just as in Fig. 429. By following the arrows, which indicate the directions of the induced e.m.f.'s, it is easy to see the necessity of arranging the connections as shown in Fig. 431, in order to have all the conductors in series.

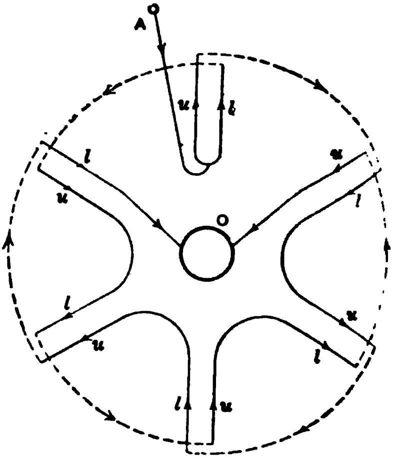

Fig. 432. Parallel connection of the groups of coils, shown in Fig. 429.

Sometimes the same frame and the same winding are used for 220-volt and 110-volt motors, the coils being connected in series for the higher voltage (Fig. 431), and in parallel for the lower voltage (Fig. 432).

574. EXPERIMENT 26-C. — Exercises with Alternating-Current Double-Layer Windings. —The properties of the winding are described in § 573. The directions for performing the experiment and the requirements for the report are similar to those in §§ 570 and 572.

2. DIRECT-CURRENT WINDINGS.*

575. The simplest type of a two-pole armature is shown in Fig. 433. A number of bars or coils are laid in the slots on the periphery of an armature core, and are interconnected so as to give a closed winding. The conductors are also connected to the commutator bars, from which the current is led to the external circuit. The connections satisfy two conditions:

(1) The winding constitutes a closed circuit in itself;

(2) The elements are so connected that the e.m.f.'s induced in them are added at all moments, and do not counteract each other.

The necessity of the second condition is self-evident. The first requirement is made clear in Fig. 434, namely, if the winding is closed on itself, it remains identical, relatively to the brushes, in all positions of the armature, so that the armature gives a direct, or *continuous* current. It will also be seen, that the armature is divided by the brushes into two parallel branches, each carrying one half of the total urrent.

576. Single-Layer Bipolar Winding. — Let us now follow in detail the connections in Fig. 433. With the polarity of the magnetic field and the direction of rotation shown there, we find by the familiar

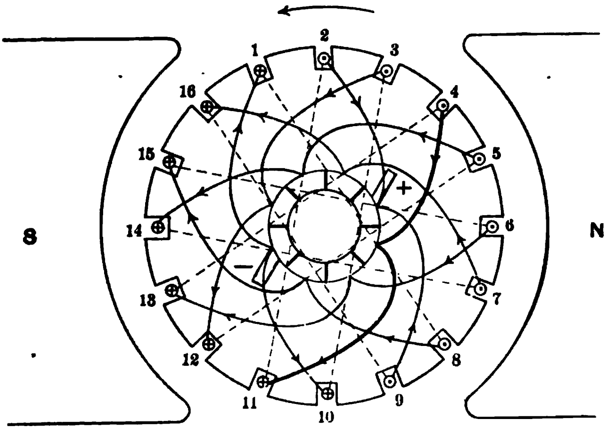

Fig. 433. A two-pole single-layer winding.

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* For a more comprehensive treatment see Kinzbrunner's *Continuous-Current Armatures.*

rule, that if the machine is acting as a generator the currents induce in the conductors under the *N*-pole are flowing toward the observe (up); in those under the *S*-pole — from the observer (down). Th is indicated in the figure by dots and crosses within the conductors.

The whole winding must be symmetrical, that is, all connectio must have the same "throw." The throw in the particular case show is equal to 7, that is to say, the conductor 1 is connected to 8 and to 1 these being the 7th conduct on each side from 1; bar 2 i connected to 9 and 11, et The throw must be an od number, in order that all th bars be connected before th winding is closed upon itsel Suppose we would take a throw equal to 6, and conne 1 to 7. Then from 7, keepin the same throw, we should ge to 13, thence to 3, etc., closin the winding on *odd* bars only. The throw must be only slightly less than one half of the number of bars. Then one comes to the first bar only after having connected together al the bars on the armature. Moreover, in no place of the armature do induced e.m.f.'s counteract one another.

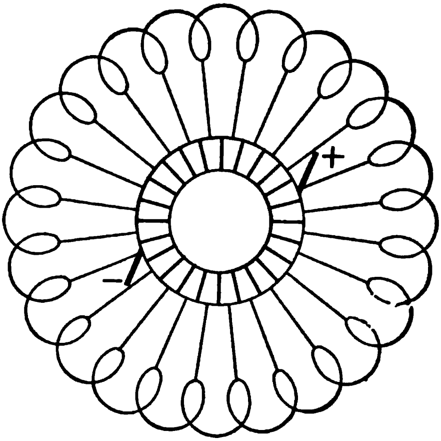

FIG. 434. Diagram showing that the armature winding is divided by the brushes into two parallel branches.

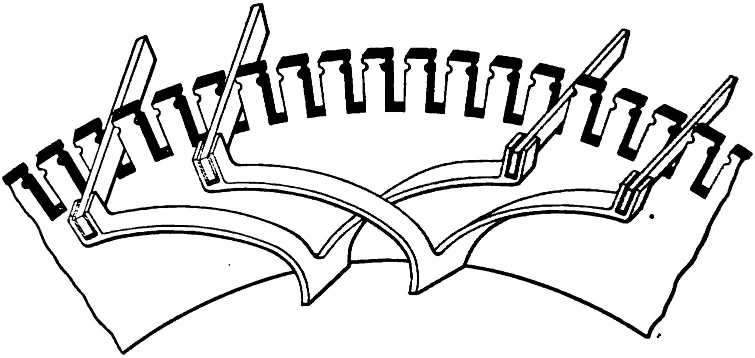

FIG. 435. End connections in a double-layer bar winding (evolute winding).

The path of the current in the armature is as follows: Current enters the armature through the negative brush and is divided into two parts.

ne flowing to 1, the other to 10. Following the first half: The urrent flows "down" through 1 and returns "up" through 8, the onnection between 1 and 8 being on the back of the armature, as ndicated by dotted lines. From 8, it flows to 15, then to 6, etc., ntil it reaches the conductor 2, and leaves the winding through the ositive brush. These connections can be represented thus:

neg. brush — 1—b—8—f—15—b—6—f—13—b—4—f—11—b—2—f
— pos. brush.

The letters "b" and "f" stand for "back" and "front," indicating whether the connection is on the front or the back side of the armature. The course of current through the other half of the armature conductors can be followed in a similar way. As mentioned before, the two halves are put in parallel by the brushes.

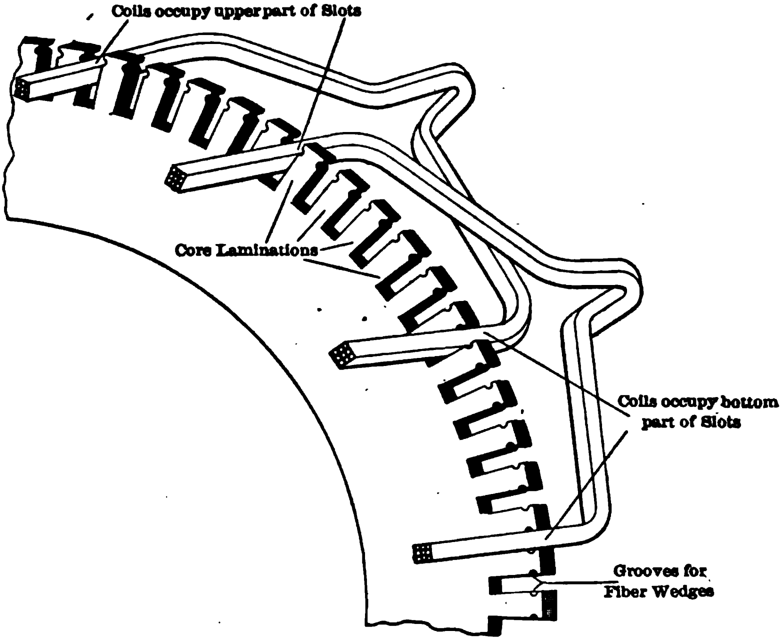
Coils occupy upper part of Slots
Core Laminations
Coils occupy bottom part of Slots
Grooves for Fiber Wedges

FIG. 436. End connections in a double-layer coil winding (barrel winding).

577. Double-Layer Bipolar Winding. — The above scheme (Fig. 433) presupposes that the conductors or the coils are placed in one layer. With modern slotted armatures the coils, as a rule, are put in two layers (Fig. 437), because this allows of a better utilization of the space available for the winding. In order to make all connections identical, one side of each coil is placed in the bottom of the slot, the other side in the top (Figs. 435 and 436).

When the winding consists of copper bars (Fig. 435) the cross-connectors are placed in planes perpendicular to the bars. Such winding is sometimes called the *evolute winding*, because of the form of the cross-connectors.

With form-wound coils (Fig. 436), end connections are usually made along the cylindrical surface of the armature. Such a winding is sometimes referred to as the *barrel winding*. The general scheme of connections remains the same as in Fig. 433, only, instead of having all the conductors in one layer, each two adjacent conductors must be imagined as placed in the same slot one on top of the other; for instance all odd conductors are in the bottom and all even ones in the top.

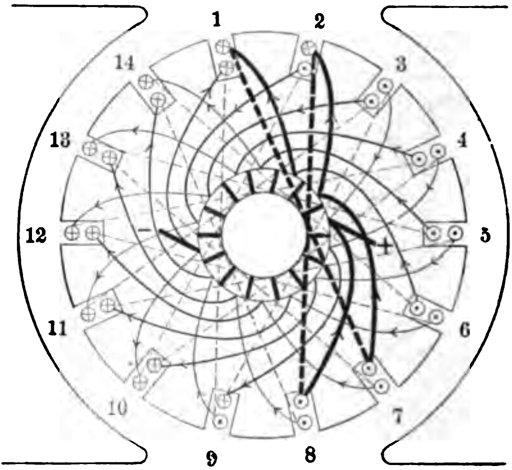

Fig. 437. A two-pole, double-layer winding with an even number of slots.

Either an even or an odd number of slots can be used with double-layer bipolar windings (Figs. 437 and 438). Actual armatures, as a rule, have many more conductors or coils than is shown in these diagrams: Therefore, putting all the coils and connections on the drawing would make it rather too complicated. This is, however, not necessary, since all the coils are connected according to a certain "throw." In the winding departments of electric manufacturing companies, drawings are used, on which but a few conductors are shown and the throw is given. The winder places the first coils as shown in the drawing, and then keeps up the right throw until all the slots are illed with coils. Then he drives in fiber wedges which hold the coils n place, and solders the connections.

578. EXPERIMENT 26-D. — Exercises with Direct-Current Bipolar Windings. — A wooden model of an armature is convenient or study of windings; it should be provided with regular teeth and lots. Ordinary cord may be used for winding, instead of wire; wooden bars with metal end-connectors are convenient for illustrating the winding shown in Fig. 435. Form-wound coils should be provided to demonstrate the barrel-winding (Fig. 436).

(1) Wind into the slots a single-layer winding, shown in Fig. 433. First wind in place separate coils, such as are shown in slots 1—8, 2—11, etc., each coil consisting of several turns. Then properly crossconnect the ends of the coils, and make sure that the winding is entirely symmetrical and closed upon itself.

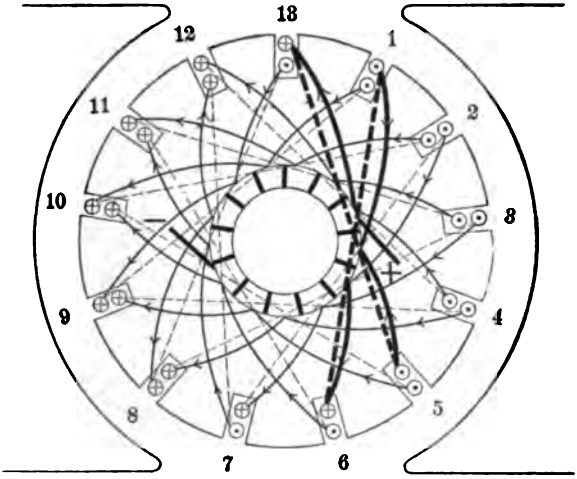

Fig. 438. A two-pole, double-layer winding with an odd number of slots.

(2) Do the same with the double-layer windings shown in Figs. 437 and 438. If the number of slots of the model is even, a winding with an odd number of slots is realized by omitting a slot, as if it were not there.

(3) Place a few form-wound coils (Fig. 436) into the slots, and make clear to yourself the way they are designed, so as to avoid crossings, and to make the winding symmetrical all around. Try also the bar winding shown in Fig. 435.

579. Multipolar Windings. — In *multipolar* machines, currents of alternate direction are induced in groups of conductors under consecutive poles. The winding is arranged so that several groups are connected either in series or in parallel. Accordingly, two types of windin are distinguished: *multiple* winding and *series* winding. The conditi of Fig. 434, namely, that the winding must form a closed circuit divid by the brushes into two parallel branches, must always be fulfille even with the pure series winding there are always two circuits, ea taking one half of the armature current (series winding is therefo sometimes called two-circuit winding). In multiple winding t number of branches in parallel is more than two. Fig. 439 gives general scheme of connections for a multiple winding of a six-po armature. There are six branches in parallel, and each coil carri but one sixth of the total line current.

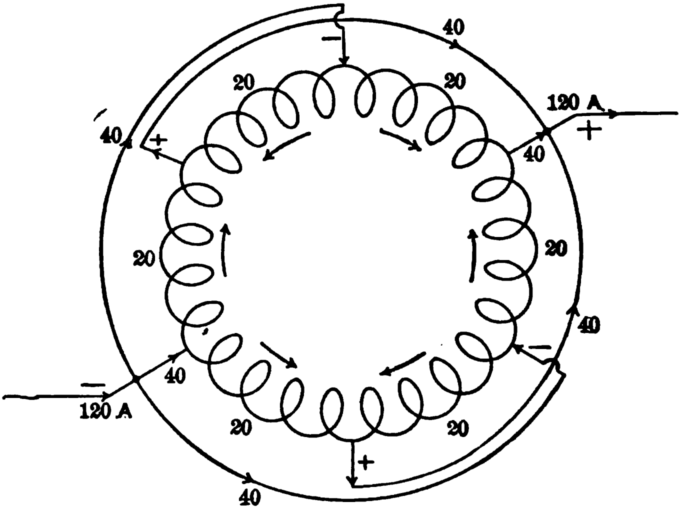

FIG. 439. Paths of current in a six-pole multiple-wound armature.

The selection of a series or a multiple winding in each particular case is substantially determined by the voltage and the current of the machine. Large currents should be avoided in armature conductors, as they necessitate heavy bars, difficult to handle and to insulate; moreover the copper loss is considerably increased by eddy currents in the conductors themselves. In this case multiple winding should be used, as it considerably reduces the actual currents carried by the armature conductors (for instance, in a 12-pole machine, currents are reduced 12 times). On the other hand, in medium-size, low-speed, 500-volt generators, such as are used for direct connection to steam-engines, the principal difficulty is to get the required voltage, without making the machine too bulky and expensive. Here, as many conductors as possible should be connected in series, and a two-circuit, or series winding is preferable. Series winding is also used in railway motors, as it enables them to have a counter-e.m.f. of 500 volts induced at a comparatively low speed.

With very large machines, say above 1000 kw., both difficulties arise at once: The currents are large enough to make necessary several branches in parallel, and at the same time the conductors under one pole are not sufficient to produce the required voltage, so that series windings must be recurred to. To meet this case a combination *series-parallel winding* has been devised. For instance, instead of using an ordinary series winding on a 1500-ampere, 500-volt railway generator, three *separate* series windings are placed on the armature, each carrying 500 amperes. Each fourth consecutive commutator bar belongs to the same winding. The brushes cover more than three commutator segments, and thus connect the three windings in parallel with respect to the external circuit, while inside of the machine each winding is a two-circuit winding.

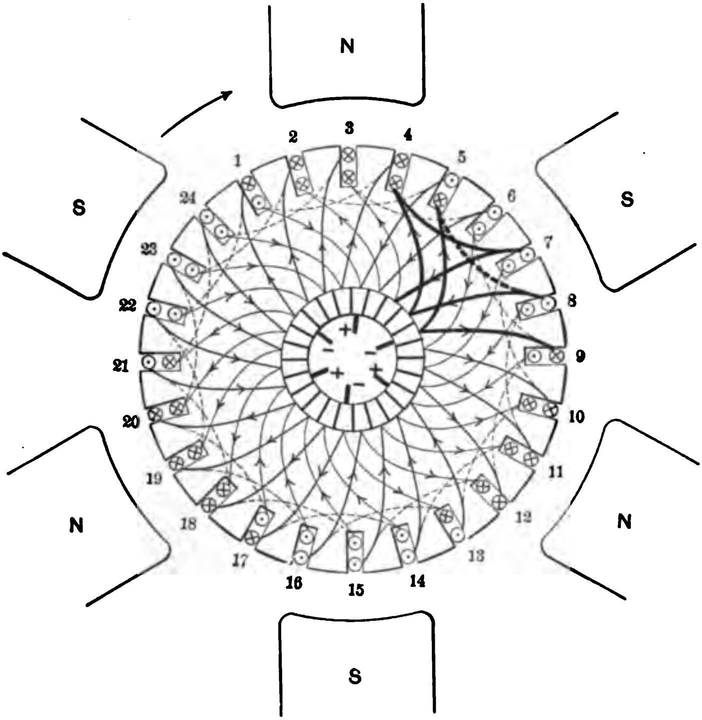

FIG. 440. A six-pole multiple-wound armature.

580. Multiple Windings. — Fig. 440 shows a six-pole *multiple winding* with two conductors per slot; a development of the winding is shown in Fig. 441, from which the origin of the name "*lap-winding*" can be seen. The winding in Fig. 440 is of the same character as that in Fig. 437 or 438, if the latter be imagined cut through in one place and spread over a large drum, so as to occupy on it only 120 degrees instead of 360 degrees. Three such windings put side by side give the six-pole winding of Fig. 440.

The number of slots in a two-pole winding can be either even or odd (Figs. 437 and 438). As a multiple-winding is a combination of several two-pole windings put side by side, the number of slots in it can also be even or odd, but must be divisible by the number of pairs of poles. For instance, a six-pole machine may be wound with 39 slots, giving 13 slots per each pair of poles. Strictly speaking, even this limitation is not absolutely necessary, and the armature may be provided with any number of slots. Under these conditions, however, the number of coils under separate poles is slightly different at different moments, which gives unbalanced induced e.m.f.'s; therefore such a disposition should be avoided.

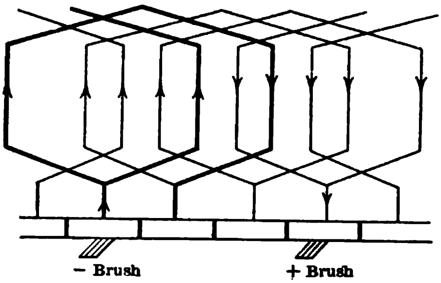

Fig. 441. Development of a multiple winding (lap winding).

The arrangement of coils is the same as in the corresponding two-pole winding. For instance, in the above example of a six-pole armature with 39 slots, the coils are placed in the slots 1—6, 2—7, etc., as in Fig. 438; this is a little less than the pole pitch. Such a "throw" insures that the e.m.f.'s induced on both sides of the coil are added, the two sides being under opposite poles. The winding is closed on itself after all the coils are properly interconnected.

581. Tables of Connections. — Windings may be conveniently represented by tables, instead of sketches or diagrams. Thus the winding shown in Fig. 440 is represented by the following table:

+ brush — brush + brush

bottom of slot $1_0-2_1-3_3-4_5-5_6-6_5-7_3-8_1-9_0-10_1-11_3-$ etc., connected on the front;

outer part of slot $5_0-6_2-7_4-8_6-9_6-10_4-11_2-12_0-13_0-14_2-15_4-$ etc., connected on the back.

Large figures denote the conductors, small figures the induced voltages.

Leaving, for the time being, the voltages out of consideration, the table reads as follows: The conductor on the bottom of the slot 1 is connected on the back of the armature (dotted lines in Fig. 440) to the conductor on the outer part of the slot 5. This latter conductor is connected in front of the armature (on the commutator side) to the conductor on the bottom of the slot 2, etc. It will be easily seen that such a table gives the diagram of connections much clearer than a complicated sketch.

The numbers, indicating the voltages, explain the electrical character of connections. During the moment shown in the diagram, Fig. 440, the following slots are under the poles:

2, 3, 4,   6, 7, 8,   10, 11, 12,   14, 15, 16, etc.;

only in these conductors are e.m.f.'s induced. We assume arbitrarily that the potential of the conductor placed on the bottom of the slot 1, is = 0, and that 1 volt is induced in each conductor under the poles.

The next conductor 5 in the table has also potential zero, since 5 is not under any pole. The potentials of the next conductors are each one volt above that of the preceding conductor, because they are under the poles, and are connected in series. The potential rises up to the conductor 8 in the top row: this conductor has a potential of 6 volts above that of the first conductor. The potential remains the same under the conductors 5 and 9 which are not under the poles; then it gradually drops again to zero at the next neutral zone.

The brushes marked on the table are placed at the points of maximum and minimum potential. It will be seen that with a six-pole multiple winding, three positive and three negative brushes are necessary in order to collect currents from all the sections of the armature.

582. Short-Chord or Fractional-Pitch Winding. — The distance between the two sides of a coil, expressed in the number of slots, is called the " throw," or the coil-pitch. In the multiple windings described above, the throw is one slot (or one half slot) less than the pole pitch. It is possible to use coils with a smaller pitch than this: such windings are referred to as having a fractional pitch, or as short-chord windings.

Take, for instance, a 12-pole machine with 120 slots, or 10 slots per pole. The pole-pitch is 10, or from the slot 1 to the slot 11. The regular coil-pitch is one less, or from the slot 1 to the slot 10. A short-chord winding is obtained by placing the coil in the slots 1 and 9 instead, and even in the slots 1 and 8. The only condition is, that the two sides of the coil should not come under the same pole; otherwise the induced e.m.f.'s would oppose each other, making the coil electrically ineffective.

The poles may be assumed to cover about 60 per cent of the periphery, so that 6 slots out of every 10 are under a pole. The pole extends from the slot 1 to the slot 7, so that by placing a coil in the slots 1 and 8, the above condition is fulfilled.

Fractional-pitch windings are used to a considerable extent; their advantages are:

(1) Shorter end-connections, therefore less copper and a higher efficiency.

(2) Armature reaction is reduced: The currents in the neutral zone flow partly in opposite directions, neutralizing each other.

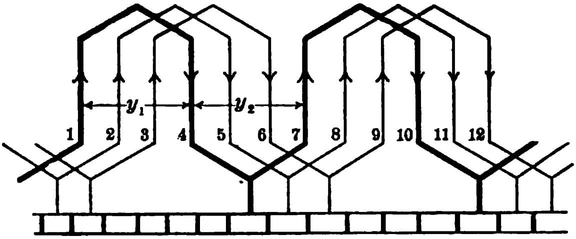

FIG. 442. Development of a two-circuit winding (wave winding).

It is recommended that the student work out a table for the above data, 120 slots, 10 poles, assuming first the throw to be equal to 9 (1–10), then equal to 7 (1–8); he should also draw a diagram similar to Fig. 440. The table will clearly show the difference between the full-pitch and the fractional-pitch windings. The diagram will explain why the armature reaction is reduced by selecting fractional-pitch windings.

583. EXPERIMENT 26-E. — Exercises with Direct-Current Multiple Windings. — The experiment is conducted on a wooden model in the same way as the experiment described in § 578. Wind coils of cord, using the type of winding shown in Figs. 440 and 441, with a full and a fractional pitch. Do this for two or three different numbers of poles, so as to become perfectly familiar with the winding. Place on the model a few machine-wound coils (Fig. 436) and bars (Fig. 435), and make clear to yourself the mechanical features of the winding, and the geometrical relations. Measure the diameter and the length of the armature core and the size of the slots.

Report. Give tables of the windings used (§ 581), for a full-pitch winding, and for a short-chord winding. Make a sketch, as in Fig. 440, for one of the numbers of poles, possible with the model. Draw, to scale, end connections for the types of winding shown in Figs. 435 and 436. The size of the coils and of the bars must be such as to fill the slots.

584. Series or Two-Circuit Winding. — The difference between the series and the above-described multiple windings can be seen by comparing Figs. 441 and 442. In Fig. 441 a conductor is connected to another one under the consecutive pole and then back to a conductor *under the same pole.* In this way, the winding under each pair of poles constitutes a circuit by itself, and the separate circuits are put in parallel by the brushes. In the series winding, shown in Fig. 442, a conductor, such as 1, is connected in series with the similarly situated conductors, 4, 7, 10, etc., *under all the poles of the machine,* and only then to the conductor 3 under the same pole as 1. In this way the conductors of the armature are all connected into one circuit, which is divided by the brushes into two parallel branches. The shape of the winding in Fig. 442 justifies the name "*wave winding.*"

A simple six-pole wave winding is shown in Fig. 443. Only two brushes are necessary, although four or six may be used, if large currents are to be collected. The winding is always placed in two layers, as in Figs. 438 and 440: a single-layer winding is shown merely for the sake of simplicity.

Conductors are connected alternately in the bottom and the top of the slots. Let s be the total number of slots, p the number of pairs of poles ($p = 3$ for a 6-pole machine, $p = 4$ for an 8-pole machine, etc.), and y_1 and y_2 the throws on the back and the front side of the armature (Fig. 442). These throws may be somewhat different from each other: the advantage of selecting them different is shown below. The condition must be fulfilled, that after having gone once around the armature, we return to the slot next to where we started. This is expressed by the equation

$$p(y_1 + y_2) = s \pm 1,$$

whence

$$y_1 + y_2 = \frac{s \pm 1}{p}.$$

If p is even (4-, 8-, 12-, etc., pole machines) the number of slots must be odd. If p is odd (6-,10,-14-, etc., pole machines) the number of slots s can be either odd or even. As an illustration, suppose that the number of conductors for a 10-pole machine ($p = 5$) is figured out to be about 500. Assuming the winding to be made of coils, of two turns each, the necessary number of slots is 125. From the above formula we see that the total number of slots s must be taken either equal to 124 or to 126: in both cases $y_1 + y_2 = 25$. Thus, we have to make $y_1 = 12$ and $y_2 = 13$, or vice versa; the throw is different on the back and on the front side. The value $s = 124$ gives the so-called *progressive* winding; $s = 126$, a *retrogressive* winding. If we should prefer to have $y_1 = y_2$, we should have to use either 119 or 121 slots, or else 129 or 131 slots.

It will be seen from the above that by making the two throws slightly different, it is possible to have a larger number of possible solutions, with the same number of slots. This not only makes it possible to design an armature closer to the results of calculation, but also to use the same armature punchings for more than one type of machine.

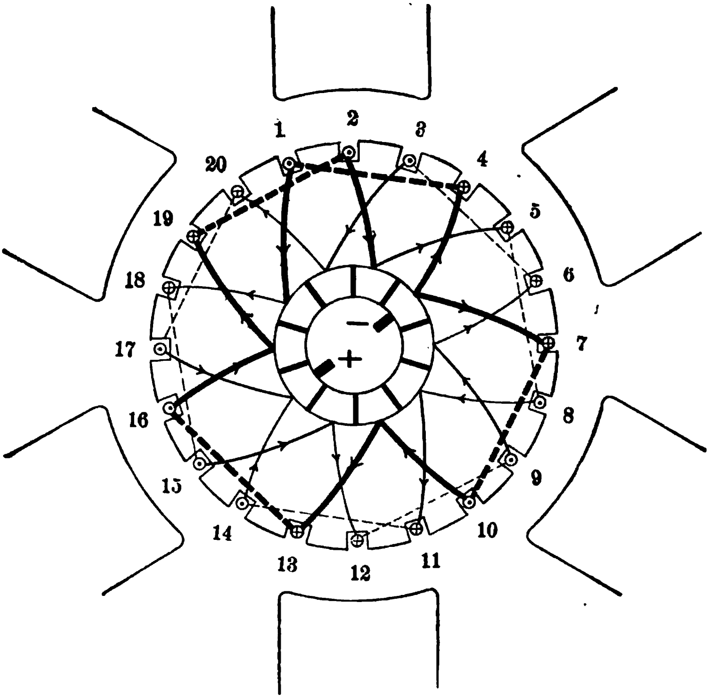

Fig. 443. A six-pole series-wound armature (single-layer).

Tables of connections, explained in § 581 in application to multiple windings, are used equally well with two-circuit windings.

585. Use of an Idle Coil. — It sometimes happens, that the punching selected for a special machine has a number of slots which does not satisfy the above formula for two-circuit windings. In such cases *an idle coil* is put in (Fig. 444), and the winding is completed in the usual manner. The idle coil gives the armature a symmetrical appearance and mechanical strength. If a standard commutator is used, there is also an extra commutator bar to which the idle coil must be connected on one side. If the commutator is made specially for the machine, with the right number of bars, the idle coil remains insulated on both ends.

In some railway-motor armatures, even of a standard manufacture, an idle coil is not to be avoided. This is the case when, for some considerations, four coils are placed in one slot, two on the bottom and two on the top. This is equivalent to using an even number of slots: But it is explained above that a two-circuit winding is possible for a four-pole machine with an odd number of slots only. Therefore one coil remains idle and is insulated on both ends.

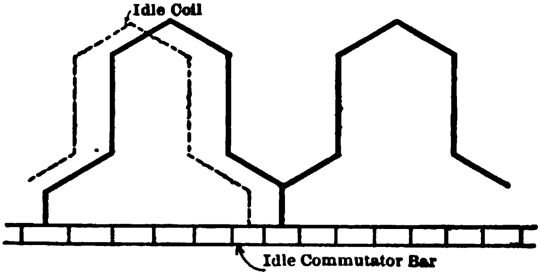

FIG. 444. The use of an idle coil.

586. EXPERIMENT 26-F. — Exercises with Direct-Current Two-Circuit Windings. — The experiment is performed in the same way as those specified in §§ 578 and 583. On the model core, place windings for two or three different numbers of poles, with equal and with different throws on the front and on the back of the armature. Place a winding, with which an idle coil is necessary. Devise a double or a triple winding (series-parallel winding), as explained at the end of § 579. Use cord of different colors to distinguish the circuits.

CHAPTER XXVII.

ALTERNATING-CURRENT WAVE FORM.

587. In a great majority of engineering calculations relating to alternating-current machinery and transmission lines, it is legitimate to assume voltages, currents, and magnetic fluxes, to vary according to the sine-law (Fig. 102). This assumption is justified, since the results of calculations come sufficiently close to the actual test data. At the same time, both the graphical and the analytical treatments of alternating-current problems are made much simpler than would be possible without the sine-wave assumption.

There are practical cases, however, in which the observed phenomena cannot be accounted for, by assuming a sine-wave distribution of currents and voltages, and where it becomes necessary, or at least desirable, to deal with the actual wave-forms of alternating currents and voltages.

An experimental investigation of the wave form of alternating currents proves to be particularly helpful in the following subjects:

(1) In the study of commutation in direct- and alternating-current machinery.

(2) In investigations of electric disturbances taking place in cables, high-tension lines, etc.

(3) In research of phenomena taking place in the electric arc, rectifiers, etc.

(4) In telephonic research.

Moreover, a knowledge of the actual instantaneous phenomena in alternating-current circuits has contributed materially to the theory of protective apparatus.

In all these cases an experimental determination of the wave form has permitted the substitution of actual facts for a mass of speculative theories which had been advanced heretofore, because of the lack of experimental means for observing rapidly-varying irregular electrical phenomena.

Determination of the wave form of alternating currents is in some respects similar to taking indicator cards on steam- and gas-engines: Both give exact information about the actual working of a machine from moment to moment. Voltmeters and ammeters give only effective or average values for a certain length of time.

Two methods are used in practice for experimental determination of wave form of currents and voltages: The *point-to-point* or *contact-maker* method, and the *oscillograph* method.

1. POINT-TO-POINT METHOD.

588. The general principle of the point-to-point method is shown in Fig. 445: NS is the field and A the armature oi an alternator, whose e.m.f. wave form is to be determined. For simplicity, the machine is assumed to be of a two-pole, revolving-armature type. It has, in addition to two regular slip rings f_1, and f_2, a third ring or contact-maker D mounted on the same shaft, and revolving synchronously with the machine. The ring D is made of some insulating material, and has a metal segment V on a part of its periphery. Two brushes, B_1 and B_2, press against this ring, and are set at such an angular distance that the circuit is closed only during a very small part of a revolution. This is sufficient, however, to charge the condenser C: when the circuit is broken, C discharges through the galvanometer G. As these charges and discharges take place many times a second, the galvanometer assumes a steady deflection, proportional to the value of the individual discharges, in other words, proportional to the voltage at the condenser terminals.

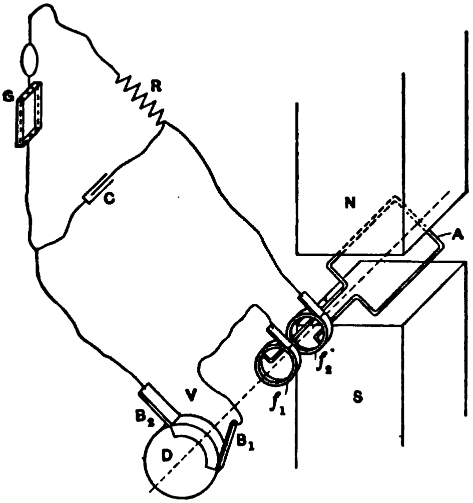

FIG. 445. The principle of determining an alternating-current wave form by the point-to-point method.

This voltage depends on the position of the brushes B_1 and B_2. If they are set in such a position in regard to the alternator field, that the circuit is closed when the induced e.m.f. of the machine is at its maximum, the voltage at the condenser terminals, and therefore the deflection of the galvanometer, have their largest values. If the brushes are set so that the circuit is closed when the induced e.m.f. passes through zero, the condenser receives no charge, and the galvanometer deflection is zero. In an intermediate position of the brushes, the deflection of the galvanometer is proportional to the instantaneous value of the voltage of the machine.

With this arrangement, the e.m.f. wave form of an alternator is taken by reading galvanometer deflections and shifting the brushes B_1 and B_2. The rocker arm, to which the brushes are fastened, is provided with an index, which slides along a circular scale divided into degrees. The brushes are shifted, say 5 degrees at a time, and the galvanometer deflection noted.

The results are plotted as in Fig. 454, giving galvanometer deflections against brush positions as abscissæ. The former are proportional to the instantaneous voltages, the latter correspond to the particular moments at which contact is closed. The curve thus plotted represents the actual wave form of the voltage of the machine. If the galvanometer constant is known, the curve may be plotted directly in volts; otherwise its effective value (square root of the mean square of the instantaneous values) is calculated and compared to the same value (voltage of the alternator) measured on the voltmeter. This gives the scale of the instantaneous values.

589. A Modification of the Point-to-Point Method. — The contact-maker D can be driven directly by the alternator in exceptional cases only, since the machine is usually not accessible. Therefore, the contact-maker is usually driven by a small synchronous motor connected to the same supply which is being investigated (Fig. 446). The speed being synchronous, this arrangement is equivalent to the one in which the contact-maker is driven directly from the alternator. The contact-maker K is somewhat different from that shown in Fig. 445, and the brushes are set side by side. The cross-hatched part is made of fiber, the remaining portion is of brass. Electrically the two arrangements are equivalent, since in both cases the current is closed for an instant during each revolution.

Another difference between the arrangements shown in Fig. 445 and in Fig. 446 is that, in the latter, instantaneous voltages are directly compared to an equivalent drop in a direct-current circuit, instead of being measured by galvanometer deflections. This does away with the galvanometer and the condenser, and makes possible the use of an ordinary direct-current voltmeter.

Referring to Fig. 446, the alternating voltage, whose wave form is to be determined, is taken from the terminals *A. C.* The circuit is closed through the contact-maker *K*, a double-throw switch *Sw.*, and a high resistance *GG*. This resistance is also connected to a direct-current supply *D. C.*, so that a constant direct-current circulates through it. By moving the contact *S* along the resistance, a position is found, in which the direct voltage is equal to the instantaneous value of the alternating voltage, given by the contact-maker. This voltage is then read directly on the voltmeter *V*.

The point of equilibrium is determined by means of a telephone receiver *T* connected into the alternating-current circuit. When the A. C. voltage does not balance the D. C. voltage, a clicking noise is heard in the telephone, corresponding to the frequency at which the current is interrupted at the contact-maker. When the two voltages balance each other, the noise disappears. To increase the sensitiveness of the receiver, a condenser *C* can be connected across its terminals. A milli-voltmeter, or any zero-reading galvanoscope, may be used for balancing, instead of the telephone receiver.

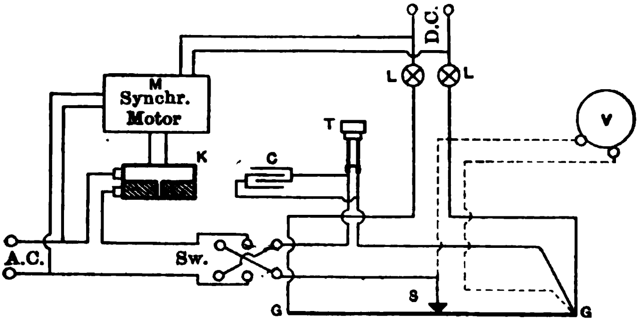

FIG. 446. The diagram of connections for balancing instantaneous values of alternating voltage by continuous voltage.

M is a synchronous motor driving the contact-maker; its armature is connected to the A. C. supply, and the field is excited with direct current. The lamps *LL*, connected in series with the resistance *GG*, are useful as a precaution in case of a short-circuit. The double-throw switch *Sw.* is necessary in order to be able to take both halves of an alternating-voltage wave, with the same direction of the direct current in the resistance *GG*.

If it is desired to determine the wave form of an alternating *current* instead of *voltage*, the general scheme of connections remains practically the same; only the terminals A. C. must be taken in this case from the ends of a non-inductive resistance connected in series with the circuit. With this arrangement, a voltage curve is taken at the terminals of this resistance, which curve has the same form as the current wave of the circuit. This procedure is analogous to using a milli-voltmeter with a shunt for measuring direct currents.

If it is desired to obtain the voltage and the current curves simultaneously, in their proper phase relation, suitable double-throw switches must be provided so as to change quickly and conveniently from one scheme to the other.

A disadvantage of the contact-maker K is that the contact lasts an appreciable time instead of being instantaneous; this is objectionable with complicated wave forms, since a considerable part of a higher harmonic may be comprised in the time of the contact. To obviate this, Professors Bedell and Ryan proposed to use a tiny stream of salt water which makes an instantaneous contact with a revolving pin. Another arrangement, proposed by Adams, is based on the use of a spring, which makes and breaks the contact in the same movement, almost instantaneously. These devices are described in detail on p. 188 of Swenson and Frankenfield's *Testing of Electromagnetic Machinery*.

590. EXPERIMENT 27-A. — Determination of Alternating-Current Wave Form with a Contact-Maker. — The purpose of the experiment is to afford familiarity with the use of the arrangement shown in Fig. 446 and to obtain curves of wave form in cases of practical importance. The synchronous motor M is started by means of an auxiliary direct-current motor, synchronized and switched on to the alternating-current supply.* Then the direct-current circuit is closed through the resistance GG. The terminals $A. C.$ are connected either across the line or across a non-inductive shunt in series with the line, according to whether the voltage or the current curve is to be taken. Then the contact S is shifted along GG until a position of silence in the telephone receiver is found. If a galvanoscope or a milli-voltmeter is used instead of the receiver, this corresponds to the zero position of the pointer. Then the voltage is read on the direct-current voltmeter V, it being at the same time the instantaneous value of the alternating voltage at the terminals $A. C.$ To get another point on the curve, the contact-maker brushes are shifted by a few degrees and the same measurements repeated. When zero point of the alternating wave is reached, the current must be reversed by means of the switch *Sw.*, and the second half-wave taken in the same way.

(*) A three-phase synchronous motor may be started directly from the alternating-current side, without excitation, as explained in § 506.

Care should be taken to keep currents and voltages constant during the whole test. If the A. C. voltage is too high to be compensated by the available D. C. voltage, a non-inductive resistance, say a few incandescent lamps, are connected across the supply, and the leads to the contact-maker are taken across a part of this resistance. Such, for instance, is the case when a curve is to be taken of a 110-volt supply, and the highest continuous voltage obtainable is also 110 volts. It is evident that the peak of the alternating curve is considerably above 110 volts, and could not be compensated on *GG*.

The following curves can be taken in the laboratory by this method:

(1) Current and voltage curves of the laboratory supply at a non-inductive load.

(2) Same, with a highly inductive load and approximately the same value of the current.

(3) The curve of the current taken by a large transformer at no load. The wave is considerably distorted because of the hysteresis phenomena in the iron.

(4) Current wave of an arc lamp, and a curve of potential difference between the carbons. Both curves have a somewhat peculiar shape, which can be explained by the fact that the arc is actually extinguished at each alternation, and the voltage has to attain a certain value before the current jumps between the carbons and starts the arc.

(5) If an experimental alternator is available, it may be of interest to take its voltage curve. Ripples similar to those in Fig. 447 will be clearly seen according to the number of teeth in the armature. These ripples constitute a higher harmonic, and in order to make them still more pronounced, some capacity is connected across the terminals of the machine, so as to produce a resonance effect.

(6) Curves of the current taken by a synchronous and an induction motor. These curves may, under certain circumstances, be considerably distorted, due to the counter-e.m.f. of the motor being of a different shape than the e.m.f. of the supply.

Have a wattmeter connected into the circuit in at least one of the above measurements, in order to compare the phase displacement actually measured by the contact-maker to that calculated from the power factor of the load.

The method itself may be learned in the taking of one curve; but the above six curves are suggested as possible exercises, and as many of them should be taken as time and laboratory facilities will permit. Moreover, the curves are of considerable practical importance, and it is interesting to take them for this reason.

Report. (1) Plot the curves obtained experimentally, and explain the irregularities, where possible. (2) Take one of the curves, figure out the effective value of the ordinates (root of mean square) and plot an equivalent sine wave; compare the effective value to that read on the alternating-current voltmeter. Determine also the ordinary mean value of the ordinates. (3) Calculate the ratios of the maximum value to the effective value, and to the mean value; compare these ratios to those for the sine wave ($\sqrt{2}$ and $\frac{1}{2}\pi$). (4) Where the curves of voltage and current were taken simultaneously, check the phase displacement from the wattmeter reading. (5) Analyze at least one of the curves into its harmonics, as explained in § 600 below.

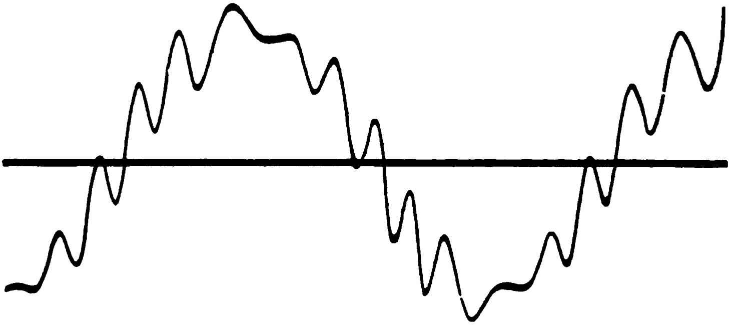

FIG. 447. Voltage curve with a prominent eleventh harmonic.

Separating the fundamental, the third, and the fifth harmonics is sufficient for the purpose, as this is merely intended to illustrate the method of analysis.

2. OSCILLOGRAPH.

591. The above-described contact-maker method has two good points in its favor: It gives fairly accurate results, and at the same time it does not require the use of special delicate devices. Its disadvantages are: Being a "point-to-point" method it is somewhat tedious; if the wave to be observed is not perfectly steady the curve obtained represents an average rather than the actual wave; the apparatus is not suited for observing rapidly-changing *non*-periodical phenomena, as, for instance, the effect of switching in or out of a long transmission line. The method is not suitable for high-frequency phenomena, because of the practical impossibility of driving the contact-maker at the required speed. For work of this kind so-called *oscillographs* are used, or instruments which give practically instantaneous wave records on a screen, or on a photographic plate.

In contra-distinction to the point-to-point method, the wave is here traced by a beam of light simultaneously with the actual changes in current: the result gives the image of an *individual* wave, instead of being an average of many impulses, as in the former method. A sample of such a curve, traced by a beam of light upon a photographic plate, is shown in Fig. 447. It represents a voltage curve actually observed on an alternator, running on unloaded cables with considerable electrostatic capacity; the eleventh harmonic is prominent.

The beam of light must be given simultaneously two movements: one along the axis of the ordinates, proportional to the instantaneous values of the current; the other — a uniform motion along the axis of abscissæ, proportional to time. These two movements, independent of each other, are produced in the oscillograph by different means, and will be described separately in the next two articles.

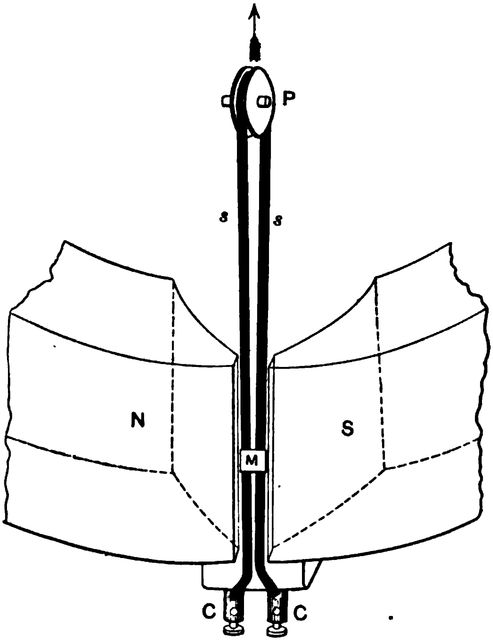

Fig. 448. The vibrating strips of the Duddell oscillograph.

592. Vibrations Proportional to Currents (Ordinates). — The principle of construction and operation of a practical oscillograph (Duddell's oscillograph) is shown in Fig. 448. In the narrow air-gap between the poles, *N*, *S*, of a powerful electromagnet, are stretched two parallel conductors, *s*, *s*, formed by bending a metal strip back on itself over the pulley *P*. A small mirror *M* is attached to the loop and serves the same purpose as a galvanometer mirror, viz., it indicates and magnifies deflections of the moving part by deflecting a ray of light. When a direct current is passing through the loop *s*, *s*, it causes one of the legs to advance and the other one to recede; the mirror is thus turned about a vertical axis. With a properly constructed instrument this deflection is made proportional to the current, so that the instrument is simply a sensitive moving-coil galvanometer. If the current is variable, instantaneous deflections of the instrument are proportional to instantaneous values of the current: This is indicated by the beam of light, reflected from the mirror on a suitable screen. The only difference between this instrument and an ordinary galvanometer is that the moving system of the oscillograph is so light and has such a short period of natural vibration, that the mirror easily follows very quick variations of the current, which an ordinary galvanometer could not follow, because of its inertia. In the most accurate high-frequency oscillographs the natural period of vibration of the loop is in the neighborhood of one ten-thousandth of a second. The moving system is immersed in a viscuous oil, which has a damping effect, and prevents even these small vibrations from showing on the curve.

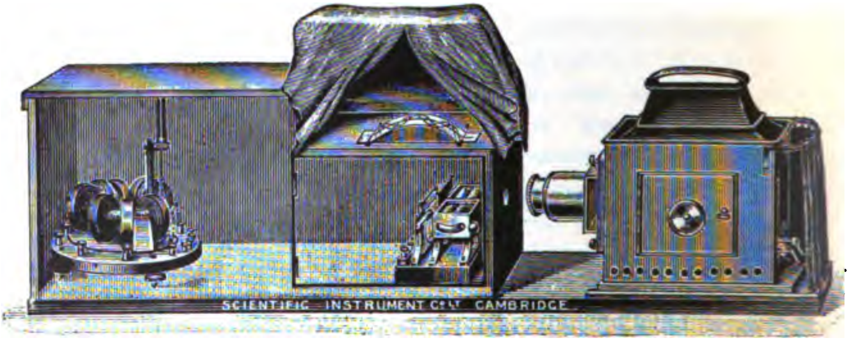

FIG. 449. The general arrangement of parts in a Duddell oscillograph with tracing desk.

With alternating currents traversing the loop *s*, *s*, the spot of light reflected from the mirror oscillates to and fro in a horizontal plane, as the current varies, and thus traces a straight line (ordinates). To obtain on the screen an image of the wave it is necessary to introduce the time element (abscissæ), as is explained below.

593. Movement Proportional to Time (Abscissæ). — Two distinct methods are used in the above oscillograph for producing a motion proportional to time, along the axis of abscissæ: (*a*) moving photographic plate, and (*b*) revolving mirror.

(*a*) *A moving photographic plate* is used when a permanent record of the observed wave is required. The vibrations of the strip *s*, *s*, being in a horizontal plane, the photographic plate must be moved in a vertical plane; this is usually done by letting the plate fall down a slide, under the action of gravity. When it passes by the exposure aperture, the beam of light traces on it a curve. The time of the exposure is so short that the speed of the plate may be assumed as constant.

It is sometimes necessary to observe alternating-current phenomena which last a considerable time, such as observations on the paralleling of alternators, the running up to speed of motors, the surges which occur in switching on and off cables, etc. Photographic records are required here, considerably longer than those which can be obtained on a single plate. In these cases a camera is mounted in place of the vertical slide. A roll of cinematograph film, such as is used for moving pictures, is driven at a uniform speed, past the exposure aperture. Records over 100 feet long may be obtained at one time, giving the complete picture of the varying phenomenon.

(*b*) *Revolving or oscillating mirror.* With this method, a second mirror is interposed in the path of the beam of light, and is caused to rotate, or to vibrate so as to impart to the beam of light a uniform motion at right angles to the vibrations due to the current. The spot of light then traces on a stationary screen a time-curve of variations of a voltage or a current, as the case may be (Fig. 447). If the variations are periodic, as in commercial alternating currents, the second mirror is synchronized, and the spot of light is caused to trace the wave form over and over again, so that it appears stationary to the eye, and may be easily observed. If it is desired to have it recorded, the beam of light is projected on a curved screen of ground glass (Fig. 449), where it may be traced by hand, or photographed.

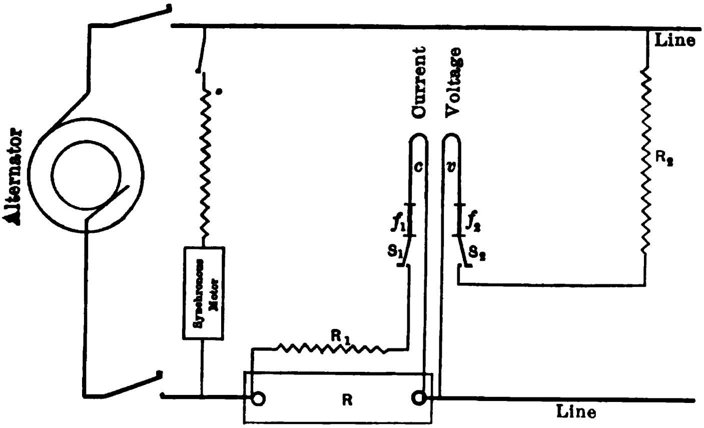

FIG. 450. Oscillograph connections for determining simultaneously a current wave and a voltage wave.

594. The Duddell Double Oscillograph. — The outfit shown in Fig. 449 represents a double oscillograph which is a combination of two single oscillographs, placed side by side. The moving part of one is connected across the terminals of the circuit and thus made to indicate the voltage wave; the other is connected as a milli-voltmeter across a non-inductive shunt in series with the circuit (Fig. 450).

The vibrating wire v, which traces the voltage wave, is connected across the line in series with a high resistance R_2; it is protected by a fuse f_2, and the switch S_2. The other vibrating wire c, which traces the current wave, is connected across a non-inductive shunt R; it is protected by the resistance R_1, fuse f_1 and switch S_1. The synchronous motor, shown in the sketch, is for driving the revolving or the vibrating mirror.

The oscillograph itself is shown in Fig. 449 to the left, its two small mirrors on the strips being illuminated by the projection lantern to the right. The light reflected from these mirrors falls on a vibrating mirror driven by a small synchronous motor, and is finally reflected on the curved screen; the vibrating mirror and the motor are shown under the tracing screen. Two curves are visible on the screen, the outfit being a double oscillograph.

The mirror is vibrated by means of a cam attached to the motor shaft. The cam is so arranged that the mirror moves uniformly for about 1½ complete periods, during which the wave form is observed it then returns rapidly to its starting point during the remaining ½ period. During the half period of return motion, the light is cut off from the oscillograph by means of a sector fixed to the motor.

Instead of a vibrating mirror, ordinary revolving mirrors may be used, provided with a sufficient number of faces, to give a suitable scale and a steady image.

When it is desired to produce waves to a larger scale, as for demonstration purposes, a large stationary mirror is substituted for the tracing screen, and the vibrating beam of light is thrown on the projection screen, or directly on the wall. Waves as high as 3 feet may be thus produced.

In taking oscillograph records, it is often desired to have the axis of abscissæ automatically marked on the record. This is done by providing a small *stationary* mirror in the oscillograph itself. This mirror gives a beam of light which is deflected by the revolving mirror along the axis of abscissæ, and traces this axis. Another way is to stretch a vertical wire before the falling photographic plate: The wire intersects the oscillating beam of light, and the axis of abscissæ is recognized on the record from the interruptions in the wave.

595. Students' Oscillograph. — The above-described oscillograph. used for accurate work, is rather delicate in handling and difficult to adjust. It is well for a beginner to start with a simpler and a more robust instrument, for instance such as is shown in Fig. 451. Its moving system is based on the same principle as that in Fig. 448, but the wires are much heavier and no oil damping is provided. This oscillograph could not be expected to show high harmonics accurately; it may even introduce harmonics of its own; but some experience in handling this instrument is a valuable preparation for work with more accurate oscillographs.

The instrument shown in Fig. 451 is a double oscillograph; it has two oscillating loops which may be used either together or separately, for instance, one for amperes and the other for volts.

The light necessary for producing the spot on the screen is obtained from a projection lantern; the rays are concentrated on the mirrors by means of suitable lenses. The revolving mirror should have about 12 faces, and may be driven by a small fan motor. The speed of the mirror can be made a multiple of the frequency of the supply to be investigated. Then the curves appear stationary on the screen, and can be traced with sufficient accuracy. In addition to two spots of light, produced by the two small mirrors on the movable parts of the oscillograph, a third spot is thrown on the screen by a third stationary mirror. When the revolving mirror rotates, this spot traces a horizontal luminous line representing the axis of abscissæ.

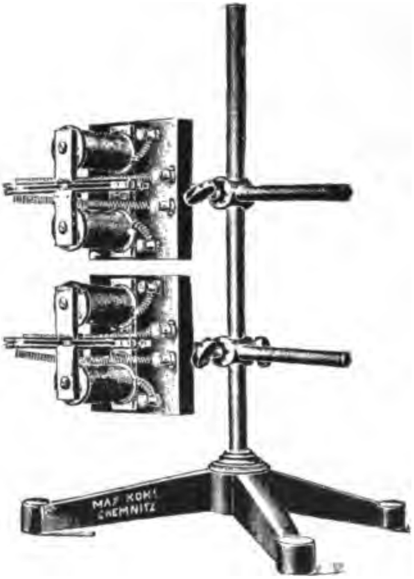

FIG. 451. A student's oscillograph.

At the beginning of the experiment the three spots must be made to coincide on the screen, with the current off. This is done by varying the relative positions of the two oscillographs on the stand, and also the position of the third mirror, which traces the axis of abscissæ. When the current is put on, and the revolving mirrors stand still, the vertical lines produced on the screen by the alternating currents flowing through the oscillograph, must be perpendicular to the axis of abscissæ. After being thus adjusted the apparatus is ready for use.

The current should not be kept switched on for a long time, either in the moving part or the stationary field of the oscillograph. The apparatus is not intended for a continuous use, and can be damaged by being overheated. The same applies to the motor driving the revolving mirror. Open all the circuits as soon as a curve is taken, and while connections are being changed for the next curve.

596. EXPERIMENT 27-B. — Determination of Alternating Current Wave-Form with an Oscillograph. — The work with an oscillograph (§§ 591 to 595) consists in taking some or all of the curves enumerated in connection with the point-to-point method (§ 590). The results must be worked out in the way indicated there. Similar curves should be taken by the two methods, as, for instance, the wave form of the laboratory supply, which is practically invariable: the results can be compared and checked with each other.

WAVE ANALYSIS.

597. It is sometimes desired to express analytically an irregular wave, such as *PQR* (Fig. 452), taken by the point-to-point method

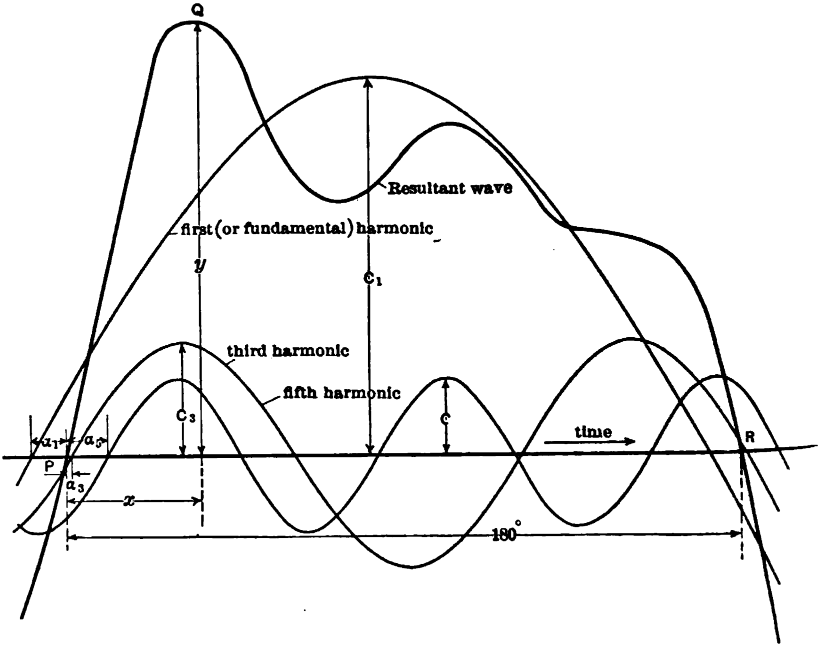

Fig. 452. A complex wave resolved into its harmonics.

or by an oscillograph. The equation of such a curve is obtained on the basis of the mathematical law, that *any periodic curve, no matter how irregular, may be represented by an infinite series of sine waves.*

one of which has the same frequency as the given curve, and the rest have frequencies which are multiples of it. Thus the wave shown in Fig. 452 is a combination of three sine-waves, or, as they are called, *harmonics;* the fundamental wave C_1, the third harmonic C_3, and the fifth harmonic C_5. A rigid analytical proof of this law (Fourier's theorem) would be out of place here. It will be easily seen, however, without any proof, that by suitably selecting the frequency and the phase position of the higher harmonics, the shape of the fundamental sine wave may be distorted in almost any desired way.

When the two halves of the irregular wave, above and below the axis of abscissæ, are identical, no *even* harmonics can be present, or harmonics having frequencies 2, 4, 6, etc., times higher than the given curve. Such harmonics would increase the ordinates of one half of the fundamental wave, and reduce by the same amount the ordinates of the other half, making the wave unsymmetrical, which is contrary to the assumption of its being symmetrical. Practically all the waves of voltages and currents, dealt with in electrical engineering, are symmetrical waves, at least those produced by electromagnetic induction. They consist, therefore, of the fundamental and the *odd* harmonics only.

598. General Equation of an Irregular Wave.— From the above considerations, it follows that the general equation of the wave PQR shown in Fig. 452 is:

$$y = C_1 \operatorname{Sin}(x + \alpha_1) + C_3 \operatorname{Sin}(3x + \alpha_3) + C_5 \operatorname{Sin}(5x + \alpha_5) + \ldots$$

the meaning of all the symbols being shown in the sketch. Opening the brackets, we get

$$y = C_1 \operatorname{Cos}\alpha_1 \operatorname{Sin} x + C_1 \operatorname{Sin}\alpha_1 \operatorname{Cos} x + C_3 \operatorname{Cos}\alpha_3 \operatorname{Sin} 3x$$
$$+ C_3 \operatorname{Sin}\alpha_3 \operatorname{Cos} 3x + \ldots\ldots\ldots$$

or

$$y = A_1 \operatorname{Sin} x + B_1 \operatorname{Cos} x + A_3 \operatorname{Sin} 3x + B_3 \operatorname{Cos} 3x + \ldots \quad (1)$$

where for any harmonic, such as the nth,

$$\left.\begin{aligned} A_n &= C_n \operatorname{Cos}\alpha_n \\ B_n &= C_n \operatorname{Sin}\alpha_n \end{aligned}\right\} \ldots\ldots\ldots \quad (2)$$

The coefficients A_n and B_n are the amplitudes of the component waves of the nth harmonic (Fig. 454). The two components of the fifth harmonic are shown in this figure, the sine component and the cosine component. The first intersects the axis of abscissæ at the same point P with the fundamental wave, the second is displaced by $\frac{1}{2}\pi$.

Having found, by the method described below, the component harmonics, the magnitude and the phase position of the total harmonic (Fig. 452) of the same order are calculated from the relations

$$\left.\begin{aligned} C_n &= \sqrt{A_n^2 + B_n^2}\,; \\ \tan\alpha_n &= B_n \div A_n\ . \end{aligned}\right\} \ldots\ldots\ldots \quad (3)$$

These formulæ are obtained from the expressions (2) for A_n and B_n. The values of C_n and α_n can also be measured graphically, as shown in Fig. 453, from the known values of A_n and B_n.

599. Expressions for Amplitudes of Higher Harmonics. — The amplitudes A_1, B_1, A_3, B_3, etc., of component harmonics are calculated on the basis of the following mathematical propositions:

(I) The integral

$$\int_0^\pi \text{Sin } mx \,.\, \text{Sin } nx \,.\, dx = 0 \,. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

except when $m = n$, in which case it becomes

$$\int_0^\pi \text{Sin}^2 nx \,.\, dx = \frac{\pi}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (5)$$

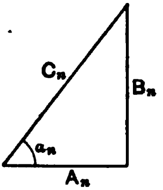

FIG. 453. The relation between a harmonic and its sine and cosine components.

(II) The same holds true when cosines are substituted for sines.

(III) The integral

$$\int_0^\pi \text{Sin } mx \,.\, \text{Cos } nx \,.\, dx = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (6)$$

whether $m = n$, or not, provided both m and n are odd numbers, as is the case in the problem under consideration. A proof of the above propositions is given in § 601 below.

In order to eliminate all the coefficients in the expression (1), except A_n, both sides of the equation are multiplied by Sin nx, and integrated between the limits 0 and π. This gives

$$\int_0^\pi y \text{ Sin } nx \,.\, dx = 0 + 0 + \;.\;.\;.\; + A_n \,.\, \frac{\pi}{2} + 0 + 0 + \;.\;.\;.\;.$$

or

$$A_n = \frac{2}{\pi}\int_0^\pi y \text{ Sin } nx \,.\, dx \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (7)$$

Similarly

$$B_n = \frac{2}{\pi}\int_0^\pi y \text{ Cos } nx \,.\, dx \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (8)$$

where $n = 1, 3, 5$, etc., according to which harmonic is desired to be calculated.

These integrals mean a summation of an infinite number of products, such as y Sin nx and y Cos nx. In applying these expressions in practice, one must be satisfied with a finite number of terms, and use an ordinary summation instead of integration.* We obtain, then:

---

* There are mechanical wave-analyzers on the market, by means of which the values of A and B are obtained by merely tracing the given wave with a point, as with a planimeter.

$$A = \frac{2}{\pi}\sum_{o}^{q} y \operatorname{Sin} nx \cdot \frac{\pi}{q} = \frac{\sum_{o}^{q} y \operatorname{Sin} nx}{\frac{1}{2} q} \quad . \quad . \quad . \quad . \quad (9)$$

$$B_n = \frac{2}{\pi}\sum_{o}^{q} y \operatorname{Cos} nx \cdot \frac{\pi}{q} = \frac{\sum_{o}^{q} y \operatorname{Cos} nx}{\frac{1}{2} q} \quad . \quad . \quad . \quad . \quad . \quad (10)$$

where q is the number of parts into which 180° is subdivided (Fig. 454). The formulæ (9) and (10) are those used in practice for analyzing alternating-current waves, as is explained below.

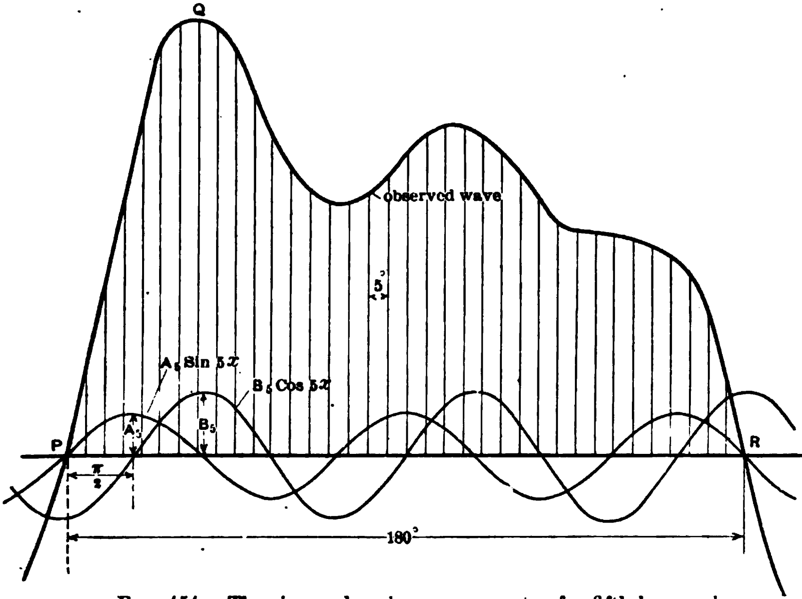

FIG. 454. The sine and cosine components of a fifth harmonic.

600. The Use of Analyzing Tables. — Suppose it to be desired to separate the fifth harmonic of the current wave PQR shown in Fig. 454. The base line PR is divided into a certain number of equal parts, so as to take into account the most pronounced irregularities of the wave. In the case shown, 180 degrees is divided into 36 parts, corresponding to a distance of 5 degrees between the ordinates, or $q = 36$. The angular distance of 5 degrees for the fundamental wave corresponds to an electric phase angle of 25 degrees for the fifth harmonic. Thus we have, according to equation (9):

$$A_5 = \frac{1}{\frac{1}{2} \cdot 36}\left\{ y_1 \operatorname{Sin} 25° + y_2 \operatorname{Sin} 50° + y_3 \operatorname{Sin} 75° + \; . \; . \quad (11)\right.$$

$$B_5 = \frac{1}{\frac{1}{2} \cdot 36}\left\{ y_1 \operatorname{Cos} 25° + y_2 \operatorname{Cos} 50° + y_3 \operatorname{Cos} 75° + \; . \; . \quad (12)\right.$$

where y_1, y_2, y_3, etc., are the measured ordinates of the curve PQR, beginning with the first division after the point P. The amplitudes A_5 and B_5 can be calculated from these expressions, and then the total fifth harmonic and its phase position may be determined, as shown in Figs. 452 and 453.

The values of sines and cosines in the equations (11) and (12) are the same with all curves, provided the number of ordinates $q = 36$ is the same. Therefore, it is convenient to have the values of sines and cosines tabulated once for all, and merely multiply them with the ordinates y_1, y_2, y_3, etc., of a given curve. Such tables for the fundamental wave, the third, the fifth, and the seventh harmonics are given below. Similar tables may be prepared for any desired nnmber of higher harmonics. Having tables of this kind, the calculation of the values of amplitudes A and B, according to the equations (9) and (10) is much simplified: The measured ordinates y are entered in column 5 and multiplied with the sines in column 1. The results are written down in columns 2 and 3; the positive and the negative products are summed up separately. Then, according to equation (9) we have

$$A_n = \frac{1}{18}\left(\sum_{\text{pos.}}{}' - \sum_{\text{neg.}}{}'\right) \quad . \; . \; . \; . \; . \; . \; . \; (13)$$

In a similar way, by using the cosines in column 6, the following expression is obtained for the amplitude B_n:

$$B_n = \frac{1}{18}\left(\sum_{\text{pos.}}{}'' - \sum_{\text{neg.}}{}''\right) \quad . \; . \; . \; . \; . \; . \; . \; (14)$$

If a large number of curves are to be analyzed, it is convenient to have the tables on pp. 632–635 mimeographed or printed so as to avoid copying the values of sines and cosines for each curve.*

Having calculated A_n and B_n for a harmonic, its amplitude C_n and the phase angle α_n are determined from the equations (3), or constructed graphically, as in Fig. 453. If A_n is positive and B_n is negative, as is assumed in Fig. 454, the angle α_n is less than 90 degrees, and the harmonic wave crosses the axis of abscissæ to the right of the given curve (Fig. 452). If B_n is positive, α_n is again less than 90 degrees, but the harmonic wave crosses the axis of abscissæ to the left of the fundamental. If A_n is negative (sine wave reversed), α_n is between 90 degrees and 180 degrees, lagging or leading, according to whether B_n is positive or negative.

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* Extra copies of these tables may be had of the publishers, Messrs. John Wiley & Sons.

601. Proof of Formulæ (4), (5), and (6).—The expressions (4), (5) and (6) may be proved by direct integration, having previously substituted a sum or a difference of trigonometrical functions for their product. Thus, we have:

$$\operatorname{Cos}(m-n)x - \operatorname{Cos}(m+n)x = 2 \operatorname{Sin} mx \operatorname{Sin} nx,$$

so that the integral (4) is reduced to

$$\tfrac{1}{2}\int_0^\pi \operatorname{Cos}(m-n)\,x\,dx - \tfrac{1}{2}\int_0^\pi \operatorname{Cos}(m+n)\,x\,dx.$$

This, after integration, gives:

$$\tfrac{1}{2}\left[\frac{\operatorname{Sin}(m-n)\,x}{(m-n)}\right]_0^\pi - \tfrac{1}{2}\left[\frac{\operatorname{Sin}(m+n)\,x}{m+n}\right]_0^\pi$$

Each of these expressions is separately equal to zero, which proves equation (4).

When $m = n$, the above expression becomes indeterminate, so that equation (5) must be proved separately. We have

$$\operatorname{Sin}^2 nx = \frac{1-\operatorname{Cos} 2\,nx}{2};$$

consequently

$$\int_0^\pi \operatorname{Sin}^2 nx.\,dx = \tfrac{1}{2}\Big[x\Big]_0^\pi - \tfrac{1}{2}\left[\frac{\operatorname{Sin} 2\,nx}{2\,n}\right]_0^\pi.$$

The second term on the right side is equal to zero; the first term gives $\tfrac{1}{2}\pi$.

This proves equation (5).

Equation (6) is proved on the basis of the relation:

$$\operatorname{Sin}(m+n)\,x + \operatorname{Sin}(m-n)\,x = 2 \operatorname{Sin} mx \operatorname{Cos} nx.$$

After integration, we obtain:

$$\tfrac{1}{2}\left[\frac{\operatorname{Cos}(m+n)\,x}{m-n}\right]_0^\pi + \tfrac{1}{2}\left[\frac{\operatorname{Cos}(m-n)\,x}{m-n}\right]_0^\pi.$$

According to the conditions of the problem both m and n are *odd* numbers (order numbers of harmonics): consequently $(m+n)$ and $(m-n)$ are *even* numbers. But cosine of an even multiple of π is always equal to 1. Therefore, each of the above terms is separately equal to zero. This proves equation (6).

APPENDIX.

The following tables of Sines and Cosines, are arranged for separating complex waves into harmonics—see Expressions (13) and (14):

FUNDAMENTAL WAVE.

1	2	3	4	5	6	7	8
Sin x	Products y sin x		No.	Measured Ordinates y	Cos x	Products y cos x	
	(positive)	(negative)				(positive)	(negative
.0872			1		.9962		
.1736			2		.9848		
.2588			3		.9659		
.3420			4		.9397		
.4226			5		.9063		
.5000			6		.8660		
.5736			7		.8192		
.6428			8		.7660		
.7071			9		.7071		
.7660			10		.6428		
.8192			11		.5736		
.8660			12		.5000		
.9063			13		.4226		
.9397			14		.3420		
.9659			15		.2588		
.9848			16		.1736		
.9962			17		.0872		
1.0000			18		.0000		
.9962			19		−.0872		
.9848			20		−.1736		
.9659			21		−.2588		
.9397			22		−.3420		
.9063			23		−.4226		
.8660			24		−.5000		
.8192			25		−.5736		
.7660			26		−.6428		
.7071			27		−.7071		
.6428			28		−.7660		
.5736			29		−.8192		
.5000			30		−.8660		
.4226			31		−.9063		
.3420			32		−.9397		
.2588			33		−.9659		
.1736			34		−.9848		
.0872			35		−.9962		
.0000			36		−1.0000		
	$\Sigma'_{pos}=$	$\Sigma'_{neg}=$				$\Sigma''_{pos}=$	$\Sigma''_{neg}=$

THIRD HARMONIC.

	2	3	4	5	6	7	8
Sin $3x$	Products $y \sin 3x$		No.	Measured Ordinates y	Cos $3x$	Products $y \cos 3x$	
	(positive)	(negative)				(positive)	(negative)
.2588			1		.9659		
.5000			2		.8660		
.7071			3		.7071		
.8660			4		.5000		
.9659			5		.2588		
1.0000			6		.0000		
.9659			7		−.2588		
.8660			8		−.5000		
.7071			9		−.7071		
.5000			10		−.8660		
.2588			11		−.9659		
.0000			12		−1.0000		
−.2588			13		−.9659		
−.5000			14		−.8660		
−.7071			15		−.7071		
−.8660			16		−.5000		
−.9659			17		−.2588		
−1.0000			18		.0000		
−.9659			19		.2588		
−.8660			20		.5000		
−.7071			21		.7071		
−.5000			22		.8660		
−.2588			23		.9659		
.0000			24		1.0000		
.2588			25		.9659		
.5000			26		.8660		
.7071			27		.7071		
.8660			28		.5000		
.9659			29		.2588		
1.0000			30		.0000		
.9659			31		−.2588		
.8660			32		−.5000		
.7071			33		−.7071		
.5000			34		−.8660		
.2588			35		−.9659		
.0000			36		−1.0000		
	$\Sigma'_{pos} =$	$\Sigma'_{neg} =$				$\Sigma''_{pos} =$	$\Sigma''_{neg} =$

FIFTH HARMONIC.

1	2	3	4	5	6	7	8
Sin 5 x	Products	y sin 5 x	No.	Measured Ordinates y	Cos 5 x	Products	y cos 5 x
	(positive)	(negative)				(positive)	(negative)
.4226			1		.9063		
.7660			2		.6428		
.9659			3		.2588		
.9848			4		−.1736		
.8192			5		−.5736		
.5000			6		−.8660		
.0872			7		−.9962		
−.3420			8		−.9397		
−.7071			9		−.7071		
−.9397			10		−.3420		
−.9962			11		.0872		
−.8660			12		.5000		
−.5736			13		.8192		
−.1736			14		.9848		
.2588			15		.9659		
.6428			16		.7660		
.9063			17		.4226		
1.0000			18		.0000		
.9063			19		−.4226		
.6428			20		−.7660		
.2588			21		−.9659		
−.1736			22		−.9848		
−.5736			23		−.8192		
−.8660			24		−.5000		
−.9962			25		−.0872		
−.9397			26		.3420		
−.7071			27		.7071		
−.3420			28		.9397		
.0872			29		.9962		
.5000			30		.8660		
.8192			31		.5736		
.9848			32		.1736		
.9659			33		−.2588		
.7660			34		−.6428		
.4226			35		−.9063		
.0000			36		−1.0000		
	$\Sigma'_{\text{pos}}=$	$\Sigma'_{\text{neg}}=$				$\Sigma''_{\text{pos}}=$	$\Sigma''_{\text{neg}}=$

SEVENTH HARMONIC.

1	2	3	4	5	6	7	8
Sin $7x$	Products	y sin $7x$	No.	Measured Ordinates y	Cos $7x$	Products	y cos $7x$
	(positive)	(negative)				(positive)	(negative)
.5736			1		.8192		
.9397			2		.3420		
.9659			3		−.2588		
.6428			4		−.7660		
.0872			5		−.9962		
−.5000			6		−.8660		
−.9063			7		−.4226		
−.9848			8		.1736		
−.7071			9		.7071		
−.1736			10		.9848		
.4226			11		.9063		
.8660			12		.5000		
.9962			13		−.0872		
.7660			14		−.6428		
.2588			15		−.9659		
−.3420			16		−.9397		
−.8192			17		−.5736		
−1.0000			18		.0000		
−.8192			19		.5736		
−.3420			20		.9397		
.2588			21		.9659		
.7660			22		.6428		
.9962			23		.0872		
.8660			24		−.5000		
.4226			25		−.9063		
−.1736			26		−.9848		
−.7071			27		−.7071		
−.9848			28		−.1736		
−.9063			29		.4226		
−.5000			30		.8660		
.0872			31		.9962		
.6428			32		.7660		
.9659			33		.2588		
.9397			34		−.3420		
.5736			35		−.8192		
.0000			36		−1.0000		
	$\Sigma'_{\text{pos}}=$	$\Sigma'_{\text{neg}}=$				$\Sigma''_{\text{pos}}=$	$\Sigma''_{\text{neg}}=$

CHAPTER XXVIII.

ELECTRIC BATTERIES.

602. ELECTRIC batteries are devices for generating electric current by means of chemical reactions. Or, in more scientific language, they are devices "for the conversion of the potential energy of chemical separation into the energy of an electric current" (Carhart)

A battery consists of two electrodes of different substances immersed in a liquid (or a paste, as in dry batteries). With proper materials, an e.m.f. is produced between the electrodes, and if the circuit is closed, the battery becomes a source of power.

The production of current is accompanied by chemical reactions in the cell itself: The plates, or electrodes, and the solution (electrolyte) are gradually changed, and finally become *neutral* with respect to each other; here the reaction stops, and the cell is said to be completely discharged or exhausted.

Some kinds of cells require renewing of the plates and of the solution in order to be able again to generate current. Others need only recharging, by an electric current sent through the battery in the opposite direction; this current reduces the elements of the cell to their original state. Batteries of the first type are called *primary batteries;* the latter, — *secondary or storage batteries.* The reason for these names is that in the first type of batteries electric current is obtained directly from the chemicals constituting the cell; whereas a storage battery merely returns the electric power previously put into it.

Primary batteries are used only where comparatively small amounts of energy are required, and the service is intermittent: For instance, in bell and telephone work; in telegraph, signal and alarm service; for igniting gas engines, etc. Storage batteries are used somewhat for the same class of work, but their principal field of application is in light and power stations, in electric railway substations, etc., where it is of advantage to store large amounts of electrical energy during periods of small demand, and use it during periods of large demand. Moreover, a storage battery is an excellent reserve in case of an accident to the generating machinery.

PRIMARY BATTERIES.

603. The following types of primary batteries are in most common use in this country:

(1) Gravity Cell;
(2) Leclanché Cell;
(3) Edison Cell;
(4) Dry Batteries.

The general features of these cells are described below; instructions for installing the cells and detailed recipes of chemicals usually accompany "wet" batteries.

604. Gravity Cell. — The positive electrode is of zinc, and is placed in the top of the cell. The negative electrode is of copper, and is placed on the bottom of the jar. The copper electrode is surrounded with crystals of copper sulphate ($CuSO_4$), known commercially as blue vitriol or bluestone; the jar is filled with water. A little sulphuric acid, or better, a weak solution of zinc sulphate ($ZnSO_4$), is added on top of the water.

During the action of the battery, zinc (Zn) replaces copper (Cu) in copper sulphate; zinc sulphate is formed in the upper part of the jar. Metallic zinc gradually disappears, and metallic copper is deposited on the copper electrode.

The name "gravity cell" is due to the fact that copper sulphate solution is kept below zinc sulphate without mixing with it, merely because of the difference in the specific gravities of the two solutions. The separating "blue line" must be kept about midway between the electrodes.

605. Leclanché Cell. — The positive electrode is again of zinc, the negative consists of carbon surrounded with peroxide of manganese (MnO_2). A solution of sal ammoniac (NH_4Cl) is used as electrolyte. When the circuit is closed, zinc (Zn) displaces ammonium (NH_4) in the solution, forming zinc chloride ($ZnCl$). The ammonium breaks up into ammonia gas (NH_3), which either unites with water or escapes, and free hydrogen (H), which is liberated at the surface of the carbon.

Without peroxide of manganese, this hydrogen would soon completely "polarize" the battery: Bubbles of hydrogen surrounding the carbon would form a layer of considerable electrical resistance and of an appreciable counter-e.m.f. In the presence of peroxide of manganese, which is rich in oxygen, hydrogen unites with the oxygen and forms water. For this reason, manganese peroxide is called a "depolarizer."

This depolarizer, being solid, does not act promptly enough, so that the cell requires periods of rest between periods of action. Therefore, the Leclanché cell is suitable for open-circuit work only, such as bell or telephone service, where the circuit is closed for short intervals, and sufficient time of rest is afforded for the depolarizer to oxidize the hydrogen.

606. Edison Primary Battery. — The elements employed in this battery are: zinc (Zn) and black oxide of copper (CuO), in a solution of caustic soda ($NaOH$). A layer of heavy mineral oil is poured on top of the solution, to prevent it from creeping up the zinc plates.

When the circuit is closed, the water of the solution is decomposed into nascent oxygen and hydrogen. The oxygen goes to the zinc plate and unites with it, forming oxide of zinc. This, in its turn, is dissolved by the caustic soda solution, forming zincate of soda (Na_2ZnO_2). The hydrogen unites with the oxygen contained in the oxide of copper plate, forming water and leaving behind metallic copper. As the oxide plate is porous, this action goes on, when the battery is in service, until the oxide plate is reduced throughout its entire mass to metallic copper in a finely divided state.

Thus, in this cell, copper oxide acts as the negative plate, and at the same time as a very effective depolarizer for hydrogen. Therefore the Edison battery is very well adapted for closed-circuit work. In fact, it has a greater capacity for work per unit weight than any other battery on the market, either primary or secondary.

607. Dry Batteries. — The solutions present in the above-described "wet" batteries are objectionable in some cases, making the battery rather difficult to handle; moreover, renewals require some skill and experience. This circumstance led to the introduction of so-called *dry batteries*, which in many instances have replaced wet batteries, and are constantly increasing in public favor.

A dry cell is set up in the factory and sealed; when it is exhausted in use, it is simply replaced by a new cell; no renewals being attempted. Strictly speaking, these batteries are not dry, but the active solution is held absorbed by some porous substance, in the form of a paste. Being sealed, the cell is as convenient for handling, as if it were actually dry.

In their chemical action the dry batteries are similar to the Leclanché cell, described above. The containing cup is made of zinc, and serves as positive electrode. The negative electrode is a vertical carbon rod in the center of the cell. The intervening space is filled with a mixture of peroxide of manganese, powdered carbon and some moisture-retaining material, such as sawdust. The whole is saturated with a solution of sal ammoniac and the cell is sealed. The principal reaction, as in the Leclanché cell, consists in the formation of zinc chloride and in depolarization of hydrogen by the peroxide of manganese. Various binding and other substances are usually added to the paste by different manufacturers, and the actual reactions are more complicated than the one described above.

608. Testing Primary Batteries. — The most important operating characteristics, for which primary batteries are usually tested, are:

(1) Internal e.m.f. and terminal voltage.
(2) Internal resistance.
(3) Amount of polarization and promptness of recovery.
(4) Useful life under given conditions of service.

These tests are described below in detail.

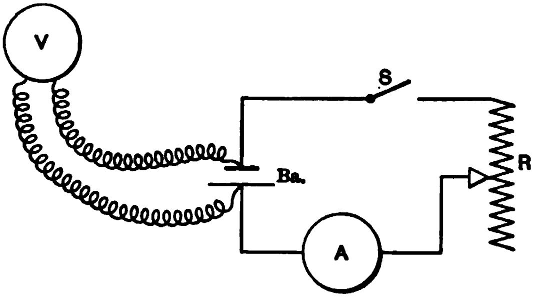

FIG. 455. Electrical connections for testing a battery.

609. Measurement of Electromotive Force. — The electromotive force on open circuit and the terminal voltage on closed circuit are measured with any ordinary voltmeter (Fig. 455), unless even a small voltmeter current appreciably alters the condition of the cell. Good modern voltmeters take but 0.01 ampere with full-scale deflection, so that this current may be considered negligible under ordinary circumstances (except in standard cells; see § 62).

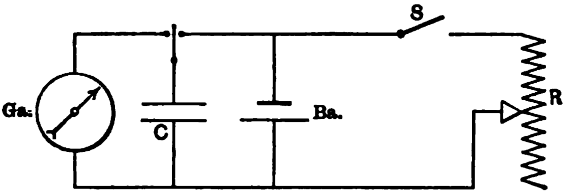

FIG. 456. Measuring the voltage of a battery by means of a condenser and a ballistic galvanometer.

If such is not the case, the voltage may be measured as is shown in Fig. 456: A condenser *C* is connected across the terminals of the battery *Ba.* under test, and charged at the battery voltage by pressing to the right the two-way key shown above the condenser. Then the condenser is discharged through the ballistic galvanometer

Ga. (§ 139) by pressing the key to the left; the deflection is noted. Then the same condenser is charged by a standard cell and again discharged through the galvanometer. The ratio of the deflections gives the ratio of the voltages compared. As the voltage of the standard cell is known, the voltage of the cell under test may be calculated.

If very accurate results are required, the voltage of the cell under test is compared to that of the standard cell by means of a potentiometer (see § 63).

610. Measuring Internal Resistance. — Suppose the e.m.f. of a battery on open circuit to be E; let the terminal voltage be e when the battery is closed on an external resistance R (Fig. 456). Then the unknown internal resistance x of the battery may be determined from the conditions that the current

$$i = \frac{e}{R} \text{ and also } i = \frac{E - e}{x},$$

or

$$\frac{e}{R} = \frac{E - e}{x},$$

whence

$$x = R\frac{E - e}{e}.$$

In reality the drop $(E - e)$ is due not only to the internal resistance, but also to the counter-e.m.f. of polarization, which increases with time (Fig. 457). Therefore, in performing the above test, it is essential *to measure the voltage e immediately upon closing the circuit,* before any polarization has set in. Besides this, the cell should have had a sufficient period of rest, in order that all previous polarization may have been removed. The measurements may be conveniently performed with the connections shown in Fig. 456. The voltage at no load is read by charging and discharging the condenser with the two-way key, having the switch S open. Then the two-way key is pressed to the right, the switch S is closed, and immediately afterward the key is again pressed to the left. The galvanometer deflection gives the terminal voltage e. The galvanometer is supposed to be calibrated, as before, with a standard cell.

The resistance of a battery cannot be measured with an ordinary Wheatstone bridge, because the e.m.f. of the battery would entirely vitiate the result. One way out of this difficulty is to measure the resistance of two identical cells connected in opposition, so that their e.m.f.'s destroy each other. Another method is to use a Wheatstone bridge with alternating currents (see § 21).

611. Polarization and Recovery Tests.—The connections are made as in Fig. 455, or as in Fig. 456, according to whether or not it is permissible to use a voltmeter (see § 609). The battery is closed on a certain resistance *R*, and terminal voltages are measured at stated intervals, with the switch *S* closed and open. The results are plotted as in Fig. 457. The curve marked "Terminal Volts" refers to the readings with the circuit closed; the curve marked "Internal e.m.f. on Closed Circuit" gives voltmeter readings immediately after the switch *S* has been opened. The current is read directly on an ammeter; the internal resistance of the battery (§ 610) is calculated by dividing the differences of the ordinates of the two voltage curves by the corresponding currents. The circuit must be open just long enough to read the voltmeter, and then closed again.

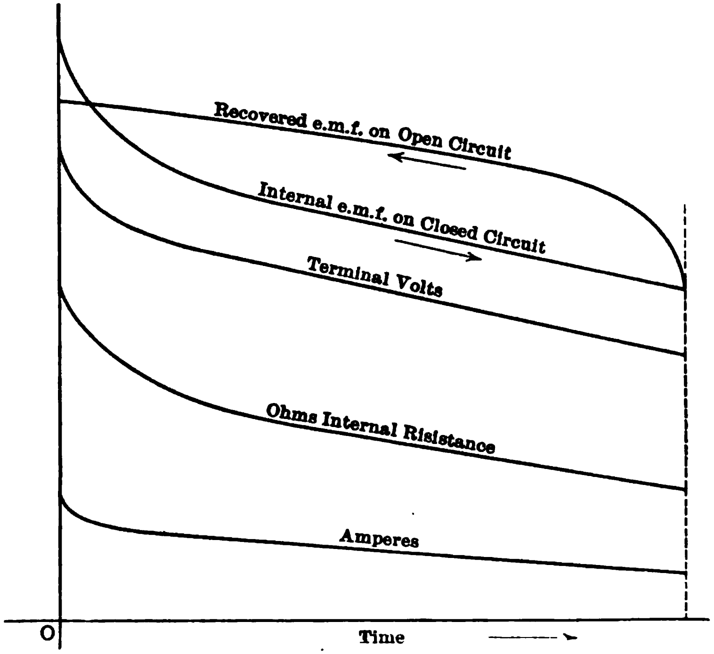

FIG. 457. Performance curves of a primary battery.

At the end of a certain period of time, say one or two hours, the circuit is permanently opened, and the battery allowed to recover from polarization. The voltage gradually rises as shown by the upper curve.

The test, in order to be conclusive, must be repeated on several cells of the same type, and with various rates of current; also with the batteries closed for different lengths of time.

612. Useful Life of Primary Batteries. — The useful life of a battery depends essentially on the character of service for which it is used; therefore the test for life must approximate, as far as possible, the actual conditions of service. If the battery is intended for closed-circuit work, it is closed during the test on a resistance equal to that of the apparatus which it is intended to supply, and is kept in the circuit until the current falls below that sufficient for the purpose.

In testing open-circuit batteries, the circuit must be periodically closed and opened, in order to give the battery time to recover from polarization. It is convenient to have, for such a test, a clock provided with contacts on its face: the minute hand carries a wiping contact which closes the circuit for a few minutes at desired intervals of time. If several cells are tested simultaneously, the circuit of each cell is closed by a separate relay. All the relays are connected in series and are operated from an auxiliary battery: the clock merely closes the circuit of this battery.

Terminal volts are plotted to days of service as abscissæ, until the end of the useful life of the cell. Such curves, plotted for competitive makes of batteries, permit the selection of the one most suitable for the service.

Some prefer to keep the current constant during the life test, instead of keeping constant resistance. An advantage of this method is that the test is shorter; at the same time it is fair, because all the competitive cells are tested under identical conditions.

613. EXPERIMENT 28-A. — Testing Primary Batteries. — The tests are described in §§ 608 to 612. The characteristics of the principal types of primary batteries may be found in §§ 603 to 607. (*a*) Measure the internal resistance of the cell by the three methods described in § 610. (*b*) Perform a short polarization and recovery test, described in § 611 and represented by the curves in Fig. 457. The life test could not be performed during a laboratory period; but it is advisable to keep in the laboratory a few cells on life test, as described in § 612, and let each student enter his readings on the permanent log.

Report. Give the internal resistance of the cells tested; plot performance curves, similar to those in Fig. 457. Make rough sketches of the batteries experimented with and mark the substances used in them; also give the principal chemical reactions in the form of equations.

CHARACTERISTICS OF STORAGE BATTERIES.

614. Storage batteries (also called electric accumulators) are devices used for storing electrical energy (in chemical form), which may be delivered at a later time. The part which storage batteries play in the distribution of electrical energy is much the same as that played by a water storage tank in water supply systems (Figs. 458 and 459). Without the tank the pumps have to supply a variable demand; their capacity must, therefore, be sufficient for the *maximum* demand. Moreover they must be operated 24 hours a day, the power consumption is much increased and the efficiency consequently reduced, to say nothing of the excessive mechanical strains imposed by sudden variations of the load. A water tank of sufficient capacity remedies all this; the capacity of the pumps needs to be sufficient for the *average* demand only, and they may be operated at practically full load. When the demand is below the average, the excess of water pumped simply raises the level in the tank. When the demand is above the average, the tank supplies the necessary excess of water into the city mains. In addition to this, the tank allows a more constant pressure to be maintained in the mains, with variable load. Similarly, in an electric power house without storage batteries, the generators have to supply the variable demand and are subjected to all the disadvantages resulting therefrom, viz., their capacity must be sufficient for the heaviest overloads which may occur, the machines must be operated 24 hours a day or at least as long as there is even the smallest demand for light and power; the engines are subjected to severe mechanical strains and are working under the most unfavorable conditions, as far as efficiency is concerned — namely, at variable load.

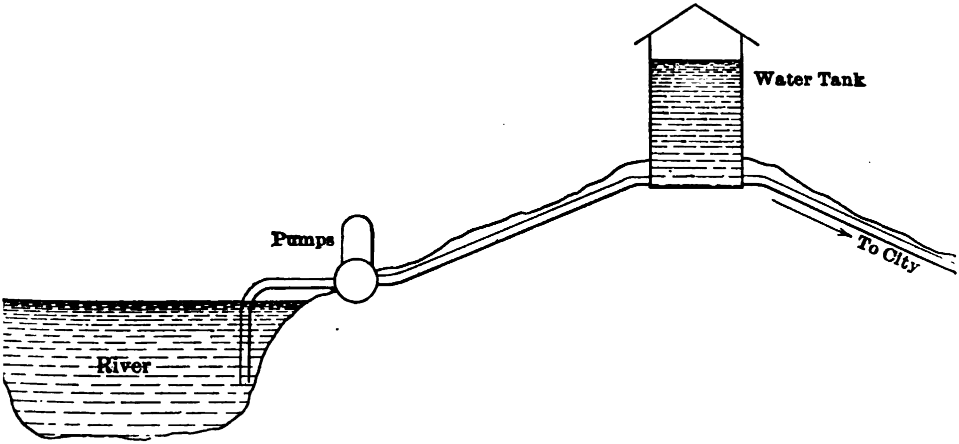

FIG. 458. A water-supply plant, analogous to an electric plant with a storage battery.

When a storage battery is connected in parallel with the generators (Fig. 459) the latter need have a capacity sufficient only for the *average* daily load, and may be worked practically all the time at this load. When the load is below normal, the excess energy is sent into the batteries, charging them. At the hours of maximum demand (peaks of the load), the battery discharges into the line in parallel with the generators. During the hours of very small demand the engines may even be shut down, the battery alone supplying the current. The efficiency of the plant is thus increased, and a steadier pressure maintained with fluctuating loads.

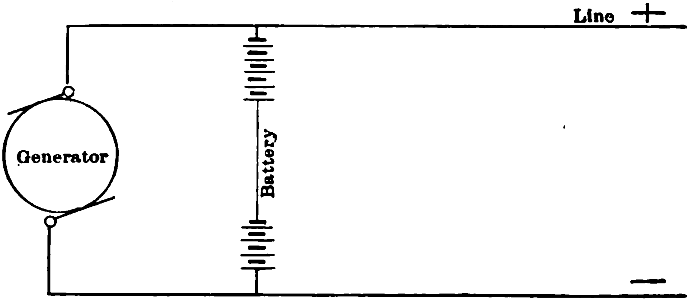

FIG. 459. A generator and a storage battery supplying a line in parallel.

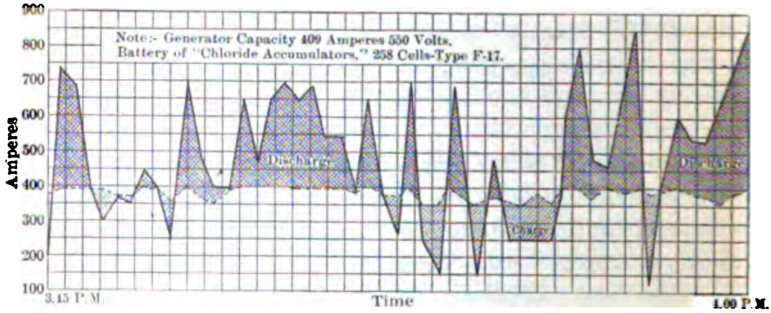
Note:- Generator Capacity 409 Amperes 550 Volts.
Battery of "Chloride Accumulators," 258 Cells-Type F-17.

FIG. 460. Effect of a battery in steadying generator load.

Fig. 460 shows the effect of a storage battery in steadying the generator load; while the total load varies between 150 and 850 amperes, the generator load is kept at an average of between 350 and 400 amperes. Fig. 461 illustrates the influence of a storage battery in maintaining a constant voltage. Without the battery the voltage fluctuates between 108 and 122 volts; the battery limits the fluctuations between 113 and

118 volts. Both diagrams represent actual curves, observed experimentally in certain power houses.

The limitations which at present prevent the universal use of storage batteries are: (1) their comparatively high first cost and depreciation; (2) additional complications resulting from extra apparatus needed for controlling and charging the batteries, and (3) the amount of care required in their maintenance. There are many cases, however, especially in city electric-railway work, where the advantages gained by the use of batteries by far outweigh the disadvantages; in such cases storage batteries are extensively used.

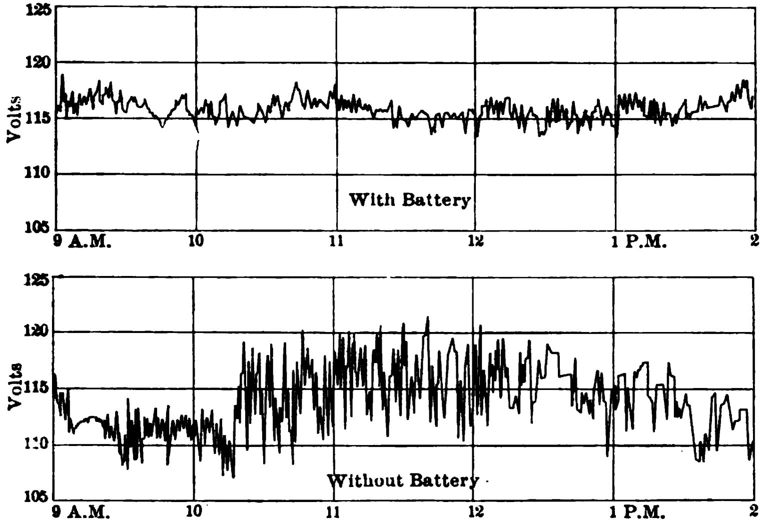

FIG. 461. Effect of a battery in steadying the line voltage.

615. Construction and Chemical Action of Storage Batteries. —

An electric storage cell is a voltaic couple, in which plates of spongy lead (Pb) and peroxide of lead (PbO_2) are used as active materials (Fig. 462). These plates are immersed in dilute sulphuric acid (H_2SO_4) which acts as an electrolyte. When the battery discharges, both active materials are partially converted into lead sulphate ($PbSO_4$), and the acid thus becomes more dilute. On charging, a reverse action takes place, the plates being again reduced to lead peroxide (positive plate) and spongy lead (negative plate). The specific gravity of the electrolyte increases to its normal value, and the battery is again ready for discharge.

These chemical changes may be represented by a formula, thus:

$$\underset{\xrightarrow{\textit{discharge}}}{\overset{\xleftarrow{\textit{charge}}}{PbO_2 + Pb + 2H_2SO_4 = 2PbSO_4 + 2H_2O.}}$$

In reality the chemical reactions are much more complicated and are hardly known at present in all details. The above fundamental equation is however, sufficient for a general understanding of the operation of storage batteries.

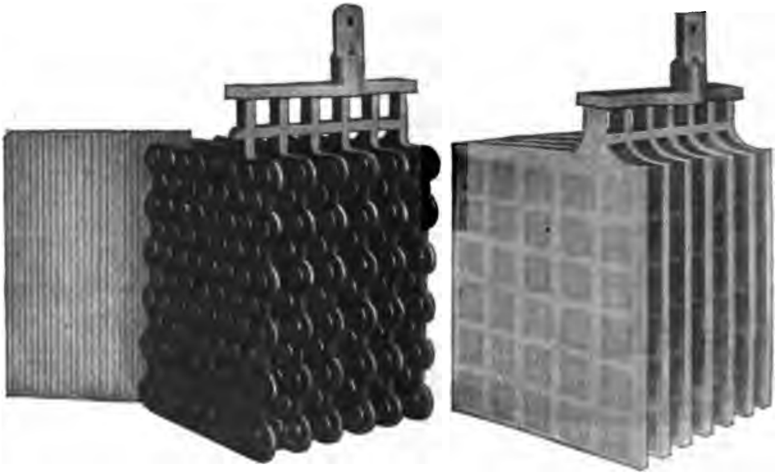

Fig. 462. "Chloride" positive and negative plates, and a separator.

The plates shown in Fig. 462 represent one of the types used by the Electric Storage Battery Co. The negative plate to the right consists of a flat chamber made of two thin perforated lead sheets, filled with active material. The positive plate to the left consists of a supporting grid with buttons rolled from lead ribbon inserted in it. A wooden separator is shown to the left.

Impurities in lead and in the electrolyte produce local chemical action which may ruin the plates. It is important, therefore, to use pure materials. The manufacturers insist in particular that chemically pure sulphuric acid and distilled water be used.

There are two types of battery plates, called Planté type and Faure type, after their respective inventors. In Planté plates the active materials, spongy lead and lead peroxide, are "formed" on the plates themselves, by successive charge and discharges, or by chemical action.

In the Faure or " pasted " plates the active materials are applied mechanically to a supporting grid; this grid is of lead; it supports the active materials and conducts the current to the terminals. The pasted materials usually require some formation by electrical or chemical processes before they are brought to their final form.

For details of construction of various types of plates and of the manufacturing processes the reader is referred to Lamar Lyndon's *Storage Battery Engineering*, pp. 103–155.

616. Voltages during Charge and Discharge. — A fully charged storage cell has an e.m.f. of a little over 2 volts on open circuit. If allowed to be discharged indefinitely the voltage will at first remain practically constant at about 2 volts, then will gradually fall off, at first slowly then more and more rapidly, down to zero (Fig. 463).

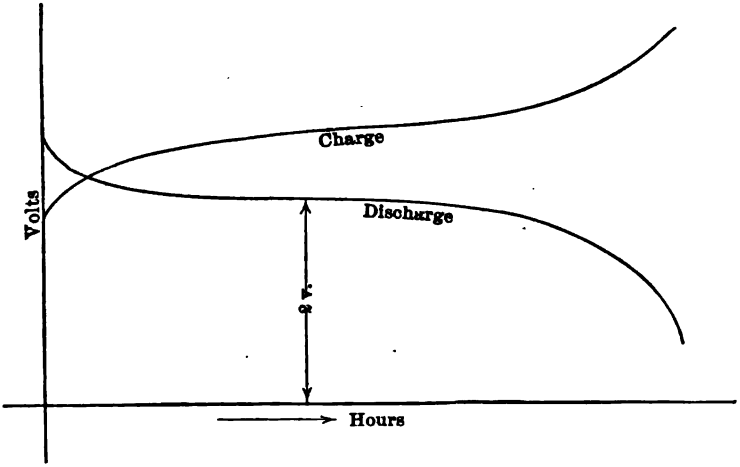

FIG. 463. Curves of charge and discharge of a storage cell.

The voltages given in the curve are supposed to be measured while a normal discharge current is flowing through the cell. The voltage drop in the cell is due to the internal resistance of the cell and to some polarization on the surface of the plates.

A complete discharge down to zero voltage would be impracticable, because for all ordinary purposes the terminal voltage of the battery must be constant within rather narrow limits. Moreover, such a complete discharge would ruin the battery. The reason for this is that lead sulphate ($PbSO_4$) which is formed during discharge is practically an insulator, and if too much of it is allowed to be formed on the plates, the reduction back to Pb or PbO_2 is very difficult, if not impossible. Enough lead or lead peroxide must remain on the plates to keep down their resistance. Otherwise, charging current cannot flow through the active material and effect a regeneration of the battery.

In practice it is considered that the battery requires a new charge when the voltage has dropped to 1.75 volts (or, better, 1.8 volts). This voltage is measured with the battery supplying a current which corresponds to the eight-hour rate of discharge (see § 619).

When the battery is being charged, the external voltage applied at its terminals must be high enough to overcome the counter-e.m.f. of the cell and to force the charging current through its ohmic resistance. At the beginning of a charge the charging voltage is a little above 2 volts per cell. As the battery becomes recuperated, this voltage must be gradually increased; until at the end of the charge it is necessary to apply about 2.6 volts in order to get full charging current through the cell. The end of the charge is also recognized by an excessive liberation of gases (boiling) due to a decomposition of water in the solution.

The best indication, however, of a complete charge is that the specific gravity of the acid has reached its maximum and remains constant. Referring to the fundamental chemical reaction, given in § 615, this means that all sulphate is liberated and the plates consist of pure lead and lead peroxide.

617. Capacity of Storage Batteries. — The *capacity* of a cell, or the amount of electricity that it can give on discharge, is measured in *ampere-hours;* a cell which can supply 25 amperes for 8 hours, before the lower limit of the e.m.f. — 1.8 volts (or 1.75 volts for some makes) — is reached, is said to have a capacity of 25 × 8 = 200 ampere-hours. Experience shows that the capacity of a cell depends essentially on the rate of discharge. The more rapid the discharge the less is the capacity; thus the above cell, if discharged at a rate of 100 amperes, would be completely discharged in one hour instead of two hours. Therefore, *in speaking of the capacity of storage batteries it is always necessary to mention the number of hours in which the battery is supposed to be discharged.* It is customary to rate stationary batteries on the basis of an eight-hour discharge, and batteries used on electric automobiles on the basis of a four-hour discharge. Storage batteries used in electric railway substations, for taking up fluctuations of the load, are usually rated on the arbitrary basis of one-hour discharge.

If a battery is intended to be discharged within a shorter period of time than the normal, its rated capacity must be reduced in a ratio usually given by the manufacturer. Roughly speaking, if the capacity is 100 per cent at an 8-hour rate, it is about 93 per cent at a 6-hour rate, 75 per cent at a 3-hour rate, and only 50 per cent at a 1-hour rate (see table in § 619).

One of the reasons for a decrease in capacity at higher rates of discharge is that the electrolyte cannot circulate as rapidly as required, thus diluting the acid in the pores of the plates before fresh acid can take its place. Another reason is, that a layer of lead sulphate is formed on the surface of the plates, preventing further action.

618. Testing Storage Batteries. — The principal points to be investigated in the performance of a storage battery are:

(1) *Behavior at discharge.*
  (*a*) Variations of terminal voltage.
  (*b*) Variations of density of the electrolyte.
  (*c*) Influence of the rate of discharge on capacity.
(2) *Behavior at charge.*
  (*a*) Variations of terminal voltage.
  (*b*) Variations of density of the electrolyte.
(3) *Electrical efficiency.*
(4) *Internal resistance.*
(5) *Weights and dimensions per ampere-hour output.*

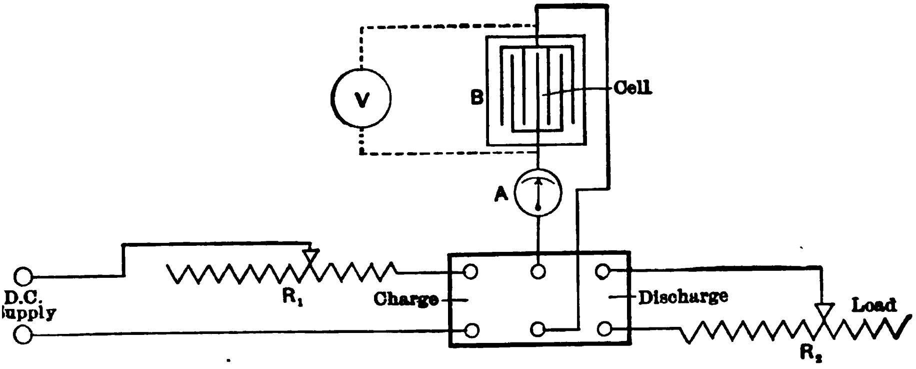

FIG. 464. Connections for testing a storage cell.

There are a few more practical tests, such as influence of temperature, loss of charge by local chemical action, durability in service, etc., which in spite of their importance cannot usually be performed in the short time allotted to students in the laboratory.

Connections convenient for the test of a storage cell are shown in Fig. 464. By means of a double-pole double-throw switch the battery *B* can be connected either to the source of power (for charge) or to a resistance (for discharge). *A* is a two-way reading ammeter, *V* is a voltmeter connected across the battery terminals.

The tests above enumerated, are described below more in detail.

619. Charge and Discharge Characteristics. — The cell under test must be fully charged before beginning the experiment on discharge characteristics. The end of the charge is best recognized by the density of the acid, which reaches its maximum and remains constant. The voltage also reaches its maximum and remains constant. The absolute values of density and voltage are usually given by the manufacturer of the cell, and may vary within certain limits.

The time of charging should not be less than three hours if the battery has been completely discharged. At a higher rate of charging, the electrolyte heats up and the liberated gases cause it to boil; with the result that active material is washed out of the plates, and the useful life of the battery is thereby reduced. Below this extreme, the amount of electrical energy necessary for charging is essentially independent of the charging rate.

It should be well noted that the acid ought to have the prescribed density when the battery is fully charged. The density may be corrected by the addition of distilled water or of acid, as the case may require: this should be done only when the battery is fully charged. *and under no other circumstances.*

After the battery has been fully charged, the switch (Fig. 464) is thrown over to the discharge side. The current is adjusted to the desired value and maintained at this value until the end of the discharge. The curve of voltage on discharge has the general aspect as shown in Fig. 463: it drops rapidly at the beginning and at the end of the discharge, and remains practically constant during the interval between. Therefore, readings must be taken every few minutes at the beginning and at the end of the run; a few check readings are sufficient for the rest of the time. Read volts, density of acid (on a hydrometer) and its temperature; stir the liquid before reading the hydrometer, so as to measure the true average density.

The constant current of discharge, multiplied by the number of hours of the test to the time when the battery is considered discharged, gives the ampere-hour capacity of the cell. This capacity, multiplied by

Hours Discharge.	Final Voltage.	Relative Value of Current.	Relative Capacity in Amp. Hours.
8	1.75	1	8 (100%)
3	1.70	2	6 (75%)
1	1.60	4	4 (50%)
⅓	1.40	8	2⅔ (33⅓%)

the *average* voltage during discharge, gives the watt-hour capacity. The test may be repeated with various rates of discharge, and the influence determined, which the time of discharge has on the capacity of the battery.

The table on the preceding page gives the voltages at which the discharge should be stopped (for "Chloride" batteries). In every case the voltage is to be measured with a discharge current, flowing at the corresponding rate.

620. Cadmium Tester. — In order to ascertain the state of charge on both plates, a cadmium tester is sometimes used. It consists of a stick of pure cadmium placed in the acid of the cell under test. It is well to have the cadmium protected by a hard rubber tube with perforations for the circulation of the acid.

At the end of the charge the voltmeter must show about 2.45 volts between lead peroxide and cadmium, and about 0.10 volt between the lead plate and cadmium. This is a more or less positive indication of the end of the charge. The voltage between the two plates is equal to the sum of the two readings:

$$2.45 + 0.10 = 2.55 \text{ volts.}$$

The same tester can be used to ascertain the end of discharge. In this case the voltages are + 1.95 and − 0.20 volts respectively; the battery voltage is

$$1.95 - 0.20 = 1.75 \text{ volts.}$$

In case it is found that one of the plates is not fully charged, the charge must be continued until the cadmium tester shows the required voltage. Or, if it is feared that an excessive charge may damage the other plate, the plate which requires additional charging may be charged in a separate cell.

A cadmium tester gives reliable indications in the hands of an experienced observer, especially when many tests are made on batteries of the same type. Otherwise it is safer to judge of the state of the charge from the acid density and the voltage.

621. Internal Resistance. — The determination of the *true* ohmic resistance of storage batteries is rather difficult because this resistance is very small, is variable, and is to some extent masked by the effect of polarization. Moreover, it is not the true resistance, but rather the *virtual* resistance of the cell that is interesting to the user, this virtual or equivalent resistance representing the total drop of voltage in the battery, due to whatever causes. The simplest method to determine the resistance R of a cell would be to observe the voltage E_0 of the battery on open circuit, and then immediately note the voltage E with a certain charging current I flowing through the battery. Then, evidently,

$$R = \frac{E - E_0}{I}.$$

A better method is to measure two terminal voltages E_1 and E_2 corresponding to two different values I_1 and I_2 of charging current. We then have

$$\begin{cases} E_1 - p - E_0 = RI_1; \\ E_2 - p - E_0 = RI_2; \end{cases}$$

where p is the counter-e.m.f. of polarization. Eliminating p and solving for R, we obtain

$$R = \frac{E_1 - E_2}{I_1 - I_2}.$$

An objection to this method is that the e.m.f. p of polarization is not quite constant with various rates of charge. Another objection is that the difference $E_1 - E_2$ is rather small; naturally, this impairs the accuracy of the result. It is advisable to perform a large number of tests with various values of I_1 and I_2 and to take an average of the calculated values of R. Experience shows that more consistent results are obtained on discharge than on charge; the same formula is used, as that given above.

Another way of measuring the resistance of a cell is the so-called "break" method which is considered by some to be more reliable. A certain value of discharge current is adjusted through the cell, and when the conditions become steady, the circuit is suddenly opened. The pressure, as shown on the voltmeter, rises instantly by a certain amount and then continues to rise *gradually* as the polarizing bubbles of gas disappear. It may be assumed with a considerable degree of accuracy that the first (instantaneous) rise in voltage corresponds entirely to the ohmic drop, since the bubbles of gas are evidently the same as a moment before when the circuit was closed. From this rise in voltage and the current formerly flowing through the battery, the internal resistance can be calculated. Suppose, for example, that the voltage rises from 1.8 volts to 1.9 volts when the switch is opened, with 100 amperes flowing through the battery. The resistance of the cell is 0.1 ÷ 100 = 0.001 ohm.

622. EXPERIMENT 28-B. —Testing Storage Cells. — The experiment is performed as described in §§ 618 to 621. The readings during charge and discharge are taken at comparatively infrequent intervals; it is possible, therefore, to test simultaneously more than one cell. One voltmeter, and one milli-voltmeter with several ammeter shunts, are sufficient for all the cells. Some of the cells may be charging while others are discharging. Tests at low rates may be continued throughout several consecutive days by different observers, who may work out the results together.

At the end of the experiment, measure all the dimensions of the cell and of its elements, so as to be able to make a drawing to scale. Determine the weight of the plates, of the electrolyte and of the complete cell. Do not keep the negative plates out of the liquid longer than necessary; they may be damaged by the action of the atmosphere.

Report.

(*a*) Capacity of the cell tested, in ampere-hours and in watt-hours at various rates of discharge.

(*b*) Corresponding efficiencies, both for ampere-hours and for watt-hours; which latter is always lower than the former, because the average voltage is lower at discharge than at charge.

(*c*) Curves of variation of voltage and of acid density at charge and at discharge.

(*d*) Virtual and true resistances of the cell.

(*e*) Capacity in ampere-hours per pound of complete cell; also per pound of plates.

(*f*) Charge and discharge rates in amperes per square foot of plate surface.

(*g*) A complete drawing of the cell.

OPERATION AND CONTROL OF STORAGE BATTERIES.

623. Storage batteries are usually connected in parallel with generators, as shown in Fig. 459. Auxiliary apparatus necessary for the operation of batteries comprises:

(*a*) Switches, circuit-breakers, measuring instruments, etc.

(*b*) Means for charging the battery, and for regulating its current and voltage on charge and discharge.

The devices mentioned under (*a*) are similar to those used with direct-current generators and motors; those under (*b*) are peculiar to storage batteries. Various methods are used for charging and controlling the output of batteries, the determining factors in the selection of the system of control being:

(*a*) Purpose of the battery;

(*b*) Its size;

(*c*) Permissible limits of current and voltage fluctuations;

(*d*) Cost of the system;

(*e*) Whether hand or automatic control is desired.

The most important systems of control in practical use are described in the following articles, beginning with the simplest system and including the most perfect automatic equipments.

624. Charging Two or Three Parts of a Battery in Parallel. — The simplest method for charging and regulating storage batteries is shown in Fig. 465. The battery is divided into two halves which are connected in series for discharging, and in parallel for charging. This is done in order to secure a sufficient voltage for charging, without affecting the line voltage, maintained by the generator. An example will make this clearer. Consider a battery intended for an ordinary 110-volt lighting circuit: The voltage of each cell at the end of discharge is about 1.8 volts; therefore the number of cells required is 110 ÷ 1.8 = 62. But the voltage necessary with this number of cells at the end of a charge is = 2.6 × 62 = 161 volts, which is far above the line voltage.

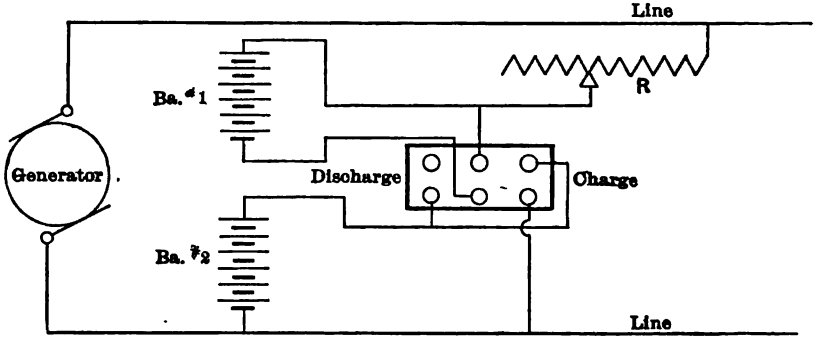

FIG. 465. Charging two halves of a storage battery in parallel.

With the battery divided into two halves in parallel, only 80.5 volts are required for charge; the excess voltage of the line is taken up by the rheostat *R*. Battery output on discharge is also regulated by this rheostat. This method, although very simple, is seldom used except in small installations, where the loss of power in the rheostat is not objectionable.

A more economical method is to divide the battery into three equal parts; let them be denoted *A*, *B* and *C*. The parts *A* and *B* are first charged in series for one-half of the time necessary for full charge; then *B* and *C* are charged in series for one-half of the time, and finally *C* and *A* for one-half of the time. Less energy is wasted in the resistances with this arrangement, although it takes longer to charge the battery. The voltage at the end of the charge is $\frac{2}{3}$ × 161 = 107 volts.

Other combinations are also possible; for instance, *A* and *B* may be connected in parallel with each other and in series with *C*. The set is charged at the full rate until *C* is completely charged. Then *C* is disconnected, *A* and *B* are connected in series, and the charge is completed.

625. EXPERIMENT 28-C. — Charging Batteries in Sections. — Wire up the two halves of a battery, as in Fig. 465, and make connections to a suitable generator. Provide a load in the form of adjustable resistances, and operate the installation under the following conditions, viz.:

(*a*) Both the battery and the generator supplying power to the line.

(*b*) The battery being charged, the generator at the same time supplying power to the line.

(*c*) The battery alone supplying power, the generator shut down.

(*d*) The generator working alone, the battery being disconnected for inspection and repairs.

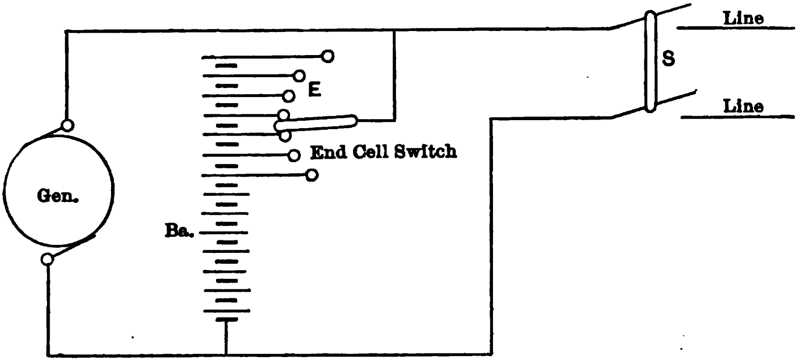

FIG. 466. Use of a single end-cell switch.

For each of these conditions select a few characteristic loads (light load, medium load, full load and overload) and take all the necessary ammeter and voltmeter readings, so as to have a complete record of the electrical relations in the circuit, with special reference to the performance of the battery. Observe voltage and current fluctuations, when the load is varied, first gradually and then suddenly.

Devise a convenient arrangement of switches for charging the battery in three parts, as explained in the preceding article. Connect the battery accordingly and observe the process of charging.

626. End-Cell Switches. — In many small installations there is no demand for current during the day. In such cases battery connections shown in Fig. 466 are used. The battery is charged during the day, when the main switch S is open; the voltage on the generator being raised to the required voltage (say 161 volts), for charging the battery. During discharge the battery voltage and output are regulated by the so-called *end-cell switch* E, by means of which more cells may be connected into the circuit, in proportion as the voltage of each cell drops during the discharge.

End-cell switches are sometimes used also in installations where charging is done by means of special machines, so-called "boosters" (Fig. 469). Storage batteries in stations and substations of electric lighting companies in most large cities are regulated by end-cell switches and charged by boosters. Large end-cell switches are sometimes operated by auxiliary electric motors, which are started or stopped either by the switchboard attendant, or automatically by a contact voltmeter.

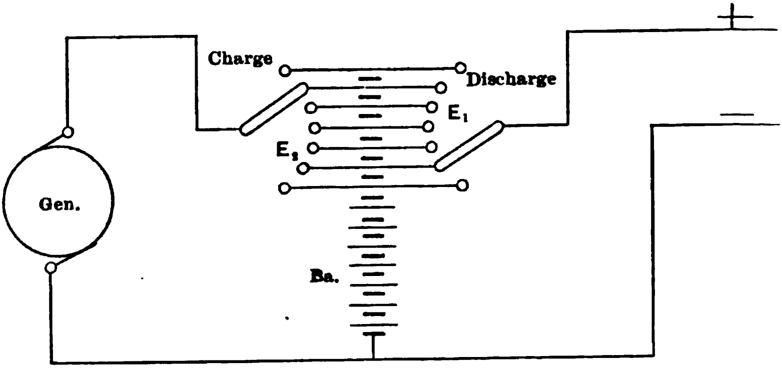

Fig. 467. Use of a double end-cell switch.

The contact on the arm of an end-cell switch must be wide enough, so as not to open the battery circuit while the arm is moved from one segment of the switch to the next. On the other hand, when the arm bridges two adjacent segments, it apparently short circuits the cell connected to these two segments, which is not permissible. Therefore, the arm contact is made in two parts, with a *protective* resistance between, this resistance limiting the current in the short-circuited cell during the instant when the arm is moved from one contact to the next.

In some cases it is not practicable to have the main switch opened, while the generator voltage is being raised for the charge; at the same time the size of the installation may not warrant the complication of a booster set. Two end-cell switches are used in such cases, as shown in Fig. 467. By means of the end-cell switch E_1 the required voltage is maintained on the line, while the charge is regulated by the generator field rheostat and the end-cell switch E_2. With this scheme, the end cells are carrying the sum of the charging current and the line current, and are therefore charged faster than the rest of the battery. Actual practice does not show, however, any disadvantage of such an arrangement, provided the charging is done during the hours of small demand.

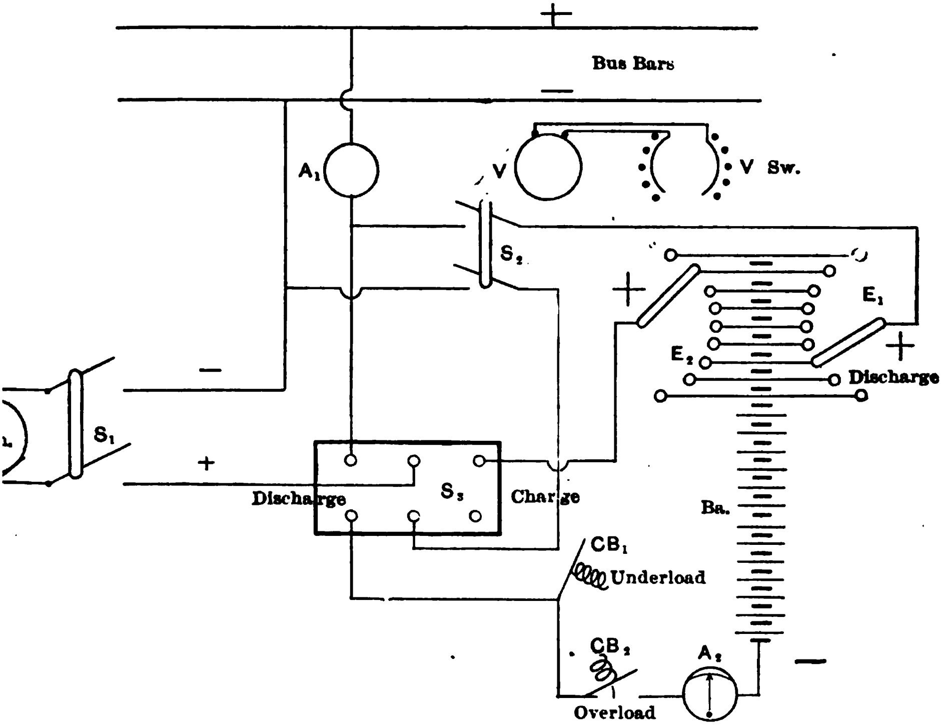

FIG. 468. Details of connections to a battery controlled by a double end-cell switch, as per Fig. 467.

It will be easily seen that more contact points are necessary with a double end-cell switch than with a single end-cell switch.

Fig. 468 shows the complete diagram of connections for the arrangement according to Fig. 467. The switch S_1 controls the generator circuit, S_2 — the battery circuit; S_3 is a double-throw switch which makes connections for charging or discharging. For charging the battery, S_3 is thrown to the right, and the switches S_1 and S_2 are closed. The charging current flows from the positive terminal of the generator, through S_3, to the charging end-cell switch E_2. Thence, through the battery, ammeter A_2, overload circuit-breaker CB_2, underload circuit-breaker CB_1, and switch S_2 to the negative terminal of the generator. At the same time the line is supplied with current through the discharge end-cell switch E_1 and the ammeter A_1. The underload circuit-breaker is used in order to prevent the battery from sending current back into the generator. An underload circuit-breaker is installed only in those battery plants in which the load is varying slowly, so that there are distinct periods of charge and discharge. In railway installations the load is rapidly fluctuating, and the battery current is reversed, sometimes every few seconds: No underload circuit-breaker could be used under such conditions.

For discharging the battery into the line, in parallel with the generator, the switch S_3 is thrown to the left; this cuts the charging end-cell switch E_2 out of the circuit, while the underload circuit-breaker CB_1 is short-circuited by the lower blade of the switch S_3. The discharge is regulated by the end-cell switch E_1 and the field rheostat of the generator. On the other hand, if it is desired to shut down the generator, the switch S_1 is opened, and the battery continues to supply current alone. This is usually the case late at night and early in the morning, when the demand for current is quite small. If the switch S_2 is opened, instead of S_1, the battery is disconnected from the line, while the generator continues to supply the power.

V is a voltmeter which, by means of the voltmeter switch *V. Sw.*, can be connected so as to show at will: the generator voltage, the battery voltage, or the line voltage. In addition, a portable 3-volt instrument is always used in storage-battery plants, for measuring the voltage of each individual cell.

627. EXPERIMENT 28-D. — End-Cell Control of Storage Batteries. — The connections with a single end-cell switch are shown in Fig. 466; with a double end-cell switch in Fig. 467, or more in detail in Fig. 468. Try both systems in actual operation, under the conditions specified in § 625.

Report actual diagrams of connections and numerical results of the test. State voltage fluctuations with sudden variations of the load, and give the relative fluctuations of the current in the generator and in the battery. Figure out the number of points necessary on the single end-cell switch and on the double end-cell switch.

628. Floating Batteries. — In plants in which considerable voltage fluctuations are not objectionable, or are unavoidable, storage batteries are often used without any means for regulating them, simply as shown in Fig. 459. This allows the battery to be freely charged or discharged with the fluctuations of the load. Such connections are used in some electric-railway substations, and also in plants containing cranes and elevators. The number of cells is selected so that, when the generators give approximately their full output, the voltage at the bus-bars is equal to the e.m.f. of the battery: under such conditions no current flows into or out of the battery.

When the load is below the average, the generator voltage is higher than that of the battery, and a charging current flows into the battery. When the generator is carrying a rather heavy load, its voltage drops below that of the battery, and the battery discharges into the line, helping the generators (or rotary converters, if it is a substation). With such an arrangement, the generator load is more constant than without the battery. The battery is never entirely discharged or fully charged, but is maintained in a medium condition. Once every few weeks it is necessary to raise the generator voltage and to give the battery a thorough charging, and even an overcharge, in order to prevent the formation of lead sulphate.

Such a *floating* battery is more effective the wider the voltage fluctuations. The voltage at the end of the feeders varies much more than in the substation, on account of the ohmic drop in the feeders: therefore, it is better to have a floating battery at the end of the line. The advantages are: The average voltage being lower than in the substation, less cells are required; the fluctuations of the voltage being more pronounced, the battery is charged and discharged within wider limits; the load on the generators or rotary converters is steadier; there is a considerable saving in line copper, since the feeders have to carry an average current only, instead of the maximum current. The chief disadvantage of placing the battery at the farther end of the feeder is that extra room and attention are required outside the substation.

629. EXPERIMENT 28-E. — Performance of Floating Batteries. — Connect a battery in parallel with a shunt-wound generator, as in Fig. 459, and select such a number of cells that the battery neither charges nor discharges at a desired load. The line must have a considerable resistance so as to represent the actual conditions of railway service; the resistance being such as to give from 10 to 15 per cent voltage drop at full load. The load, representing street cars, is connected at the further end of the line. Have an ammeter in the generator circuit, and one in the main line; and connect a voltmeter so as to measure, at will, the voltage across either end of the line.

(*a*) First disconnect the battery and use the generator alone, increasing the load from zero to a heavy overload without regulating the field rheostat. Measure line amperes and the volts at each end of the line.

(*b*) Apply the same values of load with the battery and the generator working in parallel. Observe the proportion of the load taken by the battery, and the improved conditions of voltage regulation. If the results show that the battery is charged more than it is discharged, add one or more cells, and vice versa. Repeat the experiment, until the battery gives the best performance, in other words, keeps the generator load as steady as possible.

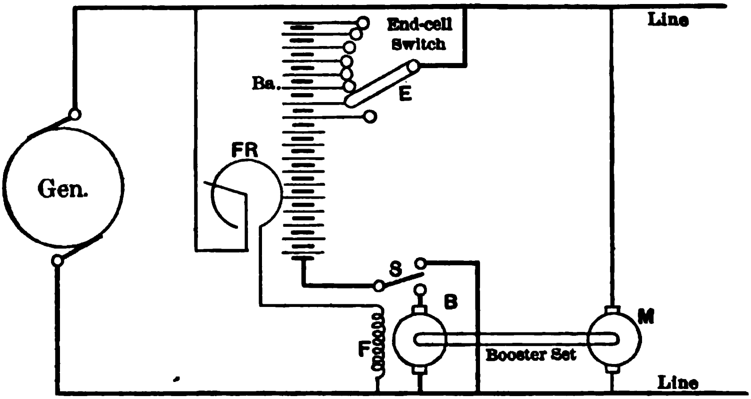

FIG. 469. Battery charged by a non-automatic booster, and discharged through an end-cell switch.

(*c*) Put a few turns of series winding on the generator field, but not enough turns to make the machine flat-compounded. Observe the difference in the performance of the battery and the new voltage regulation.

(*d*) Connect the battery at the farther end of the line and repeat the same runs, viz.: Investigate the influence of the resistance of the line, of the position of the load, and of compounding the generator.

Report. Give results in such form as to show the relative usefulness of the floating battery under the various conditions investigated.

BATTERY BOOSTERS.

630. With the exception of smaller plants in which storage batteries are charged in parallel, or by means of an end-cell switch, most storage-battery plants are provided with so-called *boosters*, or extra generators for regulating the charge (and sometimes also the discharge) of the batteries. The simplest combination is shown in Fig. 469: The booster

B is direct-driven by a shunt-motor M; the armature of the booster is in series with the battery; the fields F are separately excited across the bus-bars; E is the end-cell switch.

When the battery is discharging, the switch S is thrown up so that the booster is cut out of the circuit; the discharge is regulated by the end-cell switch E. For charging the battery, the switch S is thrown down and the booster e.m.f. raised by means of the field rheostat FR; thus giving, together with the generator pressure, a voltage sufficient for charging. This voltage is regulated, as the charge progresses, by means of the same rheostat FR. Instead of using an end-cell switch, the same booster may also be used for regulating the discharge of the battery. In this case a reversing switch must be connected into its field circuit, so that the direction of the induced e.m.f. may be changed. Some companies connect the booster field across the battery, and not across the line, as in Fig. 469; there is little difference between the two methods.

Shunt-excited boosters with hand regulation are satisfactory only in plants in which the load varies gradually and regularly, so that the battery can be charged and discharged during considerable periods of time. In railway service, where the load fluctuates within wide limits, and where charge and discharge sometimes follow each other every few seconds, it becomes necessary to have *automatic* boosters whose e.m.f. is added to or subtracted from that of the battery, according to the magnitude of the load.

Two types of automatic boosters are used at present: (*a*) *Differential boosters*, in which the booster field is varied by the addition of compounding windings (Fig. 470); (*b*) *Relay boosters*, in which the current in the shunt field is regulated by suitable relays (Figs. 472 and 475).

631. Differential Booster. — This belongs to the first of the two types of automatic boosters mentioned above, and is shown in Fig. 470. It differs from the non-automatic booster (Fig. 469) in that the end-cell switch E is omitted, and the booster is provided with an additional series-field winding F_2, which opposes the action of the winding F_1. At a certain average load, F_1 and F_2 neutralize each other, and the booster e.m.f. is $= 0$. At this load the generator voltage must be made equal to that of the battery, so that the battery neither charges nor discharges. At a heavier load the action of F_2 is stronger than that of F_1, and the booster e.m.f. is added to that of the battery, assisting its discharge. On light loads the action of F_1 is stronger than that of F_2, and the booster tends to send a charging current into the battery. This action is entirely automatic, and the booster tends to maintain a constant load on the generators, the battery taking up the load fluctuations.

Experience shows that in order to have this system work satisfactorily, it is necessary to add a third field winding F_3 (shown by dotted lines), acting in the same direction as F_2. This winding automatically corrects the voltage of the booster for the state of charge of the battery, in the following way: Suppose that the currents in the three field windings are so adjusted, that the booster e.m.f. is zero with the rated generator current and with the battery partly discharged. Then, with the same load and with the battery fully charged, the battery tends to take more than its share of load. reducing the generator current. This reduces the current in the differential winding F_3, and gives a preponderance to the shunt field of the booster. An e.m.f. is produced in the booster such as to oppose the e.m.f. of the battery and to prevent it from discharging.

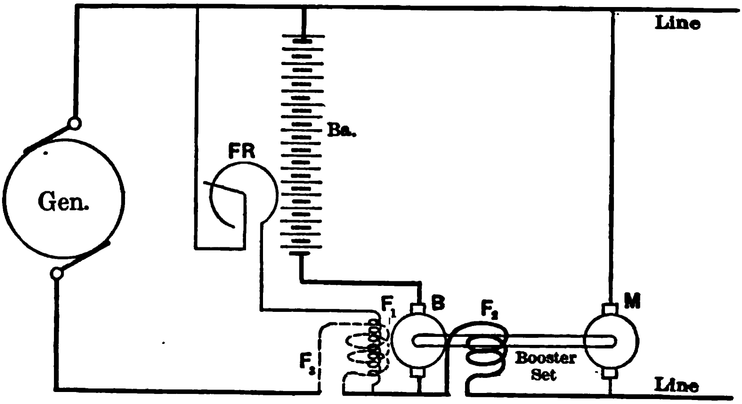

FIG. 470. Automatic differential booster.

On the contrary, when the battery is nearly discharged and does not take its share of load, the generator becomes overloaded. Then an excessive current flows through F_3 and boosts the battery voltage helping its discharge. In this way the winding F_3 helps to keep the generator load constant.

632. EXPERIMENT 28-F. — Performance of Storage Batteries Controlled by Shunt-Wound and Differential Boosters. — Connect a non-automatic booster, as shown in Fig. 469. Have an ammeter in the line and one in the generator circuit; the battery current is, at any moment, the difference of the two ammeter readings. Operate the set under the conditions specified in § 625. To represent voltage changes in the battery, that depend on the state of its charge, one or more cells may be disconnected, or more cells added to the battery. Some resistance may also be used in series with the battery, to imitate an increase in internal resistance. Add a reversing switch in the booster field and try to regulate discharge by the booster, instead of by the end-cell switch.

Take a differential booster, shown in Fig. 470, first with both windings F_2 and F_3 in the line circuit, then with the same windings connected as shown in the sketch. Compare the two arrangements with regard to their ability to maintain a constant generator output. Adjustments for the best regulation are made by the booster shunt-field rheostat, and by placing shunts around the series windings.

633. Carbon-Pile Booster Regulator. — One common disadvantage of automatic boosters with series fields, as shown in Fig. 470, is that the booster itself becomes quite large and expensive, since its frame has to accommodate three field windings. It has been sought, therefore, to have on the booster only the shunt winding, as in Fig.

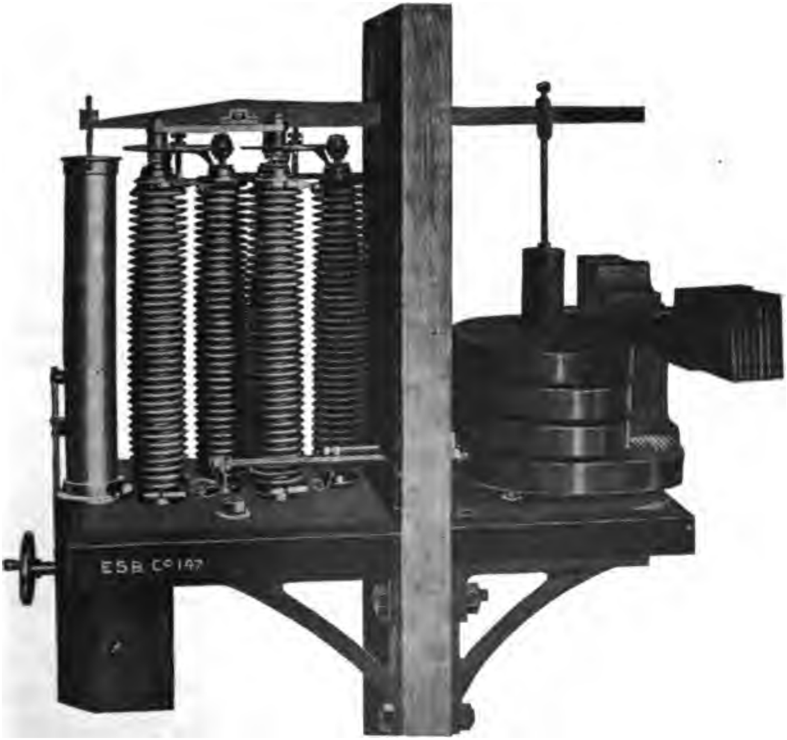

FIG. 471. Carbon-pile battery-regulator (The Electric Storage Battery Co.).

469, and to provide *outside the booster* an additional device that would automatically vary the magnitude and the direction of the current in this shunt winding, according to the load. A device of this kind, the so-called *carbon-pile regulator*, is shown in Fig. 471; Fig. 472 shows the electrical principle on which it is based. The regulator is usually mounted on the main switch-board.

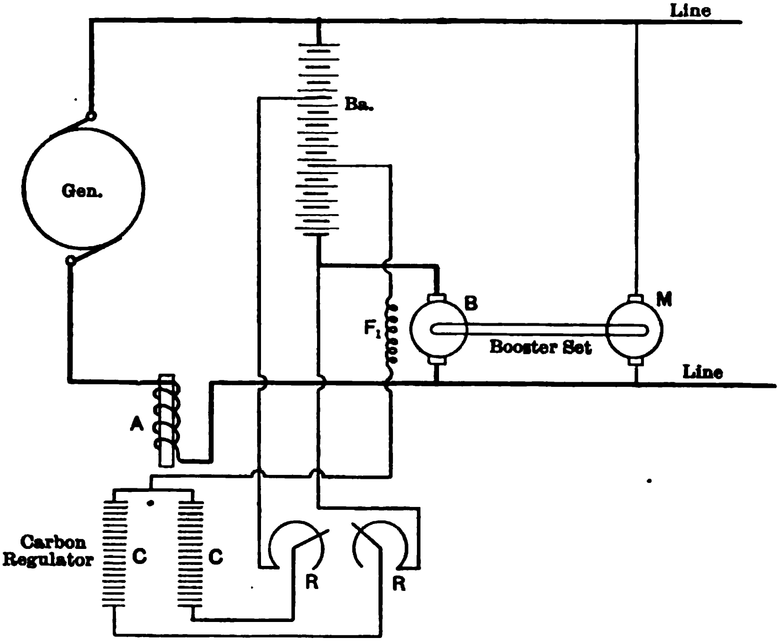

FIG. 472. Electrical connections of a booster, controlled by a carbon-pile regulator.

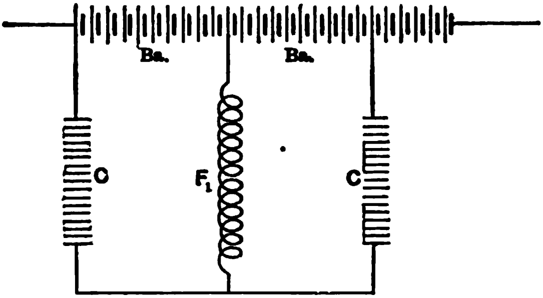

FIG. 473. A simplified diagram of connections between the booster field and the carbon regulator.

Generator current, instead of passing through a series winding placed on the booster, passes through the solenoid A of the regulator. An iron core, actuated by this solenoid, compresses more or less, through a suitable leverage, two sets of columns CC consisting of carbon disks. These carbon piles are connected to the booster field and to the battery, as shown in Figs. 472 and 473. The resistance of the piles consists chiefly of the contact resistance between the disks, and therefore varies within wide limits, with variations in the pressure exerted by the core of the solenoid A.

With the normal value of the generator current, the pressure on both piles is the same; their resistances are equal, and no current flows through the booster field F_1. The whole arrangement resembles the familiar Wheatstone bridge scheme, in which the booster field takes the place of a galvanometer. When the current is below or above normal, one column is compressed more than the other, the bridge is not balanced, and a current flows through the booster field in one or the other direction, causing the battery either to charge or to discharge.

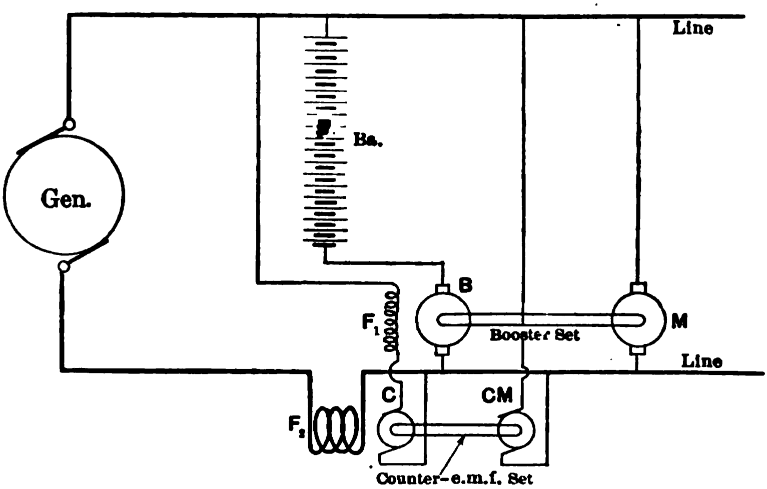

FIG. 474. Automatic booster controlled directly by a counter-e.m.f. machine.

The arrangement is entirely automatic in its action, and takes the place of a booster with two or more field windings, tending to keep the generator current constant.

In reality the connections are more complicated than those shown in Fig. 472: It would be too wasteful to have a current circulate all the time through the carbon columns, especially of such a magnitude as to be sufficient to energize the booster fields through mere unbalancing of resistances. Therefore, except in very small installations, *the carbon regulator actuates the field of a separate exciter, which in turn supplies current to the field of the booster.* The exciter being a much smaller machine than the booster, considerably less energy is lost in the regulator. The booster and the exciter are usually mounted on the same shaft and driven by a direct-connected motor.

The carbon-pile regulator is used by the Electric Storage Battery Company, chiefly in large railway plants.

634. Booster Regulation by Counter-E.M.F. — Another relay-arrangement is shown in Fig. 474. Here the booster field F_1 is automatically regulated by a small counter-e.m.f. machine C. The field F_2 of this machine is excited by the main current, similarly to the solenoid A shown in Fig. 472. At a certain desired value of this current, the counter-e.m.f. of the machine C can be made such that no current will flow through the booster field F_1. When the generator current is below this value, the booster field is excited in such a direction that the battery is charged, and vice versa.

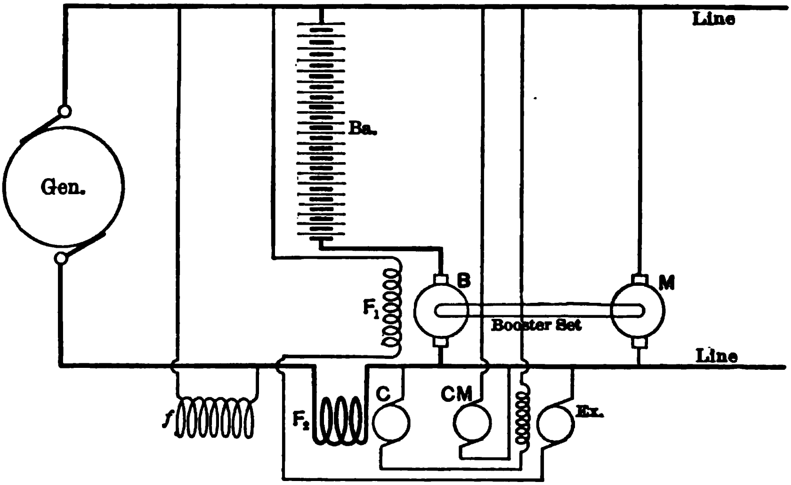

FIG. 475. Automatic booster controlled by a counter-e.m.f. machine, through an exciter (The Gould Storage Battery Co.).

Formerly the machine C was direct-connected to the main booster set and driven by the same motor. Experience showed, however, that the size of this machine can be considerably reduced by driving it separately, at a higher speed, by a small motor CM.

The size of the counter-e.m.f. set is still more reduced by introducing a second relay machine, as shown in Fig. 475. Moreover, this arrangement increases the sensitiveness of regulation, and permits the counter-e.m.f. sets to be made of the same standard size with widely different sizes of batteries and boosters. The scheme shown in Fig. 475 is the one used by the Gould Storage Battery Company.

The counter-e.m.f. machine C, instead of acting directly on the booster field, as in Fig. 474, acts on the field of a small exciter $Ex.$, which in turn controls the booster field current. The armature of the counter-e.m.f. machine is connected in series with the exciter field, across the main bus-bars. When a normal current flows through the main generator, and consequently through the field F_2, the voltage induced in the armature C just balances the voltage across the bus-bars; no current then flows through the exciter field, and the booster excitation is $= 0$. When the generator voltage is above normal, the voltage in C is higher than that across the bus-bars, and the exciter field is energized in such a direction as to assist the battery to discharge. The opposite takes place when the generator current is below normal. With proper relations, the system works so as to keep the generator load practically constant.

In some cases, the counter-e.m.f. machine has an additional field winding connected across the line and giving a constant excitation. The addition of this winding makes the system more flexible and permits of an adjustment for any desired performance of the battery. Further adjustment is made possible by a rheostat in series or in parallel with the field winding F_1, and also by shunting the main series-winding F_2 by adjustable resistances.

635. Vibrating-Contact Booster Regulator. — The success of the Tirrell regulator (see §§ 309 and 460) for controlling voltage in generators, led to the idea of applying the same principle to storage battery regulation. Such a battery regulator with vibrating platinum contacts, was recently developed by the Westinghouse Electric and Manufacturing Company. In common with the systems described in §§ 633 and 634, the booster field is energized either for charge or for discharge by a separate exciter. The same exciter, which may be either direct-connected to the booster set or driven by a separate motor, has two equal and opposite field windings, which we shall denote by A and B, connected in parallel across the main bus-bars; a third field winding C is connected across the armature terminals of the exciter and is its regular shunt winding.

A platinum contact, actuated by the main generator current, short-circuits a resistance in series with either A or B and makes the action of one of the differential windings predominant. The shunt winding

C immediately begins to build up the exciter current, which in turn effects the desired booster regulation. The tendency to over-regulate is checked by an electromagnet which immediately opens the platinum contact.

The details of the regulator are as follows: An ammeter shunt is inserted into the generator circuit, so that the drop across the shunt is proportional to the generator current. A regular milli-voltmeter movement, consisting of a moving coil, a permanent magnet and a spiral spring, is connected across the shunt. We shall denote this milli-voltmeter No. 1. Another similar movement No. 2 is connected across the booster field; the arms of the two moving coils have two platinum tips which correspond to the main contacts in a Tirrell regulator. The closing or opening of these contacts causes an electromagnet to short-circuit one or the other of the above-mentioned resistances in series with the exciter fields.

Suppose the main contact between the two moving coils be closed: this closes the auxiliary contact which energizes the booster in such a direction as to help the battery to discharge. As soon as the exciter voltage begins to build up, the movement No. 2 opens the main contact. The auxiliary contact is also opened, and returns to its zero position in which it builds up the booster field in the opposite direction, i.e., for charging the battery. This increases the generator current above normal, the main contact is closed again, etc.

In reality, both contacts are vibrating continually, and in this way maintain the generator current practically constant.

For small adjustments, the spiral spring of movement No. 1 may be tightened or loosened. For larger steps a series of resistances are provided with plugs by which the drop across the ammeter shunt is increased or decreased. Movement No. 1 is also provided with a sensibility weight. By shifting it, the regulator may be over-compounded; this means that when the external load increases, the generator load also increases by a desired percentage.

636. EXPERIMENT 28-G. — Performance of Relay-Operated Boosters. — The experiment comprises any of the systems described in §§ 633 to 635. Wire up the arrangement available for test and operate it under various conditions specified in § 625. Pay particular attention to current and voltage fluctuations when the load varies suddenly, and determine the limits within which load and voltage may be kept constant. Try the various adjustments possible with the regulator under test, and see if the generator current may be made to follow, to a certain degree, load fluctuations. Investigate regulation with a slightly different number of cells; this would correspond to the case when the battery is fully charged or totally discharged.

Report. Draw a complete diagram of connections for the system investigated, with all switches, fuses, measuring instruments, etc.; write concise instructions for starting and operating the installation. State the character of regulation obtained and per cent fluctuations from the rated current. Give your criticisms, if any, of the system.

637. Storage Batteries in Alternating-Current Plants. — The question of equalizing load in large alternating-current power plants, by means of storage batteries, gains more and more in importance. At present, batteries used in large railway and lighting plants are usually installed in substations; each substation is thus regulated separately. A better economy could be obtained by having one large battery in the generator station, even though it would necessitate extra rotary converters, or motor-generators between the line and the battery.

With fluctuating loads, such batteries have to be controlled by automatic boosters with some relay arrangement between the alternating-current line and the booster field. The requirement in most cases would be to regulate for constant kw. generator output; the current then varying only with power factor.

The problem is in rather an experimental stage, though it would seem that the systems of regulation described in §§ 633 to 635 may be made to operate successfully with alternating currents.

The Westinghouse regulator (§ 636) is particularly promising in this respect, as the only change would be to replace the ammeter shunt and the milli-voltmeter movement No. 1 by a suitable wattmeter movement. The Gould arrangement shown in Fig. 475 has also been successfully applied to alternating-current plants, by exciting the field F_2 from the main line through series transformers and a special rectifier. The device is adjusted so as to rectify only the working component of the current, thus making the effect of the booster depend on watts output and not on line amperes. A second rectifier, suitably displaced from the first, may be used for the wattless component of the current; this component, when rectified, may be used for compounding the rotary converter, in order to correct for the power factor of the load.

CHAPTER XXIX.

SWITCHBOARDS.

638. THE switchboard is an essential part of every electric plant, its object being to group together at some convenient and accessible place the necessary devices for controlling and distributing the current to the various circuits for measuring the power received or delivered, and for supporting safety devices which protect the machines and the lines.

The following laboratory exercises are intended to familiarize the student with the construction and operation of standard switchboards; also with the types and functions of the principal switchboard devices and their electrical connections.

In beginning the study of the subject, the student should first operate a switchboard in order to learn the purpose of its various devices. The next step is to study actual electrical connections and to learn to trace them out on a given switchboard. The more advanced student should himself design the connections and connect up a switchboard according to his design. This work is provided for in the exercises in assembling switchboards.

1. DIRECT-CURRENT SWITCHBOARDS.

639. Switchboards for One Direct-Current Generator.—A simple direct-current switchboard, such as are used in small isolated plants, is shown in Fig. 476. The connections are practically the same as in the diagram Fig. 265, except that no field ammeter is used as a rule; the field rheostat is adjusted so as to obtain the required terminal voltage. The switchboard itself consists of panels of marble or slate, supported by frames of angle iron or gas-pipe. The large switch in the center connects the switchboard to the generator. The handle of the field rheostat is seen above it, the rheostat itself being mounted on the back of the switchboard. The ammeter and the voltmeter are mounted near the top.

Lamps are sometimes provided for illuminating the instrument scales: the lamps are connected directly to the main cable coming from the machine, so as to light up even when the main switch is open. Two ground-detector lamps are visible in the upper corners of the panel; for their operation see § 288. Four smaller switches shown to the left and to the right are feeder switches controlling four outgoing circuits.

Each switch circuit is protected by fuses, visible under the switches. Automatic circuit-breakers are coming more and more into use instead of switches and fuses.

The main switch is connected to two horizontal copper bars, commonly called bus-bars, so that the generator power is delivered to the bus-bars. The feeder switches are also connected to the bus-bars, and in this way the energy taken from the generator is delivered to various feeder circuits.

640. EXPERIMENT 29-A.—Tracing out Connections on a Direct-Current Generator Panel. — The purpose of the experiment is to illustrate the connections and the operation of a simple direct-current generator switchboard. The principal features of construction and connections are given in the preceding article; some small variations may be met with, in switchboards of different makes.

In tracing out the connections, first estimate, as closely as possible, the destination of the various parts. Then make a rough sketch of the arrangement of the apparatus on the front of the switchboard and of the principal parts on the back, first without wiring. Finally put on the sketches all the electrical connections, and see if they are correct.

If some connections are inaccessible, or invisible, trace them out by putting current on, — for instance, through a lamp connected to an ordinary lighting supply. A galvanoscope and a dry cell, or a magneto and a bell, may also be used for tracing connections.

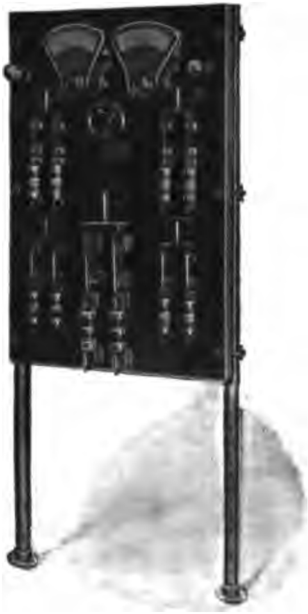

FIG. 476. An isolated-plant direct-current switchboard, for one generator and four outgoing feeders (General Electric Co.).

Having traced out all the connections, verify your understanding of their functions by actually operating the switchboard in the way in which it is supposed to be used during regular service. Make a note of the sizes and ranges of measuring instruments, switches, fuses, etc.

Report. Draw a diagram of actual connections; make a sketch of the assembly, as in Fig. 476. Give the ranges of the instruments and the size of other devices, — for instance, 75-ampere 250-volt switches, 50-ampere fuses, etc. Give instructions for operating the switchboard when starting the machine, and when shutting down the plant.

641. EXPERIMENT 29-B.—Assembling a Direct-Current Generator Panel. — The purpose of the exercise is to gain experience in designing and assembling switchboards and handling switchboard apparatus. The principal features of construction, and connections used in small direct-current plants, are described in §§ 639 and 640. Small changes may be necessary with special requirements, or with the equipment at hand.

A special wooden panel should be provided with universal clamps for mounting the equipment in any desired place. The students are expected to design the connections, to mount and to connect up instruments, switches, etc. The finished switchboard should be connected to a generator and tried in operation.

In doing this work, particular attention should be paid to placing all the devices systematically, so that they occupy a minimum space. All similar devices should be placed if possible in one horizontal row: the measuring instruments must occupy such a height as to be conveniently read by the switchboard attendant; the circuit-breakers must be placed in the top row, so as not to hit the attendant when opening the circuit. The rheostat handles are placed so that the attendant can operate them, at the same time watching the instruments; the switches are placed where the space is available, usually in the lower row. The integrating wattmeters are read infrequently, and can, therefore, be mounted at any place, even on the wall outside the switchboard if no other space is available.

All the wiring must be placed on the rear of the switchboard; the work must be done neatly, must all be visible, and be securely fastened in the right position. No slanted wires are allowed, as a rule; the wires must run either horizontal or vertical with a sharp bend.

Report. Draw the diagram of connections used, and a sketch showing the actual arrangement of apparatus on the panel. Figure out the sizes and the ranges of the apparatus to be used for a plant of a certain given capacity. The rules for doing this are given in § 645 below. Include brief instructions for operating the switchboard.

642. Direct-Current Switchboards for Two or More Generators.—Switchboard connections for two compound-wound generators are shown in Fig. 477. The two outside panels are generator panels. The middle panel is for the outgoing feeders. The left panel is shown with all the connections; the right panel is left unconnected.

The main bus-bars extend throughout the whole length of the switchboard. The negative terminals of the machines are connected directly to the negative bus-bar, through the main switches. The positive cables are connected to the corresponding bus-bar through the circuit-breakers and the ammeters. One terminal of each field circuit is taken to the switchboard, in order to have it connected to the field rheostat.

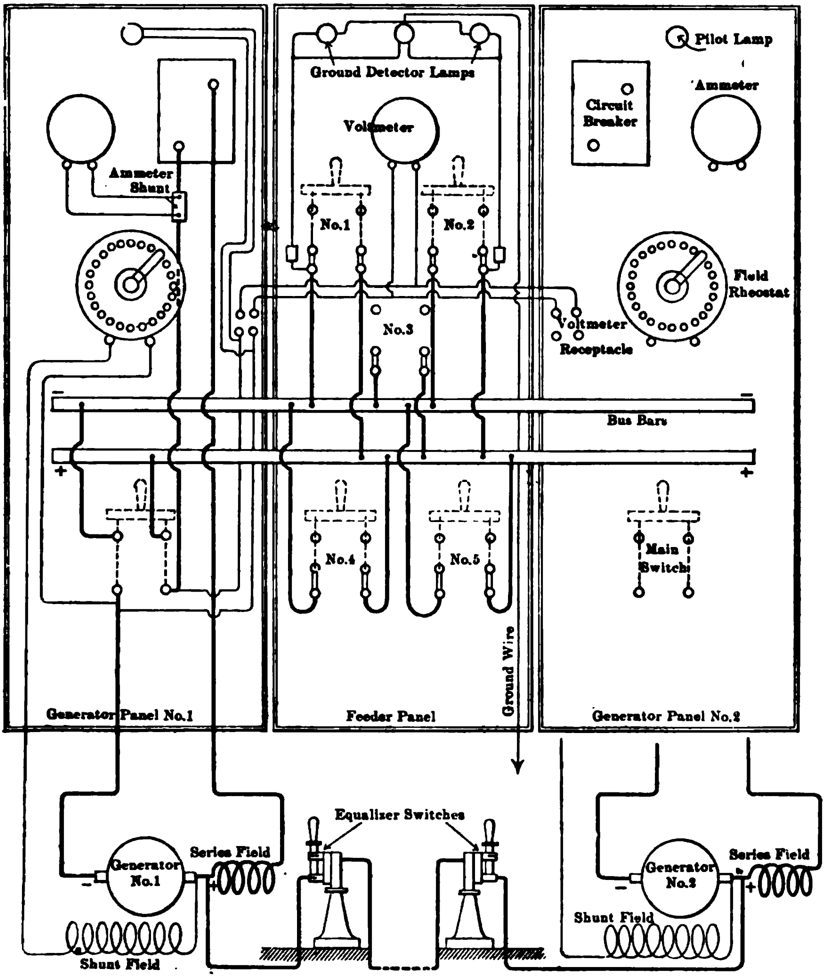

FIG. 477. Switchboard connections for two compound-wound generators and five feeders.

One voltmeter is used for both machines. It may be connected to either machine by means of a receptacle and a plug; the plug is shown separately in Fig. 478. Each generator panel is provided with a lamp which serves for illuminating the ammeter scale and also as a pilot lamp. The feeder panel has three lamps on top. The middle one is connected across the bus-bars and illuminates the voltmeter scale. The two outside ones are ground-detector lamps; they are connected as shown in Fig. 255. Five feeder switches are shown on the middle panel, each circuit being protected by fuses. Circuit breakers, taking the place of both fuses and switches, are much used at present.

The equalizing connection shown between the positive brushes of the machines is used with compound-wound generators only, its purpose being to make the two machines divide the load equally. The theory of the equalizer is given in §§ 315 and 316. It is explained there that with large machines two single-pole switches should be used, instead of one double-pole main switch, in order to avoid an inrush of current when the machine is switched in. In small plants where this circumstance is of no particular importance, the equalizer switch and the main switch are combined into one three-pole switch.

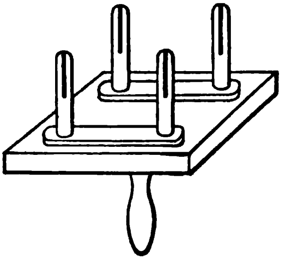

FIG. 478. Voltmeter plug.

The connections shown in Fig. 477 may be extended to any number of generators and of feeder panels. The equalizer cable runs through the station alongside the generators, and each machine is connected to it by its equalizer switch, usually mounted on a pedestal.

Integrating wattmeters are often mounted on the switchboards, the connections being as shown in Fig. 87.

Tirrell regulators are now coming into use for automatically maintaining constant voltage. Their construction and connections are described in § 309. The regulator is mounted either on the panel itself if room is available, or on a bracket on one side of the switchboard.

643. EXPERIMENT 29-C. —Tracing Out Connections on Large Direct-Current Switchboards. — The experiment is performed on a switchboard such as is shown in the diagram of Fig. 477, in the same way as experiment 29-A (§ 640).

644. EXPERIMENT 29-D. —Assembling Direct-Current Switchboards. — The experiment is performed in the same way as experiment 29-B (§ 641). For the experiment, two generators, and all the necessary instruments and auxiliary apparatus, should be available.

645. Determination of the Sizes of Switchboard Apparatus. — In the reports on some of the above-described experiments it is required to determine the sizes of switchboard apparatus for a given capacity of the plant. A few explanations may make the calculations easier.

(*a*) *Switches.* Suppose, for instance, the size of the plant to be 500 kw. and the pressure 250 volts. If there are two machines, each machine has a capacity of 250 kw., or must deliver at full load 1000 amperes. This is the maximum current which the main switches have to carry continuously. Some margin should be allowed for possible short overloads, say 50 per cent, but the exact percentage of this allowance depends entirely upon the character of the plant and the duty for which it is intended. It seems that the above figure, 50 per cent, fairly well represents the average conditions. Thus, the switches must be of a 1000-ampere 250-volt type; they must be capable of safely opening the circuit when carrying a load of 1500 amperes (in case of an emergency).

(*b*) The range of *main ammeters* is calculated in the same way as that of the switches.

(*c*) *Bus-Bars.* A table will be found in *Foster's Pocket Book,* p. 587, giving the sizes of copper bar for various current-carrying capacity.

(*d*) *Field rheostats.* Their range is usually given in ohms, and in amperes carrying capacity, with all rheostat "in," and "out." According to the size of the machines, *from* 2 *to* 5 *per cent of the main current is consumed in field excitation*; this gives the amperage of the rheostat. For instance, in the above example the maximum field current of each machine may be assumed to be about 35 amperes. Therefore, the total resistance of the field winding, when rheostat is all "out," is 250 ÷ 35 = about 7 ohms. It is customary to give the field rheostat such a resistance that the field current could be reduced to one half its full value. Thus, in our case, the resistance of the field rheostat must be also about 7 ohms, and its current-carrying parts must be so dimensioned that it can stand continually and without excessive heating, 35 amperes with the resistance all "out," and about 18 amperes with the resistance all " in." This is called in practice a " taper two to one " rheostat.

(*e*) *Overload circuit-breakers and fuses.* They should open the circuit when the current exceeds, by a certain percentage (say 50%), the full rated capacity of a machine; their size is figured out in the same way as that of the switches and ammeters.

(*f*) *Voltmeters.* Usually 110-volt plants are supplied with 150-volt instruments; 250-volt plants with 300-volt instruments, etc. This gives in each case a sufficient margin, should the voltage for some reason rise above normal.

(*g*) *Integrating wattmeters.* These are rated in volts and amperes; for instance, in the above example a wattmeter must be used whose series windings can stand about 3000 amperes as a maximum on overloads (2000 amperes continual capacity), and the potential winding designed for 250 volts.

(*h*) *Cables to generators* are figured out from tables giving their safe carrying capacity.

Designing switchboards gives an opportunity for becoming acquainted with the catalogs of leading manufacturers of switchboard supplies, also with the commercial types, notations, prices, etc. Not every size of switch or of an instrument can be found in the catalogs, and the student should use his judgment in selecting the nearest suitable type. Give, in the report, the theoretical sizes and those selected from the catalogs, using manufacturer's notation.

646. Miscellaneous Direct-Current Switchboards. — Generator switchboards described in the preceding articles do not exhaust all the types of direct-current switchboards used in practice, but they are the most important, and offer the greatest variety of equipment. With a clear understanding of generator switchboards, the construction and operation of other types of switchboards will be easily understood from the requirements of service. Among such switchboards are those used in connection with motors, rotary converters, arc lamps, storage batteries, boosters, etc.

2. ALTERNATING-CURRENT SWITCHBOARDS

647. Switchboards for One Alternator. — The connections for a simple alternating-current switchboard are shown in Fig. 479; such switchboards are used in small single-phase plants distributing current mainly for lighting. In general appearance the switchboard is similar to those shown in Figs. 476 and 480.

The generator is connected to the main bus-bars through the main switch, fuses and the ammeter. The ammeter is shown connected into the circuit through a series transformer (see Fig. 44). Series transformers are used for two reasons: (1) They permit manufacturing all ammeters of the same actual range (usually about 5 amp.) the range being multiplied in any desired ratio through a series transformer; (2) The ammeter is insulated by the transformer from the line, which may have a voltage of 2200 volts or more; this adds to the safety of the operator.

Two-phase and three-phase switchboards have similar connections, except that ammeters are usually provided in all phases, and three or four bus-bars are used, Figs. 480 and 481.

The voltmeter is shown connected across the main bus-bars; it could also be connected across the main switch, as in Fig. 477. The voltmeter is provided with a potential transformer for the same reasons for which the series transformer is used with the ammeter. A ground detector is used instead of ground-indicating lamps, because with voltages of 1000 volts and above, too many lamps in series would be required. The instrument is essentially an electrostatic voltmeter: the two line wires are connected to two stationary elements, the movable element is connected to the ground. When the insulation of the line is perfect, equal static charges are induced on both ends of the moving element; when one of the wires is grounded, the movable element is attracted more strongly to the side which is not grounded; this is shown by a pointer on the scale. The scale is calibrated in ohms or in any arbitrary units (see also § 287).

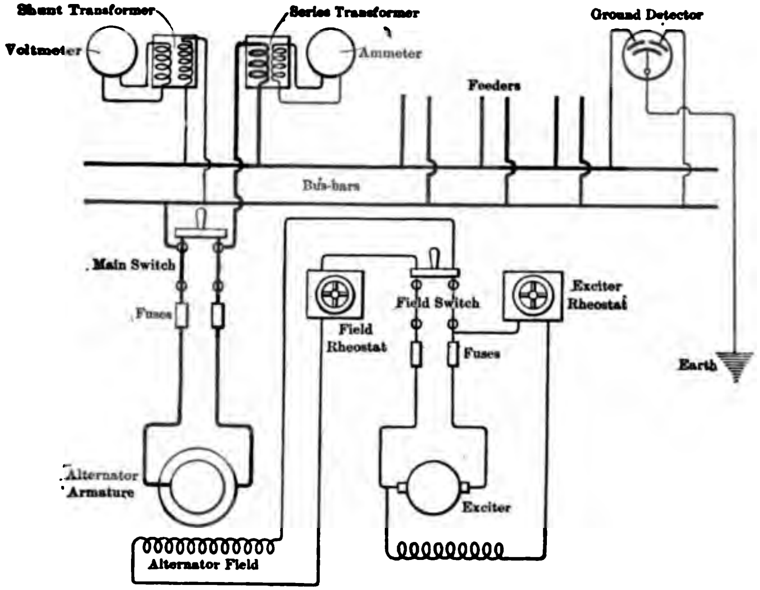

FIG. 479. Switchboard connections for one single-phase alternator.

Direct current for exciting the field of the alternator is supplied by a small direct-current machine, so-called *exciter*. The exciter may be either direct connected to the alternator, belted to it, or driven by its own prime-mover. In large installations, exciters are usually driven by separate engines or electric motors.

The main circuit of the exciter and one of the terminals of its field winding are taken to the switchboard and connected to the regulating rheostats. The alternating voltage is adjusted by regulating either rheostat. It is more economical to have as little resistance as possible in the rheostat to the left ("Field Rheostat"), voltage adjustment being obtained by suitably regulating the exciter field. Experience shows. however, that enough range cannot be obtained in this way for all conditions of load, and it is necessary to use also the rheostat to the left. The handles of the two rheostats are sometimes arranged concentrically, one within the other, as in Fig. 480, for economy in space.

648. EXPERIMENT 29-E. — Tracing out Connections on an Alternator Panel. — The principal features of construction and connections are described in the preceding article. For the conduct of the experiment and the requirements of the report see § 640.

649. EXPERIMENT 29-F. — Assembling an Alternator Panel. — The principal features of construction and connections are described in § 647. For the conduct of the experiment and the requirements of the report see § 641.

650. Switchboards for Two or More Alternators. — Two or more alternators may supply power in parallel to the same bus-bars, provided the machines are *in synchronism* with each other. This means that they must have the same frequency, and that the induced voltages must reach their maxima at the same time. Unless this condition is fulfilled, one generator will send part of its current into the other (see § 462). Thus when two or more alternators are connected to the same bus-bars, special synchronizing connections must be provided in addition to the equipment shown in Fig. 479. These connections are described in the next article.

A medium-size switchboard for two three-phase alternators is shown in Fig. 480. The middle and the left-hand panels are generator panels; the right-hand panel contains ammeters and switches for two feeders. Each generator panel is provided with three ammeters (one for each phase), a field rheostat, a three-pole main switch, and plug switches in the field circuit. The synchronizing and the line voltmeters are mounted on the bracket to the left. The synchronizing lamps and the receptacles for voltmeter and synchronizing plugs are clearly seen in the sketch.

Alternators do not always run satisfactorily in parallel, particularly when driven by gas engines. It is sometimes preferred, at least in small installations, to run the machines separately, distributing the load between them as well as possible. Two sets of bus-bars are therefore provided, one alternator being connected to the upper bars, the other to the lower ones. The outgoing feeders are connected to double-throw switches, and may be supplied with current from either machine.

651. Synchronizing Connections.—The connections for synchronizing several three-phase alternators are shown in Fig. 481. Before the main switch of a machine is closed, the machine is brought into synchronism with the other machines already connected to the bus-bars. The connections are such that the same synchronizing lamps are used with all the machines. Separate synchronizing bus-bars are provided, the lamps being connected between these and the main bus-bars. Each incoming machine may be connected to the synchronizing bus-bars through the synchronizing receptacle, by a suit-

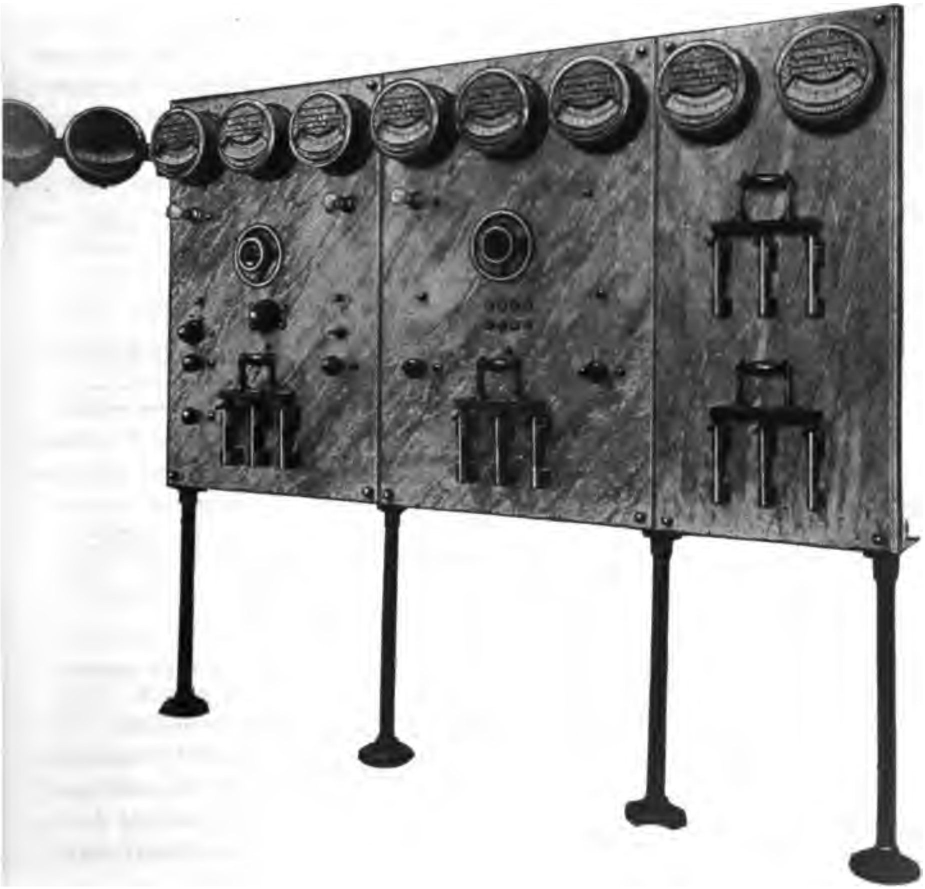

FIG. 480. A switchboard for two three-phase alternators and two outgoing lines.

able plug being inserted into the receptacle. After synchronism has been obtained the main switch of the machine is closed. The plug may be left in the receptacle, until it is needed for synchronizing another alternator.

A synchroscope (see § 464) is shown in addition to the synchronizing lamps. Both are usually provided on large switchboards, and either or both may be used as desired. In high-voltage installations the lamps and the synchroscope are connected to the bus-bars through potential transformers.

652. Voltmeter Connections. — One voltmeter is ordinarily used for indicating the voltage of any number of alternators; the connections are shown in the lower part of Fig. 481. The voltmeter is connected to so-called voltmeter bus-bars, and each machine is provided with a voltmeter receptacle, by means of which the voltmeter may be made to indicate the pressure in either phase of any machine. The corresponding plug is shown in Fig. 478. The connections to alternator No. 2 are lettered and can be easily followed. When the voltmeter plug is inserted into the four holes to the left, *a* is connected to v_1, and *b* is connected to v_2, so that the voltmeter reads the voltage in the phase *a-b*. When the plug is in the four middle holes the voltmeter shows the voltage *b-c*; with the plug in the four holes to the right, the pressure in *a-b* is indicated.

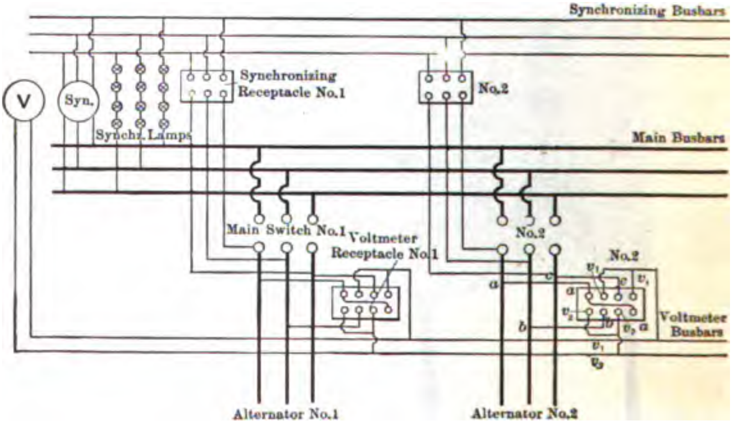

FIG. 481. Synchronizing and voltmeter connections for three-phase alternators.

A second voltmeter is connected permanently to the main bus-bars. This is necessary for watching the line voltage while one of the machines is being synchronized and the first voltmeter is disconnected from the line. Both voltmeters are sometimes mounted on a swinging arm, as shown in Fig. 480; the voltmeter receptacles are also shown on the left and the middle panels.

653. Special Instruments. — *Integrating wattmeters* are often mounted on switchboards. The connections are shown in Fig. 92 for single-phase wattmeters, and in Fig. 98 for three-phase wattmeters. As these instruments are read at infrequent intervals only, they may be mounted in any available space.

Indicating wattmeters are generally used on large switchboards, mostly for measuring the power output of each individual generator. The connections are the same as with integrating wattmeters.

Special instruments are sometimes used on switchboards to enable the attendant to follow more closely the operation of the machines; as such instruments, *power-factor meters* (§ 82) and *frequency meters* (§ 474) are in quite general use. *Recording instruments* (§ 50) are also becoming more and more popular with power-station engineers.

Tirrell regulators are now coming rapidly into use for automatically maintaining a constant line voltage. Their construction and connections are described in § 460. The regulator is mounted either on the panel itself, if space is available, or on a bracket at one side of the switchboard.

654. EXPERIMENT 29-G. — Tracing Out Connections on Large Alternating-Current Switchboards. — For a description of switchboards see §§ 650 to 653. The directions for the conduct of the experiment are the same as § 640.

655. EXPERIMENT 29-H. — Assembling Alternating-Current Switchboards. — For a description of switchboards see §§ 650 to 653. The directions for the conduct of the experiment are similar to those in § 641.

656. Remote-Control Switchboards. — With alternators of several thousand kilowatts capacity, the switches become too heavy to be operated by hand. Aside from this it is not safe to operate them by hand because of the extremely high voltages commonly used in large power houses.

For these two reasons, oil switches — controlling generator and feeder circuits — are usually placed in a safe and convenient place apart from the switchboard. They are opened and closed by solenoids or small motors connected to an auxiliary low-voltage circuit. The operator has the control of the auxiliary circuit on the main switchboard, and opens and closes the main switches by operating small auxiliary switches. Tell-tale indicators or lamps are provided, so that the operator may see whether a certain switch is open or closed.

With large capacity machines field rheostats are too bulky to be mounted behind the switchboard. They are usually placed under the switchboard gallery and operated from the gallery by sprocket wheels and chains.

Relays are provided in large power houses for automatically opening the main switches on overloads, or in case the current becomes reversed.

657. Miscellaneous Alternating-Current Switchboards.— Generator switchboards, described in the preceding articles, do not comprise all the alternating-current switchboards used in practice. However, they are the most important and offer the greatest variety in equipment. With a clear understanding of generator switchboards, the construction and operation of other alternating-current switchboards will be easily understood from the requirements of service. Among these types, the principal are those used for controlling synchronous and induction motors, rotary converters, transformers, arc-lamps, etc.

CHAPTER XXX.

CONTROLLERS AND REGULATORS.

658. An electric controller in the specific sense of the word is a device for starting, regulating speed of, and reversing, electric motors. The controllers described in this chapter are for use with direct-current motors only. Induction motor starters and regulators are described in § 523; starting and regulation of synchronous motors is explained in §§ 467 and 470.

As indicated by Fig. 284, which shows a complete diagram of connections of a shunt-wound motor, a variable resistance must be used for starting. A second resistance is used for field control, if variable speed is desired. If it is necessary to operate the motor in both directions, a double-throw switch is substituted for the single-throw main switch; to reverse the motor, the armature leads are interchanged.

But the use of three separate devices could not be tolerated in practice, such a system being awkward and complicated. It is particularly objectionable in cases where motors are started and reversed many times a day, or are intrusted to incompetent persons. Operating the above three devices in a wrong order would cause an accident: For instance, closing the main switch when the starting resistance is all "out" is sure to blow the fuse, and may, under certain circumstances, do considerable damage to the motor. Therefore, controllers and starters must either be combined in one device, or be so mutually interlocked, that it would be impossible to operate them in a wrong order.

A study of such improved starting and regulating devices is the subject of this chapter.

1. MOTOR STARTERS AND REGULATORS.

659. Starters with No-Voltage Release.—A common type of starter for shunt-wound motors is shown in Fig. 482, in front and side views. Its automatic feature consists in an electromagnet (no-voltage release) which holds the starting arm in the running position as long as the coil is energized from the line. Should the main switch be opened, a fuse blow, or the power be shut off for any reason, the starting arm is released, and returns to its "off" position under the action of the spiral spring at its pivot.

The starting resistances are placed in a box provided with ventilating slots. Taps from resistances are taken to the contact buttons on the face plate. The release coil is wound on an iron core, and is provided with two pole-pieces. The starting arm has an iron armature attached to it, which is held by the pole-pieces in the running (extreme right) position of the arm.

The armature of the motor is connected across the circuit in series with the starting resistance. The field and the release coil form another circuit. When the motor is standing still the starting arm is in the position shown in the sketch, and the main switch is open.

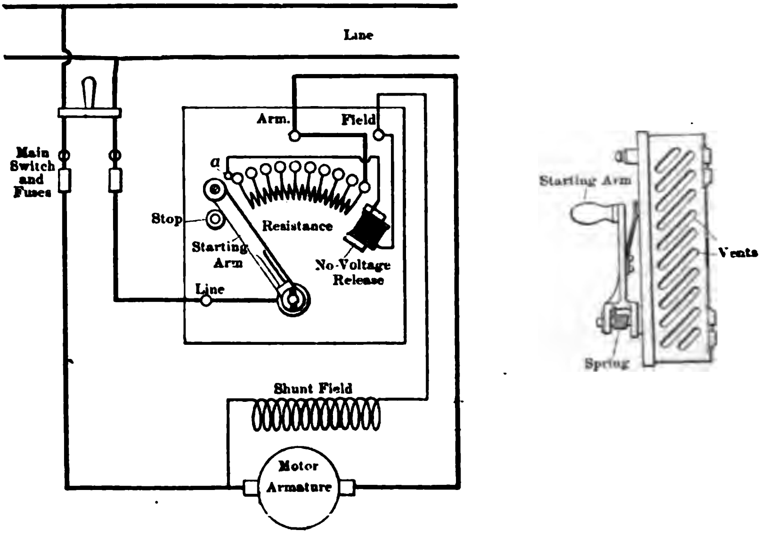

Fig. 482. Motor starter with no-voltage release.

To start the motor, the main switch must be closed first, and then the starting arm moved slowly to the right. On the first notch all of the starting resistance is connected in series with the armature; as the arm is moved farther, the resistance is gradually cut out of the circuit, until on the last notch the armature is connected directly across the line. In this position the starting arm is held by the release electromagnet.

To stop the motor the main switch is opened. This deënergizes the release coil, and the starting arm flies back to its zero position, under the action of the spiral spring, seen in the side view to the right. The same happens, should the field circuit be open for any reason, or the power be shut off. Thus, whenever the motor is stopped, the starting resistance is *automatically* introduced into the motor circuit, protecting the motor for the next starting.

The small auxiliary contact *a* connected to the first starting button serves two purposes: (1) It permits energizing the field of the motor a moment before the armature circuit is closed, thus allowing for the time lag due to a considerable inductance of the field winding. (2) It takes up the spark when the starting arm flies back to zero position; after the contact is sufficiently burnt by this spark, the button *a* can be easily replaced.

It will be seen from the diagram that the part of the starting resistance cut out of the armature circuit is automatically introduced into the field circuit. This is of no practical importance, since the starting resistance is many times lower than the resistance of the field winding of the motor.

In some cases, the motor field winding and the no-voltage release coil are connected independently across the line, instead of being in series with each other. This is done, for instance, when it is impossible to put the required number of ampere-turns on the coil in any other way, especially with high voltages.

In the above starting box the arm is returned to its zero position if it be left on an intermediate notch, so that the starter cannot be used for regulating the motor speed. In some cases, however, this regulation is required, and the starting arm is allowed to remain indefinitely in any position. For instance, regulators used with small fan motors, where economy is a secondary consideration, are so arranged.

If it is desired to have a no-voltage release with such regulators, the starting arm is provided with a sector having slots on its periphery. The release coil has a pivoted armature carrying a hook on one end. This hook engages in any of the slots of the sector, and holds the arm in a desired position, as long as the coil is energized. When the power is off the armature is released and the spiral spring returns the arm into the "off" position.

660. Multiple-Switch Starters.— With large motors it becomes difficult to insure a good contact between the buttons and the arm, with the construction shown in Fig. 482. A good solution is to use separate switches for each step, as shown in Fig. 483. The connections are similar to those shown in Fig. 482, including the automatic no-voltage release. The switches are mutually interlocked, so that they can be closed in a certain order only. The first switch to the left, when closed, is held in position by the no-voltage coil, provided the latter is energized. This first switch pushes out of the way a stop which otherwise prevents the second switch from being closed. Now the second switch may be closed; this being done, the stop of the third switch is lifted up, etc. Should the power be cut off, the no-voltage coil releases the first switch, and then all the switches open at once.

Some details of construction may be seen in the side view to the right. The switch s_1 is shown closed, the switch s_2 open. The switches are provided with curved copper brushes *c* which close the circuit between a bus-bar *n* and contacts *m*, connected to the sections of the starting resistance. The retaining hooks *d* and their locks are also clearly seen in the sketch.

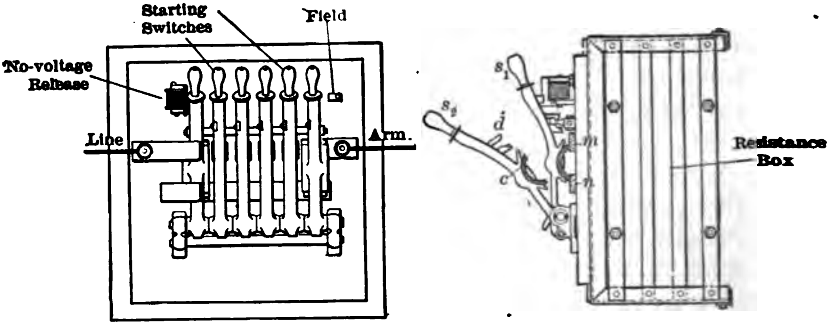

Fig. 483. Unit-switch type motor-starter with no-voltage release.

661. Electrically Operated Starters.

— Instead of operating a starting-rheostat by hand, as shown in Figs. 482 and 483, the rheostat may be operated electrically, by solenoids energized from an auxiliary circuit.

The principal cases in which starters are operated electrically (Fig. 484) instead of by hand are:

(1) Where it is necessary to start a motor from a distance, for instance, from an elevator car.

(2) Where it is desired to have an acceleration independent of the operator's will, as in large electric cars.

(3) Where the motor is started and stopped automatically, — for instance, by a float in a water tank.

With the arrangement shown in Fig. 484, closing the main switch energizes the solenoid, whose plunger moves the starting-arm upward, cutting out the resistance. The rate at which the arm is moved is regulated by the dash-pot. At the end of the travel, an auxiliary contact shown in the sketch is broken by the arm, and some resistance, usually a lamp, introduced into the solenoid circuit. The current in the coil is thereby reduced to a value just sufficient to hold the starting-arm in the upper running position. When the power is off, or the main switch is opened, the solenoid is deënergized, and the starting-arm falls by gravity to its zero position.

It is not necessary to run the main line to the place from which the motor is started: An electrically operated main switch, Fig. 485, may be placed near the solenoid starter, and only small service wires run to the operator's place. Closing the service switch energizes the solenoid *C* of the main switch, which in turn closes the main circuit. This energizes the starter solenoid and the starting-arm begins to move (Fig. 484).

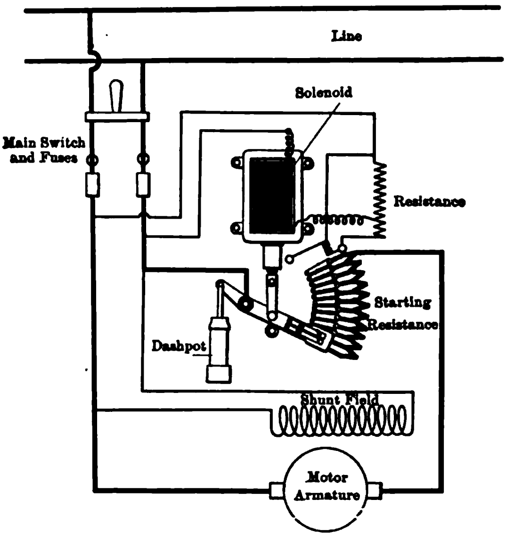

FIG. 484. An automatic motor starter.

This arrangement is used with motor-driven pumps and air compressors. The circuit *aa* to the solenoid of the main switch is closed

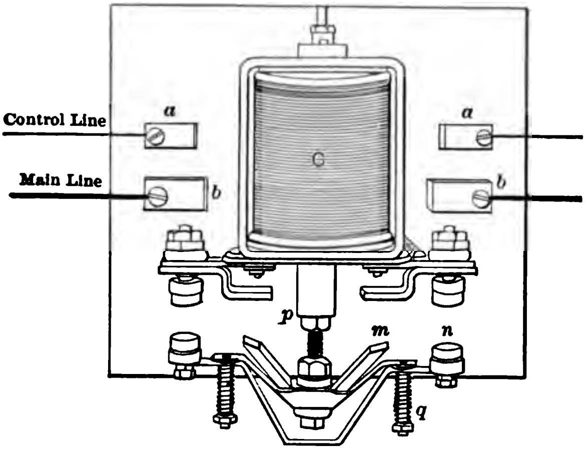

FIG. 485. An electrically operated switch for remote control.

automatically by a float switch, or by a pressure indicator, when the water level in tank or the air pressure is beyond certain limits. This automatically starts and stops the motor, as needed.

With large motors, or when the connections are more complicated, several unit switches, such as the one in Fig. 485, are used, instead of a starting-arm. This is the system applied in many electric elevators (see Fig. 498); also in the multiple-unit system of electric train control (Fig. 509).

662. EXPERIMENT 30-A. — Operating Direct-Current Motor Starters with No-Voltage Release. — A starting-box should be provided, of a type shown in Fig. 482, and a suitable motor to be operated in connection with it; also an ammeter, a voltmeter, and means for loading the motor.

(1) Wire up the motor and the starting-box, and practice starting and stopping. Make clear to yourself the order in which the main switch and the handle of the rheostat should be operated, and to what extent the arrangement is "fool-proof"; also what would happen if operations were performed in a wrong order.

(2) Explain why the handle does not fly back immediately after the main switch is opened; prove the explanation by an experiment. Determine the minimum line voltage at which the coil can hold the arm. Interpose pieces of thin paper between the coil and its armature. and observe the effect on the magnetic attraction.

(3) Apply a certain brake load and start the motor by moving the rheostat arm at a certain definite speed, for instance, using a metronome. Read instantaneous values of line amperes and volts across the armature every few seconds. With three observers, after some practice, it is possible to take readings on an instrument every two seconds: One man signals at the proper time, another reads aloud the scale indications, the third records the readings. A much better way is to use high-speed recording ammeters and voltmeters (§§ 52 and 701), if such are available. Repeat the same experiment with different rates of starting, and with different values of the load.

(4) Measure total resistance of the starting-box, and the resistances of separate steps. This may be done by putting a steady current through the rheostat, and taking voltage drop between adjacent buttons.

(5) If electrically operated starters are available, such as shown in Figs. 484 and 485, connect up and operate them, in order to clearly understand their action. Take a few readings, as before, in order to characterize the devices by their performance.

Report. Draw diagrams of actual connections used during the experiment. Explain your findings concerning the operation of the no-voltage release coil. Give curves showing the variation of motor volts and amperes during the period of starting; show the influence of the load and of the rate of starting. Plot the resistance of steps and the total resistance of the rheostat to steps as abscissæ; explain the reason for which the resistances of various steps are made different.

663. Overload Release. — In the above-described starters, no provision is made for protecting the motor against overloads, except the fuses in the main circuit. This is a satisfactory solution in cases where overloads occur but seldom; but, with frequent overloads, fuses are expensive and inconvenient. An automatic circuit-breaker is preferably used in such cases, in place of fuses, or in connection with them. A circuit-breaker is an automatic switch opened by a coil connected in series with the main circuit. When the current exceeds a certain limit, the coil attracts an iron armature, which trips and opens the switch.

Where a fuse and a circuit-breaker are used in conjunction, the fuse is supposed to take care of long but moderate overloads, while the circuit-breaker operates on sudden excessive overloads. A continual overload is certain to heat up and melt a fuse, but it may not give enough momentum to the armature of the circuit-breaker to trip it. On the contrary, a short but heavy overload may not melt a fuse but is able to communicate a considerable momentum to the armature of the circuit-breaker.

There is some demand for starters having an overload feature incorporated in them, instead of separate circuit-breakers or fuses. A device of this kind is shown in Fig. 486. In addition to the parts shown in Fig. 482, it has a second arm *k*, which closes the main circuit and is connected by a spiral spring to the starting-lever, so that the two arms are impelled towards each other. Under ordinary conditions, the overload lever *k* is prevented from flying up by a lock *t*, and thus keeps the main circuit closed between the jaw *j* and the starting-arm.

Should the line current exceed a certain limit, the series coil *c*, shown in the lower part of the starter, attracts its plunger upwards; the same strikes the lock *t*, releases the overload lever *k*, which opens the circuit. The lever has no handle attached to it, so that in order to return it to its former position and close the circuit, it is necessary to return the starting-arm to its "zero" position, and then start the motor again. This feature prevents starting the motor with the resistance cut out of the circuit. The underload or no-voltage release acts in the same way as in Fig. 482, and is independent of the overload feature.

The overload release may be made to operate at any predetermined current (within certain limits), by raising or lowering the plunger by the knurled head *r*. The setting is read in amperes on the scale.

Another type of starter with an overload release is shown in Fig. 487; only the lower part of the face plate is shown, the rest being identical with the construction illustrated in Fig. 486. The use of the second arm is avoided here: When the current exceeds a certain limit, the overload coil *c* attracts the iron armature *i*, which strikes and closes the auxiliary contact *p*. This short-circuits the no-voltage release coil, and the starting-arm flies back to "zero" position. The armature *i* may be set by means of the screw *r* to open the circuit at a desired value of the current.

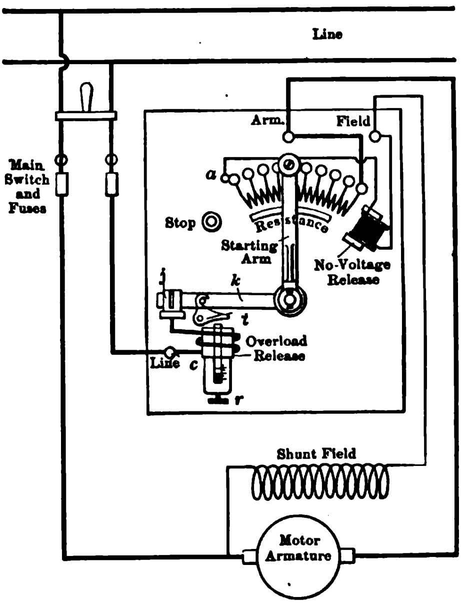

Fig. 486. A motor starter with no-voltage and overload release.

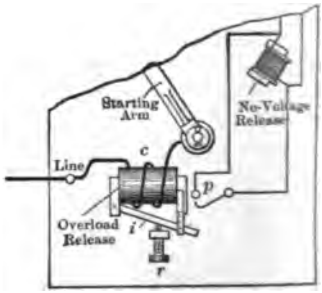

Fig. 487. Another type of overload release (see Fig. 486).

It is generally agreed that the device shown in Fig. 486 is more positive in its action than that shown in Fig. 487, but the latter is less expensive.

664. EXPERIMENT 30-B. — Operating Direct-Current Motor Starters with Overload Release. — Two types of starters should be investigated: (*a*) Double-arm starter, as in Fig. 486, and (*b*) single-arm starter, Fig. 487. It is well to test, in addition, an ordinary starter with no-voltage release (Fig. 482), using a separate overload circuit-breaker.

(1) Connect the devices in succession to a motor, and practice starting. Observe the action of the overload protection. Make clear to

yourself that no wrong move is possible, except with deliberate intention.

(2) Calibrate the overload attachment in amperes. For this work use an ordinary load rheostat instead of a motor; the main current can be kept more constant.

(3) Make tests of the influence of the "time element" on the action of the overload attachment. Adjust a certain current through the coil *c*, and then increase the current by a certain per cent, first gradually, then instantaneously: observe the difference in the operation of the tripping mechanism. Perform this experiment with different values of current, and with different percentages of increase.

(4) Compare the action of fuses and of a circuit-breaker on slow and sudden overload; obtain, if possible, definite numerical results.

Report. Give a diagram of the actual connections used. Describe the influence of the time element, and the difference in the action of fuses and circuit-breakers. Give your opinion as to the relative advantages of the types of starters investigated, and describe peculiarities of operation, if any.

665. Field Control. — In the above-described starting rheostats, no provision is made for regulating the field current of the motor, in other words, for varying its speed. If speed control is required, an additional rheostat must be connected into the field circuit. But it must be remembered that the motor should always be started with as strong a field as possible, in order to get a good starting torque, without an excessive rush of current. Therefore, the starting and field rheostats must be suitably interlocked, either mechanically or electrically.

A popular device of this kind is shown in Fig. 488: the lower row of contacts is connected to the starting resistance, the upper row to the field rheostat. A double lever is provided, the outside arm being for the field contacts, the inside one for the starting contacts (see side view to the right). The outside arm only is provided with an operating handle. In starting, the two arms are moved together, but the field arm is electrically inoperative, because the field current flows directly through the starting-lever, the bar *b*, the solenoid, and into the field of the motor. At the end of the starting period, the starting-lever is attracted and held by the no-voltage release coil, while the field lever may be moved back to increase the speed of the motor. The upper row of contacts is now operative, since the starting-lever no longer touches the short-circuiting bar *b*, but rests on the blind button *d*.

Opening the main switch releases the starting-lever, which flies back, strikes on its way the field lever, and both levers are returned to the "zero" position. It will be clearly seen from the above description, that it is impossible to start the motor with weakened field. An overload feature, such as is shown in Fig. 487, may be added to this starter.

In some cases it is preferred to have a field rheostat separate from the starter. In one device of this kind the field rheostat is kept normally short-circuited, by the armature of an electromagnet: This insures starting the motor with full field. The short-circuit is not removed by energizing this electromagnet, since the armature is too far from it to be attracted. But when the field-regulating arm is returned to its "zero" position, corresponding to the strongest field, it pushes the armature against the electromagnet, and removes the short-circuit. After this, any amount of resistance may be cut into the field circuit, if desired, and the motor speed increased. But as soon as the main switch is opened, the short-circuiting armature of the field rheostat is again released and the field rheostat short-circuited, independent of the position of its operating arm.

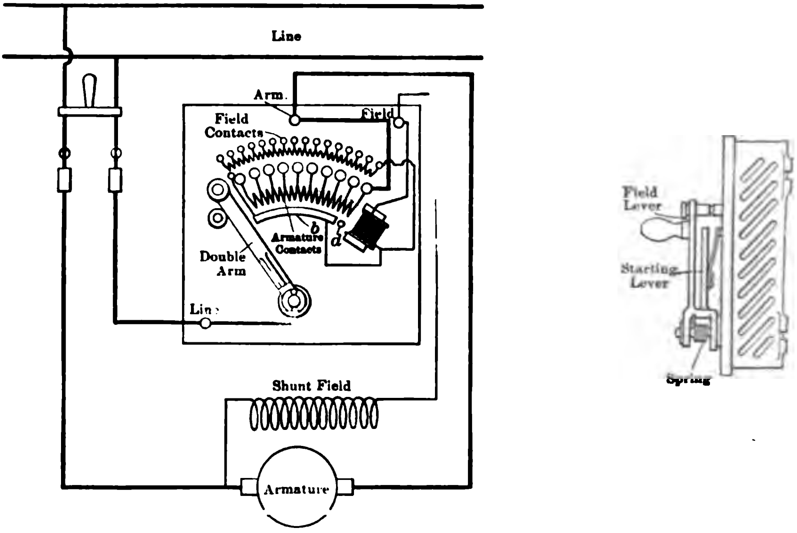

FIG. 488. A combination motor starter and speed regulator (Cutler-Hammer Mfg. Co.).

666. EXPERIMENT 30-C. — Operating Motor Speed Regulators. — A device, such as is shown in Fig. 488, should be provided; also an ordinary starter, as in Fig. 482, with a separate field rheostat. An ammeter should be connected into the field circuit.

(1) Connect the speed regulator to a motor and practice operating it; make clear to yourself the automatic features of the device.

(2) Measure the speed and field current of the motor with several positions of the regulating handle.

(3) Measure the total resistance of the field rheostat, and the resistances of the separate steps; also the resistance of the motor field.

(4) Connect the common type of motor starter and a separate field rheostat in place of the combination starter and regulator. Devise an electrical or mechanical interlocking arrangement, which would prevent starting the motor with weakened field.

Report the connections used, and the automatic features of the device tested. Plot curves of motor speed, field current, and total resistance in the field circuit, to rheostat steps as abscissæ. Describe your own automatic arrangement, devised to prevent the motor from being started with weakened field.

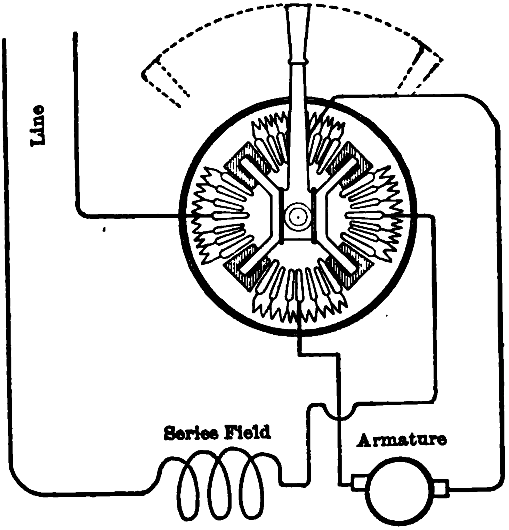

FIG. 489. Crane controller.

667. Crane Controllers. — The principal points of difference between the duties which a crane controller is called on to perform, and those performed by the regulators and starters described above, are:

(1) Series motors are used on cranes, so that there is no separate field circuit (Fig. 285).

(2) No automatic feature or interlocking is required, the operator himself watching the performance of the motor constantly.

(3) The controller must be reversible.

Face-plate controllers (Fig. 489) are generally used for crane service. On large three-motor cranes, three such controllers are placed side by of the line passes through the finger *a*, the strips z, x_4, x_3, x_2, and x_1 to the finger *e*; thence, through the whole starting resistance, to the armature of the motor and out to the other terminal of the line. On the second notch the finger *d* touches the strip x_2, and part of the starting resistance, that between *d* and *e*, is cut out. On the third notch still more resistance is cut out, and finally on the fourth notch the current flows through *a*, z, x_4 and *b* direct to the armature, without any starting resistance in series: this is the running position of the drum.

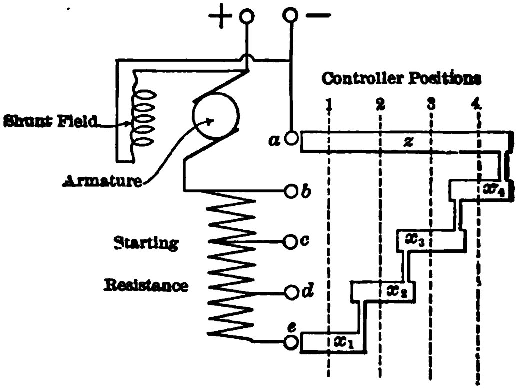

FIG. 490. Starting connections.

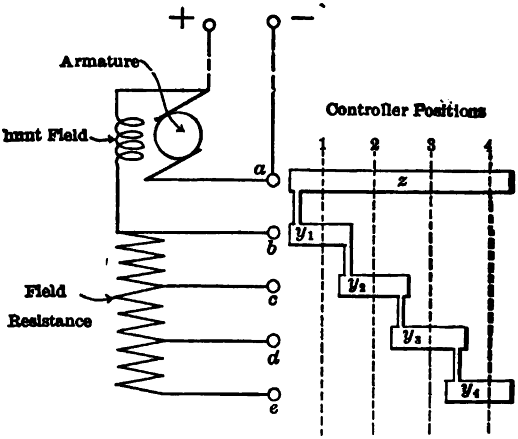

FIG. 491. Field-control connections.

Fig. 491 represents controller connections for speed control, by means of a variable resistance in the field circuit; for the sake of clearness the starting connections are omitted. On the first notch the field is excited directly across the line without any resistance in series with it. This gives the strongest field, and therefore the lowest speed. On the second notch the resistance between the fingers *b* and *c* is inserted into the circuit, on the third notch that between *b* and *d*, etc., until on the last notch all of the field resistance is put into the circuit, and the motor runs at its highest speed.

Connections for reversing the motor are shown in Fig. 492. When the drum is in the "Forward" position, the current from the positive terminal flows through *a*, x_1, x_2 and *b* to the armature terminal A_1, and thence returns to the line through the terminal A_2. When the controller handle is in the "Reverse" position, the current passes through *a*, y_1, y_2 and *c* to the armature terminal A_2, thus flowing through the armature in the opposite direction. Therefore, the motor now runs in the opposite direction, the field connections not being reversed.

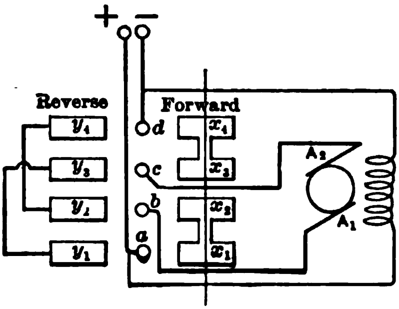

FIG. 492. Connections for reversing.

Controller connections for operating a motor on a three-wire system (§ 318) are shown in Fig. 493: In the position marked "Half-speed" the armature is connected between the positive and the neutral (±) wires; at full speed it is connected between the positive and the negative terminals.

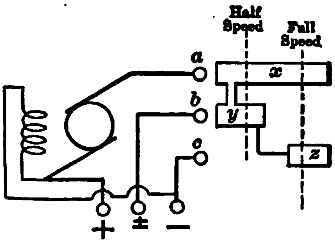

FIG. 493. Three-wire connections.

671. Examples of Actual Connections. — With a clear understanding of the above simple diagrams, the following out of the connections in Figs. 494 and 495 will hardly offer any difficulties. Fig. 494 represents the connections in a single-voltage reversible controller: The motor shown is compound-wound, but the series field has nothing to do with the operation of the controller, which can just as well be used with a shunt-wound motor. The controller has two starting positions, 1 and 2, in which the armature resistance is gradually cut out of the circuit; the other positions are for field control.

The line, the armature leads, and one of the terminals of the field are connected to the fingers corresponding to the long strips, and are in the circuit for all positions. The "Forward" strips and the "Reverse" strips are so connected, that the current flows through the armature in opposite directions. The shorter upper strips are for cutting out starting resistance in steps. The lower diagonal strips serve for gradually introducing resistance into the field. Fewer steps are provided on the "Reverse," because it is not supposed to be used in regular operation.

The controller shown in Fig. 495 is designed for a three-wire system. No starting-notches are provided; the starting resistance is automatically cut out in one step by a spring-actuated contact *c*, which is released by the controller, and retarded by a dash-pot, shown in the upper right-hand corner of the sketch. This arrangement prevents a rush of current due to the operator passing the first contact too rapidly.

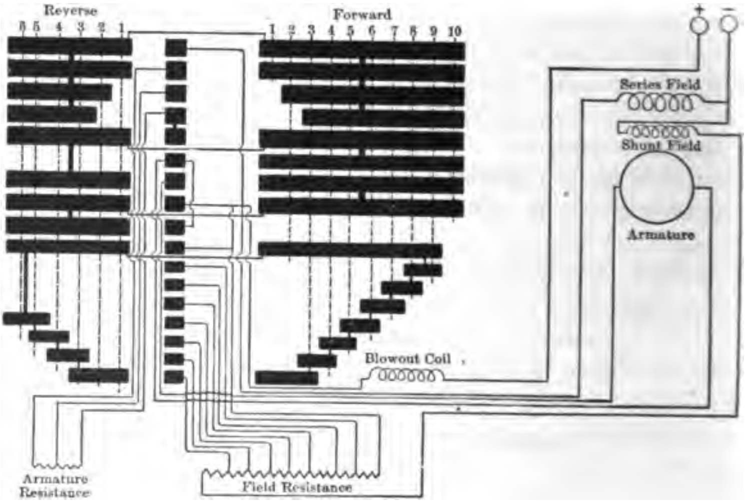

Fig. 494. Connections of a reversible machine-tool controller.

In the "Forward" direction, the motor may be operated both on high and low voltages; the "Reverse" strips are connected to the low voltage only. In most cases it is sufficient to operate the motor in the "Reverse" direction at moderate speeds only.

The blow-out coil shown in both sketches is for the purpose of extinguishing the arc at the finger strip which finally opens the circuit. The coil is connected in series with the main circuit and produces a strong magnetic field in the place where the last contact in the controller is broken. This field shifts the arc outwards, increases its length, and in this way blows it out (Fig. 506).

672. Experimental Controller. — In studying connections and experimenting with an actual controller, the student is handicapped by the fact that the controller is all wired up, and some of the wiring is not accessible. Moreover, the controller is usually intended for a specific duty only, and cannot very well be used for various purposes.

It is therefore advisable to have in the laboratory an *experimental* controller, especially adapted for exercises in wiring. No permanent connections should be made between the strips on the drum, but each strip should be provided with one or more binding posts so that the student may establish any desired connections himself. Some of the strips must be long, others short, and arranged stepwise, for gradually cutting in or out of resistances. Such a controller, if properly designed, is very useful for a study of the operations explained in § 670.

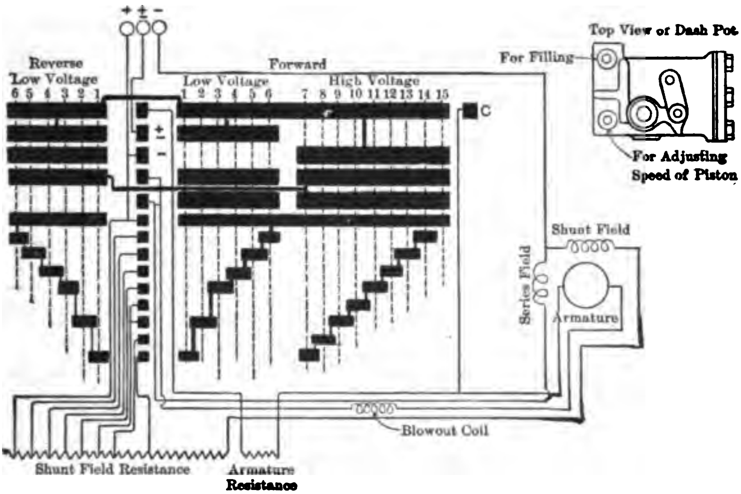

FIG. 495. Reversible machine-tool controller for three-wire service.

The controller should be mounted horizontally in order to be more accessible, and should have no cover, save that there must be a board on which the fingers are mounted. It is not advisable to have a blow-out coil in connection with it, in order to keep the device as simple as possible. The student should be given an opportunity to study the action of a magnetic blow-out on a separate electromagnet.

673. EXPERIMENT 30-E. — Wiring a Machine-Tool Controller. — The purpose of the experiment is to illustrate the connections shown in Figs. 490 to 495. A special experimental controller should be used, described in § 672, and the student is supposed to wire up the controller so as to suit certain given requirements. The experiment may be performed in the following order:

(1) Connect up the controller for starting a shunt motor in one direction only, without field control;

(2) Add the necessary connections for regulating the field;

(3) Supplement the scheme by connections for reversing the motor;

(4) Wire up the controller complete for running forward and reverse, on three-wire system.

A shunt-wound motor should be provided, and operated in connection with the controller: this is the best check on the connections. Have an ammeter in the armature circuit, and one in the field circuit; also a voltmeter across the armature terminals. Measure the speed of the motor with various positions of the controller handle.

At the end of the experiment remove all the connections in the controller, so that the next students to use the controller may have the benefit of designing their own connections.

Observe separately the action of a blow-out coil, in a circuit formed with considerable inductance, by interrupting the circuit at a suitable place. At first use a separately excited electromagnet, or a permanent magnet, and observe its action on the arc: Reverse the direction of the main current, also the polarity of the magnet, and in each case notice the effect. Then take a regular blow-out coil, connected in series with the main circuit, and investigate its action. In performing the experiment with the blow-out coil the student must be careful to have the wires well insulated. The e.m.f. of self-induction, when the circuit is opened, may reach several hundred volts and give one an unpleasant shock.

Report. Give the actual connections used and numerical data on the performance of the motor. Describe the experiment with the blow-out coil.

3. ELEVATOR CONTROLLERS.

674. The essential parts of an electric elevator are shown in Fig. 496. The driving engine may be placed either in the basement or overhead. It consists of a hoisting-drum driven by an electric motor, usually through a worm-gearing (except in very high-speed elevators). The car, the counter-weights and the ropes are clearly shown in the sketch, as well as a centrifugal safety device mounted on the top beam. The movements of the car are controlled by the handle in the car itself: The handle operates the stationary controller shown behind the motor. The controller makes the necessary connections between the motor, the line and the resistances.

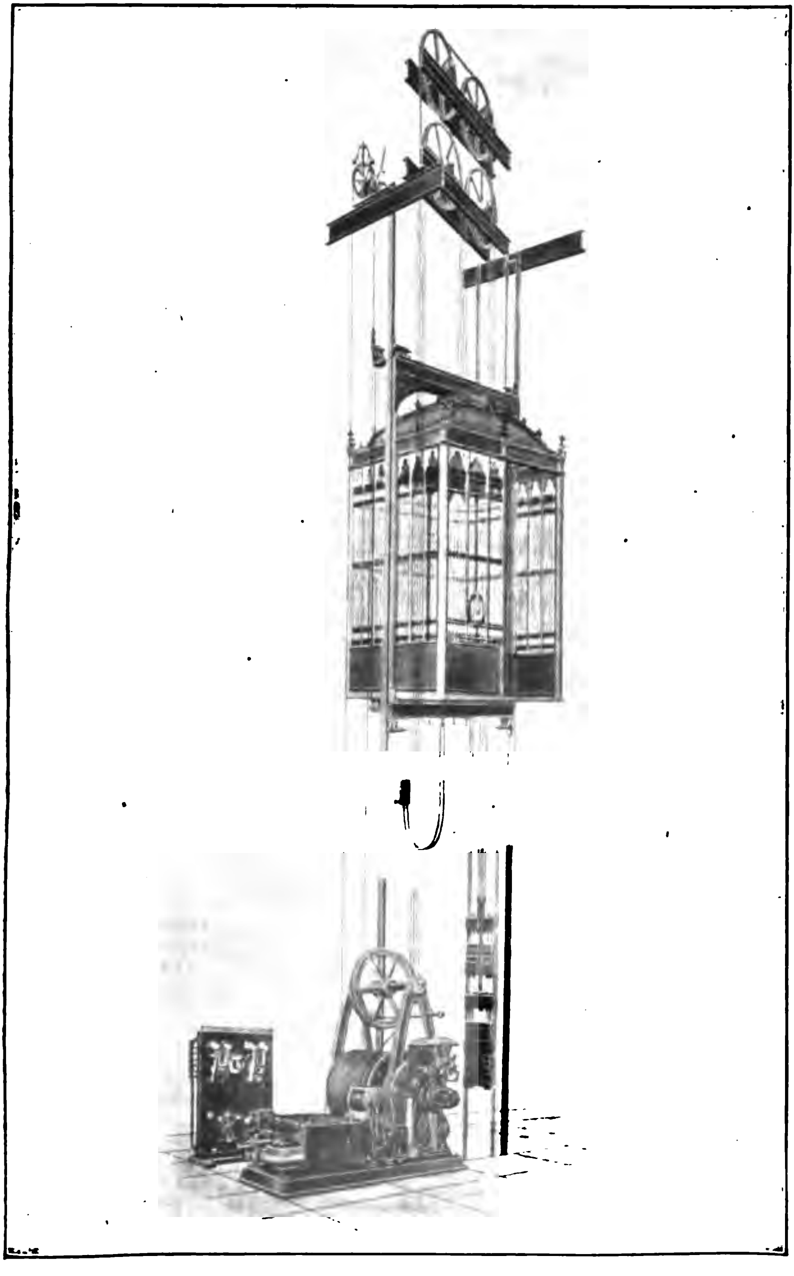

FIG. 496. Otis electric elevator.

The motor is provided with a powerful mechanical brake, shown between the motor and the gear-case. The brake is automatically applied when the motor circuit is opened: When the power is "on" the brake is released by a solenoid connected in series with the main circuit. An additional braking effect is obtained by short-circuiting the motor on resistances which make it act as a generator. In large elevators an additional mechanical brake is provided for emergency purposes. The safety devices consist of:

(1) Stop-motion switch.
(2) Slack-cable switch.
(3) Two hatchway limit-switches.

The stop-motion switch is shown in front of the gear-case, mounted on the drum shaft; it automatically cuts off the circuit at the limits of travel, should the operator neglect to do so. The slack-cable switch is an emergency device intended to operate, should the car become jammed in the hatchway while descending. As soon as the drum has slacked off the cable, the switch is opened stopping the motor. The hatchway limit-switches open the main circuit should the car for any reason over-run, either at the top or bottom. These switches operate only in case the "stop-motion" switch on the drum should fail to stop the car, thus constituting an additional emergency device.

The controller operating the motor is actuated from the car either mechanically by ropes, as shown in Fig. 497, or electrically, as in Figs. 496 and 498. The electric control may be either non-automatic (handle control) or automatic (push-button control).

Mechanical control is used only with low-speed (usually freight) elevators. High-speed passenger elevators are provided almost exclusively with non-automatic electric control. Push-button control is used in residence elevators, where no regular operator is kept, and the machine is operated by the passengers themselves; also in so-called "dumb-waiters," or small elevators for carrying goods between floors, when no operator rides with the car.

We shall now describe these three types of elevator control more in detail.

675. Mechanically Operated Controllers. — A representative elevator controller of this type is shown in Fig. 497. It consists of a motor starter shown to the left, and of a combination reversing switch mounted to the right. The motor is heavily compounded, as usual with elevators, in order to obtain a powerful starting torque; at the same time, the shunt winding keeps the speed within reasonable limits. The starter is operated by the motor itself, through a worm-gear and a magnetic clutch, visible on top of the sketch. The reversing switch is operated by a rope from the car; it has three positions: "off," "up," and "down."

Assuming the car to be at rest, when the operator pulls on the rope in a certain direction, first the reversing blades of the switch connect the motor armature so as to run in the proper direction, then the line contact to the left is closed through the segmental ring, and finally the other side of the line is closed by the carbon contact between the poles of the blow-out. The motor starts with all the resistance "in."

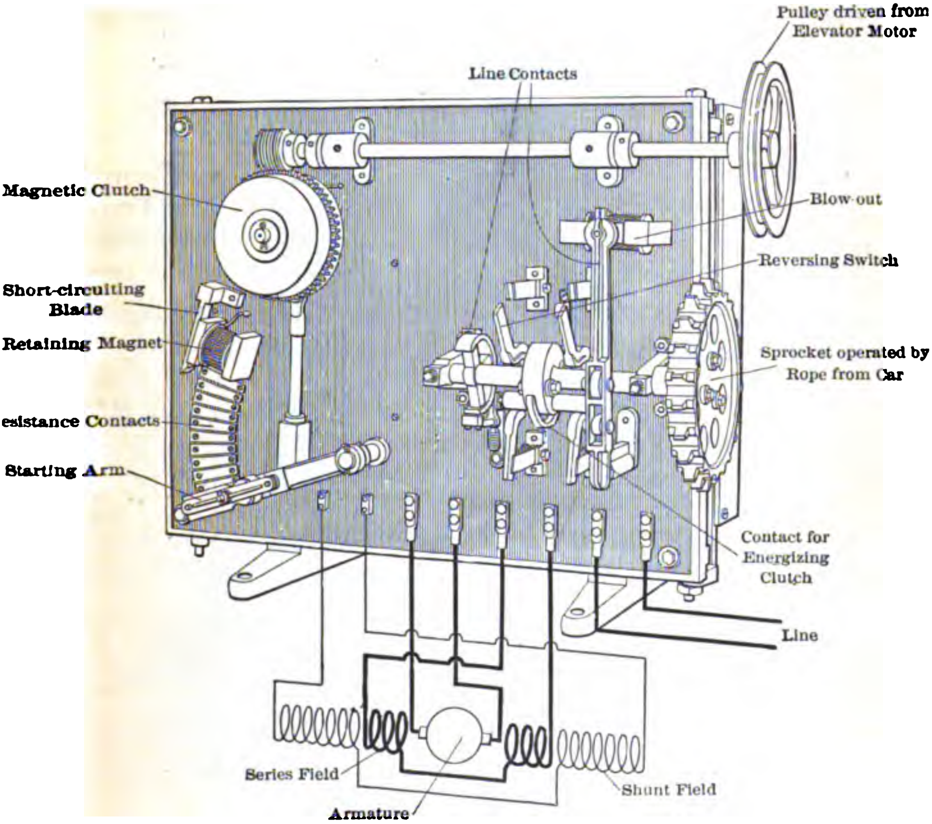

FIG. 497. Mechanically operated elevator controller (The Automatic Switch Co.).

At the same time the contact ring on the shaft of the switch closes the circuit of the magnetic clutch, and energizes it. The horizontal shaft, shown on top of the controller, is belted to the motor, and begins to rotate as soon as the motor starts. When the clutch is energized, the shaft is mechanically connected with the starting-arm, which begins to move upward, gradually cutting out the starting resistance.

When the starting-arm touches the contact marked "Short-circuiting blade," it bridges the clutch and deënergizes it. The arm is held in the upper position by the retaining magnet, as in ordinary starters, such as in Fig. 482. The clutch is deënergized so as to prevent wear due to friction: one part of it continues to revolve as long as the motor is running. Opening the main switch deënergizes the retaining magnet. and the starter-arm drops by gravity.

The sprocket wheel operating the main switch is also connected to the automatic limit device on the winding-machine. In this way it is not only actuated by the operator to start and to stop, but is also automatically opened, as soon as the car approaches the limit of travel in either direction.

The actual electrical connections on the back of the face plate may be easily understood from the above description of the parts and the functions of the controller.

676. EXPERIMENT 30-F. — Study of Mechanically Operated Elevator Controllers. — In performing this experiment it is advisable to have the controller and the hoisting-engine disconnected from the elevator car, so as to operate them separately. This is safer for the beginner, and also more profitable, since combinations may be tried, impossible when the car is in motion. An elevator car can be entrusted only to a mature and experienced student. The necessary load on the hoisting-engine is obtained by putting a Prony brake on the drum, or on a pulley mounted by its side. Perform the experiment in the following order:

(1) Wire up the controller and the motor, as per blueprint, which usually accompanies the controller; also study the permanent connections.

(2) Put current through some of the controller circuits, without allowing the motor to revolve: make sure that the clutches, the brake, the retaining-coil, etc., operate properly.

(3) Start the motor, and adjust the regulation so as to obtain the lowest possible speed.

(4) Observe the operation of the "limits" and of the safety devices in the elevator shaft, and on the machine itself.

(5) Take a few readings with the motor running at different loads. and also when starting and stopping. Read current, speed, e.m.f. across the brushes, seconds time necessary for the operation of controller parts, action of the brake, etc.

677. Non-Automatic Electric Control. — Controllers, operated electrically by a master switch in the car, are shown in Figs. 496 and

498. Such controllers consist of individual electromagnets, which, being energized, close the line and the motor circuits in the required order. The functions of the different parts of the controller shown in Fig. 498 are as follows:

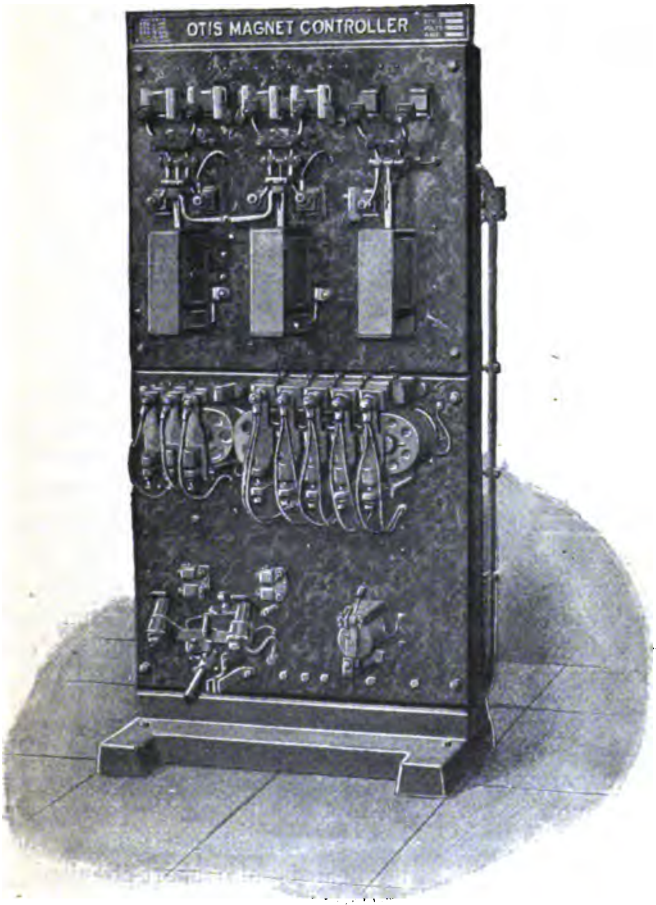

FIG. 498. Electrically operated elevator controller.

Upper row: The two solenoids to the left close and open the main line-circuit and connect the armature of the motor for the "up" or "down" motion of the car. The solenoid to the right is open with the low speed of the car, and is energized when the car switch is moved in the position corresponding to the high speed.

Middle row: The right-hand magnet with five contacts is the accelerating switch, which automatically short-circuits the starting resistance in steps: The magnet to the left, with three armatures, is the so-called "load magnet." It automatically brings the car to a stop at the top and bottom landings at the same point, independent of the load of the car.

Lower row: The switch to the left is an underload circuit-breaker, or, as it is called in this case, a potential switch. The coil which holds it closed is connected across the main line, and in its circuit are connected several switches which act as safety devices. Should trouble develop, the switch flies open and breaks the main circuit. The small magnet to the right is a relay switch, which establishes contacts for the load magnet and for the magnetic brake of the machine.

The motor with which this controller is intended to be used is heavily compounded, as in Fig. 497, and in addition has an "extra" shunt-winding which is operative on the low speed, and when stopping the car. The car switch has two positions for both "up" and "down" directions: One contact is for the low speed, the other for the high speed of the car. The operation of the controller is as follows:

Consider the car somewhere in the middle of the hatchway and the car switch in its zero or center position. No coils on the controller are energized, except that of the potential switch: this switch remains closed all the time as long as the elevator is in regular operation. The first movement of the car switch energizes either the "up" or the "down" coil on the reversing-switch in the upper right corner of the controller: the armature of the motor is thereby connected to the line, in the desired direction. The closing of the armature circuit also excites the brake coil at the motor and releases the brake. The motor and the car are set in motion.

Now the accelerating magnet (second row, to the right) is brought into play: Its magnetizing coil is connected across the armature terminals, and, as the counter-e.m.f. of the motor increases, the several arms of the magnet are operated one after another, short-circuiting the starting resistance. Finally the series field is also short-circuited, and the motor continues to run with the shunt winding only. The magnet armatures are set at different distances, so that they respond in succession to the gradual increase of the potential of the armature. The second movement of the car switch energizes the slow-and-fast-speed magnet, in the upper right-hand corner, with the result that the upper contacts of its switch short-circuit some resistance in series with the motor armature, while the lower contacts open the circuit of the "extra" shunt field. The field of the motor is thereby weakened, while the voltage at its terminals is increased. The car is accelerated to its full speed, either up or down.

The first movement of the car-switch handle from the full running position towards the stop or center position, interrupts the circuit of the slow-and-fast-speed magnet, and the "extra" field is again inserted in the circuit, as well as some additional resistance in series with the armature: This brings the motor to a lower speed. The second movement of the car switch, which brings it to the center, interrupts the reversing-switch circuit, thereby interrupting all of the controller circuits, with the exception of the potential switch. This cycle of operations may be performed as often and as quickly as necessary.

The potential switch is equipped with an auxiliary contact, to which is connected one side of the brake circuits; this allows the brake solenoid to become deënergized the instant the potential switch is opened. Thus, when the safety switch in the car, the extreme hatchway limits, or the slack-cable switch are opened, deënergizing and opening the potential switch, the brake has its current instantly cut off, and becomes operative. At the same time, when the potential switch flies open, it connects across the armature terminals of the motor a "stop resistance." The motor acts then as a generator, and this braking effect aids in bringing it quickly to a stop.

The "load" magnet in the middle row, with three armatures, permits stopping the car at the top and bottom landings at exactly the same point, independent of the magnitude of the load. This is important, since elevators are generally installed with very small overhead room, and very small pit room: Care should be taken therefore to prevent a possibility of running into the overhead work, or against the bottom. This is a serious problem with high-speed elevators, since the load makes a considerable difference in the speed and distance in which the car can be brought to rest. The above load magnet solves this difficulty in the following way:

As the car nears the top or bottom landing, it opens a switch, which introduces some resistance in series wth the armature, and a high resistance in parallel to the armature. The motor slows down and varies its speed according to the load, because of the resistance in series. The heavier the load or the armature current, the larger is the drop in the resistance, consequently the lower the voltage at the armature terminals. After this load variation of speed has had time to establish itself, a second circuit is closed automatically which energizes the coil of the three-contact "load" magnet. According to the load, or to the potential resulting from the speed of the motor, more or less of the load magnet armatures are pulled in, short-circuiting more or less of the resistance in parallel to the motor armature. The slow-down effect resulting from this consequently depends upon the load: The heavier a descending load, the more contacts are closed, the greater is the reduction in the resistance of the parallel circuit, and the greater is the reduction in speed. By taking a sufficient number of contacts on the load magnet, any desired exactness of automatic stop for different loads can be attained.

678. EXPERIMENT 30-G. — Study of Electrically Operated Non-Automatic Elevator Controllers. — See § 676 for directions.

679. Automatic Electric Elevators (Push-Button Control). — Automatic control of elevators is used in places in which a limited use of the elevator does not warrant keeping a special attendant, as, for instance, in residences. Automatic control is also used in the so-called "dumb-waiters," or small elevators intended for carrying goods between the floors of a store or a factory. A bank of push buttons is provided on each floor, and also in the car, if it is intended for passenger service. Each bank consists of as many buttons as there are floors; an extra "stop" or emergency button is also provided.

Thus, if a person on the second floor of a building wishes to send something to the fourth floor by means of the dumb-waiter, when the car is at rest on the first floor, he presses the button No. 2; this closes an auxiliary circuit on the controller, which in turn starts the elevator motor. The car is brought to floor No. 2 and automatically stopped there. The person opens the hatchway door, loads the car, closes the door again, and presses button No. 4. This sends the car upward to the fourth floor, and it stops there automatically opposite the hatchway door, without any further attention on the part of the person who has sent it.

The controller is, in its principle, similar to that described in § 677: it also consists of unit switches, each performing a specific duty. The switches are closed by solenoids, operated by the push-buttons. When the car arrives at the desired point, the solenoid circuit is automatically opened by the "stop-motion" device, which is connected mechanically to the motor. A detailed description of circuits used by different companies would be out of place here; each controller is accompanied by a blueprint of its connections and by detailed instructions for wiring and operation. The understanding of these diagrams may be assisted by keeping the following points in mind:

(1) The push-button circuit is closed through all the hatchway doors and the car door in series, so that the car cannot be started unless *all* the doors are closed. This is an important precaution for the safety of those using the elevator.

(2) A "stop" or emergency button is always provided; pre this button instantly stops the car in whatever position it hap to be. After this its destination may be changed by pressing the corresponding push-button.

(3) After having pressed a button it is not necessary to hold it until the car ends its travel: as soon as the corresponding floor solenoid on the controller is energized, it closes not only the main circuit, but also an auxiliary contact, which keeps the solenoid circuit closed, around the push-button.

(4) The push-buttons, and the solenoids which they operate, are so interconnected that when one of them is in operation no other circuit can be closed, until the car has finished its travel.

(5) If two push-buttons are pressed at once neither solenoid operates. This can be accomplished, for instance, by having a resistance in series with the push-button circuit: when two push-buttons are pressed down the voltage drop in this resistance becomes so large, that the current is no longer sufficient for operating the solenoids.

The most important features of the power circuit are the same as with non-automatic controllers: When the main circuit is closed the magnetic brake is automatically released. The motor starts as a compound-wound machine; when a certain speed is reached, an accelerating magnet gradually short-circuits the starting resistance, and finally the series winding. When the main circuit is opened, the brake is automatically set.

680. EXPERIMENT 30-H. — Study of Push-Button Operated Elevator Controllers. — For directions see § 676.

CHAPTER XXXI.

ELECTRIC RAILWAY WORK.

681. EXPERIMENTS and devices described in this chapter are intended to illustrate certain features peculiar to the operation of electric railways. At the same time it is believed that the principles involved are of sufficient interest to warrant the experiments being performed by the general student of electrical engineering. The subject matter is treated under four headings:

(1) Drum-Type Electric-Car Controller;
(2) Multiple-Unit Control of Trains;
(3) Acceleration and Retardation Tests on Cars;
(4) Air-Brakes.

1. DRUM-TYPE ELECTRIC-CAR CONTROLLER.

682. The street-car controller (Fig. 499) accomplishes electrical connections necessary for starting the car, stopping it, and varying its speed. All these operations are performed by the motorman, by simply turning the handle through a certain number of positions, or "notches." The use of the controller is made as simple as possible, so that the motorman may devote most of his attention to the road and to signals.

The principal parts of the controller are two revolving drums: The large drum to the left is for starting and speed control; the small one to the right, for reversing the motors; each drum is provided with an operating handle. The drums are provided with copper strips which rub against stationary contact fingers. The fingers are connected to the motors, resistances, trolley, ground (car truck), etc. In different positions of the drum, various fingers are connected by the strips on the drum in different combinations, so as to produce the desired control.

The other parts of the controller shown in the cut, are: a blow-out coil for extinguishing the arc, cut-out switches for disconnecting a disabled motor, an insulating frame with partitions, and the cover.

683. Operating Features of the Controller. — The electrical connections in a controller may be best understood from the functions which it is obliged to perform. Electric cars are usually equipped with two or four series-wound motors; in the latter case the two motors of each truck are permanently connected in parallel, and for the purposes of regulation may be considered as one motor. In starting the car, the armatures of the motors, their fields, and some additional resistance are all connected in series, in order to limit the rush of current. As the car acquires speed, the starting resistance is gradually cut out, because the counter-electromotive force of the motors prevents an excessive rush of the current from the line. When all of the resistance is cut out, the two motors run in series *at half speed,* for, if the line voltage is 500 volts, each motor has only 250 volts at its terminals (Fig. 500). To increase the speed, the motors must be connected in parallel, so that each has 500 volts pressure at its terminals. Before this is done, some starting resistance is again inserted into the circuit, to avoid a sudden jerk and a rush of current in changing from half speed to full speed. Then, as the car goes faster and faster, this resistance is again gradually cut out, until the two motors are connected directly across the 500-volt line in parallel, and the car runs *at full speed* (Fig. 501).

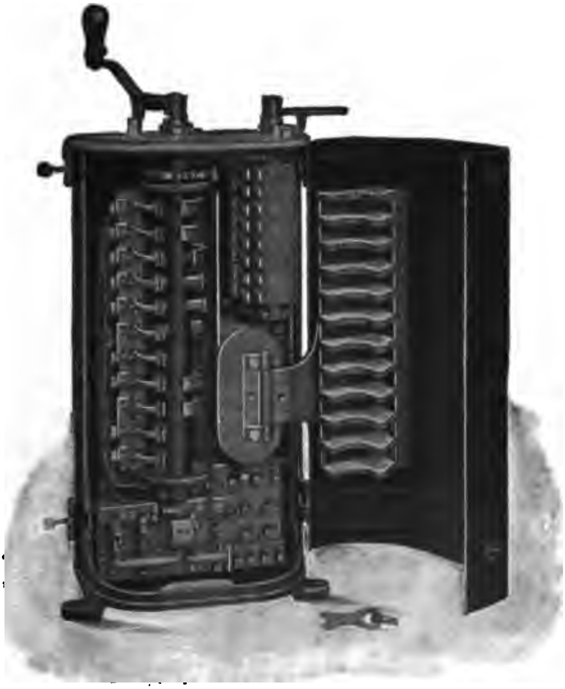

FIG. 499. Electric car controller.

In stopping the car, the motorman first shuts off the power and then applies the brakes. Three kinds of brakes are in use at present: hand brakes, air brakes, and magnetic brakes. Each car is necessarily provided with a hand brake, either for regular service or for emergency. Cars in large cities, high-speed suburban and inter-urban cars, cars running on roads with long and steep grades, etc., are provided in addition with air-brakes (described below, beginning § 704). In a few cities electric, or so-called *magnetic*, brakes are used (§ 692).

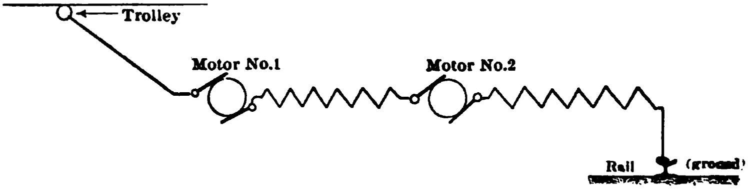

FIG. 500. Motors in series.

Thus, the controller must give two principal combinations of the motors, — in series and in parallel. For each of these combinations intermediate positions are provided, with more or less resistance in the circuit. A complete diagram of connections in a street-car controller, with all details, including a magnetic brake, is shown on the folded plate below. But, before attempting to follow in detail this comparatively complicated system, the student should clearly understand the three simple diagrams shown in Figs. 502, 503 and 508.

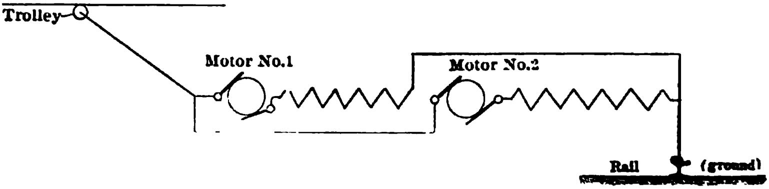

FIG. 501. Motors in parallel.

The complete diagram is a combination of these three schemes: as far as possible, the same letters are used in the elementary plans and in the complete diagram.

684. Connections for Starting. — In describing controller connections it is customary to develop the drum and to indicate the positions of the motorman's handle by vertical dotted lines, as shown in Fig. 502. In this way the connections for any position of the controller may be easily followed out on paper. The connections for starting the car, with the motors in series, are shown in Fig. 502.

When the controller handle is on the first notch, the row of fingers T, R_2, E_1 and A_1, touch the drum contacts along the vertical dotted line 1. The path of the current is as follows:

The current from the trolley flows through the circuit-breaker $C.B.$ to the contact finger T and thence through the strips a_1 and a_2 to the finger R_2. Since the fingers R_4, R_5, and R_6 do not touch the corresponding strips a_3, a_4 and a_5, the current flows through the whole resistance R_2R_6, through motor No. 1 to the finger E_1. Thence through b_1 and b_2 to motor No. 2, and finally to the ground G.*

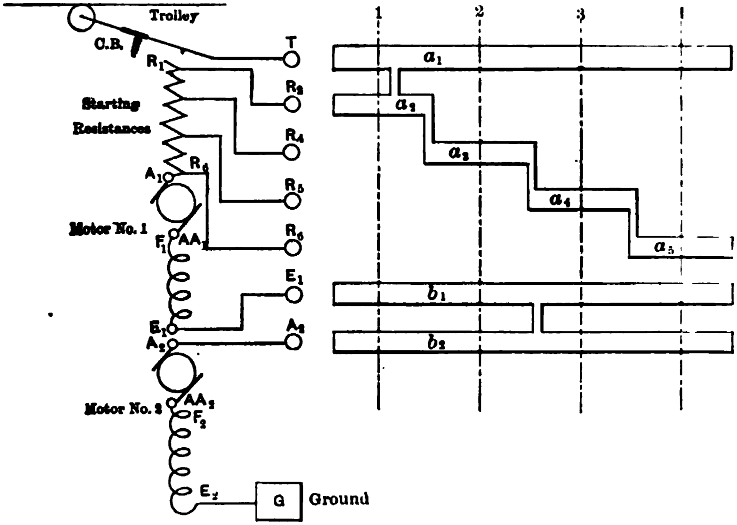

FIG. 502. Controller connections with motors in series.

On the second notch the current finds a shorter way from T through the strips a_1, a_2, and a_3 to the contact finger R_4, so that part of the resistance between R_2 and R_4 is cut out; in other respects the current follows the same path as before. On the third notch the contact R_5 becomes operative; finally, on the fourth notch the current from T follows directly through a_1, a_2, a_3, a_4 and a_5 into the armature of the first motor, and the whole resistance R_1R_6 is cut out of the circuit: this gives the *full series* (half-speed) connection.

(*) Only *one* wire (trolley wire) is used in electrical-railway circuits, the current returning to the generating station (or substation) through the rails; the ground on the car is obtained by connecting the wire G to the truck. The current flows through the truck, axles, and wheels, to the rails; one of the bus-bars in the station is also connected to the rails.

Positions 1, 2 and 3 are called the "resistance points"; 4 is th "running" position. The motorman is not allowed to ride for an length of time on these "resistance points," but must use them fo starting only. The starting resistances, used on cars, are not larg enough to be permanently connected into the circuit; they would ge excessively hot and burn out in a short time.

It is well to remember that a diagonal chain of contacts on a controller diagram, such as a_2, a_3, a_4, a_5, usually signifies a gradual cutting in or out of resistances.

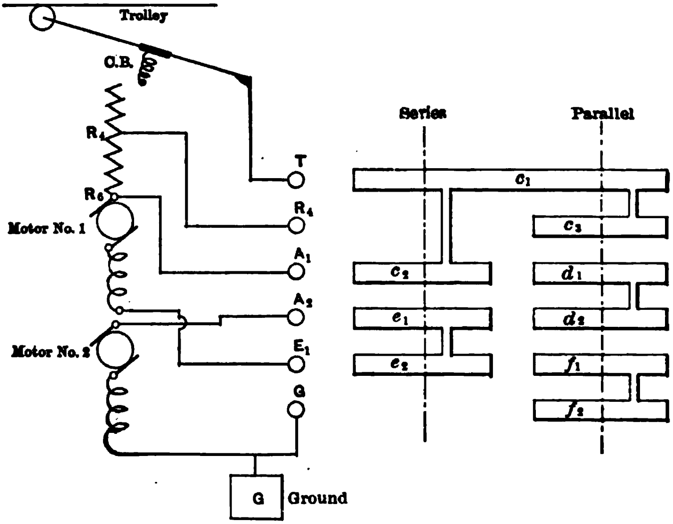

FIG. 503. Change from series to parallel.

685. Change from Series to Parallel.—To change from half speed to full speed the motor connections have to be changed from series to parallel, and some resistance temporarily introduced into the circuit. The controller fingers and strips that are operative in this change are shown in Fig. 503. With the motors in series, the current from the trolley finger *T* flows through c_1 and c_2 into the first motor; then through e_2 and e_1 to the second motor, and to the ground. On the first "parallel" notch the current from *T*, through c_1 and c_3, flows to the resistance R_4R_6, and then *is divided* into two halves: One half follows the path — motor No. 1, E_1, f_1, f_2, G, to the ground; the other half — finger A_1, d_1, d_2, A_2, motor No. 2, to the ground. On the next notches (not shown) the resistance is gradually cut out, until the motors are in "full parallel."

This change involves breaking the circuit; as the connections are changed hundreds of times during the day, the resulting sparks would quickly deteriorate the fingers. Therefore, intermediate points are provided for changing from series to parallel, without opening the circuit. The principle is shown in Fig. 504: The upper line (*a*) represents the connections on the last notch, with the motors in series and the resistance short-circuited; the last line (*e*) corresponds to the first parallel position, same as in Fig. 503. The intermediate connections are shown on the three sketches (*b*), (*c*) and (*d*) between these: First, some resistance is introduced into the circuit; then motor No. 2 is short-circuited (or, which amounts to the same, motor No. 1 is grounded around it). After this, the circuit of the second motor is opened, and the connections changed from series to parallel. These changes are made in the positions 6 to 8 of the controller handle (see folded plate). These positions are called transition points, and the motorman should pass them without stopping.

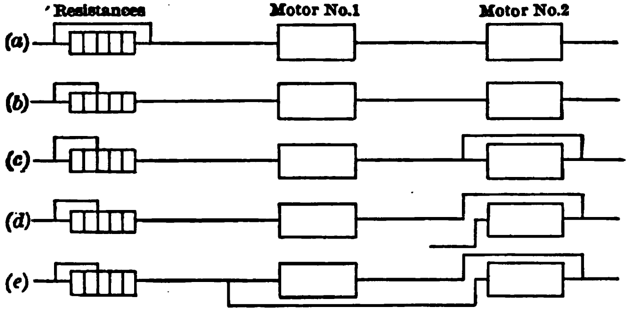

FIG. 504. Change from series to parallel without opening the circuit.

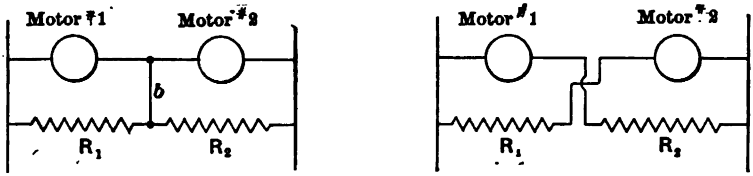

FIG. 505. "Bridge" method for changing from series to parallel.

686. Bridge Connections. — The above method of changing from series to parallel has a disadvantage that the current, and consequently the torque of the motors, change suddenly, producing an unpleasant jerk during acceleration. This is obviated in the so-called bridge connection recently put into practice. The principle of the bridge connection is shown in Fig. 505: The sketch to the left corresponds to the last notch, with the motors in series; the sketch to the right gives the connection on the first notch, with the motors in parallel. After all the resistances have been short-circuited and the motors are connected in series across the line, two equal resistances, R_1 and R_2, are connected across the line. The value of the resistances is such that they take about the same current as the motors. This doubles the current taken from the line, but does not affect the torque of the motors, which are in a separate circuit. Now the bridge connection *b* is established between the motors and the resistances: No current flows through this bridge, both its extremities being at the same potential. With this bridge, motor No. 1 and the resistance R_2 may be considered as one circuit, and motor No. 2 with the resistance R_1 as another circuit. Separating these two circuits gives the diagram shown at the right. The motors are now connected in parallel, with some protective resistance in series with each. The change is accomplished without breaking the circuit, and with practically no change in current through the motors, since hardly any current flows at first through the bridge *b*.

Bridge connection is used on large cars, especially those provided with multiple-unit control (see § 695): with this arrangement, no jerk is felt during the change from series to parallel, and starting is accomplished smoothly, even with high rates of acceleration.

687. Other Features of the Drum Controller. — The connections described above are incorporated in the complete diagram of the controller shown on the folded plate. A glance at this diagram will show, however, that the connections are more complicated than a mere combination of the series-parallel control. The complications are caused by the addition of the following parts:

(1) A reversing drum for backing the car.

(2) Cut-out switches for disconnecting a disabled motor.

(3) Blow-out magnet for extinguishing electric arcs.

(4) Contact points for shunting motor fields, to obtain a higher speed.

(5) Contact points for magnetic brake.

In addition to these, the requirements of construction and insulation necessitate a somewhat different disposition of the controller fingers and of the drum strips, than that described above. However, keeping in mind the above two fundamental schemes (Figs. 502 to 504), the following up of the actual connections will hardly offer serious difficulties.

The five above-enumerated features are taken up below, more in detail.

M
S
AN
R

688. Reversing the Motors. — The motors are reversed by interchanging the armature leads; a separate drum shown to the right, in Fig. 499, is provided for this purpose. The development of this drum is shown in the upper part of the large diagram (folded plate), to the right of the development of the main drum. The reverser is, in principle, an ordinary double-throw switch. The leads A_1 and AA_1 come from the terminals of the armature of motor No. 1; terminal 12 is connected to one end of its field winding, terminal 20 to the resistances. In the "forward" position of the reversing drum, terminal 12, or the field of motor No. 1, is connected to AA_1, while terminal A_1 is connected to terminal 20, or to the resistances. In the "reverse" position of the drum, terminal 12, or the field of the motor, is connected to A_1, while AA_1 is connected to 20, or to the resistances. The connections are identical in the two cases, except that the armature leads are interchanged. Similar connections are provided for the other motor, by means of the fingers 11, AA_2, A_2, 17 (see also Fig. 492).

A reversing drum is not supposed to be operated with the power "on"; for this reason no blow-out coil is provided. The two controller handles (Fig. 499) are so interlocked mechanically, that the reverser can be operated only when the regulating handle (the left-hand handle) is in its "off" position. Thus the motorman is forced to first open the circuit by means of the regulating handle, before he can reverse the motors. Moreover, the interlocking is such, that the regulating handle cannot be moved when the reverser is in its middle or "off" position. It is easy to see that without this feature, the interlocking would not be complete.

A third feature of interlocking is that the reversing handle cannot be removed, except in the "off" position. When the motorman leaves the car, he is supposed to take this handle with him. To do so, he has first to bring his regulating handle to the "off" position; only then can he turn the reverser drum into the neutral position and remove the handle. All the circuits are thus broken, and no one can start the car, except by having an identical handle.

689. Cut-Out Switches. — Switches for disconnecting a disabled motor are shown at the bottom of Fig. 499; in the folded plate they are shown below the reversing drum. It will be seen from the diagram of connections that if motor No. 1 is disabled and the motorman opens the corresponding switch, a by-pass is offered to the current through the auxiliary contact marked "3 and R_6" to motor No. 2, while the field, the armature and the shunt circuits of the disabled motor are opened. If motor No. 2 is disabled and its switch opened, a by-pass to the ground for the other motor is offered by the auxiliary contact

G, behind the switch, while the field and the armature circuits of the disabled motor are opened.

690. Blow-Out Coil. — The blow-out coil is shown in Fig. 499 below the reversing drum. It is intended for blowing out the electric arc between the controller fingers and the drum strips, every time the circuit is broken. Its application is based on the fundamental fact (Fig. 506) that an electric current, being placed in a magnetic field, tends to move *across* the field; in other words, so as to cut the lines of force. The pole-pieces *MM* are placed so that the lines of force are perpendicular to the direction of the arc between *A* and *B*, and tend to move the arc outside, increasing its length and finally rupturing it. The blow-out coil is one of the most essential parts of the controller: it must be remembered that the circuits of a street car are broken thousands of times a day, and without a blow-out arrangement the sparks would deteriorate the contacts in a very short time.

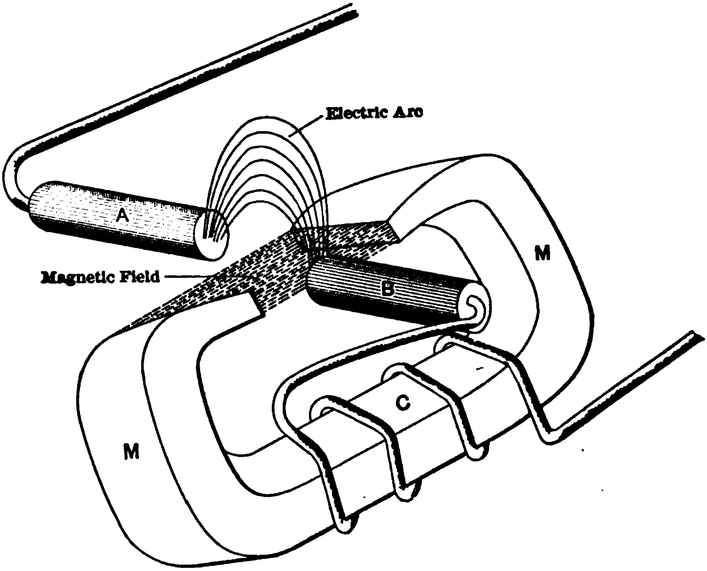

Fig. 506. The action of a magnetic blow-out.

The coil of the blow-out magnet is usually energized in series with the line, so that its action is stronger the heavier the current to be ruptured. In the folded plate, the blow-out magnet is shown in the lower left-hand corner. It will be noted, that in this case the coil consists of two sections, one operative when the power is "on," the other when the magnetic brakes are applied.

691. Shunting Fields. — The speed of direct-current motors may be increased by weakening their fields (§ 329); in series motors this is done by shunting the fields by certain resistances. This method had been used in the earlier days of electric traction, but has been abandoned of late years on account of sparking at the commutator, due to weakened field. However, with the advent of the compensated motor (§ 340), in which the armature reaction is greatly reduced, shunting of fields seems to gain in favor again.

Two positions with fields shunted, are provided in the controller, shown on the folded page: Position No. 5 is that with the motors in series, position No. 12 with the motors in parallel. The shunting resistances are permanently connected to the motor fields on one side, while the other ends of the resistances are connected to the controller fingers L_1 and L_2. In the positions Nos. 5 and 12 these fingers become operative, closing the shunts and thereby weakening the fields of the motors.

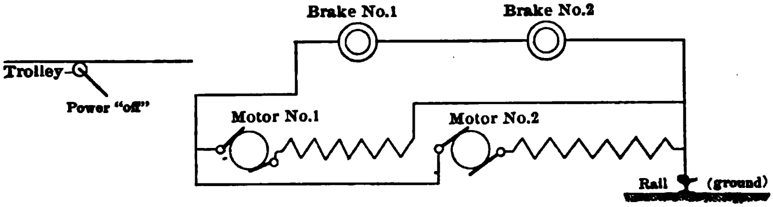

FIG. 507. Connections with magnetic brakes "on."

692. Magnetic Brakes. — The principle of magnetic braking is substantially as follows (Fig. 507): The motors are disconnected from the trolley line, and their fields and armatures are connected in such a way that they act *as generators*, utilizing the acquired momentum of the car for supplying the energy necessary for producing current. The car is provided with brake shoes made in the form of electromagnets, which are placed either near the rails or near iron disks mounted on the axles. When the current, generated by the motors, is sent through these electromagnets, they become energized and press themselves against the rails, or the disks on the axles, producing a powerful braking effect. The intensity of braking is regulated by the amount of resistance in the circuit.

Formerly a method of electric braking was used, which consisted in closing the motors, acting as generators, on resistances, in which the stored energy was dissipated in the form of heat. This method of braking has been abandoned, because it is liable to overload the motors: they have no time to get cooled off, even while the car is coasting down a long grade.

Controller connections for magnetic braking are shown in Fig. 508: The two motors are connected in parallel by the strips i_1 and i_2 on one side, and by h_2 and h_3 on the other; the current generated in both motors flows through h_1 and the resistances R to the strip g_1 and thence through the contact-finger B to the brakes; the circuit is completed through the ground G. The three positions shown in the sketch differ only in the amount of the resistance R in the circuit. Brake connections on the large diagram will be understood from this simple scheme.

There are two more points to be noted:

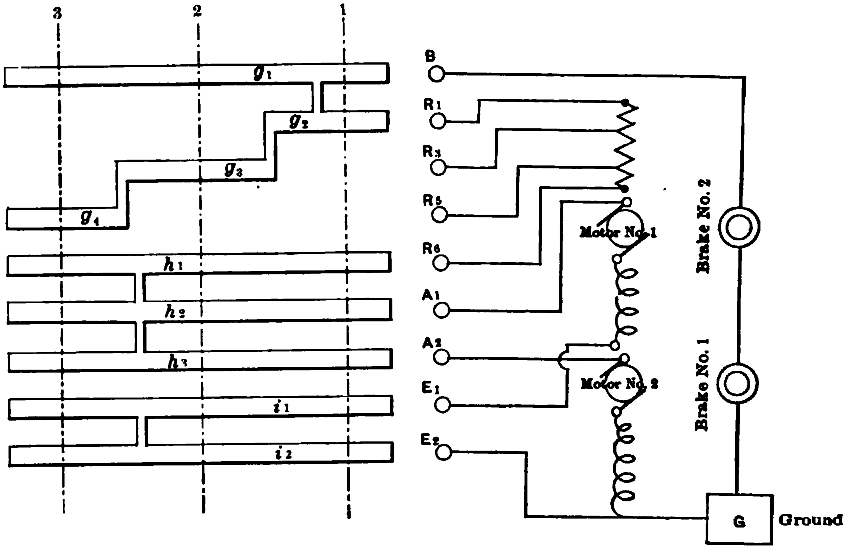

FIG. 508. Controller connections for magnetic brakes.

(1) *The brakes must be demagnetized* before power is put on, because of residual magnetism which keeps them attracted, even after the current has been shut off; therefore, additional contacts *B* and *DR* (" brakes " and " demagnetizing resistance ") are provided on the first power notch. By means of these contacts, a small current is sent through the brakes in the direction opposite to that in which they are usually magnetized. This destroys the residual magnetism, and the brakes are released. The corresponding scheme of connections is shown separately in the lower right-hand corner of the sheet.

(2) *A current-limiting electromagnet* is connected into the brake circuit, as shown in the lower left-hand corner of the diagram. When the braking current rises beyond a certain limit this electromagnet closes the limit switch which short-circuits the fields of the motors:

this reduces the main current, and the electromagnet again releases the switch. No circuit-breaker or fuse could be used for this purpose, because the device must be *self-restoring.*

693. Acceleration Limit. — In order to prevent the motorman from moving the controller handle too fast at the start, special devices are sometimes used, which limit this speed to a certain predetermined value. A simple apparatus of this kind, the so-called "Automotoneer," is based on the pawl-and-ratchet principle; every time, after the motorman has moved the handle by one notch, he has to give it a slight backward movement in order to release the "dog." This makes it impossible to move the handle above a reasonable speed.

In another device, the handle is connected to the controller drum through a heavy spiral spring. By moving the handle the motorman winds up the spring, but the motion of the drum is retarded by a dash-pot, which may be set to operate at any predetermined speed. The motorman may thus throw his handle at once around to the last parallel position without effecting the speed at which the drum follows it.

A third method is to regulate the speed of the controller drum by a current relay. The drum is governed by a magnetic clutch operated by a relay, having its coil connected into the main circuit. The clutch holds the drum until the current has fallen to a predetermined value, after which the drum is allowed to go to the next notch.

It is claimed that considerable economy in power consumption, and a better schedule, are obtained by using the above-described devices, since they limit the rush of current at the start and permit of a more uniform acceleration.

694. EXPERIMENT 31-A. — Operating an Electric-Car Controller. — The laboratory equipment for this exercise should consist of a regular drum-type series-parallel controller, and of two street-car motors which may be connected to it. It is well to have the two motors either belted or direct-connected to each other, in order to make them run at the same speed.

It is hardly feasible to operate this equipment on a 500-volt circuit, because the motors, having no load, would run away. The performance of the controller is studied just as well on an ordinary 110-volt circuit, at which voltage 500-volt motors run at a moderate speed even at no load.

The student is expected to connect up the controller to the motors and to the power supply, and to operate it as if it were on an actual car. A blueprint of connections usually accompanies each controller. The connections may be different from those shown on the folded plate.

but with a clear understanding of the above diagram and of the principles explained in §§ 682 to 693, the student will find no difficulty in understanding the connections in any street-car controller.

The points, to which particular attention should be paid during the operation, are:

(1) Interlocking of handles.
(2) Disconnecting a disabled motor.
(3) Operation of the blow-out coil.

Report. Give the actual diagram of connections, or at least any parts of it which are different from those described above. Explain the mechanical details of the interlocking devices. Describe peculiarities, if any, observed in the operation of the controller and of the motors.

2. MULTIPLE-UNIT CONTROL OF TRAINS.

695. As long as electric cars were operated singly, the ordinary hand-operated drum-type controller, described above, was entirely satisfactory. But with the advent of electric trains, such as are operated on elevated, underground, and some inter-urban roads, new problems of car control presented themselves. The first electric trains consisted of one motor car and several trailers, as on steam roads, but this method was soon found objectionable. Not only is the acceleration obtained too low for fast service, but the trailers cannot be operated separately during the hours of small traffic, thus depriving the electric system of one of its great advantages, — flexibility.

It was soon perceived, that the right solution was to have trains consisting of *several motor-cars* operated by a motorman from the front car. This brought up the question of controllers by which all cars on the same train could be operated simultaneously, or each car be used separately, if so desired (Fig. 509). It would not be practicable to carry heavy electric currents through the train and to distribute them to different motors; each car must collect its own current through a trolley, or through contact shoes from the third rail. Therefore, each car must have its own controller, operated by some electrical or mechanical means by the motorman on the front car.

Two sources of power are available for such control: electric power from the line, and compressed air stored for the brakes. The first method is used by the General Electric Co., the second method by the Westinghouse Electric and Mfg. Co. This system of control, with which each car is a separate unit, and at the same time any number of cars may be operated in multiple, is called the *multiple-unit control.* The controllers themselves do not need to be in the form of a drum, since they are not operated by hand. They usually consist of separate

" contactors " or "unit switches " (Fig. 509) which are closed and opened either by electromagnets, if electric control is used, or by stems of pistons, in the pneumatic control. In either case, auxiliary wires are provided, running through the train and controlled by the motorman.

In the system used by the General Electric Co., the auxiliary circuit is supplied from the line, and it energizes the operating solenoids of the contactors. In the Westinghouse electro-pneumatic control, the valves which admit compressed air into the contactor cylinders are operated electrically. The necessary current is supplied from small 14-volt batteries, through the auxiliary wires running through the train.

696. Principles of Operation of a Multiple-Unit Controller. — The principle of operation of a General Electric multiple-unit controller may be gathered from Fig. 509. There are two distinct circuits, both fed from the trolley: the power circuit is shown by heavy lines, the control circuit is shown by light lines. Two motor cars of a train are shown, the motorman being supposed to be on car No. 1.

To simplify the explanation of the principle of the control, each car is assumed to have but one motor, with but three positions of control: (1) Circuit open, (2) circuit closed through the starting resistance, and (3) the motor connected directly to the line. When the contacts at *A* and *B* are open, the motor circuit is open; closing the contactor *A* sends a current through the motor, in series with the starting resistance; when the contactor *B* is closed, the resistance is short-circuited and the motor connected directly to the line.

The contactors *A* and *B* on all the cars of a train are operated from two train wires, either of which may be energized from the master controller, operated by the motorman. In the position of the master controller, shown in the diagram, all the circuits are open. When the controller drum is moved to the position 1, the upper train wire is energized, causing the operation of the contactors *A* throughout the train. When the master controller is moved to the position 2, the lower train wire is also energized; the contactors *B* are closed on all cars. Returning the master controller to its zero position opens all the contactors. With this arrangement, a train consisting of any number of cars may be operated by the master controller of any car; at the same time each car may be operated singly, if so desired.

Two contactors only, on each car, are shown in Fig. 509; in reality the connections are much more complicated, because about 15 contactors are required on each car in order to perform all the necessary connections for starting two or four motors, gradually cutting out resistances, changing from series to parallel, and reversing. It would lead to needless complications to give here a complete diagram of connections for a multiple-unit controller. Moreover, such a diagram would be of little practical value, since the connections for various types greatly vary in details, so that a blueprint must be supplied

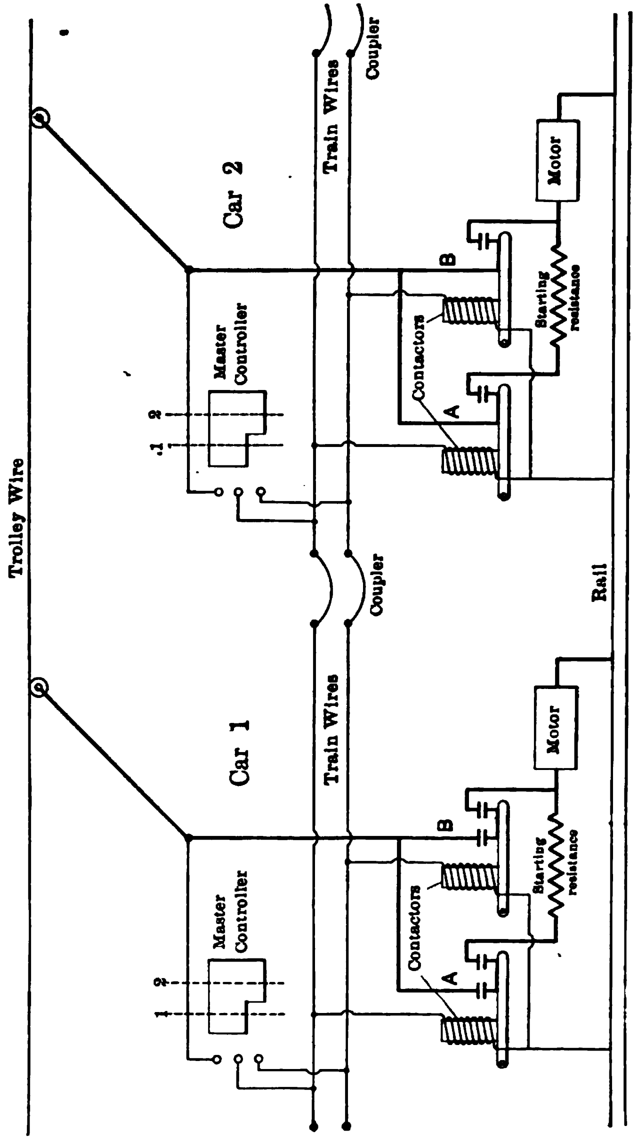

FIG 509. Diagram illustrating the principle of the multiple-unit control of trains.

with each controller. A diagram of the general disposition of parts in a car is shown in Fig. 510 and is explained in the next article. With a clear understanding of this diagram and of the principle illustrated in Fig. 509, the student should find no difficulty in tracing out the actual connections on a given controller.

In the Westinghouse electro-pneumatic controller the contactors, such as *A* and *B* (Fig. 509), are actuated by compressed-air cylinders. The auxiliary, or the control circuit may be supposed to be fed from a small storage battery. Energizing one or the other of the train wires admits compressed air into the corresponding contactor cylinders, operating the pistons. The admission values are electrically controlled.

697. Disposition of Parts of a Multiple-Unit Controller in a Car. — The general arrangement of the parts of an electrically operated controller is shown in Fig. 510. The power circuit is again shown by heavy lines, the control circuit by light lines; several wires of either circuit, running together, are combined in a cable. All the contactors of the car are placed in a box, which is connected to the source of power, and to the motors, through the reversing switch (reverser). The motor rheostats are also connected to the corresponding contactors. In this way any desired connections are established between the trolley, the motors, and the rheostats. The reverser, which is also electrically operated, determines the direction of motion of the car.

The auxiliary circuit comprises (*a*) the train-control cable, running through the whole train, and (*b*) the car-control cable through which the contactors of a particular car are energized. When the master controller on the front car is set in a certain position, definite wires of the train-control cable are energized, and they energize, through the " connection boxes," the corresponding wires of the car-control cables on all the cars. The latter wires operate certain contactors, and thus establish the required connections in the power circuit.

Each car is provided with two master controllers, as it may be desired to operate it from either end. A cut-out switch and fuses are provided on each car-control cable, in order to be able to disconnect a disabled equipment. The control rheostat, shown in the diagram, contains resistances in series with the windings of the contactor solenoids; these resistances reduce the control currents to the required value.

698. Details of Operation. — The most noteworthy feature of the multiple-unit controller is *automatic acceleration.* The motorman may throw his handle in the "full parallel" position, corresponding to the maximum speed, from the very start: with an ordinary drum controller this would either open the circuit-breaker or produce a most undesirable jerk of the car. But the unit switches, or contactors, are electrically interlocked, so that they can pick up in a certain succession only. Moreover, as soon as the line current exceeds a certain limit, the current-

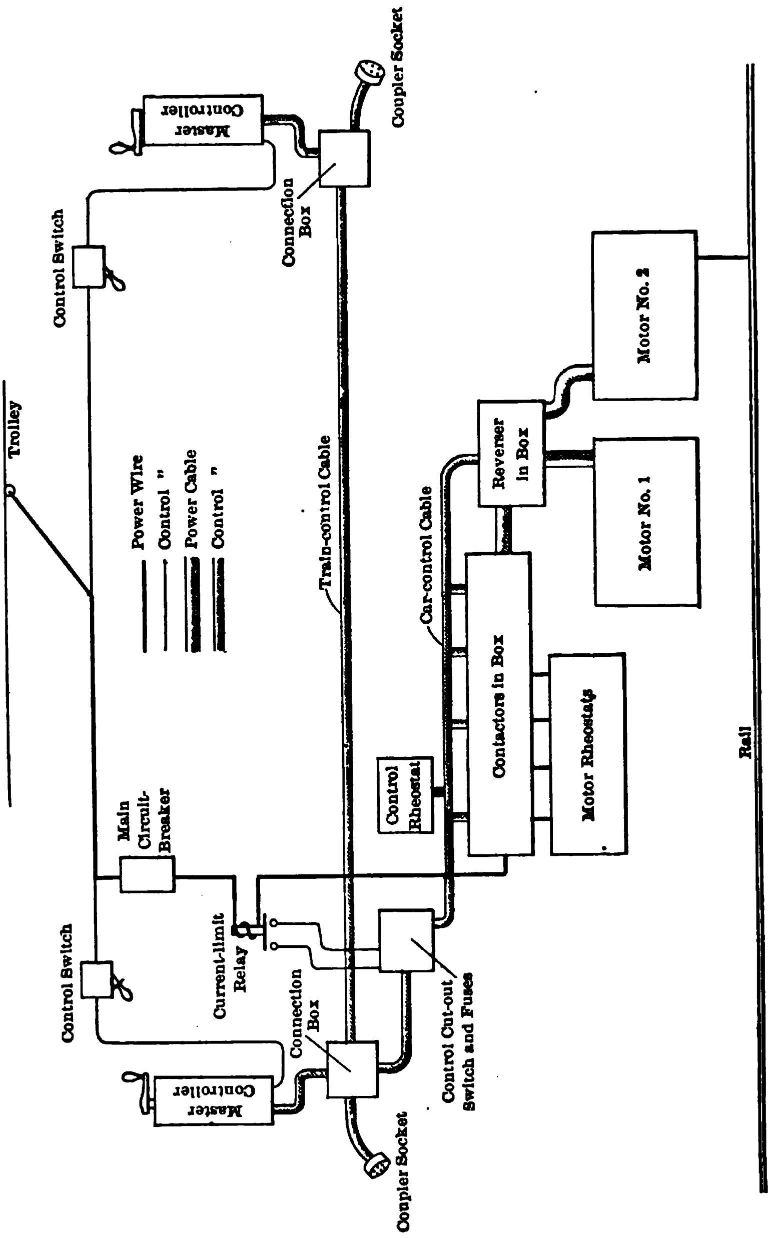

Fig. 510. Diagram of a car equipped with a multiple-unit controller.

limit relay, shown in Fig. 510, prevents any further change or connections, at the same time holding up the contactors already closed.

The master controller has two handles, one for speed control, the other for reversing the motors; in this respect it is similar to the hand-operated controller, shown in Fig. 499. As the gradual cutting out of resistances is done automatically by the contactors themselves, there are but four positions of the regulating handle:

(1) Motors in series, with all resistance "in."
(2) Full series.
(3) Motors in parallel, with all resistance "in."
(4) Full parallel.

The interlocking of the handles is the same as explained in § 688.

In the first position of the handle, one of the control wires leading into the reverser box is energized, and the reverser is thrown into the "forward," or the "reverse" position, according to the position of the reversing handle of the master controller. In this movement the reverser closes an auxiliary contact which completes the circuit of the coil of the so-called "series" contactor. The contactor is energized and closes the motor circuit with all the starting resistances in series. The train starts, but no more contactors are brought into play, as long as the master controller is on the first notch.

By moving the handle into the second position, the motorman closes the circuit of the so-called "accelerating" wire, which energizes a *second* contactor; this contactor short-circuits part of the resistance in series with the motors. At the same time, this second contactor closes the circuit of the solenoid of a *third* contactor, which short-circuits some more resistance in the motor circuit. This process goes on, until all the resistance is short-circuited, and the motors are in full series.

The speed at which the successive contactors are lifted up, is controlled by the above-described current-limit relay. When the first contactor in the second position of the handle is closed, it automatically shifts the circuit of its own solenoid to that of the preceding solenoid, making it thus independent of the "accelerating" wire. If the main current is too heavy, the accelerating circuit is opened by the current-limit relay, preventing the next contactor from being energized. This does not open, however, the contactor already in the circuit, because its solenoid is now connected around the accelerating wire. As soon as the current drops to its normal value, the current-limit device again closes the circuit of the accelerating wire, which energizes the next solenoid, etc. The plunger of the relay has some lost motion, so that it does not open the accelerating circuit, before the solenoid has been transferred to the other circuit.

In the third position of the operating handle of the master controller, additional control wires are energized, other contactors are brought into action, and the connections of the motors are changed from series to parallel. Bridge connections are used (§ 686) with this type of unit-multiple controller. In the fourth position of the motorman's handle, operations are performed similar to those in the second position, but with the motors in parallel.

It is clear then, that the controller handle may be moved into the fourth position from the very start, because the contactors can be energized in a certain order only, and the rate, at which the successive contacts are closed, is controlled automatically by the current-limit relay.

699. EXPERIMENT 31-B. — Operating a Multiple-Unit Controller. — The purpose of the experiment is to give practice with the circuits, construction and operation of this system of control. The principles explained in the preceding articles apply to an electric as well as to an electro-pneumatic controller. An exact diagram of connections always accompanies a controller. It is out of the question to have all the wiring done during one or two laboratory periods, but the most important connections should be made by the student himself, in order to become familiar with the wiring of the controller.

Connect the controller to two railway motors and operate it under the conditions under which it is operated in practice. Observe the action of the interlocking and safety devices. If possible, put a certain load on the motors, and also have them belted to a fly-wheel to imitate the inertia of the car. Measure the total time of operation of the contactors, also approximately the moments at which the new contactors operate. Note the corresponding fluctuations of the current. Vary the load, the inertia, the setting of the current-limit relay, and observe the difference in the operation of the controller. Make sketches of the contactors, of the reverser, the relay and the master controller.

Report the actual connections, or at least the details by which they differ from those described above. Draw rough sketches showing the construction of the most important parts of the controller. Give the results showing the time of starting, fluctuations of current, effect of various settings of the current-limit relay, etc.

3. ACCELERATION AND RETARDATION TESTS ON CARS.

700. Car schedules on long suburban and inter-urban lines depend essentially upon the maximum speed which the motors can develop in a given car and with the profile of the road. On the contrary, car schedules within city limits depend to a considerable extent on the rate of *acceleration* and deceleration (retardation) possible with a given equipment. Stops within city limits are so frequent, that sometimes a larger part of the time is taken by accelerating and stopping the car, than in running it at a uniform speed.

A study of car acceleration and deceleration is thus important for determining the schedules and the power consumption to be expected with a given equipment. Experiments on actual cars are performed from time to time by operating and manufacturing companies, and the results are used as a basis for designing new equipments. Numerous tests of this kind were performed in 1904 under the supervision of Profs. Norris and Swenson, acting for the Electric Railway Test Commission of the Louisiana Purchase Exposition. The very interesting results of these tests may be found in the printed report of the commission.

In order to observe the performance of an actual car, with the action of its inertia, friction, air resistance, influence of grades and track curvature, etc., a long experimental track and a specially equipped car must be available. All that can be attempted along these lines, in the laboratory, is an approximation on a small scale to actual conditions; and a demonstration of instruments, used in actual car tests for recording amperes, speed, distance, etc. At the same time it is believed, that some acquaintance with the methods and devices used in acceleration and retardation tests, as well as some practice in working out the results, constitute a good preparation for the performance of similar experiments on a larger scale. Moreover, even such a laboratory experiment helps one to understand the most important factors affecting the practical operation of electric cars.

701. Description of Apparatus. — A simple laboratory arrangement for acceleration and retardation tests, which may be built at a moderate cost and which well serves the purpose, is shown in Fig. 511. It consists of a small series motor, belted to a shaft which carries one or more heavy fly-wheels and a pulley provided with a Prony brake. The motor represents the motor equipment of a street-car; the fly-wheels play the part of the car inertia; the Prony brake load supplies the frictional, grade and other resistances of the car. The set can be run under various electrical and mechanical conditions, and the results observed may be applied, at least qualitatively, to actual cars.

Speed and current vary so rapidly during the period of acceleration, that it is hardly possible to take sufficiently accurate readings with an ordinary tachometer and an ammeter. Therefore, recording instruments must be provided, similar to those used in testing actual cars. In the apparatus shown in Fig. 511 a roll of paper is driven at a uniform speed by a chronograph. The pencil *A* traces amperes, the pencil *T* marks seconds of time, and the pencil *D* marks distances run by the car. These three pencils are actuated as follows:

The pencil *A* is connected to a cord fastened at the other end to a handle *h* mounted on the ammeter. This ammeter is connected into the motor circuit and indicates amperes input into the motor. An observer follows with the handle *h* the indications of the ammeter pointer as closely as possible, and in this way transmits them to the pencil *A*. The pencil *T* is operated by the electromagnet *t* whose circuit is closed once a second by the pendulum *S*. The pencil *D* is actuated by the electromagnet *d*, whose circuit is closed once during each revolution of the fly-wheel shaft. With this arrangement, amperes are plotted directly to time as abscissæ, while from the marks made by the pencils *T* and *D* the time-distance curve or the speed curve of the car can be easily constructed.*

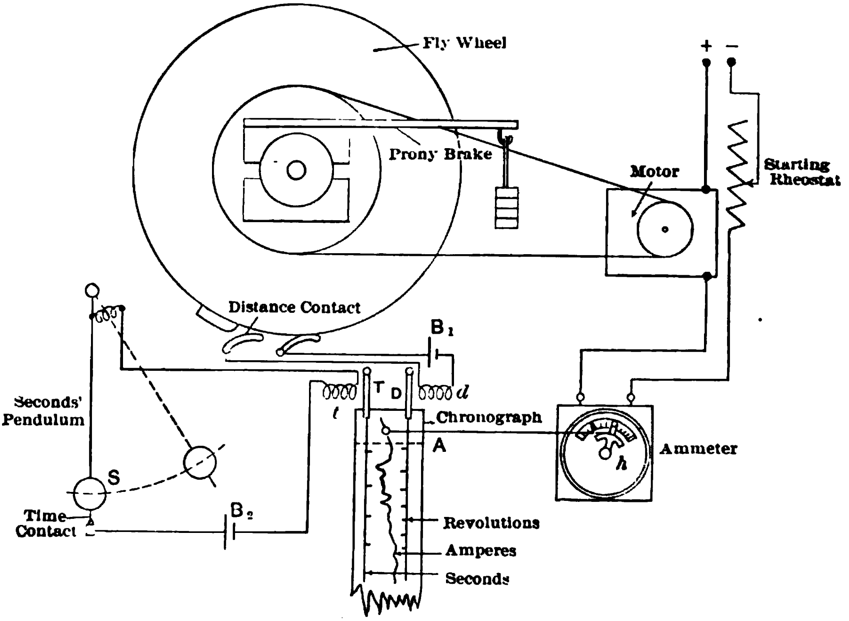

FIG. 511. A laboratory apparatus for study of acceleration of electric cars.

702. Car Performance Curves. — Car performance curves, in their elementary aspect, are represented in Fig. 512. In the most general case they consist of four parts: (1) acceleration period, (2) running at full speed, (3) coasting, and (4) braking. The toothed part of the current curve corresponds to the first part of the acceleration period, when the controller resistances are gradually cut out. With a two- or four-motor equipment this part consists of two distinct portions — motors in series and motors in parallel. The smooth part of the current curve during the period of acceleration corresponds to the increase in speed at full voltage. During the second period (full speed) the current is constant, if the profile of the road is uniform; then the current is cut off, the car is allowed to coast for a reasonable length of time, and finally the brakes are applied in order to stop it at the right place.

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* For some other special instruments of this kind see Chapter III in Ashe and Keiley's *Electric Railways*, — Train Recording and Indicating Instruments.

The distance covered by the car (counting from the preceding stop),

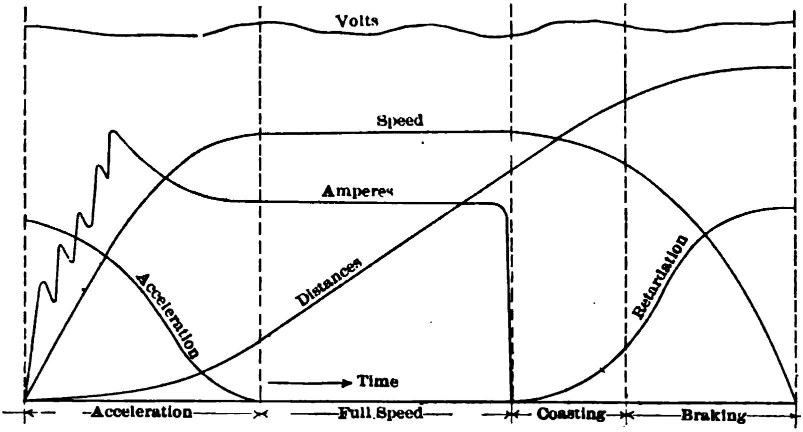


FIG. 512. Car performance curves.

increases at first slowly, then faster, and when the car reaches its final speed the distance increases proportionally to the time. During the periods of retardation the distance again increases more slowly. The speed curve reaches its maximum in the middle portion and gradually decreases to zero at both ends. The ordinates of this curve are proportional to the tangents of the distance-time curve ($v = ds/dt$).

Acceleration is positive during the first part of the run, and negative during the last two periods. When the car is running at full speed the acceleration is $= 0$. The ordinates of the acceleration curve are proportional to the tangents of the speed curve (accel. $= dv/dt$).

703. EXPERIMENT 31-C. — Acceleration and Retardation Tests on an Electric Car. — The experiment consists in getting data for the four parts of the above performance curves (Fig. 512). These parts are then put together to form complete performance curves, from which certain deductions may be drawn in regard to the equipment and the possible schedule.

(1) Begin the test by calibrating the Prony brake for different speeds; in other words, determine the values of torque which must be applied to the pulley in order that the motor may reach certain final speeds. Then begin taking data for the first part of the performance curves (acceleration). Use recording instruments, such, for instance, as are shown in Fig. 511, and take several runs, varying the following factors:

(*a*) The period of time during which the starting resistance is gradually cut out. Move the rheostat handle as evenly as possible. The more slowly the resistance is cut out, the more time it takes to reach the final speed, but the less is the rush of current; the subsequent heating of the motor and the waste of power are thus reduced.

(*b*) The final speed attained by the motor: this is done by properly setting the Prony brake according to the calibration, as mentioned before.

(*c*) The pulley ratio on the motor shaft and the fly-wheel shaft: in application to actual cars this means changing the gear ratio between the motor and the axle. The lower the ratio (the nearer to each other the diameters of the gears), the higher the speed of the car, and the heavier is the duty required of the motor.

(2) The second part of the curves (running at full speed) does not require any extra test, since the necessary data can be obtained from the previous runs by continuing them until the speed and the amperes input become constant.

(3) The necessary data for the third part of the curves (coasting) are obtained by bringing the fly-wheel to the highest safe speed and then observing the gradual decrease in speed due to the frictional resistance. This experiment should be performed with the same values of brake load as were used in determining the preceding curves. Instead of recording the speed with a chronograph, it is preferable in this case to use a tachometer, or else to use the motor itself as an electrical tachometer. After the set has been brought up to the desired speed, the current is cut off, and the motor field excited with a small current from an independent source. The voltages thus induced are proportional to the instantaneous values of speed, and may be read on a suitable voltmeter.

(4) The last part of the curves (braking) are taken in a similar way. The brake can be applied to the fly-wheel; or else the Prony-brake load can be increased to give the necessary braking effect. Repeat this run with a few different values of brake pressure, in order to see the effect of this factor on car schedule.

It is also desired that the student try to stop the fly-wheel by converting the motor into a generator and closing it on suitable resistances (starting resistance can be used for this purpose). See that the armature leads are reversed in the brake position, so as to have the right direction of residual magnetism in the field. At first, the motor must be closed on a comparatively high resistance, in order not to overload it. As the speed decreases, the resistance is gradually cut out of the circuit, and finally the motor can be short-circuited on itself. Have an ammeter and a voltmeter in the circuit, so as to observe the electrical relations. Take ampere-time and speed-time curves with various rates of cutting out the resistance; or else, take curves for different values of "braking current," regulating the resistance so as to keep the current in each case as nearly constant as possible.

Report. The results of the experiment should be applied for determining some of the schedules which the equipment under test can give in commercial service. Assume a certain distance between the stops, and plot the curves shown in Fig. 512, selecting such experimental data as to have this distance covered in minimum time. This means cutting out starting resistances as fast as possible, and applying the maximum braking pressure immediately after the power has been shut off; in other words, running as long as possible at maximum speed and leaving out the coasting period. Assume a certain length of stops, say 5 seconds or thereabouts, and take the ideal and the simplest case that the car performs a regular schedule of the above runs and stops all day. Figure out the average commercial speed with and without stops, kw.-hrs. power expenditure per unit distance covered by the car, and the average effective current (square root of the mean of the squares of current). This latter should not be larger than the rated current of the motor; otherwise the temperature of the motor will rise beyond the safe limit, after a few hours run.

Should it result, that the above acceleration and speed overload the motor, the schedule should be modified so as to reduce the power input, of course at a sacrifice of the average speed or of the number of trips per day. Take a lower rate of acceleration, introduce a reasonable amount of coasting and reduce the brake pressure. Make these changes by trials until a satisfactory schedule is arranged, with as high an average speed as possible.

As another alternative for bringing the schedule within the capacity of the motor, the gear ratio can be reduced, so as to decrease the maximum speed. Compare the schedule and the power consumption attained in this way with those obtained with the motor geared for a higher speed (with the same average commercial speed).

Numerous performance curves taken from experiments on actual cars may be found in the above-mentioned *Report of the Electric Railway Test Commission.*

4. AIR-BRAKES.

704. One of the most important items in the equipment of a car is the brake. Most of the electric cars in common use are equipped with *hand-brakes*, in which the brake shoes are forced against the wheels by a system of levers operated by a handle under the control of the motorman. The general tendency towards an increase in the weight and size of cars has caused hand-brakes in many cases to be inadequate for control. This has resulted in the introduction of *air-brakes*, in which the shoes are pressed against the wheels by means of a piston operated by compressed air.

Compressed air may be supplied either by a compressor on the car itself, or the car carries an air-storage reservoir which is replenished at certain intervals from station tanks. The car compressor may be driven either by a separate electric motor, or directly from a car axle. A separate motor compressor on each car is the best solution, if the road can afford to so equip the cars.

705. Straight-Air Brake Equipment. — The general arrangement of parts of a simple air-brake equipment, as used on single electric cars (not on trains), is shown in Fig. 513. The principal parts, clearly seen in the figure, are: The motor-compressor, the compressed-air reservoir, the brake cylinder, and the operating (or motorman's) valves, one on each end of the car. To apply the brakes, the motorman turns the handle to such a position, that compressed air from the reservoir flows into the brake cylinder. This causes the piston inside of the cylinder to move and set the brakes. To release the brakes, the operating handle is moved to such a position that the air reservoir is cut off from the brake cylinder, and the latter is connected to the atmosphere (through a muffler, in order to deafen the disagreeable noise of the escaping air). As the air pressure in the cylinder is reduced, the piston travels back under the influence of an opposing spring inside of the cylinder.

The motorman's valve has four positions: The extreme left position of the handle is the "release" position, in which the brake cylinder is connected to the atmosphere, and the air reservoir is closed. The next position is the "running" or the "lap" position; here all the ports are closed, so that if there was any air in the brake cylinder it remains there, as long as the handle is in the "lap" position. The next position corresponds to the so-called "service application" of the brakes: the brake cylinder is disconnected from the atmosphere and is put in communication with the compressed-air reservoir, through a small opening. The air flows to the brake cylinder, and the brake shoes are gradually applied to the car wheels. When the car has sufficiently slowed down, the handle is returned to the lap position, in order not to increase the pressure further. The extreme right position of the operating handle gives an "emergency application" of the brakes; the air reservoir is connected to the brake cylinder through a large port, so that the full air pressure is instantaneously applied to the wheels.

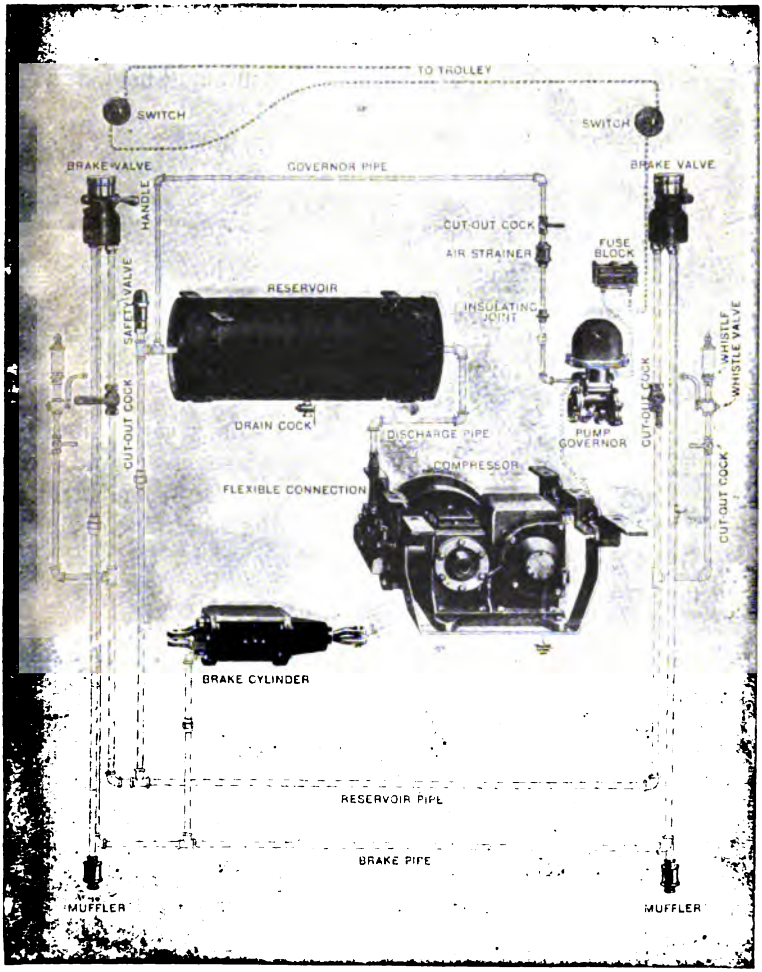


FIG. 513. Westinghouse straight-air brake equipment.

This gives a very unpleasant sudden stop, and should be used in extreme cases only, for instance, in order to avoid a collision, to save a life, etc.

The electrical connections in such an air-brake equipment are very simple: The current from the trolley passes through two snap-switches, one on each platform, through a fuse block and the pump governor to the motor. The motor is series-wound and has no starting resistance whatever; the current passes through its armature and field, and thence to the ground with the rest of the car current.

The pump governor, shown in the diagram, automatically starts and stops the compressor-motor, and in this way maintains the air pressure in the reservoir within the required limits. The governor consists of a snap-switch, actuated by the piston of a small air cylinder, connected to the air reservoir. When the pressure has reached its upper limit the piston moves against a spring and opens the electric circuit, stopping the motor. When the air pressure reaches its lower limit the spring returns the piston to its former position, and closes the circuit; the motor starts again and raises the air pressure.

The other parts of the equipment are: A safety valve, limiting the pressure in the main reservoir to a predetermined value; and a duplex pressure gauge (with two hands) on each platform; one hand shows the pressure in the reservoir, the other, in the train pipe. The signal whistles, used on inter-urban cars, are operated by the air from the same main reservoir as the brakes.

706. EXPERIMENT 31-D. — Operating Straight-Air Brakes. — An equipment should be provided for this experiment, similar to that shown in Fig. 513, and conveniently arranged so that all the parts are accessible for study. Before beginning the experiment, carefully study the piping and the electrical connections of the outfit. Then the compressor set must be started and the pressure in the main reservoir brought up to the required value. This will make the equipment ready for operation. The work should be conducted as follows:

(1) Practice operating the motorman's valve so that you know exactly all the positions and what operations they represent; also observe the variations in pressure, without taking any exact readings.

(2) Make a study of operating conditions of the motor-compressor and of the main reservoir. Assume a certain schedule which the car is supposed to perform; say, a stop every n minutes, requiring an application of the brakes, and a certain number of seconds x that the brakes must be "on." Actually perform this schedule during 15 or 20 minutes, and observe as accurately as possible the following: per cents

time during which the compressor is operating, and at rest; amperes and volts that it takes to operate the motor: variations of pressure in the main reservoir; the pressure at which the governor starts the motor and at which it opens the circuit. Plot the results to time as abscissæ.

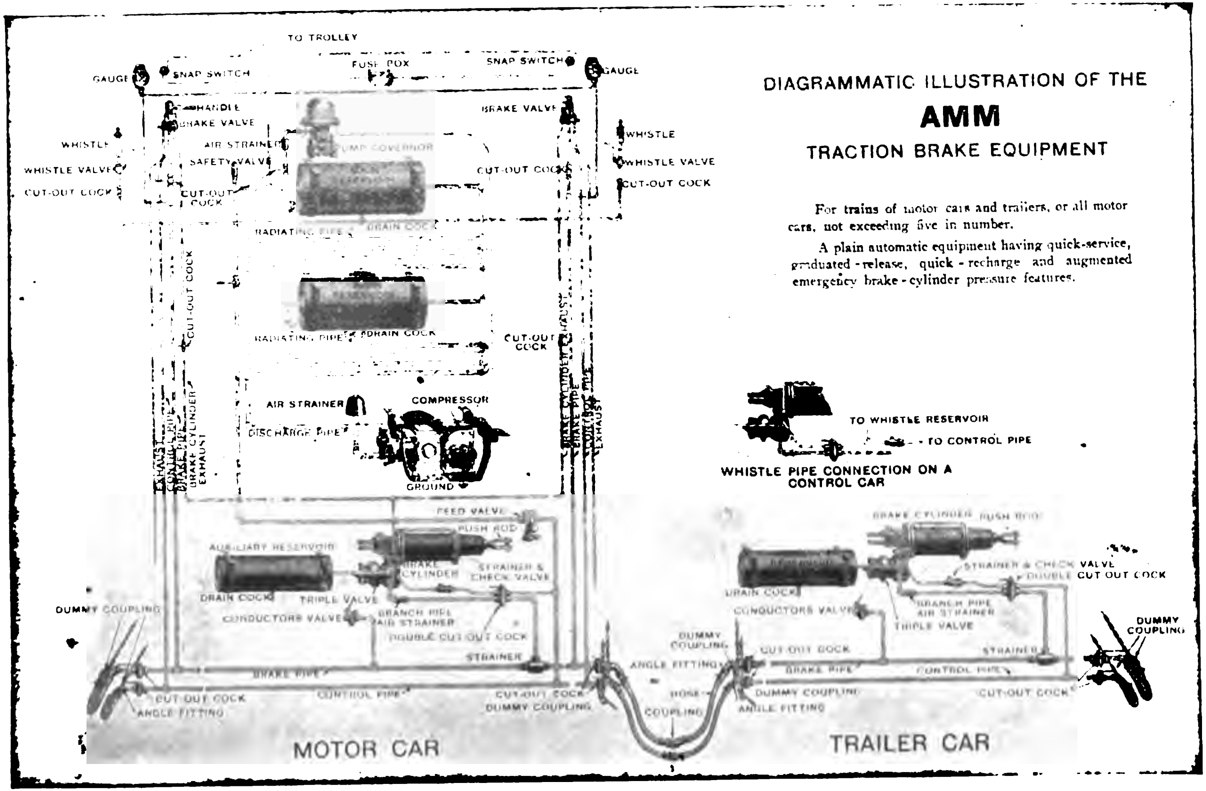
(3) Take pressure-time curves showing how the pressure changes in the different parts of the equipment when the motorman's valve is moved in a certain position, or a complete cycle of handle operations is performed. Read the gauges, always beginning with the same pressure in the main reservoir. Plot the pressures to seconds time as abscissæ, also mark the time of the beginning and the end of the travel of the brake-cylinder piston.

(4) Measure with a spring balance the brake-shoe pressure at different air pressures in the brake cylinder. Check the results with the theoretical pressures figured from the piston area and the leverage ratio.

707. Automatic Air-Brake Equipment. — The above-described air-brake equipment, so-called *straight-air system,* is no longer used on steam railroads, notwithstanding its simplicity; even on electric roads it is in some cases being replaced by the more complicated *automatic air-brake equipment.* A drawback of the straight-air system is that with a train, consisting of several cars, an appreciable time elapses before the air pressure is transmitted along the train pipe. Therefore, the brakes on the front cars are set earlier than on rear ones, and the latter have a tendency to run into the front cars. Another important drawback of the straight-air system is, that should there be a leak in the train pipe, or should the train be pulled asunder, the brakes can no longer be applied; this might lead to a serious accident.

These disadvantages are eliminated in the automatic air-brake system, with which the brakes on all cars, even with the longest freight trains, are set practically simultaneously. Moreover, should there occur a break in the train pipe, or a serious leak, the brakes are applied *automatically,* and the train stopped.

These advantages are obtained by the addition of an auxiliary air reservoir on each car (Fig. 514). Compressed air for brake application has to flow but a short way between this auxiliary reservoir and the brake cylinder on the same car; this insures a simultaneous application of the brakes on all the cars. The auxiliary reservoirs are recharged from the main reservoir during periods of release. Each auxiliary reservoir is operated by a so-called *triple valve,* placed on each car near the brake cylinder. This valve automatically performs the motorman's duties for each car; it sets the brakes, releases the brakes, and recharges the auxiliary reservoir. *The triple valve assumes one of these three positions* (hence the name *triple* valve) *according to the pressure in the*



DIAGRAMMATIC ILLUSTRATION OF THE
AMM
TRACTION BRAKE EQUIPMENT

For trains of motor cars and trailers, or all motor cars, not exceeding five in number.

A plain automatic equipment having quick-service, graduated-release, quick-recharge and augmented emergency brake-cylinder pressure features.

Fig. 514. Westinghouse automatic air-brake equipment.

train pipe. The motorman on the front car has only to establish a certain pressure in the train pipe; the triple valves on all the cars operate accordingly.

The triple valve is so constructed that increasing the pressure in the train pipe releases the brakes, while with the straight-air brake system this would set the brakes. With an increased pressure the triple valve is moved to such a position that it connects the brake cylinder to the atmosphere. Reducing the pressure in the train pipe gives a preponderance of pressure to the auxiliary reservoir. The air in the latter shifts the valve out of the way, puts itself into communication with the brake cylinder, and sets the brakes. For further details of the operation of the triple valve, see § 709 below.

708. Motorman's Valve. — With the automatic-air systém there are five different positions of the motorman's operating valve: release, running, lap, service application and emergency application. As the motorman faces the valve, the position farthest to his left is the release; the other positions follow to the right in the order named.

The purpose of the release position is to provide a large and direct passage from the main reservoir to the train pipe, in order to insure a quick release and recharging of the auxiliary reservoirs. With the valve in this position, a "warning port" discharges air from the main reservoir into the atmosphere with a considerable noise, attracting the motorman's attention, if he neglects to move the valve handle to the running position. This is necessary, because the pressure in the main reservoir must be higher than in the main pipe; it is about 90 pounds in the former and only 70 pounds in the latter.

In the running position of the valve, the main reservoir is connected to the train pipe through a special feed valve, placed under the motorman's operating handle. This valve lets the air pass into the train pipe when the pressure in this pipe is below 70 pounds, and thus supplies the leaks.

In the lap position all the ports are closed and thus made inoperative. This position is used after the service application of the brakes, in order to keep the brakes set in the desired position, preventing an interchange of compressed air.

In the service-application position, the train pipe is connected to the atmosphere through a small port; the reduction of air pressure effects the movement of the triple valves to the right, and the brakes are set.

The emergency position is distinguished from the preceding one by offering a much larger passage to the air into the atmosphere; this causes a nearly instantaneous application of the brakes throughout the train.

Some motorman's valves are provided with two service positions: one for slow application, for instance to prevent acceleration on grades, the other for quick service application, such as would be used for rapid stops with fast city schedule.

709. Triple Valve. — The action of the triple valve may be understood from its cross-section shown in Fig. 515. The valve shown there is the simple and original form of the device, which has now been superseded by the so-called quick-action triple valve. The quick-action feature will be mentioned later on.

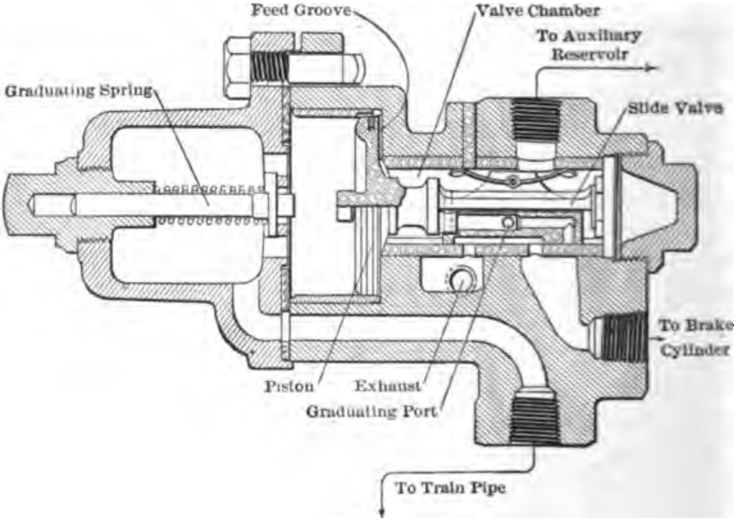


Fig. 515. Westinghouse triple valve.

The triple valve is connected to the auxiliary reservoir and to the brake cylinder on one side, and to the train pipe on the other side (see Fig. 514); it has also an opening to the atmosphere (exhaust). The necessary connections are accomplished by the slide valve, which is operated by the piston to its left. This piston is in turn actuated by the difference in pressure in the auxiliary reservoir and in the train pipe.

When air is admitted to the train pipe, through the motorman's brake valve, the piston is forced to the right-hand end of its stroke. In this position the feed groove is uncovered, and the air is permitted to flow past the piston into the auxiliary reservoir. The feed groove is made of such a size that about sixty seconds are required to fully charge an empty auxiliary reservoir. In this position of the piston, the brake cylinder is connected to the atmosphere through the cavity in the slide valve, the brakes being thus released. It should be noted, that only in this position can the auxiliary reservoir be recharged.

Now, if the train-pipe pressure be reduced four or five pounds, the preponderance of pressure on the auxiliary reservoir side of the piston will cause it to move to the left until it strikes the flexible abutment. In this position, known as *service application,* a graduating port in the slide opens a small passage from the valve chamber to the brake cylinder port: The air from the auxiliary reservoir then flows into the brake cylinder, until the air pressure in the auxiliary reservoir is reduced slightly below that in the train pipe. As there is a small amount of lost motion between the piston stem and the slide valve, the slight excess of pressure now on the train-pipe side, moves the piston inwardly far enough to close the graduating port, but has not sufficient power to overcome the friction of the main slide valve. The triple valve is now in the *lap position,* with all communications cut off between the train pipe, auxiliary reservoir, brake cylinder and the atmosphere.

This cycle of operations may be repeated with a resultant increase of the brake-cylinder pressure and a decrease of the auxiliary reservoir pressure, until their point of equalization is reached; after this any further reduction of the train-pipe pressure is not only useless but wasteful.

On the other hand, a sudden reduction of the pressure in the train pipe causes the piston to move to the left with such force, that it compresses the graduating spring and uncovers the brake-cylinder port. This opens a direct passage between the auxiliary reservoir and the brake cylinder, giving an almost instantaneous application of the brakes, necessary in emergencies.

Experience has demonstrated, that the above-described triple valve does not act quickly enough on long trains, giving the same trouble, as had been experienced with the straight-air brake system, namely, the rear cars running into the front ones. This fault has been remedied by the introduction of the so-called *quick-action triple valve,* in which a direct passage is provided between the train pipe and the brake cylinder. This passage is usually closed by a valve, but is opened in the emergency position of the slide valve by an ingenious mechanical combination. The opening of this passage accomplishes two results: (1) Venting of air from the train pipe helps to reduce pressure in it, and so assists the action of the brake on the next cars. (2) A pressure of 60 pounds is obtained in the brake cylinders, instead of 50 pounds, obtainable from the auxiliary reservoirs.

It is claimed that in a 50-car train but three seconds elapse between the movement of the engineer's valve and the application of the brakes on the last car, when the above quick-action attachment is used.*

710. EXPERIMENT 31-E. — Operating Automatic Air-Brakes. — The principal parts of the equipment are described in the three preceding articles. The minor parts may be easily understood from the diagram shown in Fig. 514. In performing this experiment, it is well to have a dummy motorman's valve and a dummy triple valve, showing a cross-section of these devices. The dummy valves should be connected mechanically to the actual valves so as to follow their movements. This enables the student to see clearly the relative positions of the various slides and ports. For further instructions, see § 706.

(*) For more recent improvements in triple valves, namely, the graduated-release and the quick-recharge features, see a paper by Mr. C. C. Farmer in the *Proceedings of the Air-Brake Association*, 1906, p. 126.

CHAPTER XXXII.

ELECTRIC HEATING AND WELDING.

1. ELECTRIC HEATING.

711. Among minor applications of electricity, heating and cooking by electric current are growing continuously in importance. It seems necessary, therefore, to become familiar with the operation of electric heating devices, and with the numerical relations governing transformation of electrical energy into heat. It may be stated at the outset that, from the standpoint of pure heat economy, it is ordinarily cheaper to heat a room or cook with gas or coal, than by electric current: In the latter case the thermal energy of coal or gas must first be communicated to the steam-boiler, then be converted into mechanical and electrical energy in the power house, transmitted through a line, and finally converted into heat again, instead of being directly used in a stove. However, gas light is also cheaper than electric light, and yet lighting by gas is gradually giving place to electric lighting. Here, as in the case of electric heating devices, *indirect advantages*, and *convenience, or comfort* gained, overwhelm the mere difference in price, and make the application of electric current for heating desirable and rational. Moreover, in places where cheap water power is abundant, and coal is scarce, heating and cooking by electricity are cheaper than by coal or gas.

712. Principal Heating Devices. — The principal forms of electric heating and cooking devices, now in use, are as follows:

(1) Street-car heaters, and radiators for heating rooms.

(2) Kitchen utensils: food, water, coffee and tea heaters and boilers, chafing dishes, cook stoves, etc.

(3) Soldering irons, laundry-, factory-, tailors'-irons, etc.

(4) Curling-iron heaters, cigar lighters, heating pads, etc.

An electric kettle is shown in Fig. 516; it has a double bottom accommodating a resistance, in which heat is produced. This particular device has three terminals, making it possible to have either the whole winding or part of it connected in the circuit. This gives three values of heating: intense heat, medium heat, and low heat.

Fig. 517 shows an electric soldering iron; its construction is clear from the sketch. An electric flatiron is represented in Fig. 518; it can be connected to an ordinary lamp socket, and heats very quickly after the current is turned on.

All these devices, however different in form and application, are based on the familiar Joule's law, that the rate of generation of heat, developed by the passage of a current through a conductor, is equal to $I^2 R$ watts, where I is the current in amperes, and R the resistance of the conductor in ohms. Any amount of heat can thus be generated by electric current, by suitably selecting I and R, provided the conductor and the vessel can withstand the temperatures resulting.

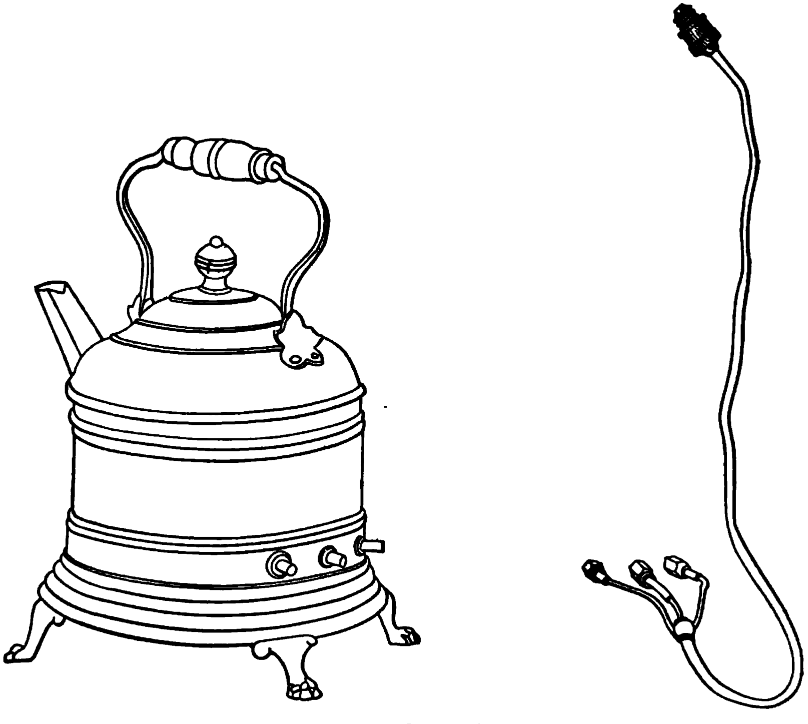


Fig. 516. Electric kettle.

The introduction of electric heating and cooking devices is very desirable for electric operating companies, since it gives them some day-load, especially in residence districts. In the business part of cities, day-load is supplied by motors in industrial establishments, by elevators, fan motors, etc., while in residence districts practically all of the demand comes during the evening, and the machines in power houses, necessarily large enough to carry the evening peak of the load, are running uneconomically during the day, at a very light load. The day-load can be increased by stimulating the use of electric heating devices. In other words, electric cooking and heating devices tend to improve the "*load factor*" of the power houses, or the ratio between the average and the maximum load of a station; this ratio plays an important part in the economic operation of central stations.

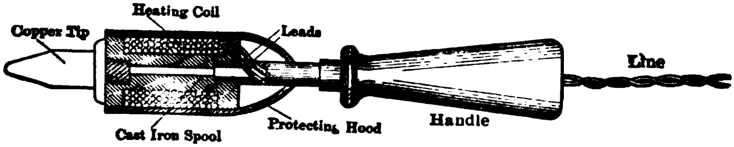


FIG. 517. Electric soldering iron.

713. Forms of Heating Elements. — Materials possessing high specific resistance are used for heating elements, in order to generate large amounts of heat within comparatively small space. Such elements consist either of wire or of metal strips, and may be entirely surrounded by air, insulated with mica, or embedded in special enamel. An original type of heating element, shown in Fig. 519, consists of a strip of mica covered with a thin layer of non-oxidizable metal, which is firmly secured to the mica by a process of firing. The conducting strip is covered by another piece of mica, and the whole is inclosed in a protecting metal case. Such elements are placed inside of utensils nearest to the place where heat is required.

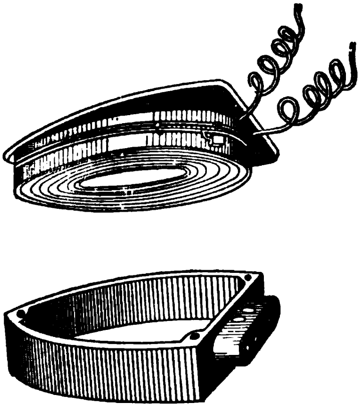


FIG. 518. Electric flatiron.

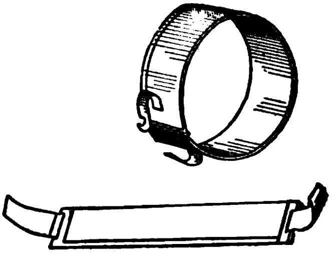


FIG. 519. Prometheus heating element.

714. Conversion of Watts into Heat Units. — In testing electric heating utensils one has to know numerical relations between watts input, and the output expressed in heat units. Confusion sometimes arises in the conversion, because the electrical unit "watt" refers to one second of time, while the heat units do not imply time. Correct results are obtained by using either "watt-seconds" (joules) or "thermal units per second."

The fundamental numerical relation is as follows: *When one kilowatt, or* 1000 *watts, of electrical energy are continually absorbed in a resistance, an amount of heat is liberated, equal to* 0.948 *British thermal units* (*B. T. U.*) *per second, or* 240 *gram-calories per second.* Remembering, in addition, that one horse-power is equal to 0.746 kw., we have all the necessary data for a theoretical conversion of electrical energy into heat, or mechanical power. The relations are somewhat more complicated in practice because of the losses which occur in the transformation of one form of energy into another.

715. EXPERIMENT 32-A. — Testing Electrical Cooking Utensils. — A water heater and a cook stove may be selected as representative of this class of devices. In the water heater, heat is communicated to the water directly, while in the cook stove it must be first communicated to the vessel containing the water. The experiment consists in determining the relative and absolute efficiencies of the devices.

(*a*) Measure the total electrical input necessary for bringing a known quantity of water to the boiling-point; the useful output is equal to the number of heat units required for bringing the water to the boiling-point. The ratio of output to input (both reduced to the same units) represents the efficiency of the device. During the process of heating, read temperature rise several times per minute, also the approximate temperature of the outside walls of the heater, and the distribution of heat by means of a suitable pyrometer, or an electric thermometer.

(*b*) It is interesting to investigate the influence of the supply voltage on the efficiency and power consumption of the devices; also on the amount of time that it takes to bring the same quantity of water to the boiling-point. This time depends essentially on the initial temperature of the water and of the vessel itself: In order to get comparable results, be sure to start all the runs under the same conditions.*

(*c*) Another question to be investigated is the comparative cost of cooking by gas and by electricity. An ordinary gas stove and a gas meter should be provided for this purpose; the same quantity of water. as was used in the electric heater, is brought to the boiling-point by using a gas heater, and the number of cubic feet of gas consumed is determined by the meter readings. The price of gas in different localities varies from $0.80 to $1.50 per 1000 cu. ft.; the price for electric current is between 8 and 15 cents per kilowatt-hour. On the basis of these figures a comparison of cost can be readily made. As was stated above, however, the difference in price alone does not decide the relative advantages of using gas or electric current.

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* Great care should be exercised in handling electric cooking devices: the boiler must be filled with water, before the current is put on. Otherwise, it is easy to burn out the heating resistance. Current must be turned off as soon as a run is completed.

There is one test which in spite of its great practical importance must be omitted in the laboratory: This is the "life test," or the determination of the number of hours of service which a heating device can stand before its heating element is burned out. For obvious reasons such a test could not be performed in a student's laboratory.

Report. Plot curves of temperature rise and of distribution of temperature in the devices investigated. Give data in regard to their input, output, and efficiency. State influence of supply voltage on time necessary for bringing a given amount of water to the boiling-point. Figure comparative cost of heating by gas and by electricity, for the average prices of both, prevalent in the locality.

716. EXPERIMENT 32-B. — Testing an Electric Radiator. — An electric radiator is one of the few devices in which the efficiency is equal to 100 per cent, theoretically as well as practically: All of the electrical input is converted into heat, and this is used for raising the temperature of the surrounding air. The only test, which can be performed on an electric heater, consists in determining the temperature-time curve of the device, and in measuring the total amount of time which it takes for the heater to reach its final temperature. In performing this test read temperatures at various places of the heater, and the temperature of the air at a certain distance from the heater; also note the changes in current due to the variations in the resistance of the heating elements.

Tests relative to the determination of the size of heaters for given conditions, important in the subject of Heating and Ventilation, may also be profitably performed, assuming certain requirements of service.

Report the results of the tests: Figure out theoretically what should be the resistance and the current consumption of a 110-volt air heater, sufficient to raise the temperature of Q cu. ft. of air at a rate of t degrees Fahrenheit in one hour. The air is assumed to be at the ordinary room temperature and atmospheric pressure; moreover, it is supposed that there is no loss of heat through the walls.

717. EXPERIMENT 32-C. — Testing an Electric Soldering Iron. — The purpose of the experiment is to illustrate the use and the advantages of an electrical soldering iron (Fig. 517); also to determine the cost of its operation. The current is switched on and the input measured every few seconds, until a temperature is reached sufficient for soldering. Have a sufficient supply of small pieces of wire, brass ribbon, etc., and keep on soldering them for a sufficient length of time, in order to determine the cost of operation per joint.

Report time and power consumption for the first joint, and the same for consecutive joints. Figure out the cost of operation on the basis of a certain price per kilowatt-hour, say ten cents.

718. EXPERIMENT 32-D. — Testing an Electric Flatiron. — The distribution of heat, of importance to the users of electric flatirons (Fig. 518), is determined by means of a suitable pyrometer. The amount of time necessary to reach the required temperature, is determined by testing, say every half minute, the ability of the iron to scorch papers; such records are sometimes called *shadowgraphs*. Get impressions while heating the iron, and also in cooling it: one of the good characteristics of a flatiron is that it should hold heat as long as possible.

Report. State the amount of time necessary for heating the iron and the watt-hours required for reaching the working temperature. Give the obtained distribution of temperature, and inclose samples of shadowgraphs. Figure out the cost of operation per hour, on the basis of the price per kilowatt-hour, prevalent in the locality.

2. ELECTRIC WELDING.

719. The welding of two pieces of metal is a process by which they are made to cohere strongly, or to hold together as a single piece, when powerfully pressed together. To weld such metals as iron, brass and copper, they must be heated up to very high temperatures, near those at which they become incandescent.

Since metals can be heated to any desired temperature by passing through them a sufficiently large electric current, the idea naturally arose of applying electricity to the process of welding. The advantages of using an electric current rather than an ordinary coal fire, are: The joint can be heated much more quickly; the temperature is easily adjusted to any desired degree; greater cleanliness and a better weld result from the absence of impurities due to the gases of the fuel.

720. Description of an Electric Welder. — The operation of the electric welder may be understood from Fig. 520. The device is, in principle, an ordinary alternating-current transformer: The primary winding P may be wound for any commercial voltage (100 volts, 200 volts, 500 volts, etc.); the secondary S consists of one turn of heavy copper bar. The secondary circuit is completed by the pieces M to be welded; they are fastened in the clamps CC and pressed against each other by the weight W. If the apparatus is intended to be used continually, the secondary turn is made hollow, and cooling water is circulated through it.

The reason for using alternating current for welding, and particularly for using a transformer of this construction, is as follows: The resistance of two pieces to be welded is very low, but it takes a heavy current to bring them to the required high temperature. Thus, a *heavy current at a low voltage* is needed for welding. The easiest way to produce such currents is by means of a transformer, since by selecting a proper ratio of the number of turns, primary and secondary, the primary current can be increased any desired number of times, with a corresponding reduction of the voltage. Suppose, for example, the primary winding to consist of 250 turns, and to be connected to a 500-volt supply. With a single turn in the secondary the secondary voltage will be only 2 volts. At the same time, with the primary current of, say, 10 amperes, the secondary current will be 2500 amperes, which is sufficient for welding comparatively large pieces.

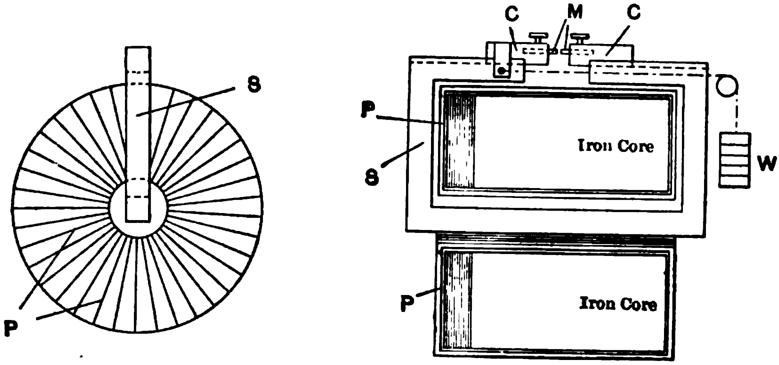

FIG. 520. Electric welder.

To secure uniform results, the following automatic feature is added to the above welder. The weight W is adjusted so as to get the best results for a given operation, and an auxiliary electrical contact is placed near the movable clamp C. The operator clamps the pieces to be welded and closes the primary circuit. As soon as the joint becomes soft and the clamp begins to yield, it closes the above contact. This actuates the circuit-breaker, and opens the primary circuit. In this way overheating and waste of power are avoided, and the apparatus can be intrusted to an inexperienced man.

721. Principal Applications. — Electric welding is used to a considerable extent in various metal industries, especially for articles manufactured in large quantities, where the same operation is performed continually. Carriage tires, wheel spokes, axles, etc., are economically welded by this process; pieces of wire and cable are joined in a continuous length; heavy pipes and bars are locally heated to be bent to any desired shape. It is possible to obtain an electrically welded joint of nearly the same mechanical strength as the welded bars themselves.

Large electric welders are used to some extent for joining together rails on electrically operated railways. Rails are commonly used as a return circuit for electric current operating the cars. Therefore a good electrical contact between the rails is essential in order to reduce the voltage drop and the losses; also to prevent electrolysis of water and gas-pipes by stray electric currents. The usual method of connecting rails is by means of *bonds*, or copper strips of different shape. Some engineers prefer to have a *continuous* rail by welding all the joints; electric welders are used for this purpose. A discussion of the comparative advantages of bonding and welding rails is out of place here, and the student is referred to the special electric-railway literature.*

A track-welding car, which is an ordinary motor car provided with a large welding transformer and a motor-generator set, for converting the 500-volt direct current from the trolley wire into the alternating current necessary for welding, is required for welding rails. The two ends to be welded are clamped into the transformer on the track and pressed together by hydraulic pressure of a hand pump. Then the current is put on and the welding effected, as described above for smaller pieces.

722. EXPERIMENT 32-E. — Exercises in Electric Welding. — The student should begin the experiment by welding together two pieces of a metal rod of a certain size and material. This should be done at different values of the applied voltage, beginning with the lowest voltage at which the welding can be accomplished. The variables of practical interest in this test are: The amount of time required to weld a joint; kilowatt-hour input; average current and watts and their fluctuations; power factor; quality of the weld and its mechanical strength. Having an ammeter, a voltmeter and a wattmeter in the primary circuit, all the necessary electrical data can be readily obtained.

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* It may be mentioned here that electric welding now has a serious competitor in Goldschmidt's thermit, which is an aluminum compound burning at a very high temperature. A mould is made around the place to be welded, and the molten thermit is poured into the mould. The temperature thus obtained is sufficient to weld two pieces of rail.

A testing machine should be provided for measuring the tensile strength of the joint and comparing it to that of the material itself. The pressure exerted on the pieces during the welding may also affect the results, and should be noted and varied within certain limits. In order to get reliable results, the main switch must be opened the very moment the welding is affected, as would be the case in an automatic welder. It may occur that minimum time corresponds to one voltage, minimum power to another, etc.; the most advantageous voltage can then be selected in each case, according to the importance ascribed to these different factors.

After reliable average data have been obtained with samples of a certain size, take samples of the same material, having different cross-sections, and weld them, in order to see the influence of the cross-section. Try also to weld samples of different materials.

Report the results as to power consumption, time, tensile strength, influence of cross-section, etc., and state your conclusions. Figure out cost per weld at a certain rate for power supply.

723. EXPERIMENT 32-F.—Test of a Welding Transformer.— The purpose of the experiment is the determination of the electrical efficiency of a welding transformer (Fig. 520), or of the proportion of the watts input, which is actually converted into heat at the joint. This is by no means a simple problem; the relations in the secondary circuit being such as to permit no very accurate measurements of volts and amperes, with ordinary commercial ammeters and voltmeters, nor usually of any wattmeter readings. Moreover, the resistance of the secondary circuit is so low, that connecting an ammeter into it appreciably changes the current and the voltage relations. The following method, without any claim to accuracy, gives at least a general idea of the electrical relations in the secondary circuit of a welding transformer.

(1) Measure the secondary voltage on open circuit. This can be done either by using an ordinary alternating-current voltmeter, connected through a step-up shunt transformer (because of the very low voltage); or else, a low-reading hot-wire ammeter can be used as a voltmeter, with some resistance in series if necessary. If the primary winding of the transformer is accessible, or if its number of turns can be determined in some other way, compare the ratio of voltages with the ratio of turns, primary and secondary, as a check.

(2) Measure the ratio of currents, primary and secondary. For this purpose the secondary must be short-circuited through a known low resistance, wound non-inductively and the volts drop across it measured, say with a hot-wire instrument, calibrated as a voltmeter. This

permits one to figure out the secondary current, corresponding to a certain primary current, in other words the current ratio of the transformer. See if this ratio is nearly equal to the inverse ratio of the number of turns, as in ordinary transformers.

(3) Now put into the welder two pieces to be joined together, and read primary volts and amperes, and secondary volts across the joint. From the test (1), secondary induced volts are known; from the test (2), secondary amperes can be calculated. With these data at hand, both the efficiency of the secondary alone, and of the welder as a whole, can be determined. Volts read across the joint, times secondary amperes, give watts developed at the welding place, or useful watts. The ratio of these to the watts input gives the efficiency of the transformer. The ratio of the voltage across the joint to the voltage induced in the secondary gives the efficiency of the secondary, because it shows which part of the induced voltage is available for welding.

As a check, the student should measure the primary resistance by direct current, and determine the iron loss by the wattmeter method (Fig. 185). Then, knowing the primary copper loss and the iron loss, the input into the secondary can be calculated. Subtracting from it the measured output, the difference gives the secondary losses. These losses are comparatively high, because of unfavorable electrical relations in the secondary circuit, and also on account of a comparatively high contact resistance between the secondary itself, the clamps, and the pieces to be welded.

No particular accuracy is expected from this efficiency test; this, however, does not diminish its value. There are many cases in practical engineering, in which only approximate results are obtainable. And yet the engineer has to design, build, and use apparatus on this uncertain foundation.

CHAPTER XXXIII.

ELEMENTS OF TELEPHONY.

724. THE two most essential parts of every telephone apparatus are *the transmitter* and *the receiver*. Their construction and connections may be understood from the general scheme illustrating the transmission of speech (Fig. 521); the sketch represents two subscriber stations connected by a line. The transmitter (Fig. 522) consists of a chamber filled with granulated carbon, between two carbon electrodes. The front electrode is fastened to a thin metal diaphragm; the back electrode is rigidly fastened to a metal bridge. The transmitter circuit is closed through a battery, as shown in Fig. 521. When

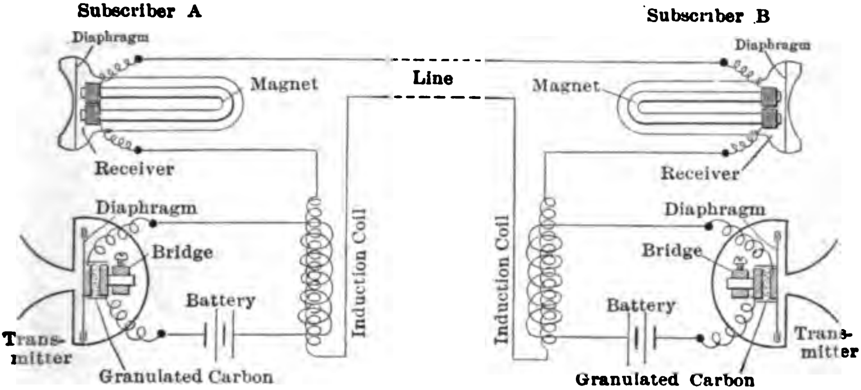

FIG. 521. Diagram illustrating transmission of speech.

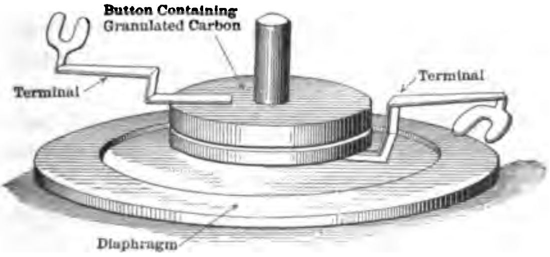

FIG. 522. Telephone transmitter.

a person is speaking into the mouthpiece of the transmitter, and the sound waves strike the diaphragm, it is set in corresponding vibrations, and periodically compresses and releases the granulated carbon in the chamber. The resistance of the circuit is thus varied, and corresponding rapid variations occur in the current, giving the effect of small oscillating currents, superimposed upon current of a constant value.

These oscillating currents, acting through the induction coils, produce secondary currents which are sent through the line and reach the receiver of the other subscriber, where they are converted back into sound waves. The receiver (Fig. 521) consists of a sheet-iron diaphragm placed near the poles of a steel magnet. The poles are surrounded by coils of wire through which pass the incoming talking currents. These small variable currents change the magnetism in the pole-pieces and cause the diaphragm to vibrate accordingly, thus reproducing the sounds spoken at the sending station. A typical receiver is shown in Fig. 523: The magnet is made of *hardened* steel so as to retain its initial magnetism permanently. On the contrary, the polar projections are made of very soft iron, in order to respond more easily to small changes in magnetism.

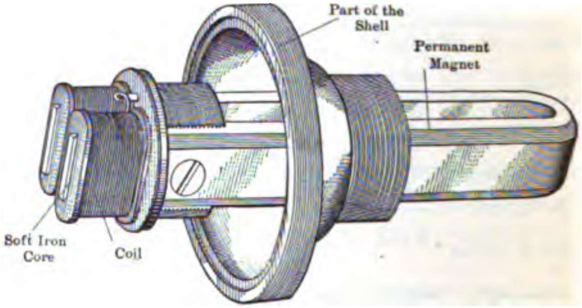

Fig. 523. Telephone receiver with the diaphragm removed.

725. Induction Coil. — Oscillating currents produced in the transmitter are sent to the line through the so-called *induction coil* (Fig. 521). This is merely a static transformer the purpose of which is (*a*) to raise the voltage of the "talking" currents, (*b*) to liberate them from the direct-current component. The purpose of raising the voltage is to effect a more efficient transmission through long lines. The direct-current is " sifted out," because it is useless in producing sound waves in the receiver, and should be confined to the local transmitter circuit.

A typical induction coil is shown in Fig. 524: It has a primary winding consisting of comparatively few turns, and a secondary winding of many turns of very fine wire. The windings are placed in inductive relation one to another, on an iron core made up of fine iron wires, to prevent eddy currents. Such an induction coil differs from ordinary transformers, used in connection with light and power transmission, in that its magnetic circuit is open, instead of being closed (compare Figs. 314 and 315). A closed magnetic circuit is not needed in telephonic transmission, on account of very low magnetic densities used; it would be rather harmful, increasing hysteresis loss and distorting the speech.

In simple house telephones, where current is transmitted only a short distance, induction coils are sometimes omitted, and the transmitters connected directly to the line.

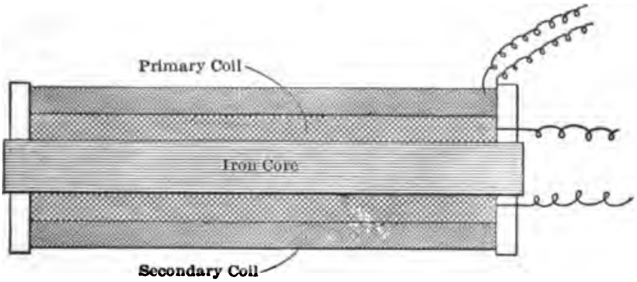

FIG. 524. Induction coil for telephone work.

726. EXPERIMENT 33-A. — Adjusting Transmitters and Receivers. — The purpose of the experiment is to afford familiarity with the construction and operation of the two most imporntat parts of telephone apparatus — receiver and transmitter (§ 724). A special transmitter and a receiver should be provided for this purpose, arranged so that they can be easily taken apart, for study and adjustment. The transmitter is adjusted by varying the amount of the granulated carbon, the quality of carbon, and the pressure on it. In addition to this, the quality of speech depends on the magnitude of the current flowing through the instrument.

The receiver is adjusted by varying the distance between the pole-pieces and the diaphragm. A well-adjusted receiver should be used for listening, while the transmitter is being adjusted, and vice versa.

Report. Draw sketches showing cross-sections of the instruments used for study and adjustment. Give your results with regard to the adjustment of the devices, and the factors affecting the quality of speech.

727. Classification of Telephone Installations.—Telephone installations differ widely in the number of subscribers served, and in the distance through which speech is to be transmitted. The following general subdivision indicates the principal cases met with in practice.

(1) House, and intercommunicating telephones, used in residences, schools, offices, etc.: Such installations contain but comparatively few instruments, and these are connected at will by the subscribers themselves, without any operator.

(2) Private branch-exchanges, installed in large offices, factories, hotels, etc., where instruments are interconnected by an operator, and may also be connected for conversation with the city and long-distance telephone systems.

(3) City and long-distance telephone systems, covering many miles and serving possibly hundreds of thousands of subscribers.

Telephone installations may also be divided into those using local batteries, and into central-energy telephones. This latter distinction may not be important from the standpoint of the subscriber, but it materially affects the construction and the connections in the system. In local-battery telephones the current necessary for operating transmitters is supplied by dry or wet batteries placed in each telephone set. In systems using central energy the exchange is provided with a large storage battery, from which the current necessary for conversation is supplied. Further distinction between these two systems is made clear in the course of the chapter.

728. Signaling. — In addition to a receiver and a transmitter, each telephone set must be provided with a device for signaling and being signaled. In small house-installations ordinary electric bells with push-buttons are used for the purpose: The bell is either connected to the transmitter battery, or, if the distance between the stations is comparatively great, a separate battery of a larger number of cells is used for signaling. Battery bells cannot be used for signaling over any considerable distance, and are replaced by alternating-current bells, or so-called ringers. Current for these ringers is supplied by special generators, commonly called magnetos, because their fields are supplied with permanent magnets instead of electromagnets. Such a magneto is placed in the exchange, and the operator uses it in calling up subscribers.

Calling up the " Central " is done in different ways in local-battery telephones and in central-energy telephones. In the first system, each telephone set is provided with a small magneto-generator. By turning the crank of the generator, the subscriber sends currents through the line to the so-called "drop" in the exchange. The drop shutter is released, displaying the subscriber's number; this calls to the operator's attention that the subscriber wishes to speak to her.

In central-energy systems the "central" is called up by simply taking the receiver off the hook. This closes the subscriber's circuit and lights up a small lamp, which calls the operator's attention to the fact that a connection is desired. Further details in regard to signaling are explained in the later sections of the chapter.

729. Ringer and Magneto.—An alternating-current bell, also called a ringer, or polarized bell, is shown in Fig. 525 to the left. It consists of a permanent magnet *NS*, one of whose poles is periodically strengthened and the other weakened, and vice versa, by alternating currents sent from the magneto-generator, shown to the right. This unbalancing of forces causes a soft-iron armature to vibrate periodically; a hammer attached to the armature strikes alternately two gongs and produces the ringing.

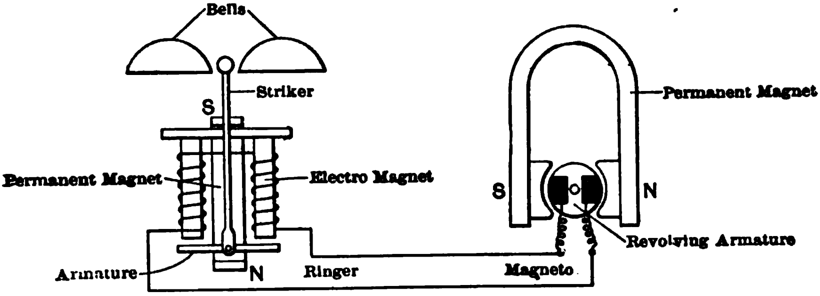

FIG. 525. Magneto-generator and polarized bell.

The generator, shown to the right, is a two-pole machine, with permanent magnets, and a shuttle-shaped armature. Magnetos in subscribers' sets and in small exchanges are hand operated, the speed being multiplied through gears: In large exchanges, ringing generators are usually driven by electric motors, and are running continuously; the operator merely connects the generator to the desired subscriber's line.

One detail in the construction of magnetos deserves to be mentioned: A magneto, connected across a telephone line, would shunt a part of the talking currents, and thus lower the efficiency of speech transmission. Therefore, magnetos are provided with some form of the so-called "generator cut-in," a device which keeps the armature circuit open, except when the crank is turned. These devices are based either on centrifugal force; or else, by turning the crank, the shaft is moved longitudinally and the contact is closed. In magnetos connected in series with the line the armature must evidently be normally short-circuited, except when in use. The devices used for short-circuiting the armature are called "magneto shunts," and are based on the same two principles, mentioned above.

730. Magneto-Telephone Connections. — Standard connections in subscribers' sets, with local battery and magneto call, are shown in Fig. 526. The battery circuit is closed only when the receiver is off the hook: this is done automatically by the hook itself, which is therefore called the *switch-hook*. It is shown separately in Fig. 527: when the receiver is on the hook, the lower contact, connected to the ringing circuit, is closed, allowing the "central" to call up the subscriber. When the receiver is off the hook, the ringing circuit is open, while the upper contact, completing the battery circuit, is closed. With the connections according to Fig. 526, two contacts must be closed, when the receiver is off the hook. This is accomplished by adding one more spring and an extra contact to the switch-hook shown in Fig. 527.

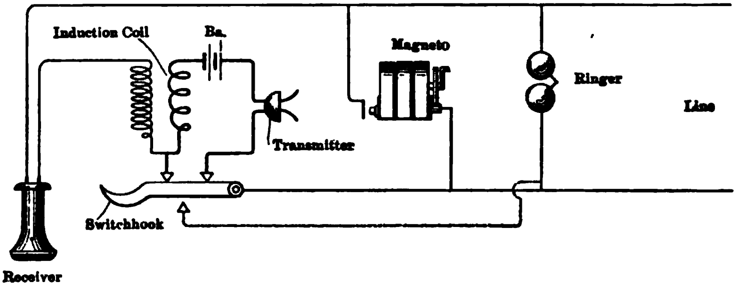

FIG. 526. Connections in a magneto telephone set.

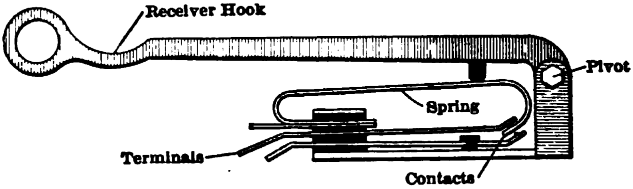

FIG. 527. Switch-hook

The ringer is sometimes left across the line all the time, in order to simplify the switch-hook connections. If the resistance and the inductance of the ringer are sufficiently high, only a very small part of the high-frequency talking current passes through it, and the efficiency of speech transmissions is hardly affected. The magneto-generator is also bridged across the line, but its circuit is always open by the generator cut-in, except when the subscriber turns the crank (§ 729).

An assembly of the parts shown in Fig. 526 is illustrated in the perspective view in Fig. 528: the sketch needs no further explanation. The ground connection shown, is used in selective signaling (§ 745).

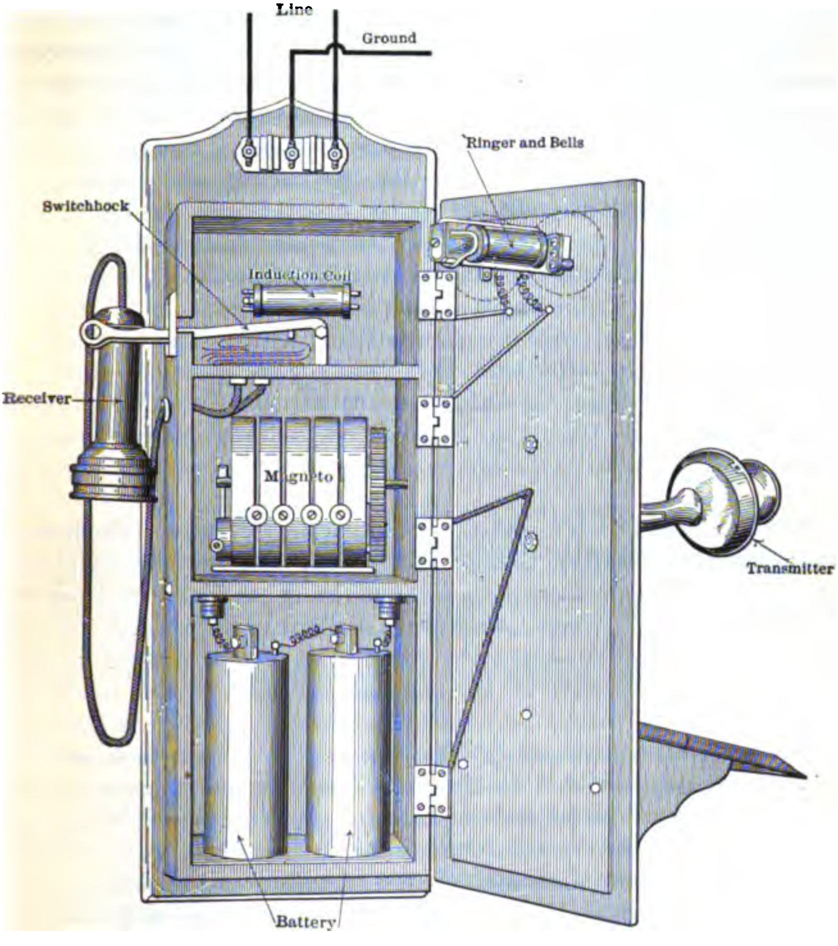

FIG. 528. Magneto-telephone set.

731. EXPERIMENT 33-B. — Connecting Magneto-Telephones. — The student is given the separate parts shown in Fig. 528, and he is expected to connect them up as in Fig. 526, with possibly slight changes, and to obtain satisfactory signaling and talking.

732. House Telephones with Battery Call. — In small house-installations the magnetos and the polarized bells may be omitted, and ordinary electric bells, operated from the same batteries which supply current to the transmitters may be substituted. The student can easily plan the necessary connections, on the basis of Fig. 526, by omitting the magneto and the induction coil.

Three conditions must be fulfilled. (1) When the receiver is on the hook, the talking circuit is open, and the bell is connected to the line, so that the subscriber may be called up. (2) When the subscriber presses the push-button, leaving the receiver on the hook, his battery is connected to the line, in order to ring the bell of the other station; at the same time his bell must not be in the circuit. (3) When the receiver is taken off the hook, the transmitter, the battery, and the receiver must be connected in series across the line. To give a hint as to the proper connections, it may be mentioned here that the battery. the transmitter and the receiver are permanently connected in series, the circuit being open at the upper contact of the switch-hook. The lower contact of the switch-hook is connected to the bell. The push-button normally closes the bell circuit, but, when pressed down, it opens the bell circuit, and closes the battery across the line. The switch-hook itself is connected directly to the line. With these instructions the student will have no difficulty in designing the proper connections.

733. EXPERIMENT 33-C. — Connecting House-Telephones Provided with Battery Call. — The student is given the telephone parts necessary for two stations, and he is expected to connect them up and to obtain satisfactory talking and ringing. It is recommended that he design the connections before coming to the laboratory, on the basis of the explanations given in the preceding article.

734. Intercommunicating Telephones. — In some cases, several telephones, installed in a building, are used for intercommunication without being connected to the city telephone system. When the number of such private telephones is not greater than 20, it may be preferable to run 20 wires through all the subscriber-sets so that each subscriber may connect his telephone with any of the remaining nineteen subscribers, without a central switchboard and an operator. A plan of connections is shown in Fig. 529: ordinary magneto telephones are used, or battery-bell telephones, and a switch is provided by means of which each subscriber can connect his telephone, to the desired subscriber, and call him up. Instead of using two wires for each subscriber, only one wire is used, the connection being completed through a common return wire, for all subscribers. Thus, with n subscribers, the total number of wires needed is $(n + 1.)$ Plugs and jacks are sometimes used, instead of button contacts and levers.

It will be noticed that the numbering of subscribers is different on different sets: The contacts on set No. 1 are: 1, 2, 3, 4; those on the set No. 2 are: 2, 1, 3, 4; etc. This is done in order to make the "home" position the first on the dial; this makes it easier for the subscriber to return the lever to its proper position after he is through talking. Unless he does so, his telephone is disconnected from his line-wire, so that he cannot be called up by any other subscriber.

This necessity for the subscriber to remember that he must return the lever to the home position, is one of the troubles encountered in the operation of intercommunicating systems: it is only natural to hang up the receiver and to forget to return the lever to the proper position. This drawback is obviated by mechanically connecting the selective lever with the switch-hook, so that hanging up the receiver automatically returns the lever to its home position. In one of the best systems of this kind, invented by Mr. Ness, the selective lever is under the influence of a spring: when the subscriber hangs up his receiver, the spring is released, and the lever snaps back to its home position. In addition to this automatic feature, the Ness system has some other noteworthy details, which are described in the next article.

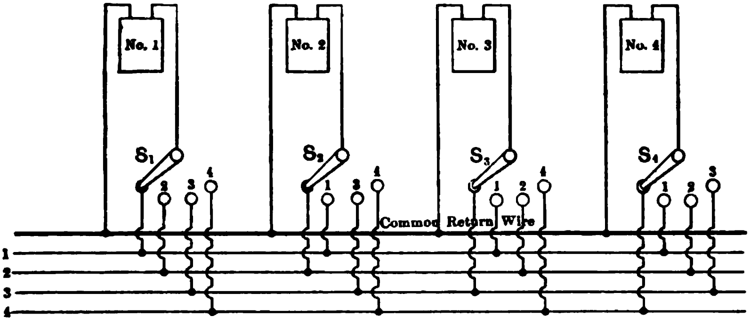

FIG. 529. Principle of intercommunicating telephone systems.

735. The Ness Intercommunicating System.—The electrical connections are shown in Fig. 530: A local battery is used for talking with each set, while one common battery is provided for ringing. The reason for this is that when the telephones are scattered over a considerable distance, more cells are required for ringing than for operating the transmitters. With n subscribers the total number of wires is $(n + 2)$, there being one common talking wire and an extra call wire.

In the position, shown in Fig. 530, subscriber No. 1, having originated the call, is supposed to be in conversation with subscriber No. 2. To call up subscriber No. 2, subscriber No. 1 moves the selective switch L

to the position 2, and presses the button on the end of the switch. By doing so he sends a current from the common calling battery through the call wire and the line wire 2, into the bell of subscriber No. 2, provided that the receiver of subscriber No. 2 is on the hook. Then, after both receivers have been taken off the hook, the bell circuits are opened at the contacts *c*, shown under the switch-hooks, and the talking circuits are closed through the contacts *a*, *b*, above them. The speech transmission takes place through the wire 2 and the common talking wire. When the receiver is hung up, the spring returns the selective switch *L* to its home position, and the switch-hook opens the talking circuit; at the same time the bell is connected to the line, making the set ready for the next call.

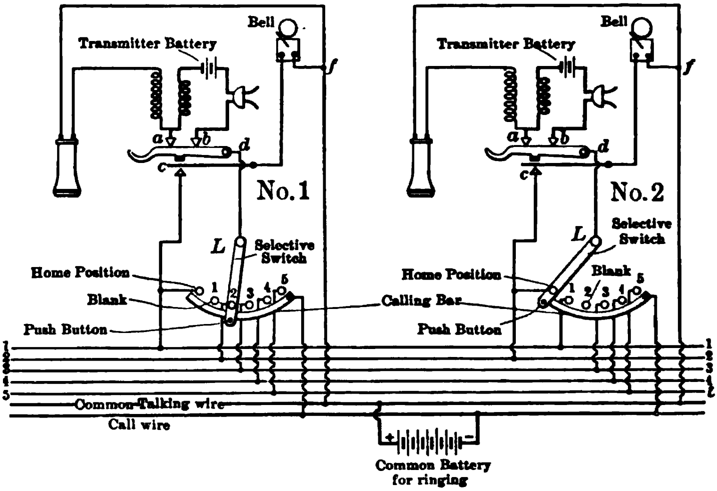

FIG. 530. Connections in Ness intercommunicating telephones.

It will be noted that the contacts of all the subscribers in this system are marked in the same order, and not as shown in Fig. 529. This is made possible by providing a separate "home" contact and leaving blank the contact corresponding to the number of the subscriber. For instance, on set No. 1 the contact 1 is left blank, on set No. 2 the contact 2 is left blank, etc. The advantage of this arrangement is that all the sets are interchangeable, and all are connected to the lines in the same way. Such an arrangement can be used with any intercommunicating system, whether automatic or not.

It is not absolutely necessary with intercommunicating telephones to have the battery call: The magneto call may be used if desired, leaving local batteries for talking. In some cases, one common battery is used for both ringing and talking. The only characteristic feature of intercommunicating telephones is that the subscriber is enabled to make the connections himself, without calling up the operator.

736. EXPERIMENT 33-D. — Connecting Intercommunicating Telephone Sets. — Connect up a few intercommunicating sets, following the general principles outlined above; explicit directions and blueprints of connections usually accompany the apparatus. If the set is intended for separate transmitter batteries at each subscriber, change the connections so as to use one common battery. Also, see if the connections could be changed so as to use the magneto call, instead of the battery call. Notice what occurs when a subscriber forgets to hang up his receiver; also when two subscribers try to call up the same subscriber, or when a third subscriber attempts to call up one of the subscribers engaged in conversation.

737. Magneto-Telephone Exchange. —The function of a telephone exchange, or the "central," as it is commonly called, is to suitably connect the subscribers, whose lines terminate on the switchboard of the exchange. The two wires coming from each subscriber terminate in a contact device, called the "jack" (Fig. 531). A "drop," shown in same figure, and placed above the jack, bears the subscriber's number. The drop consists of an electromagnet, and a shutter which is held in its position by a soft-iron armature. By following the connections it will be seen that when the subscriber sends alternating currents through the line, by turning the crank of his magneto, the electromagnet of the drop attracts its armature; this releases the shutter, calling the operator's attention that a connection is desired. She inserts a plug (shown to the left) into the jack and connects her telephone to the subscriber's line, uttering the familiar "Number, please." Having received the desired information, she inserts another plug, connected to the same cord-circuit with the first plug, into the jack of the subscriber to be called up; then she rings his bell by pressing a handle which connects his line to the exchange generator. At the end of the conversation the subscribers are supposed to give one or two short rings, as a signal that they are through. These signals operate in the exchange the so-called "clearing-out" drop: as soon as the shutter of this drop is released, the operator pulls out the plugs from the jacks, disconnecting the subscribers.

Following this general description of the devices used in the exchange, the separate parts will now be described in greater detail.

738. Switchboard Connections.— The jack, Fig. 531, is a double-pole contact in which a subscriber's line is terminated. The terminal connecting to the tip of the plug is made in the form of a spring; the other contact connecting to the sleeve of the plug is made into a ring. The winding of the drop is connected on one side of the line, on the other side to an auxiliary contact, which bears on the spring contact of the jack, when the plug is not in the jack. Thus the drop circuit is normally closed, enabling the subscriber to call up the operator. When she inserts the answering plug into the jack, the tip of the plug lifts the spring and opens the drop circuit. At the same time a connection is established, through the plug and the cord, between the line and a special key called the "ringing and listening key" (Fig. 532). The left side of this key is connected to the operator's telephone set, while the right side is connected to the ringing generator and to the calling plug. When the handle of the key is in the central position, the

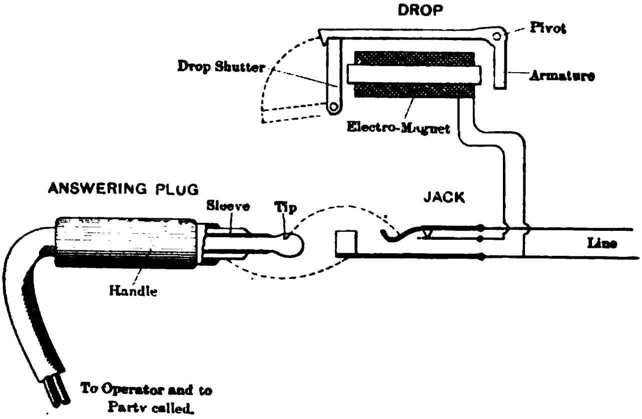

FIG. 531. Jack, drop, and plug, in a magneto switchboard.

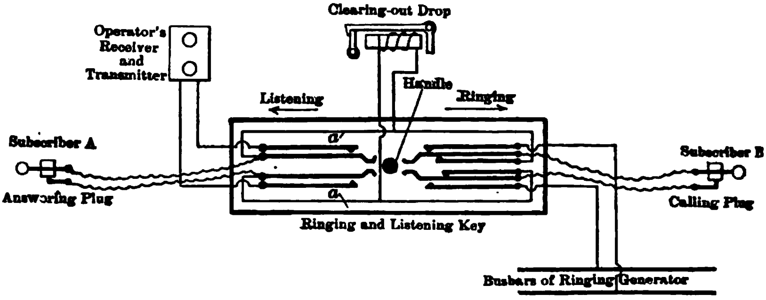

FIG. 532. Connections to ringing and listening key.

answering plug is connected directly to the calling plug *B*. This is the ordinary position during the conversation of two subscribers. By moving the handle to the left the operator connects her telephone to the cord circuit: this is called the "listening" position. The handle is in this position when asking the number of the party desired; also when listening for conversation, if the operator has any reason to believe that the parties forgot to "ring off."

When the handle is turned to the right, or the "ringing" position, the calling plug is disconnected from the answering plug and is connected, instead, to the generator bus-bars. This position is used for calling up a subscriber after the connection has been completed.

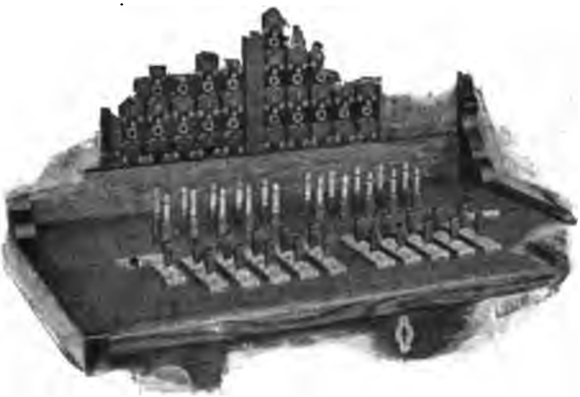

FIG. 533. Part of a magneto switchboard (Kellogg).

The clearing-out drop is shown on top of the sketch: it is connected directly across the cord circuit *aa* and is thus actuated by current impulses sent from either subscriber at the end of the conversation. It is not necessary to have as many pairs of plugs as there are pairs of subscribers connected to the exchange, because it never happens that all the subscribers use their telephones at once. Experience shows, that ten cord-circuits are sufficient for each hundred subscribers connected to the exchange: this permits twenty per cent of the subscribers to be connected at a time. This number is never exceeded even during the busiest hours of the day.

A part of a magneto switchboard is shown in Fig. 533. The jacks and the drops are mounted on the vertical frame, the plugs and the ringing and listening keys on the shelf before it. The crank of the magneto is visible in the lower left-hand corner of the cut, under the table.

739. EXPERIMENT 33-E.—Wiring a Magneto Switchboard.—An experimental switchboard should be provided in the laboratory, with drops, jacks, listening and ringing keys, etc., so arranged that the necessary connections can be easily made by the student. A few subscriber sets are to be connected to the switchboard, and the practical operation of the system studied. The explanations given in articles 724 to 738, together with Figs. 521 to 533, give all the necessary information for performing this experiment.

Report. Make sketches of actual devices used on the switchboard; give diagrams of connections if they differ from those given above. Draw in detail the connections of the operator's set.

CENTRAL-ENERGY TELEPHONE SYSTEM.

740. The above-described magneto telephones have two drawbacks:

(1) Batteries in substations require care and renewals.

(2) Calling up the central by turning the crank of a magneto is too much trouble for a busy man.

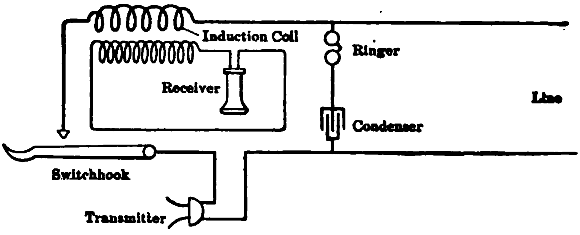

FIG. 534. Connections in a central-battery telephone set.

These inconveniences led to a gradual introduction of *common-battery* or *central-energy telephones;* practically all telephone exchanges of any importance are now equipped according to this system (see also § 728).

One of the simplest schemes of connections of a subscriber's set, in a system using a common battery, is shown in Fig. 534: When the receiver is taken off the hook, the line circuit is closed, and the battery in the exchange supplies the subscriber's transmitter with the current necessary for talking. The incoming alternating currents of high frequencies are transmitted to the receiver through the induction coil. The ringer is connected across the line and is protected by a condenser from the passage of direct currents; alternating currents for ringing pass easily through the condenser (§ 127).

Another scheme of connections is shown in Fig. 535: It differs from the previous diagram in that the receiver circuit is also opened when not in use, and is protected by a condenser against the direct current supplied to the transmitter circuit. There is little difference between the two schemes: both are simple and efficient, and both are widely used.

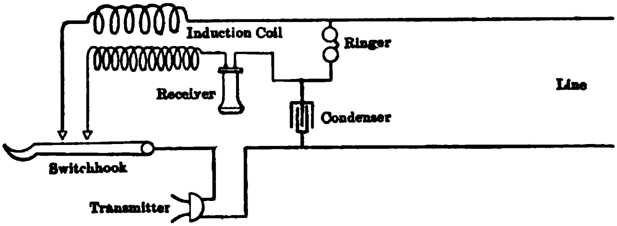

FIG. 535. Connections in a central-battery telephone set.

741. Battery Connections.—The common battery in the exchange must be connected so as to supply the subscribers with direct current for the transmitters, and at the same time offer a considerable resistance to the passage of high-frequency oscillating currents, which, if they found an easy path through the batteries, would be lost to the receiving station. The arrangements used for separating direct current from "talking" currents are based either on the action of inductance, or on that of capacity. An inductance offers an easy passage to direct-currents, but effectively prevents the passage of high-frequency oscillating currents. On the other hand, a condenser offers an infinite resistance to the passage of direct current, at the same time offering a comparatively easy path for high-frequency alternating currents.

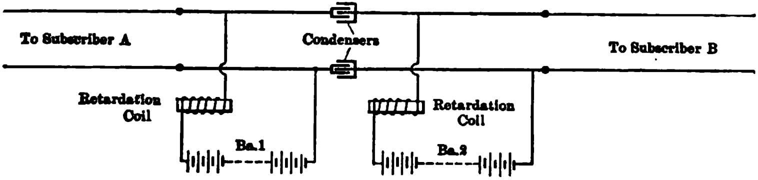

FIG. 536. Separation of talking currents from battery currents by means of condensers and retardation coils.

One application of these principles is shown in Fig. 536: Two batteries are provided in the exchange, one for the "calling" side, another for the "answering" side of the cord circuits. Condensers are interposed in the cord circuits, and the batteries are protected by so-called *retardation coils.* The latter are ordinary inductance coils, intended to prevent talking currents from passing through the batteries. With this arrangement, direct current flows from the batteries to the corresponding transmitters, while talking currents find a direct path, from the line *A* to the line *B*, through the condensers.

With the arrangement, shown in Fig. 537, the same result is accomplished by means of a "split repeating coil." Direct current from the positive terminal of the battery flows to the point *m*, whence it is divided into the line *A* and the line *B*, returning to the negative pole of the battery through the point *n* of the split repeating coil. Oscillating currents flowing through the wires of the two subscribers magnetize the iron core of the coil in the opposite directions; therefore, the coil exerts no "choking" action between the two subscriber stations. On the contrary, oscillating currents, trying to flow through the battery, magnetize both sides of the coil in the same direction, thus creating a high-reactance path. The advantage of this arrangement is that only one battery is used, and no condensers are required.

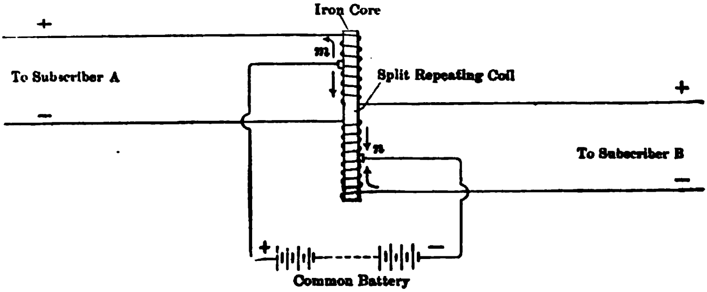

Fig. 537. Separation of talking currents from battery currents by means of a "split" repeating coil.

742. Operations Necessary for Connecting Two Subscribers. — The action of a common-battery switchboard may be understood by referring to Fig. 538. It is desired that the student should first understand the operations without following up in detail the actual wiring connections. When the subscriber *A* takes his receiver off the hook, the line lamp L_1, placed over his jack, lights up, calling the operator's attention to the fact that a connection is desired. She inserts one of her answering plugs into the jack *A*, presses the listening key *k* to the left, and calls for the number desired. The number being given, she inserts the calling plug of the same cord circuit into the jack of the subscriber *B* to be called, and presses the key to the right, thereby ringing the bell of subscriber *B*.

As soon as the calling plug has been inserted into the jack *B*, the supervisory lamp l_2 lights up, and remains lighted until the subscriber

B takes his receiver off the hook. Now the two subscribers are conversing, and all the switchboard lamps, belonging to these subscribers, are extinguished; the connection is indicated by the inserted plugs.

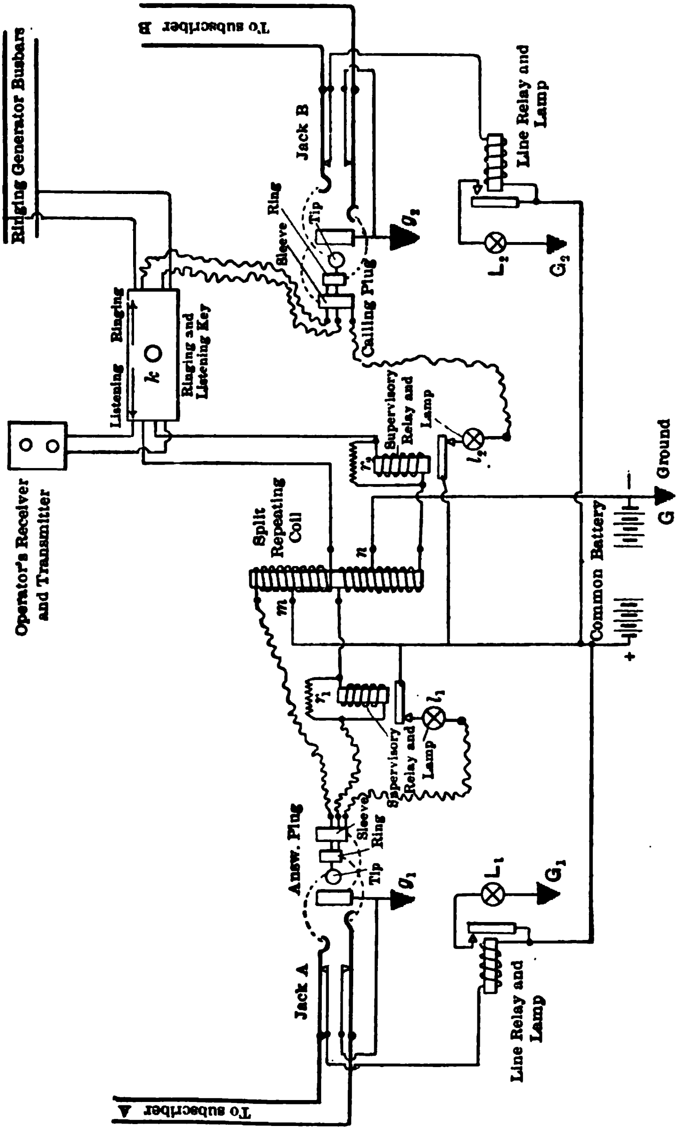

FIG. 538. Connections between two subscribers, in a central-energy exchange.

As soon as one of the subscribers hangs up his receiver the corresponding supervisory lamp lights up; when both supervisory lamps l_1 and l_2 are lighted up, the operator knows that both receivers have been hung up. She pulls out the plugs and thereby disconnects the subscribers. If for any reason it is necessary for her to listen for the conversation, she may do so by pressing her listening key *k* to the left.

743. Common Battery Switchboard Connections. — Having understood the operation of the switchboard, shown in Fig. 538, the electrical connections may be followed more in detail. When subscriber *A* takes his receiver off the hook (Fig. 534), the current from the positive pole of the common battery flows through the line relay under the jack *A*, the line, and his apparatus back to the station, and to the ground g_1. The circuit is thus completed since the negative pole of the battery is permanently grounded. The relay attracts its armature and closes the circuit of the line lamp L_1, whereupon the lamp lights up through the ground connection G_1. Inserting the answering plug into the jack *A*, opens the relay circuit through the auxiliary springs, shown inside of the main contacts, and thus extinguishes the lamp.

The plugs used with this system have three contacts, instead of two, as shown in Fig. 531. These three contacts are referred to as tip, ring, and sleeve. Through the tip and the ring the talking circuit is completed, while the sleeve contact merely connects the supervisory lamp l_1 into the circuit. The battery connections for talking are the same as shown in Fig. 537. The connections to the listening and ringing key are similar to those shown in Fig. 532, and need not be repeated here.

The supervisory lamp l_1 lights up between the positive pole of the battery and the ground g_1; its circuit is opened by the armature of the supervisory relay shown above the lamp; it is also opened when the answering plug is not in the jack *A*. The purpose of this lamp is to indicate to the operator that the subscriber's receiver is on the hook; in other words, that he is not using his telephone. The supervisory relay is inserted in the main talking circuit and holds the lamp circuit open as long as direct current is flowing through the line *A*; in other words, as long as the receiver is off the hook. When the lamp lights up it is extinguished again by pulling the answering plug out of the jack, thereby breaking the connection to the ground, through the sleeve contact.

The operator's set in the above-described switchboard consists of a receiver, a transmitter, and an induction coil. The receiver and the secondary of the induction coil are connected in series, and connected to the listening key through a condenser, also in series. This condenser protects the operator from getting an undue click in her receiver when she throws the listening key in. The operator's transmitter and the primary of the induction coil, in series with it, are all the time connected across the main battery, through a suitable resistance. The transmitter and the induction coil are bridged by a condenser, in order to offer an easier path for oscillating currents.

744. EXPERIMENT 33-F.—Wiring a Small Common-Battery Switchboard.—The student is given the necessary jacks, lamps, relays, cord circuits, etc.; also a few subscriber sets, and an operator's telephone. The experiment consists in connecting up and operating the switchboard as described in previous articles. Having obtained a satisfactory working of the system, try to determine the amount of time necessary for making a connection, from the moment when the first subscriber takes his receiver off the hook, to the moment when the second subscriber answers the call.

Report. Give rough sketches of the devices used, and the exact scheme of connections employed during the experiment, at least where they differ from Fig. 538. Give a diagram of connections of the operator's set. Mention the adjustments of relays, etc., if any.

745. Party Lines. — In many cases it is desirable for economic reasons to have more than one subscriber connected to the same line. Such lines are called *party lines;* their use involves some additional problems in regard to signaling. All party-line systems used can be divided into non-selective and selective. With a non-selective system the ringing current operates the bells of all the subscribers connected to the same line, a code being necessary to distinguish between the subscribers (number of rings, short and long rings, etc.). In selective systems, only the bell of the subscriber wanted rings, leaving the other subscribers on the same line entirely unaware of the call.

In the most common of the selective systems, two subscriber sets are connected in parallel across the line, but the ringer of one of the sets is connected between the positive line and the ground, while the other ringer is connected between the negative line and the ground. The ringing generator in the exchange may be connected at will, either between the positive line and the ground, or between the negative line and the ground; thus only one bell can ring at a time. The ground takes no part in the talking circuit, the main current flowing as usual between the positive and the negative lines. Therefore, when one of the subscribers is talking, the other party connected to the same line can listen to the conversation. There is, however, very little likelihood of this, since he does not hear the call, and is not aware of the conversation on his line. The technical expression for this is: "talking metallic, ringing through the ground."

On four-party lines, *biased* ringers are used, which respond to either positive or negative current-impulses only. Of the four subscribers, bridged across the same line, two have their ringers connected between the positive line and the ground, one ringer being biased for positive impulses, the other for negative impulses. The other two ringers are connected between the negative line and the ground, and are biased for opposite impulses. The generator in the exchange is provided with a two-part commutator so as to give uni-directional impulses. In this way four combinations for ringing are made possible, and either of the four subscribers may be called up, without disturbing the other three.

The four subscribers connected to the same line are usually marked in the telephone directory with the same number, with a distinguishing letter. As, for instance:

Subscriber 127-*a* has the ringer on the right wire, positively biased.
Subscriber 127-*b* has the ringer on the right wire, negatively biased.
Subscriber 127-*x* has the ringer on the left wire, positively biased.
Subscriber 127-*y* has the ringer on the left wire, negatively biased.

A biased ringer has the same construction as that shown in Fig. 525, but is provided with a spring, which normally holds the armature against one of the pole-pieces. Thus, it cannot respond to the current impulses which would tend to bring it closer to this pole, while it responds to the opposite impulses, by overcoming the tension of the spring.

746. EXPERIMENT 33-G. — Wiring a Party-Line Switchboard. —Wire up and operate a switchboard arranged for two-party lines, and for four-party lines, with selective signaling, as explained in the previous article. Note that condensers cannot be used in series with the bell, as in Figs. 534 and 535, when the ringers are biased. This is because a condenser transforms a uni-directional current into alternating current. This brings in the difficulty that the line circuit is closed all the time, a current flowing through the bells and the ground. To prevent this current from operating the line relays (Fig. 538), the resistance of the bells is made sufficiently high, and the relays are made comparatively insensitive (marginal adjustment of relays).

747. Large Telephone Systems. — When there are more lines in an exchange than one operator can handle, more switchboard sections and more operators are added. As far as answering a call is concerned, the operations are the same as in the case of the simple switchboard, described before. Each operator is assigned a certain number of subscribers whose calls she answers. A new problem arises when the subscriber wanted has his jack on another section of the switchboard, outside the reach of the operator. This is taken care of by providing, within the reach of each operator, so-called "multiple" jacks of all the subscribers connected to the exchange. Thus, instead of using a long cord circuit for the answering plug and inserting it into the principal jack, the operator uses a multiple jack of the same subscriber, which she finds either on her own section of the switchboard, or on one of the adjacent sections.

The use of several jacks in multiple, belonging to the same subscriber, brings in a new feature, namely, the so-called "busy" test. Without multiple jacks the operator can see at once if the subscriber wanted is already connected with some other party; but with multiple jacks she may not know that one of the jacks belonging to this subscriber has been "plugged in" by another operator, perhaps at the other end of the room. The switchboard connections are therefore so arranged, that when she touches the nearest part of the jack with the tip of the plug, she hears a click in her receiver, if the line wanted is "busy." This is because a telephone line in use is "alive": hence, making a new connection brings in a change in current and produces the above click.

With very large telephone systems the multiple connections become so complicated that it is more advantageous to use two or more separate exchanges, interconnected by a number of "trunk lines." The number of the trunk lines must be sufficient to handle the traffic during the busiest hours of the day. Dividing one exchange into several smaller exchanges has also the advantage that the subscribers' lines become shorter. When a connection is desired between two subscribers connected to the same exchange, the operations are the same as with the above-described multiple switchboard. If the subscriber wanted is connected to another exchange, the operator merely receives the call and transmits it to a special operator in the other office, who completes the connection.

Private-branch exchanges constitute a special case of the above scheme. The operator of the private branch completes the connection between subscribers connected to the branch. When, however, a connection is desired outside of the branch, she transmits the call to an operator of the main exchange, who in turn completes the connection, or, if necessary, transmits it again to another exchange.

The next development of telephone work is the long-distance transmission between cities. This traffic is handled by special operators, so-called "toll" or long-distance operators, who make the necessary connections. General features of connections may be understood from the above description of small exchanges. Special details and the corresponding laboratory experiments are outside of the scope of this work, which does not claim to give any but general information about telephone practice. Those specially interested in telephony are referred to comprehensive treatises by Kempster B. Miller, A. V. Abbot, and others.

748. EXPERIMENT 33-H. Influence of Resistance, Inductance and Capacity of the Line on Speech Transmission. — Connect up two telephones, to operate satisfactorily as regards talking and signaling; gradually increase the resistance of the line, until the words cannot be distinctly heard. Note a few values of resistance in the line, and mark the corresponding quality of speech as: excellent, good, fairly distinct, hardly intelligible, poor, etc. Repeat the same experiment with inductance in the line, instead of the resistance; also with a combination of resistance and inductance in series.

If large condensers are available, they may be connected across the line, either in one place or in various places, in combination with the resistance in the line itself. This will show the effect of distributed and concentrated capacity, which sometimes impairs the quality of speech, particularly with long telephone cables. Having appreciably affected the transmission by means of capacity, connect up some inductance in series with the line so as to counter-balance the effect of capacity by producing electric resonance (§ 131). The use of inductance for counteracting the capacity effect in telephone lines has been proposed by Prof. Pupin; telephone lines provided with distributed inductance coils are now known as "loaded" lines.

Appreciable disturbances in telephone lines are caused by light and power transmission lines paralleling them. Magnetic fields, produced by heavy alternating currents, induce secondary currents in the telephone lines; this impairs the quality of the speech and sometimes makes talking practically impossible. String a power line in the laboratory, parallel with the telephone line, and observe the disturbances produced. First use alternating currents and vary their strength, frequency, the distance between the wires, and the mutual arrangement of the telephone and the power lines; observe in each case the effect on the transmission of speech. Investigate the effect of *transposition* of the telephone wires, and of the power wires, so as to compensate for the effect of induction. Try this in particular with a three-phase line. Then use direct current in the power line, suddenly vary its value, and observe clicks and sounds produced in the telephones. Finally use one wire as a common return (ground) for the telephone currents and the power currents; observe the effect produced on the quality of speech.

The above combinations are those met with in practice, and are intended to illustrate the difficulties with which telephone engineers have to contend in the operation of their lines.

Report the connections used in the experiment, the values of power current, the distance between the wires, with other factors, and the extent to which they were found to affect telephone transmission. State the effect of resistance, inductance, and capacity, and of their combinations.

CHAPTER XXXIV.

SAFETY OF ELECTRIC PLANTS.

749. Electrical machinery and wiring, if improperly or carelessly installed, may become a source of annoyance, fire hazard, and danger to life. In order to minimize the chance of such disturbances, and to make electric installations reliable and safe, a code of rules and regulations was drawn up in 1897, by the united efforts of fire insurance companies and various engineering and business associations, interested in the development of electrical industry. These rules, commonly known as the "National Electrical Code," or as Fire Underwriters' Rules, are at present recommended and adopted by the following associations:

American Institute of Architects.
American Institute of Electrical Engineers.
American Society of Mechanical Engineers.
American Institute of Mining Engineers.
American Street Railway Association.
Associated Factory Mutual Fire Insurance Companies.
Association of Edison Illuminating Companies.
International Association of Fire Engineers.
International Association of Municipal Electricians.
National Board of Fire Underwriters.
National Electric Light Association.
National Electrical Contractors' Association.
Underwriters' National Electric Association.

The rules are yearly revised by the delegates of the above associations, so as to be up to date with the progress of the art. They are published in book form, and may also be found reprinted in various pocket-books and text-books (for instance, in *Foster's Pocket-book*). A very convenient edition, with explanatory notes and examples, is that of the Associated Mutual Fire Insurance Companies.

It must be understood that these rules have no legal power, as do the corresponding rules in some countries; but being adopted by practically all the insurance, electrical, mechanical, architectural and allied interests of the country, their authority is acknowledged by all reliable manufacturers, contractors and consulting engineers, who strictly adhere to them. Moreover, it is difficult at present to insure a building, unless its electrical installation is put up in accordance with the National Electrical Code, or some similar rules.

In all contracts for electrical work it is advisable to introduce a clause to the effect, that all work must be done strictly in accordance with the National Electrical Code, and that no fittings shall be used, not found in the latest edition of the *List of Approved Fittings* (published semi-annually). This clause insures good workmanship and materials, and in case of a litigation one has the support of the competent and impartial National Board of Fire Underwriters.

750. Subdivision of the Rules. — The general plan of the National Electrical Code is as follows:

CLASS A. — STATIONS AND DYNAMO ROOMS. Includes central stations; generator, motor and storage-battery rooms; transformer substations, etc. Rules 1 to 11.

CLASS B. — OUTSIDE WORK, all systems and voltages. Rules 12 to 13A.

CLASS C. — INSIDE WORK: —

General Rules, all systems and voltages. Rules 14 to 17.
Constant-Current Systems. Rules 18 to 20.
Constant-Potential Systems: —
    General Rules, all voltages. Rules 21 to 23.
    Low-Potential Systems, 550 volts or less. Rules 24 to 34.
    High-Potential Systems, 550 to 3500 volts. Rules 35 to 37.
    Extra-High-Potential Systems, over 3500 volts. Rules 38 and 39.

CLASS D. — FITTINGS, MATERIALS, AND DETAILS OF CONSTRUCTION, all systems and voltages. Rules 40 to 63.

CLASS E. — MISCELLANEOUS. Rules 64 to 67.

CLASS F. — MARINE WORK. Rules 68 to 83.

With the exception of the Classes E and F, which are of less importance for general work, the rest of the rules can be subdivided in two large sections: (1) *installations* (classes A, B and C), and (2) *fittings and supplies* (class D). In the latter class rules will be found prescribing details of construction of switches, fuses, lamp sockets, wire, etc., *per se*. The other classes contain regulations as to how these supplies must be installed, and which types are permitted in various cases.

The *List of Approved Electrical Fittings*, published semi-annually as a supplement to the National Electrical Code, contains the names of firms, trade marks, types and styles of fittings, manufactured in accordance with the Fire Underwriters' rules. When purchasing or specifying such supplies, it is not necessary to compare them each time with the National Electrical Code, since they are manufactured in accordance with the requirements of this code.

In view of the importance of the National Electrical Code, it is essential for every electrical engineer to become familiar at least with the general arrangement of the rules, and with their principal requirements. The simplest way to begin this work is to go over an electrical installation with the Underwriters' rules in hand, and to compare different parts of the installation with the codes as though assuming the duties of a fire-insurance inspector.

In accordance with the above subdivision of the rules, the laboratory exercises are also divided into two parts: Inspection of supplies, and inspection of installations.

751. EXPERIMENT 34-A.—Inspection of Supplies.—This work is covered by CLASS D of the National Electrical Code. The following are the supplies most used, and on the quality of which the safety and the reliability of service essentially depend: Insulated wire and cables, conduits, cleats and knobs (for supporting wire), switches, cut-outs and fuses, cut-out cabinets, rosettes and lamp sockets. Samples of these supplies should be provided in the laboratory, and the students are to inspect as many of them as time will permit. The inspection consists in taking a device apart, or at least opening it if necessary, and in comparing the details of its construction and the materials used, with those specified in the corresponding section of the Underwriters' rules.

If the device only partly satisfies these rules, state which rule is not complied with, and suggest the changes necessary to make it "approved." Suppose, for instance, that you have to decide about a switch: you will find in the Code rule 51 relating to switches. Sections *a* and *b* of this rule refer to *all* types of switches, and it must first be decided, whether the particular switch satisfies the conditions specified there. Then, if it is a knife-switch, further requirements will be found under the sections *c* to *k*; if it is a snap switch, the rules *l* to *t* apply to it. For knife-switches you will find, that they must be mounted on non-combustible bases, that the hinges must be provided with spring washers in order to insure good contact, that the separation between the parts of opposite polarity must be not less than a certain limit, etc. See if all these requirements are fulfilled in the switch under inspection, and if not, state whether in your opinion the switch should be absolutely condemned, or if it could be used after certain changes.

Report your conclusions, referring to the rules by number (for instance, 51*m*), and give free-hand sketches suggesting some changes in the devices inspected.

752. EXPERIMENT 34-B. — Inspection of Installations. — The work consists in inspecting some actual wiring and installed machinery, and in comparing the installation with the requirements of the National Electrical Code. The following classes apply here:

A. Stations and Dynamo Rooms.
B. Outside work.
C. Inside wiring.

In order to perform the work intelligently and with safety, the general layout of the plant must be known, at least in its general features, such as source of power, the places of the main connections and cut-outs, voltage of the supply, etc.

The rules covering installations are too numerous to be learned in a few laboratory exercises; moreover, they are not all of the same importance. It is desired that the student should first get experience in the most important requirements of the National Electrical Code, namely, those relating to:

Generators (Rule 1);
Switchboards (Rule 3);
Motors (Rule 8);
Transformers (Rules 11 and 13);
Outside wiring (Rules 12*a* to 12*h*);
Inside wiring (Rules 14 to 17; 21; 24 to 28);
Arc lamps (Rule 29).

During the inspection of wiring be sure not to come in contact with live high-tension wires; also do not use screwdrivers, wrenches, etc., which may cause a short-circuit. Above all, do not open or close any switches, without permission from the proper authorities.

As this work of inspection does not require the use of any apparatus or current, part of the inspection can be done outside of the regular laboratory time. It is most earnestly recommended, that the student spend as much time as possible in inspecting the available installations, with the National Electrical Code in hand. This will give him one of those most valuable experiences, which no books can supply, and which enables a man to distinguish good workmanship from a "cheap job."

Report. State what machines and wiring have been inspected, and which rules found to be complied with. Where you think the installation is not in accordance with the National Electrical Code, state so, referring to the rule number, and suggest the necessary changes.

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